Networks, Signals and Systems

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Lecture - 20

Graph Theory (Contd….)

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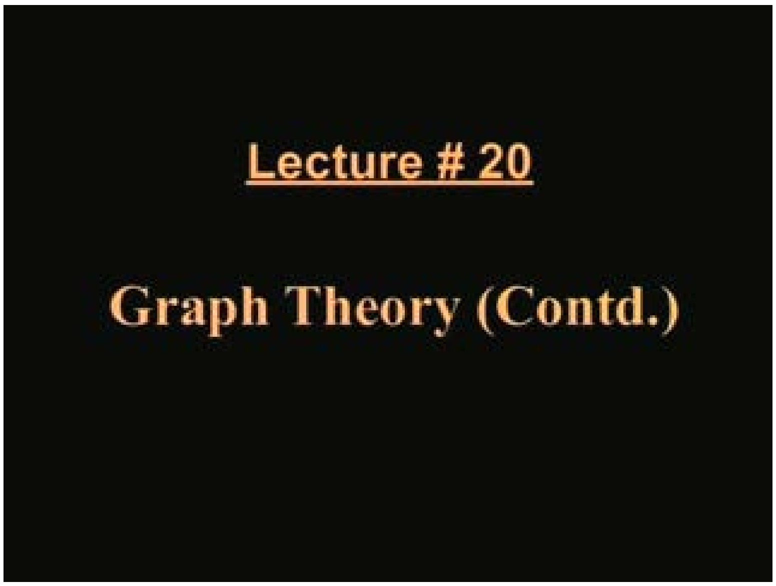

Okay good afternoon friends, we shall continue with our discussions on graph theory. Yesterday, we had taken up a problem the A matrix was given please correct it there is a minus sign here. I forgot to write that 1, minus1, 0, 0, 0, 0, 0, minus 1 and so on. So we shall work it out here, so from A matrix if you apply those transforms what will be the possible next transform I can change the sign of this okay, I can change the sign of the second row so this will become 1, minus 1, 0, 0, 0, 0 then 0, 1, 0, 1, 0, 1 okay then 0, 0, 1, 1, 0, 0, 1, 0, minus 1, 0, minus 1, 0 okay. Then from here if I subtract this this one was sign changed from the previous matrix then if I subtract row 1 from row 4, A_2 row 1 you subtract it from row 4 you get 1, minus 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1 then 0, 0, 1, 1, 0, 0 then this minus this will give me 0 then 1 is that all right, then minus 1, minus 1 then 0, 0 sorry minus 1, 0 is that okay 1, minus 1, 0, minus 1, 0 and then what you do this is having 1 here. See if I subtract, if I subtract this from this 2 minus 4 row 2 minus row 4 what you get A_3 1, minus1, 0, 0, 0, 0 then 0, 1, 0, 1, 0, 1 then 0, 0, 1, 1, 0, 0 then 2 minus 4 will give me 0, 0 next guess please.

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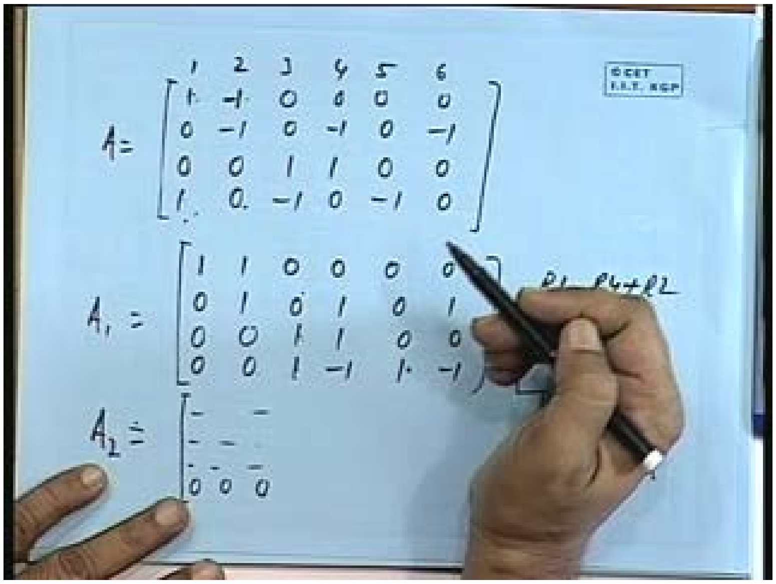

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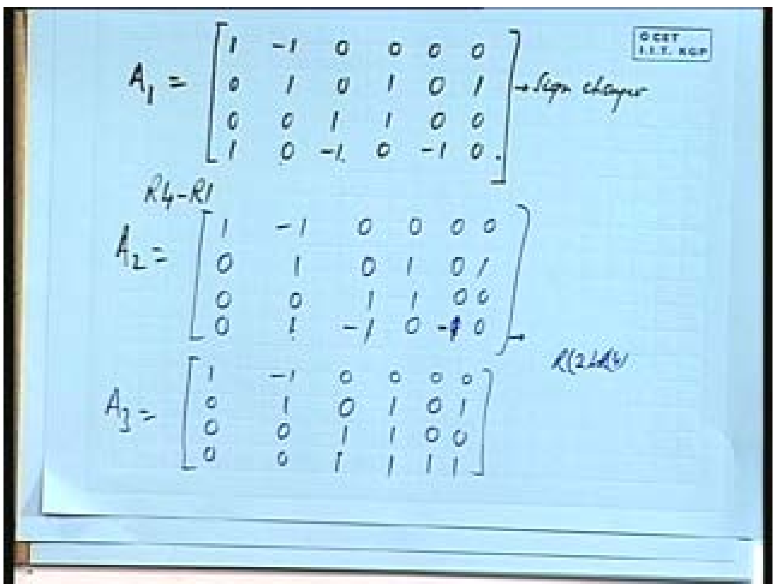

So if I change the sign I am subtracting 4 from 2, so that will become plus 1, then 1, then plus 1 then 1 is that okay. Now if I subtract 4 from3 or 3 from 4 and replace the 4th row what you get A_4, A_4 will be 1, minus1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1 then 0, 0, 1, 1, 0, 0 and then 0, 0. I am subtracting this from this so 0, 0 then 1, 1 is that all right, so how is that Echelon structure. So 1, 2, 3, 4, 5, 6 let us see the original graph if this is a 5 node system including the reference then what it was like 1, say this is node 1, suppose this is node 1, element 1, this is node 1 and these are the elements nodes 1, 2, 3, 4 fifth one is the reference all right.

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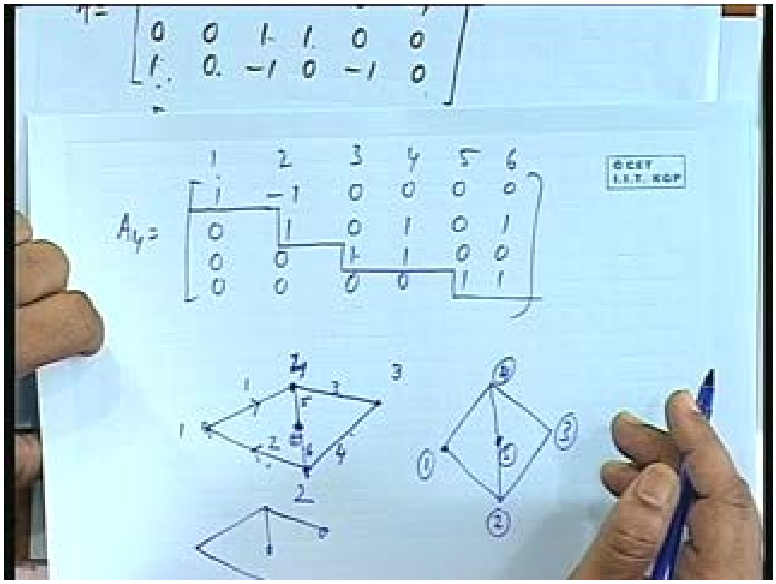

So what would be the structure of the graph the overall graph, let us see can you reconstruct the graph from the given a matrix we discussed last time. This was the original matrix so from a matrix can you construct this 1, minus 1, 0, 0, 0, 0, so this is node 1, element 1 is going okay then element 2 is coming in all right, no other element is connected to node 1, node 2, node 2 is connected to element 2, node 2 is connected to element 2 mind you this is also this is not the original A matrix, this is actually A matrix with some sign change otherwise both of them cannot be minus 2 then node 2 is okay element 2 is connected between 1 and 2. Let us see element 3 is between 3 and 4, so if 3 is between element 3 is between nodes 3 and 4 element 2 element 2 sorry element 1 is between 1 and sorry 1 and 4.

So this is 4, this is 2 okay then element 3 is between 3 and 4, 3 and 4 okay element 4 is between 2 and 3 the graph is going down sir sorry okay, so it was 1 and 4, 1 and 4, 1 and 2, 1 and 2, element 2, then 3 and 4, 3 and 4, then 2 and 3, 2 and 2 and 3 this is element 4, this is element 3 then element 5 is between 4 and 5, so between 4 and 5 this is element 5, this is node 5 and between 2 and 2 and 5 this element number 6, 2 and 5 element number 6 this is node 5 okay. So this was the actually the structure okay 1, 2 sorry 2, 4, 3, 5, now these are the elements for forming the tree what are the elements that we have found here 1, 2, 3 and 5. Let us see 1, 2, 3 and 5 this is the possible tree is that okay I could have add 1, 2, 4 and 6 also or 1, 2, 3 and 6 that also could have been possible but 1, 2, 3, 4 will not be possible here 1, 2, 3, 4 will give you 1, 2, 3, 4 a closed loop without having any connection to this node that would not have been a possible tree it would would have been a closed loop okay.

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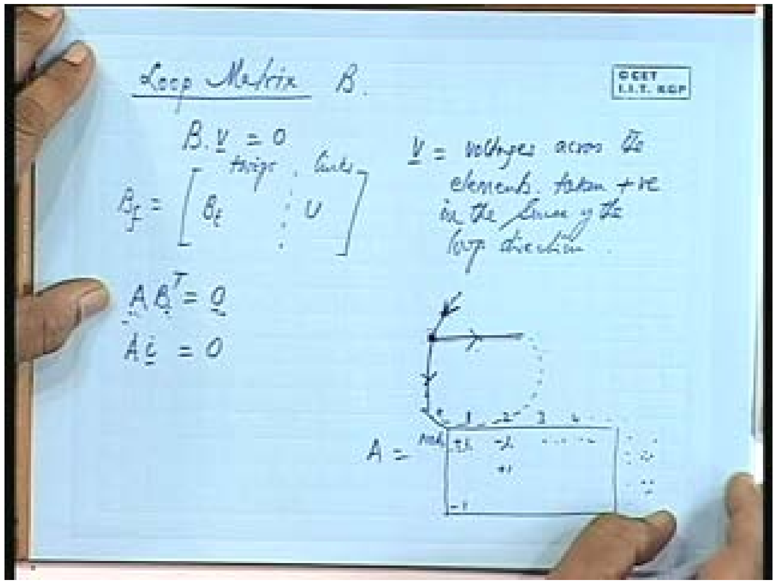

So 1, 2, 3, 4 will not be it is not possible for us to get an Echelon form with 1, 2, 3, 4 nodes by any manipulation, is that clear, so this is how we develop a possible tree. Now once again let us come back to loop matrix, some relations we shall derive for the loop matrix, we have observed we observed earlier that any matrix loop matrix B multiplied by voltage v is equal to 0, where v is the vector of voltages, vector of voltages across the elements. So if there are say 5 elements then v_1, v_2, v_3, v_4 this will be the elements of the voltage vector and this will be taken positive in the sense of the loop, taken positive in the sense of the loop direction.

So fundamental loop matrix, we discussed last time it will have 2 components we can put the twigs on 1 side and the links on the other side okay twigs and links if you put 1 link at a time then this will be a unity matrix, is it not, if we put one link at a time and then call that as loop 1, loop 2, loops are numbered with the number of the link, link 1, link 2, link 3. Similarly, we call them loop 1, loop 2, loop 3 so that will give me a unity matrix in this portion okay. Now we also established this relation a into b transpose equal to a null vector is it not because the elements of A, if you remember what is an incidence matrix A, A will be consisting of the elements connected between nodes and you are taking plus sign for the currents which are for for the elements which are going out from the node and entering the node will be given a minus sign all right.

So you will find a into i is equal to 0 will that be all right. You consider from a node suppose these are the direction of the currents we have taken i_1, i_2, i_3 so from a matrix these are the elements elements 1, 2, 3, 4, 5, 6 and so on and these are nodes. So element 1 will be connected between say node 1 and node 5 and so on element number 2 depending on the arrow direction it may be minus 1 and plus 1 here.

So if I take the sum of these currents i_1 into plus 1, i_2 into minus 2 minus 1 and so on suppose i_2 is in this direction so at this node first row multiplied by the current vector will give me 0 that is Kirchhoff's current law, is it not i_1 plus i_2 plus i_3 all the currents going out if they are taken as positive should be equal to 0 if there is a current coming in the opposite direction it will be negative, so and that is reflected in the A matrix.

So basically a into i is equal to 0 is our well known Kirchhoff's current law okay. Now we have also discussed about this product a into b transpose equal to 0 all right. When you consider a particular node and a corresponding loop here if you consider the elements of the loop matrix there can be only 2 elements which can come into play in a particular loop, this element will not be included in this loop. So in a particular loop where a node is included there can be only 2 elements in that particular node which will be involved in the loop and if the loop direction is like this, this is positive, this is positive for A matrix elements whereas this will be positive and this will be negative for the B matrix elements, is it not.

So if I take A into B, for example if you consider this particular node then nodes are having there are many other element connected to the node but for a particular loop where there will be 2 non-zero entries it will be this one and this one, this element does not come into the loop. So even if there are 1s or minus 1s in the a matrix row the corresponding elements here will be 0s only there are 2 elements involving these 2, 2 non-zero elements here and what will be their directions what would be their signs if this is plus 1 minus 1 this will be minus 1 minus 1 or plus 1 plus 1, so that product sum is 0, is it not.

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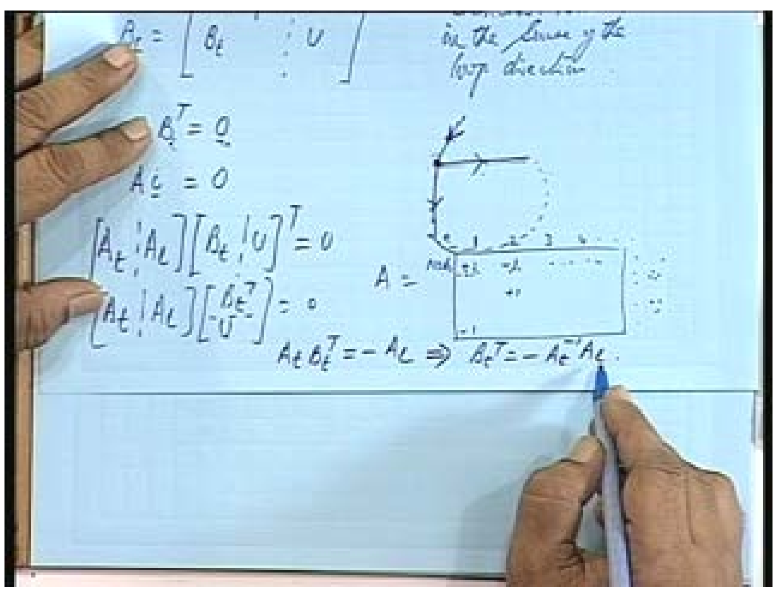

So this also we established last time by breaking up A okay into a_t and a_l and B into b_t and unity matrix we got a_t partitioned a_l multiplied by b_t transpose u equal to 0 from where, we finally wrote see A_t into B_t transpose plus A_l is equal to 0. So A_t B_t transpose equal to minus A_l which give us B_t transpose equal to minus A_t inverse A_l, A_t inverse A_l okay this is what we got last time. I am just repeating some of the points that will be of some relevance now. Now let us come to cut set matrix, what will be the relationship with this matrix, cut set relationship with b and a, q_f cut set matrix, once again if you have a graph say may be like this I am just taking an example I can have a cut set like this okay.

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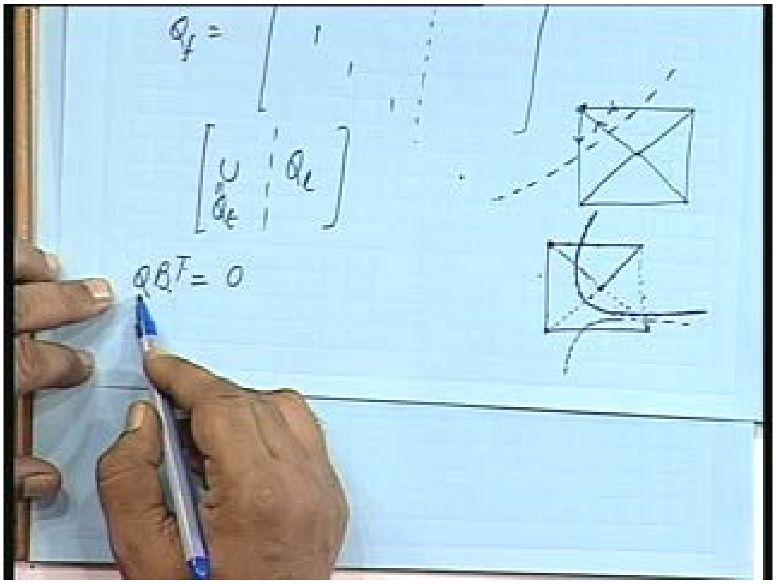

So this may be the directions of different elements. Now depending on the choice of the tree if you would have chosen the tree as this say suppose this is a tree then this would have been a twig a link this would have been a link is that all right. So if I want to have a cut set I can how do I cut this suppose I want to I select this or I want to I select this so this can be a cut set, is it not I cut one twig at a time. So this will not be a possible cut set if I choose the tree like this because 2 elements of the tree will be eliminated in a cut set in a fundamental cut set I am talk about a fundamental cut set there can be cut only in one of the tree elements, so this is a possible cut set, fundamental cut set okay.

Similarly if I want to cut this, if I want to cut this twig I should cut like this, I cannot cut like this then 2 element will be cut okay, so whenever you are cutting 1 element of the tree at a time okay, 1 element of the tree at a time then then what will you observe suppose we take twigs the tree elements on this side link elements on this side if I call the first tree element I can number them accordingly first tree element when it is being cut this is cut set number 1, so cut set 1 this will have 1 here no other twig will be cut so I will get similarly for cut set 2 only the element number 2 will be cut so it will give me what a unity matrix in this side in this side. So it will be unity matrix and the remainder I will call it q_l okay which will be containing all other elements involving the links okay. For example, in this cut set what will be the elements here they will be corresponding to this one this one this one and this one is it not if I consider this cut set cutting this tree element then this, this, this and this these 4 dotted lines will be coming here 4 dotted lines means 4 link elements okay.

So that will be giving me the sub matrix q_l okay, so q_t is u and rank of this this is an n by n because how many twigs can you cut if you are having n, n plus1 nodes then how many elements are there in a tree n and this is cut 1 at a time, so this will be n by n so the rank of q_f is n is that all right number of nodes minus 1 that will be the rank of the fundamental cut set matrix okay. Now you see q there is an interesting relationship between q and b, q into b_t is equal to 0 can you prove this q into b_t let us see what it means when we are considering say this particular matrix and this particular tree when we consider a cut set I am cutting 1 twig at a time okay. There can be a loop this loop okay. Let us consider this cut set okay now with this tree element being cut so this can be a possible loop because we are taking the product of loop matrix transpose and q matrix all right.

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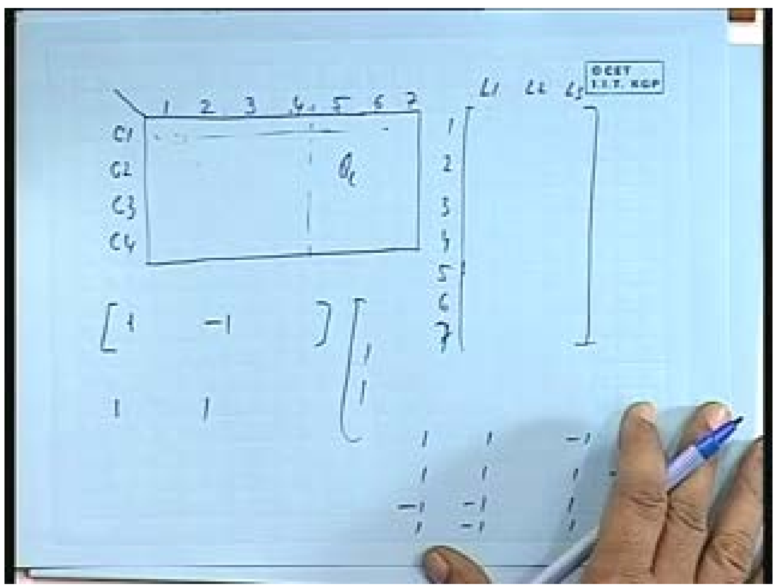

Now what does it mean q suppose the elements are q matrix will be elements 1, 2, 3, 4, 5, 6 suppose it is a 7 element network it is like this all right out of which I have chosen. Suppose there are4 nodes here so c_1, c_2, c_3, c_4 baring the reference node there are 4 nodes left all right. So that is the order n, so n by n or arrange the twigs number them in such a way that the twigs are coming as element number 1, 2, 3, 4 okay. So this is the unity matrix and this is q_l okay and b transpose will have1, 2, 3, 4, 5, 6, 7 all the elements here okay.

Now let us see what will be the elements of this actually this need not be arranged I am just getting a general relationship q into b_t element of this and elements of this what will be the total products what will be the product sum which one will be multiplied with which number on this side in a cut set, in a cut set 1 twig is being cut for the tree and there are other elements but in the B, A, in the B matrix what will be the values of this there can be only 2 elements forming a loop, 2 elements which will be non-zero, which will be included in this, is it not. When I write the loop matrix say this is loop 1, loop 2, loop 3, loop 4 and so on there are so many loops depending on the number of links okay so that will be1, 2, 3, loop 1, loop 2, loop 3 these are the fundamental loops is it not depending on the number of links. So in loop 1 if I put 1 link at a time then that will form1 closed loop so you will be cutting only 2 elements 2 elements on the loop which will be involved in the cut set other elements of the cut set they are all links but you are taking only 1 link at a time for a loop is it not.

So either this 2 will be non-zero others are all 0 this will not come in this loop this link though it is cut by the cut set though it is an element of the cut set non-zero element of the cut set plus 1 or minus 1 but in the loop it is not existing. So in the loop there are only 2 elements which are non-zero is it not and now you see the direction of the cut set for example suppose this is the direction of the tree, then the direction of the element of the tree, then the direction of the cut set. Suppose this is the element of the link then the cut set direction is taken as the direction of the twig so the cut set direction is this so for the cut set this is plus, this is minus, next element is minus, this elements whatever be the directions they do not come into picture when I talk about the products of this okay.

So in the loop this is plus 1 this is plus 1 whereas in the cut set this is plus 1 this is minus 1 all right. So what you get 1 element is plus 1 is minus so for as the cut set matrix is concerned and on this side both are plus 1 say if I take the product sum it become0 this is 1 possible combination, had it been in the other direction say both are plus 1 in the cut set elements both are plus 1 then in the loop 1 will be plus the other 1 will be minus.

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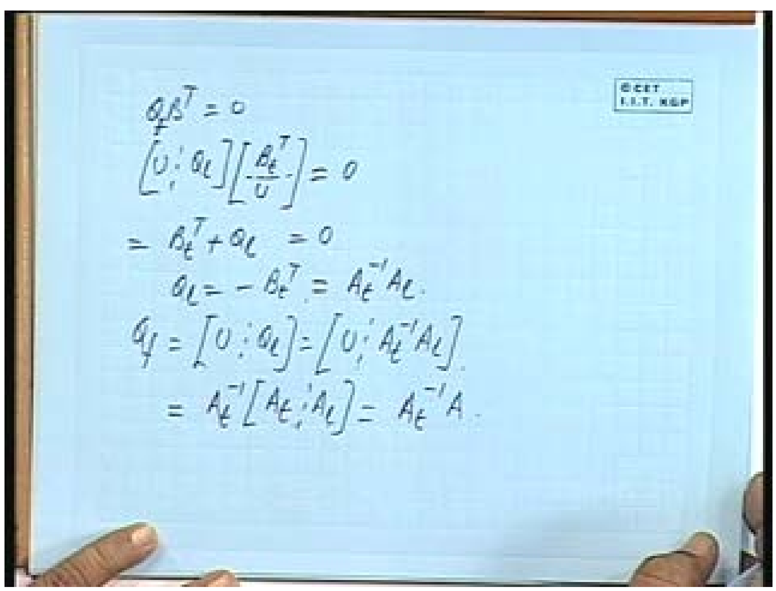

So it will be 1 and minus 1 here so if it is1, 1 it will be1, minus1 here again n product will be 0 did you get my point. So there are possible combinations 1, 1, minus 1, 1, 1, 1, 1, inus 1, minus

1, minus 1, 1, minus 1 okay or 1, minus 1, 1, 1 and so on. This kind of combinations will be there so that the final product sum is 0 okay. So q into b_t is0 all right, now let us expand this q into b_t equal to 0, what is q? Q is u partitioned q_l is it not we got q we will consider only q_f this is a general relation for any q okay if you take the fundamental cut set then q_t and q_l q_t is unity so I am writing unity into q_l and what was b_t, what was b_t if you remember yes, how did you write b_t b transpose b_t transpose and u was it like this so bt transpose plus q_l is equal to 0 this is equal to 0 is it not therefore q_l is how much b_t transpose okay and how much did you get as minus b_t transpose yes a_t inverse a_l, a_l okay. So q_f is therefore u partitioned q_l and I can write this as u partitioned a_t inverse a_l is that okay yes sir if I multiply by a_t if I multiply by a_t I will take a_t inverse outside inverse into at partitioned a_l which means nothing but a a_t and a_l combined together is a, so it is at inverse A is that okay.

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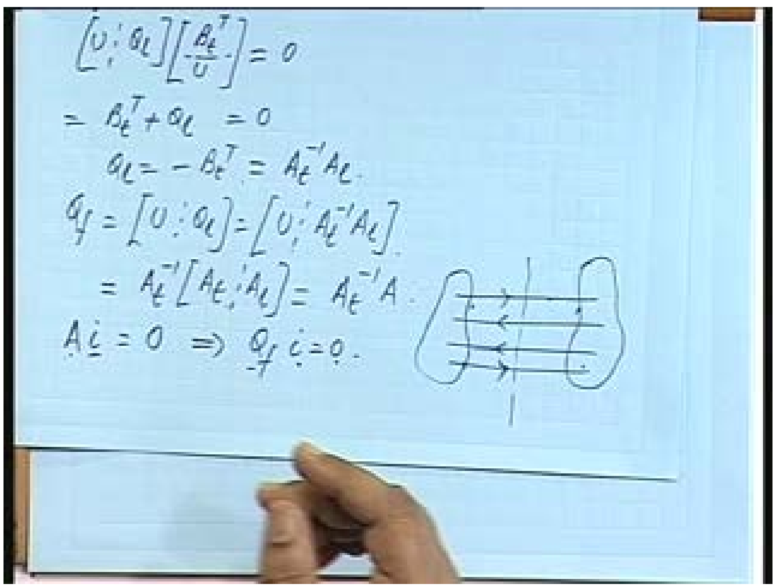

Now what is a into current vector I Kirchhoff's current law is it not this is giving the incidence matrix, relationship with the elements and the nodes so that gives me this relationship. So if I multiply by i therefore q_f into i is equal to 0 is that all right, what does it physically mean inside loop no q_f is a cut set matrix. So suppose I have a cut so currents going here here here like this say there are many currents flowing through this so currents flowing through only these elements will be non-zero other elements there can be other elements here but they do not come into picture so far as this cut set is concerned so this plus this algebraic sum of this currents will be 0, what does it mean, it means that if I have a cut set like this suppose these are 2 sub networks connected by these lines we are physically cutting the linescutting suppose cutting means you are imagining scissor cut.

So the total sum of currents is equal to 0 it is an extension of Kirchhoff's current law, Kirchhoff's current law start I mean talks about a node, nodes say the currents coming out of the node is equal to 0. Now we replace the node by a sub network currents coming out of a system if I partition it if I cut it if I imagine this is one are this is another area connected by many lines then current coming out of a particular system is equal to 0 this is precisely we use in power systems in2 sub networks suppose 2 states, 2 areas are connected by different tie lines.

So the total amount of current going out of the tie line will be equal to 0, is it not, it is an extended concept of Kirchhoff's current law. So q_f into i equal to 0 now we would like to discuss something about formation of the q matrix from a matrix before we go into that we will be discussing a little bit about Echelon form some transformations, yes please.

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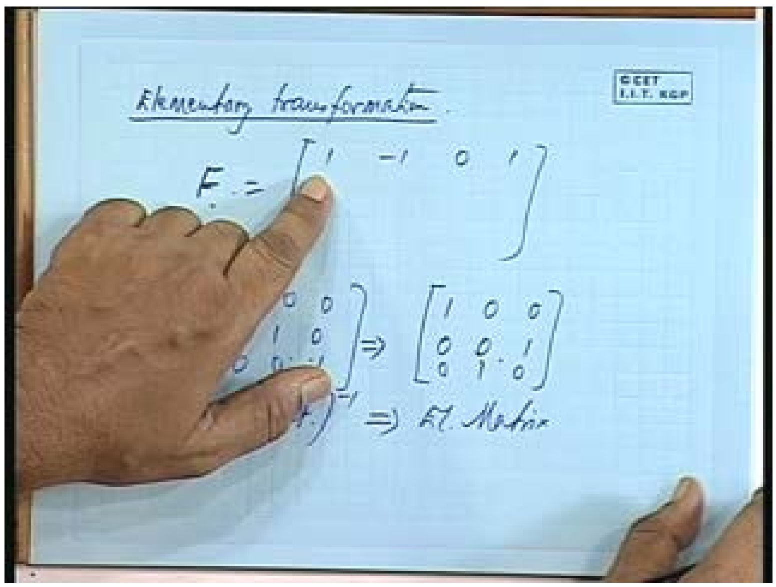

Elementary operation or elementary transformations suppose you are having any matrix f okay hello, hello elements of the matrix f is it not coming all right okay, this pens are drying up okay so suppose f is any matrix 1, minus 1, 0, 1 and so on when we try to make those operations changing the rows or interchanging or changing the sign.

Similar operations can be done by elementary transformation we call it. You consider a unity matrix say, this is the identity matrix okay. Suppose I interchange these 2 rows, row 2 and 3 then what do I get 1, 0, 0, 0, 0, 1, 0, 1, 0 all right. This is an elementary matrix that is from the identity matrix by interchanging the rows we can always generate an elementary matrix. An elementary matrix its inverse is also another elementary matrix, another elementary matrix. So elementary matrix inverse is also another elementary matrix, thank you.

So instead of interchanging rows etcetera on f matrix you can also do that interchanging on the unity matrix get an elementary matrix and multiply f with the original matrix by this elementary matrix, you will be getting the same n product all right. Let us sir why do we do this transformation pardon, elementary transformations, we will see what did you I mean what use it has because we have already used it earlier in the formation of the tree mat tree elements in is not in the formation of the tree for selecting the elements we have gone through this operation. So we shall be using this theory of elementary matrix transformation for the formation of the q matrix from a matrix can you generate q matrix this is what actually we are aiming at. So we will take it up later on right now let me just demonstrate this how a simple multiplication by the elementary matrix we will generate the new transformed matrix of f.

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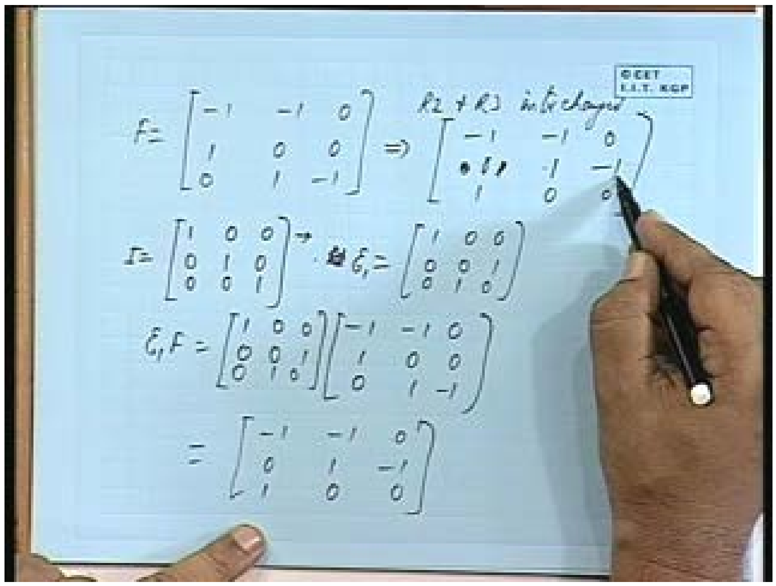

Suppose f is given I am taking just as an example a very simple matrix minus 1, minus1, 0 okay 1, 0, 0 then 0, 1 minus 1, so we want to interchange rows 2 and 3, row 2 and 3 interchanged. So what you get minus 1, minus 1, 0, 0, 1 sorry 0, 1, minus 1, 1, 0, 0 okay what is an identity matrix 1, 0, 0, 0, 1, 0, 0, 0, 1 and if I make row 2 and row 3 interchange elementary matrix I will call it Epsilon matrix 1, 0, 0, 0, 0, 1, 0, 1, 0. Now if I instead of doing this operation on f matrix I am doing it on e matrix, identity matrix all right.

Now what is e_1 into f what is e_1 into f, 1, 0, 0, 0, 0, 1 and then 0, 1, 0 multiplied by minus 1, minus 1, 0, 1, 0, 0, 0, 1, minus 1 okay what will be the n products minus 1 you are getting minus 1, minus 1, 0, 0, 1, minus 1, 1, 0, 0 its same as this one that means the interchange operation which we were doing on f is as good as multiplying f by an elementary matrix with the same operation true this is one elementary matrix that means by instead of doing it on the original matrix f, I will be multiplying by different elementary matrices different elementary matrices I will get the same f sir wont we think that we do one extra operation here apparently so you will find yes, so in this process what we are trying to achieve that q matrix, what was q matrix equal to q q, finally q matrix was a_t inverse a is it not a_t inverse a so q_f is basically u and a_t inverse a_l all right that was q_l. So if by transformation of this a t a matrix somehow if we generate a unity matrix here from a, if we go to q if I can make, if I can make a unity matrix here then this will automatically be generated. We will see that later on, so if you remember from a matrix we started for making a tree we are just getting the Echelon form if we try to make a go a little further to get a unity matrix in that side that means a select a tree, select a tree and then try to get a unity matrix there then rest of it will be q_1, we will get the transformed a will become q.

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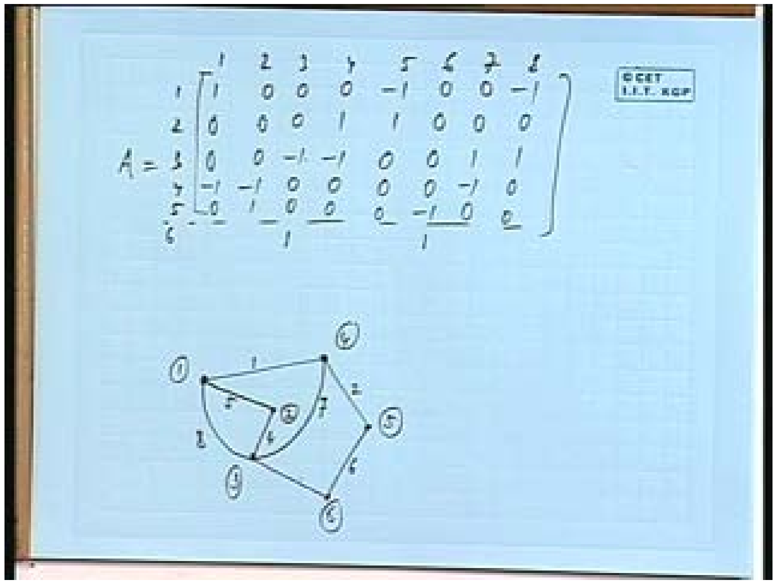

So this is what our aim will be before we go into that operation I thought it will be better if we can work out 1 or 2 simple examples, some kind of a tutorial exercise on whatever we have done so far okay. There is a simple question can you draw the graph a is given can you draw the graph I will read out the values 1 this 0 okay 1, 0, 0, minus 1, 0 then 0, 0, 0, minus 1, 1 then 0, 0, 0 a 0, 0, minus 1, 0, 0 then 0, 1, minus 1, 0, 0 then minus 1, 1, 0, 0, 0 then 0, 0, 0, 0, 4, 0 s minus 1 then 0, 0, 1, minus 1, 0 then minus 1, 0, 1, 0, 0 it is quite an involved network so there are 1, 2, 3, 4, 5 how many nodes are there sir 6 , there is a reference node why because in column 3 you will find there is 1, 1 missing column 3 and column 5, 6.

So if I call it 1, 2, 3, 4, 5, 6, 7, 8 it is an 8 element vector node 1, 2, 3, 4, 5 then 6^{th} one is the reference, so 6^{th} one will be this one plus 1 and this one plus 1 rest are all connected inside. So node 1 node 1 it is okay element 1 is between node 1 and node 4, 1 and 4, element 1, element 2 is between 4 and 5, element 2 we will put the arrows later on node a element number 3 is between 3 and 6 between some 6 and 3 then element number 4 is between 2 and 3 okay.

Let me put it here element number 4 is between 2 and 3 element number 5 is between 1 and 2 between 1 and 2 element number 5, element number 6 is between 5 and 6, this is 6 element number 7 is between 3 and 4 between 3 and 4 that is element number 7, element number 8 is between 1 and 3 is that all right. So this is the graph you can always draw like this and then put the arrows depending on the plus 1 and minus 1 signs how do we decide the resistances between with sign is like plus 1 plus 1 see 1, 1 so both of the there can be only a_2 elements 1 plus 1 and 1 minus 1 so find out where the1s are missing there is only 1 element here plus1 is missing. So that will be connected to the 6th node which is missing reference will equivalent to other node also in the third column, possible then you have to select whether it is a or a a matrix I asked you whether it is a matrix or a a matrix suppose it is given as some incidence matrix is it the complete incidence matrix or that reference matrix normal incidence matrix that we deal with without the reference okay.

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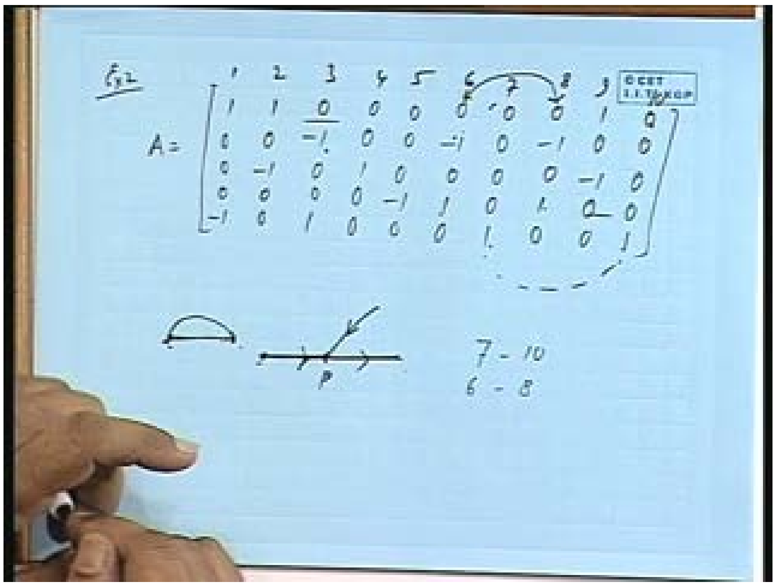

So since in these 2 columns we find these 2 elements are missing, so there is a another row another node 6th node, so we plot accordingly okay so this is the complete graph. Next question is find different trees using the matrix and then check the results from the figure, find different trees so from this matrix you have to do that operation by that Echelon forms you have to find out the possible trees there can be many trees and this is one example.

Next we take another example there is an incidence matrix a which is given here 1, 0, 0, 0, minus 1, 1, 2, 3, 4, 5 elements, next 1, 0, minus 1, 0, 0, next 0, minus 1, 0, 0, 1 then 0, 0, 1, 0, 0 okay 1, 2, 3, 4 then 0, 0, 0, minus 1, 0 okay then 0, 0 sorry 0, minus 1, 0, 1, 0 then 0, 4, 0s and 1 okay, then 0, minus 1, 1, 2, 3, 4, 5, 6, 7, 8th row 1, 2, 3, 4, 5, 6, 7, 8th row, 0, minus 1, 0, 1, 0 then 1, 0, minus 1, 0, 0 then 0, 0, 0, 0, 1 question is without drawing the graph, without drawing the graph indicate which branch is arranged parallel and which are in series, did you get my point? There is a network I am not drawing any particular graph for this 2 elements are in parallel which elements are in parallel and which are in series in series means what? 6 8 there is no other interconnection here okay.

Suppose this is some node p then there are 2 elements which are coming like this which are in series means what does it mean at that node, at that node there will be a plus 1 and minus 1 and these elements are connected to some other nodes then only in series and okay thank you, if these 2 nodes are in parallel same starting and finishing nodes. So do you find any such see this and this see 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10, so 6th and 8h ye minus1 and are they in parallel and then 7 and 10, seven and10 7 and10 are in parallel is it not they are having the same 1, 1, minus1 is in minus 1 is in reference ya possible minus 1 is in the reference.

Okay so 7 and 10 and 6 and 8 all right they are in parallel and what about series at the same node there will be plus 1 and minus 1 at the same node there will be plus 1 and minus 1 2 and 4, node2 and 4 node2, element 2 and 4 node 3 but what about element 9, element 2, 4 and 9, so there will not going from the , are they in series then? Are they in series then? Not in parallel suppose there is a branching like this < like this will you call it in series no so is there any such element here, no any such element in this node 1, 1, 1, no is there any such element here no, is there any such element here, no not even here. So that kind of a node is not present here if it is possible then there will be only one plus 1 and 1 minus 1 in a particular node all right, more than 2 elements will valid that condition okay. We will stop here for today and we will continue with that Echelon form and that elementary transform in the next class, thank you very much.

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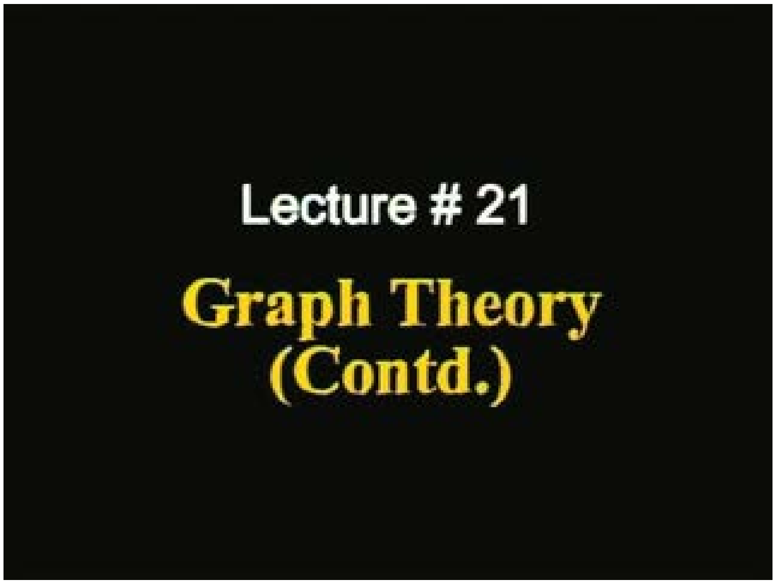

Good morning friends, today we shall be discussing about computer generation of topological matrices. Now generation of the matrices b_f and b_f is quite a formidable task. Here see how to generate A, now A we started of with a_0 element matrix and then we filled it up with plus 1 or minus 1 depending on the interconnections between various elements. Now for q_f it is if you remember it is a_t inverse into a, a_t inverse into a and b_f was unity matrix and a_t inverse a_l okay sorry b_f transpose b_f transpose was minus of a_t inverse a_l and a unity matrix all right. So competition of this will require competition of a_t inverse.

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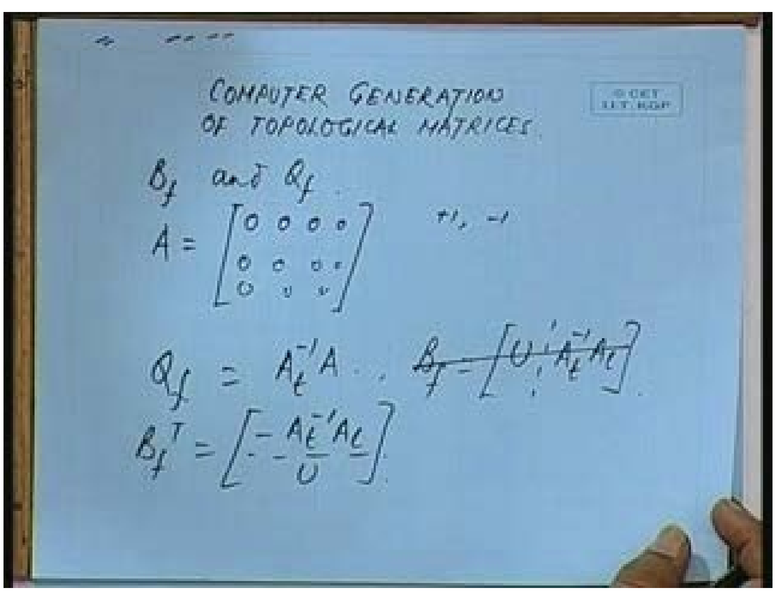

Now if you are having say 50 or 60 node system, 50 or 60 node system then it is it is not advisable to go for the inversion of such a matrix because a_t will have the dimension 50 by 50 or 60 by 60, all right. So if a_t.