Networks, Signals and Systems
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Lecture - 19
Graph Theory (Contd…)

Good morning friends, last time we are okay, good morning friends last time we are discussing the basics of graph theory. So we have defined what tree, co tree, branches twigs etcetera mean okay utilization to network graphs. Now we will continue with this and we will find out the different relationships between the standard matrices. We also discussed about cut set now what are the cuts I just forgot to mention last time, what is a proper cut and what is an improper cut. Suppose this is one graph of a network I will chose an arbitrarily the orientation of different elements these are the nodes will be like this, you can have cuts along any chosen line okay.

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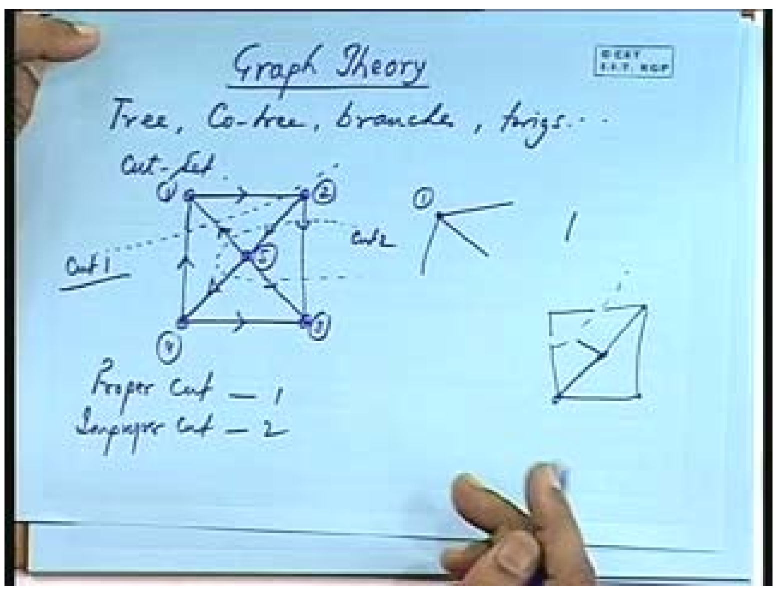

So these are the elements being cut, one may chose a cut like this also that is also cut I can cut it with a scissor in any way I like but if after a cut when you are putting I say when you are separating the 2 parts of the entire network then the parts must be connected to a node all the cut parts, the cut branches must be connected to a node there should not be a part like this then this is a proper cut. So this is a proper cut you see if I call these node numbers 1, 2, 3, 4 and 5 this is cut 1 by cut 1 these 3 elements are appearing like this they are connected to node 1 and others are connected to node 2, 3, 4, 5 is that all right. So node 2 this branch is cut into 2 parts, so one branch will be connected to node 1 the other part is connected to node 2. So node 2, node 5, node 3, node 4 and so on these are the cut parts.

So this is the line of cut but if I consider this cut, if I consider this cut you see this element will be hanging it is connected no nodes all right it is an unconnected portion. So this is an improper cut all right, so when we consider cut sets will be considering only the proper cuts. So proper cuts will be cuts like 1 cut 1 and cut 2 is improper cut okay. We also discussed about the network relationships that is established by the matrix a was the overall matrix we had the elements on this side 1, 2, 3, 4 and so on and these are the nodes 1, 2, 3, 4, 5 etcetera you have observed for each element that is a plus 1 symbol we have assigned a plus 1 symbol when a particular element goes out from a node say from node 1 if a particular element is going out then node 1 will be assigned plus 1 sign and wherever it enters that will be given minus 1 sign.

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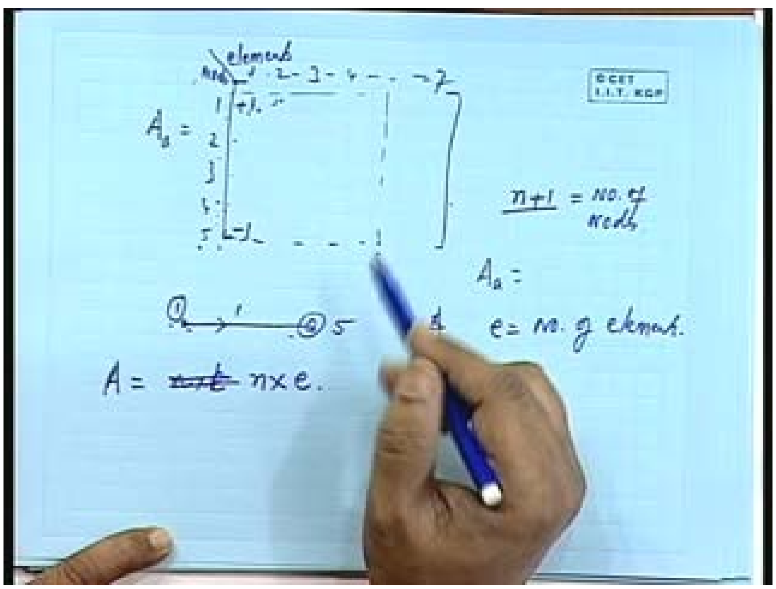

So say element number 1 this is element number 1, it will have a plus 1 sign here suppose this is node number 5 then it will be having a minus sign here so like that for all the elements there will b e_1 plus 1 and 1 minus 1. An element can be connecting only it can connect only 2 nodes it cannot connect 3 nodes so there will be only 2 non-zero elements others will be all zeros. Now if you add up all the terms in a particular column it will be 01 plus 1 and minus 1 and rest are all 0, so if we consider any of the rows, any of the rows, it is not an independent one, independent number of rows will be one less than the total number of nodes you are having 5 rows say, if you have a 5 node system like this there can be many more nodes. Suppose you consider this 5 node system and these are numbers say 1, 2, 3, 4 like that so each element will be connecting only 2 nodes. So there will be a plus 1 sign here a minus 1 sign here.

Okay so this a a matrix what will be its rank what will be its rank can it be 5 suppose there are 7 elements here so it cannot be more than 5 by 5 square matrix at the most can be of size 5 by 5. So the rank at the most could be 5 now if you find that one of the any particular row is in depend it is not independent means this can be generated with the help of the other 4 rows then the rank will be less than 5 because its determinant will be 0. So it is not linearly independent, so we resort to a matrix a taking one of the rows any one of them as a reference, any of the nodes as reference and deleting that particular row is that all right.

So A will be known as the incidence matrix whose dimension will be suppose there are n plus 1 number of nodes, number of nodes okay suppose n plus 1 is the number of nodes then n will be the order of a, n by n, n by total number of branches, total number of elements say there are 7 elements 7 by 4 that will be the dimension of A. Okay e is the number of elements, so matrix a is having say in this particular case a dimension of 4 by 7 if there are 7 elements so what will be the rank of A it can be at the most 4, at the most 4. Let us see the nature of matrix a now a I can partition I can always select a particular tree, I can always select a particular tree and arrange the elements of the tree first and then the links later 1, 2, 3, 4, 5, 6, 7. I can choose a tree in such a way that the tree elements say may be 1, 2, 5, 6, 7.

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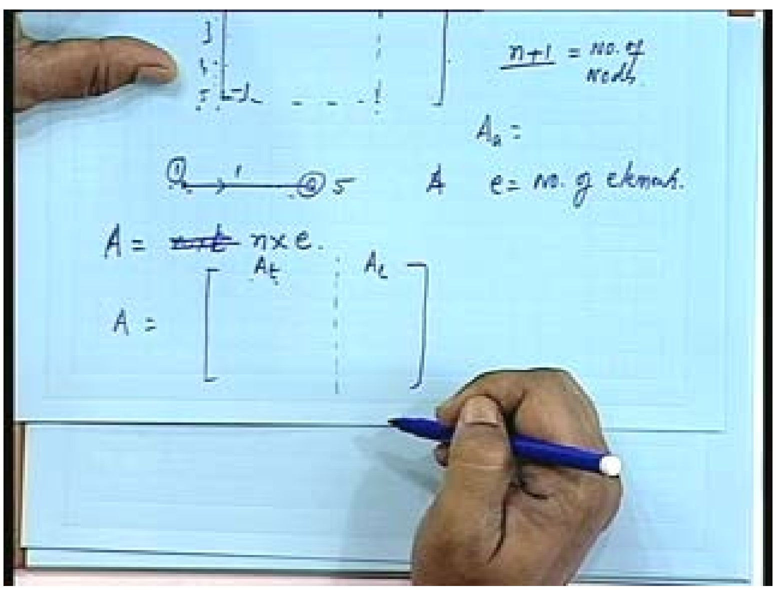

I can choose a tree of number a the element numbers 1, 2, 5, 6, 7 then I will put them first and then the rest of the elements which we discussed last time at links that can be put separately. So a mat matrix a can be partitioned into A_t and A_l, A_t refers to the tree elements, A_l refers to the link elements alright. So this is not unique because the tree is not unique tree can be anything I can choose any tree there can be many possible trees okay, now in a tree for each column you have got say these are the tree elements you have got once at least 1, you will get a matrix a.

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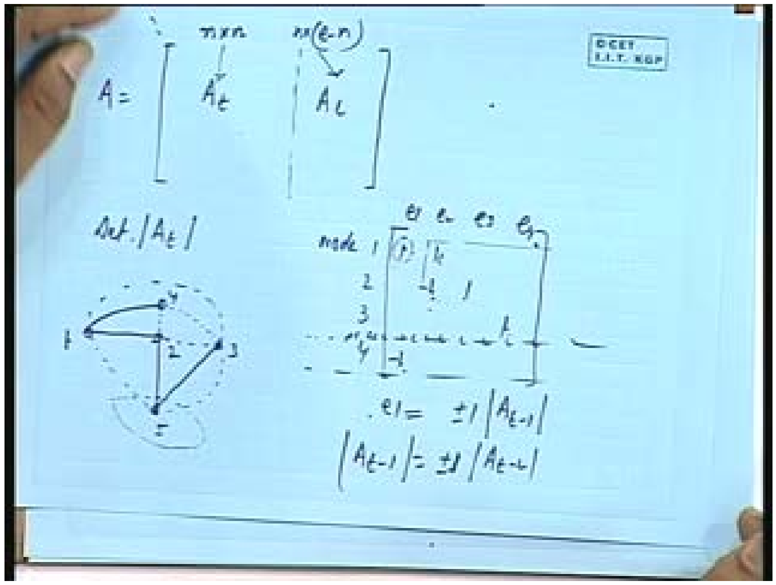

I have got A_t A_l this is n by n, this matrix n by n where what would be the number of elements in a tree if you are having n plus 1 as the number of nodes if there are n plus 1number of nodes, how many elements are there in a tree 1 less n, only n, 1 less than number of nodes, is it not. So if you are having the number of element in a tree as n then the tree matrix that is A_t this sub matrix will have the order n by n okay is that all right. So rest of the elements will be e minus n here, so it will be n by e minus n okay number of elements can be any thing you can have large number of elements, highly interconnected network you can have.

So what would be the determinant of this a A_t that will be determining the rank of the matrix that is n and what is this determinant A_t what will be the determinant A_t it consists of plus 1 and minus 1. Suppose we arrange the tree elements in such way this is node 1, 2, 3, 4 okay suppose this is a tree, so these are the links, possible links there can be any more I mean a many more connection there can be a connection here also okay. Suppose this is a tree these are the nodes given I choose this as the reference. So it will be 1, 2, 3 only but this is sorry 1, 2, 3, 4, this is 1, this is 2, this is 3, this is 4 and this is 5, this is the 5 nodes and 1, 2, 3, 4 these are the nodes baring that reference one. Now here we will find 1 to 4 both plus 1 and minus 1 will be included okay 1 to 2 in this particular case say 1 to 2 again the second element will have plus 1 and minus 1, 2 to 5, 5 since 5 is eliminated so there will be 1 plus 1, so in the third element there can be a plus 1, I am not putting the 0s rest are all 0s.

Similarly for 3, node 3 there will be one either plus 1 or minus 1 okay depends on the direction you choose and for 4 again there is an element. So for element no sorry these are the elements so 1, 2, 3, 4 there are 4 elements okay these are the elements, element 1,element 2, element 3, element 4 okay. So element 1 so all of them are having at least 1element which is 11 non zero element there can be 2 non zero elements also 1 and minus 1 like this, for this, for this element between 1 and 2, 1 and 4.

So this is having 2 non zero element, this is having 2 non zero elements others are having 1 non zero element okay if I take the determinant of this say it will be one multiplied by the determinant of the 3 by 3 matrix there suppose I choose this one first e_1 first which is a non zero element it can be plus 1 or minus 1. So corresponding to this the determinant of A_t will be plus minus 1 into determinant of the element with 1 less order okay I, if I call it A_t minus 1 basically that will be n by n minus 1 by n minus 1. In that again if I chose the next element which is having a plus 1 in the second column all right so bring that first then it will be again plus minus 1 into A_t minus 2 that will be equal to determinant A_t minus 1, so like that since there is always a non zero element non zero element and there will be write up to the end that means all the nodes are connected with some element or other so you can choose always the nodes corresponding to their position such that this will be always plus minus 1 into plus minus 1 into plus minus 1into so the determinant will be always the overall determinant will be always plus minus 1, did you get my point, what therefore we are trying to arrange is a matrix A such that we get all plus minus 1 along this it may be plus minus 1 plus minus 1 plus minus 1 and so on I have picked up nodes all right

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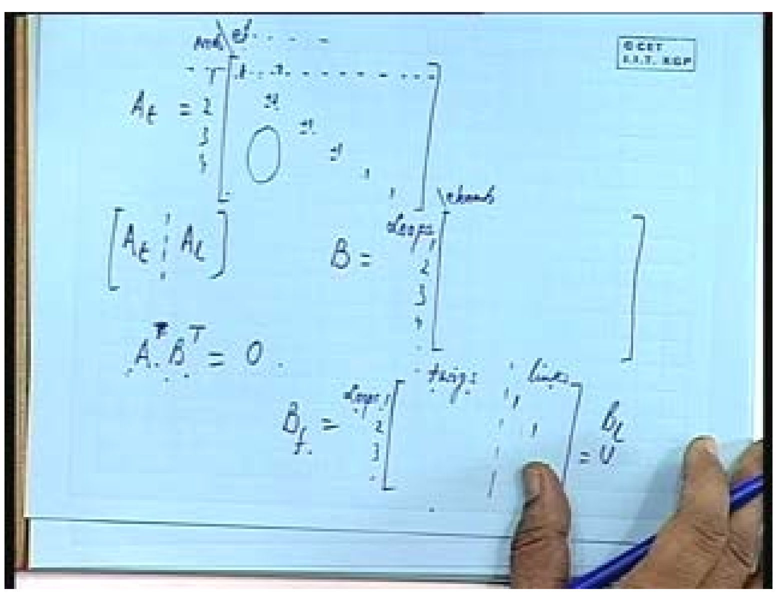

Suppose this is node 1 this is element 1, I have picked up the nodes such that in the second position which ever is having the second element as non zero I place that as a second element here okay I can either pick up the elements in that way or I can change the order of the nodes order of the nodes in such a way that 1, 1, 1, 1 they will come appear they will appear like this I can try to get a 0 here.

Here there can be plus minus ones or zeros this will give me a determinant which is 1 okay. Now after partitioning a into this 2 sub matrices A_t and A_l we discussed about matrix B that is the loop matrix if you remember you you consider a any tree first then put one link at a time then that will be called a fundamental loop after the selection of a tree you choose a particular link so that is one fundamental loop. If the second loop comes, a second element comes here then there can be a loop like this but that will not be a fundamental loop because fundamental loop is created out of a tree, there will be only one element at a time there can be many loops but all of them are not considered as fundamental loops fundamental loops means in a fundamental loop just one link will be present this is one loop okay if there is another element all right connecting these two nodes then that will be another fundamental loop but if there is a link here and there is another link here that will be a loop but that will not be a fundamental loop because that is consisting of more than one links. So this is how we define the fundamental loop.

So if you consider B with loops 1, 2, 3, 4 need not be always fundamental loop any loop and you are having the elements here then what did we establish A say if I multiply this sorry a transpose, a transpose b, what does it mean a transpose b, was it a transpose B A B transpose or AB transpose either way even take transpose of that also okay AB transpose we prove that is equal to 0 okay, what does it mean physically? Let us see this are the elements here these are the elements here and these are the nodes. So the elements that will be connected here these are the nodes rest are all 0 and the 0 may not be sorry may not be, so you are having many of these say node 1, 2, 3, 4 and so on so each node will have a plus 1 sign and minus 1 signs are connected may or may not be there and if I take B transpose, B transpose.

See a matrix A will consists of plus one signs and minus 1 signs for any element if that element is involved in a loop then in the loop basically we are considering the loop like this there are elements how many links are present <a_side>one link one link at a time okay so when you go along this from a particular node from a particular node there can be large number of element coming out but in a loop only 2 elements will be coming into picture. Now one is going out suppose this is going out, this is also going out then in the loop this will have a positive sign, this will have a negative sign.

So in the a matrix though both of them are positive in the loop matrix, one is positive the other one is negative and other elements connected to matrix A will not be coming into picture in the corresponding elements in the B loop all right. So whenever we are multiplying a with b transpose we are considering the elements b transpose means the elements connected in a particular loop involving that node all right. So only 2 elements will come into picture in a particular loop there can be another loop considering this element and another third element connected to this node okay. So ab transpose is always 0 irrespective of the type of loop.

We shall be considering only the fundamental loops, what is so big about fundamental loops? If I consider b fundamental means if we consider a tree, if we consider a tree then what would be the elements of a tree and elements of the links in the b matrix. Can you partition b matrix into b_t and b_l, so let us see what b_t and b_l will look like okay. So b twigs okay twigs means the branch of a tree and links, this is the loops 1, 2, 3 and so on. Now first loop first loop in a fundamental loop matrix in the first loop you are taking the first link all right, the second link I am calling as second loop, so if I number the loops like that then this will be 1, 1, 1 like that is it not. So that will be a unity matrix and identity matrix so for b fundamental if I partition the matrix b into twigs and links then the b link sub matrix will be an identity matrix sorry unity matrix, is that all right. I hope this point is clear for example suppose I call I call this one as element number 1, the link number 1 then this is a loop is it not, this is a loop I call that as loop number 1, so loop number 1 here we will have 1 in the links that can be anything here I am not bothered, so far as this element is concerned this is involved in this this is included in this loop.

Similarly, if I consider this as the second loop, second link and this as the second loop then this link is involved in this loop so loop 2, we will have element number 1 of this link there cannot be any other element present in a loop all right because one link at a time is forming the loop so in a fundamental loop you will have a unity matrix here if it is not a fundamental loop then there can be many number of links okay.

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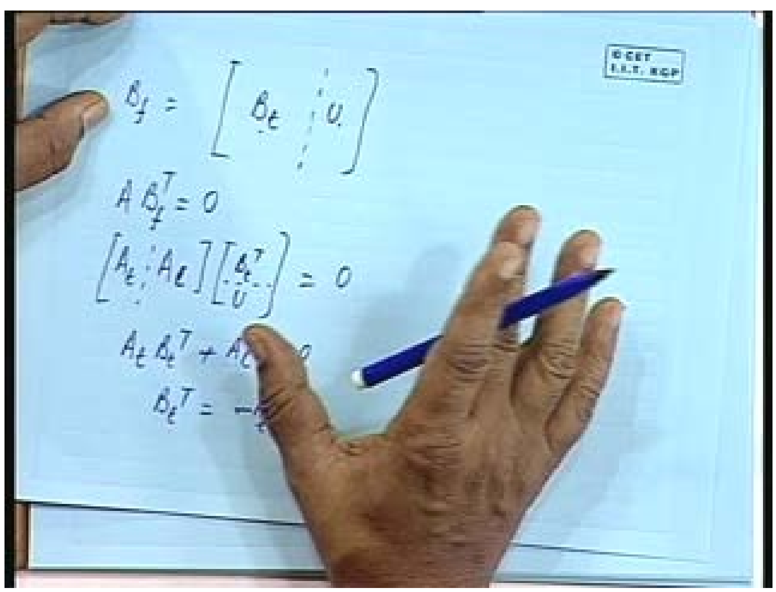

So bl is unity therefore matrix b_f is b twigs and a unity matrix. So what is a b transpose now b this relation ship is valid for any b, b_f is only a class of class with in b unity matrix, identity matrix which one unity matrix yes yes its the same thing. so $ab_f{}^t$ is also 0 I can write a as A_t A_l partition it into 2 sub matrices b_f^t if I take the transpose of this okay so A_t b_t transpose plus A_l is equal to 0 all right. So what is b_t transpose if I transfer it on this side it is minus A_l and sorry A_l then A_t inverse okay.

So you can calculate b_t so from the a matrix itself from the a matrix itself you can can you can calculate b matrix b f matrix a matrix you partition then you take the transpose of the sorry take the inverse of the sub matrix at post multiply by A_l take the negative sign so that gives you bt transpose. So the transpose of the whole thing will give you b_t once you know b_t this you fill up with you so that gives you b matrix so loop matrix fundamental loop matrix can be created out of the incidence matrix a. Now we shall go to what we discussed last time about an algorithm for generating any possible tree, generation of tree. As I told you this is not a unique tree, tree can be anything they can be n number of possibilities. So this algorithm is also going to give you many possible trees it all depends on how you handle the different operations and how you enter the data.

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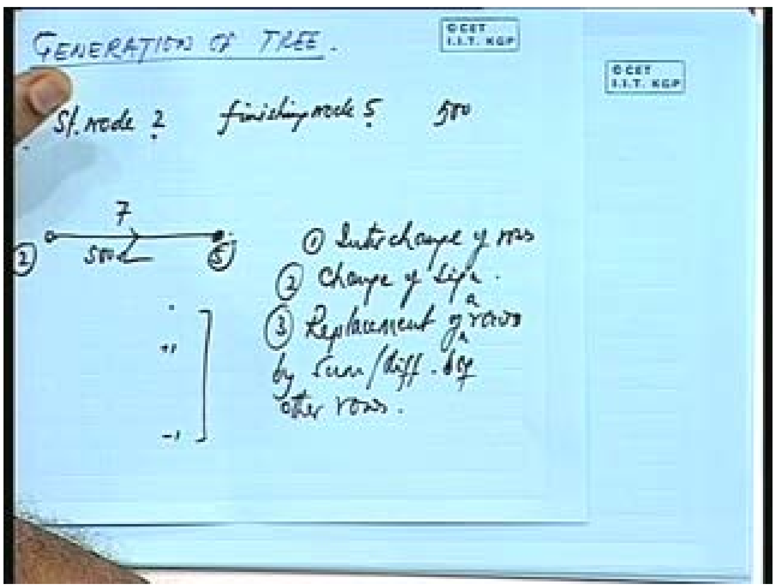

So the first step is normally we give the input as say it is a resistance element so we write r then the starting node say starting node can be two finishing node or ending node may be 5 before this some times we give the element number say it may be element number 7, this is the type it may be resistance, inductance, capacitance etcetera. So this is between node 2 and node 5 you have an element number 7 the data has to be entered like this finishing node 5 and then element value resistance can be say 500 means, 500 ohms okay. So the moment you define this if it is l, it will be in Henry, if it is c, it will be in farads and so on. So the units are decided by the type of element that you entered here any way after the date, data is entered it will be only this 2, 5 and 7 which is of importance to us.

So far as the network graph is concerned because, we are not bothered how 2 nodes connected whether it they are connected by a resistance or an inductance or resistance inductance combination what is 7 indicates the unit in the lab okay in the network, in the network graph it represents element number 7 because when you have given an orientation, an oriented connected oriented graph means you give a particular orientation as well as a number to the element you also number the nodes from any network when you draw the graph you number the nodes and the elements because finally you will be hand handling matrices. So the node number and the element number numbers are relevant in your matrix A and B if you see the rows and columns element number 1, 2, 3, 4 so number 7, element number 7 what is it? Which is connecting say node number 2 and 3 node number 2 and 3 may be connected by element number 4, is it a resistance, is it an inductance. So that must be specified in the while entering the data okay.

So in the A matrix initially we will be starting with all 0 elements okay as and when the data comes, so we will start with element number 1, 2, 3, 4 whatever be the way of numbering them after that you sequentially go by the element number and their specifications. So element number seven when it comes to 7, element number 7 it will fill up starting node 2, so 2 will be given a plus 1 sign, 5 will be given a minus sign. So earlier they are all with zeros so 0 plus 1, 0 minus 1, so that will be modifying the matrix a, so this how you keep on entering the data. Now after entering the data what operations should we carry out, so there are 3 standard operations we call them elementary operations which are used here which will be very subjective when we choose the particular operation it is not a unique 1, 1 is interchange of 2 rows, you can interchange any 2 rows say 1 with 3 or 1 with 4, I choose 1 with 4, you choose 1 with 3 so you will get 2 different transformed matrices.

So matrix A is gradually transformed by various operations, one is interchange of rows next you can change the signs that means plus will be replaced by minus, minus will be plus so change of sign and then the third one is replacement of a row by addition, subtraction all these and combinations of these operations of any other rows all right. So replacement of a row by the sum or difference of other rows, replacement of rows of a row by sum or difference of other rows okay. Now these 3 elementary operations we shall be using in transforming a matrix.

Let us take up an example then it will be clear what we are aiming at suppose we have but then one may ask to what end finally what is you aim you keep on doing it what is your aim? Our aim is to generate a matrix suppose these are the rows 1, 2, 3, 4, 5, 6, 7 and these are the nodes 1, 2, 3, 4 okay. Now we shall try to form a matrix form which is known as Echelon form which means there will be zeros say it will be step like structure step like structure here we have ones below this they are all 0 and the elements here can be a's where a can be either 0 or 1, I am not bothered 0, 1 or minus 1 okay. It can be anything but here to the left of this it is all 0 then wherever this steps begin these are the nodes which can form a possible tree. So here it is 1, 2 next one is 5 and then 6 did you get my point this is called an Echelon form. So we are going to row Echelon form.

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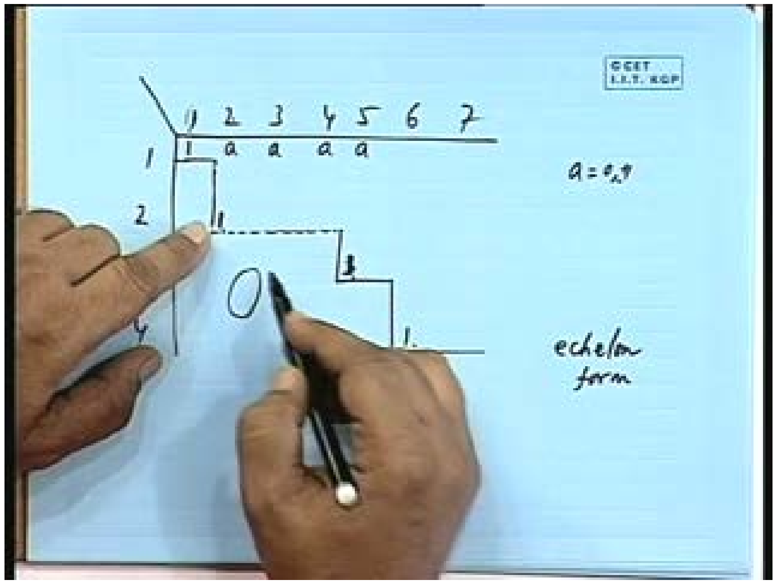

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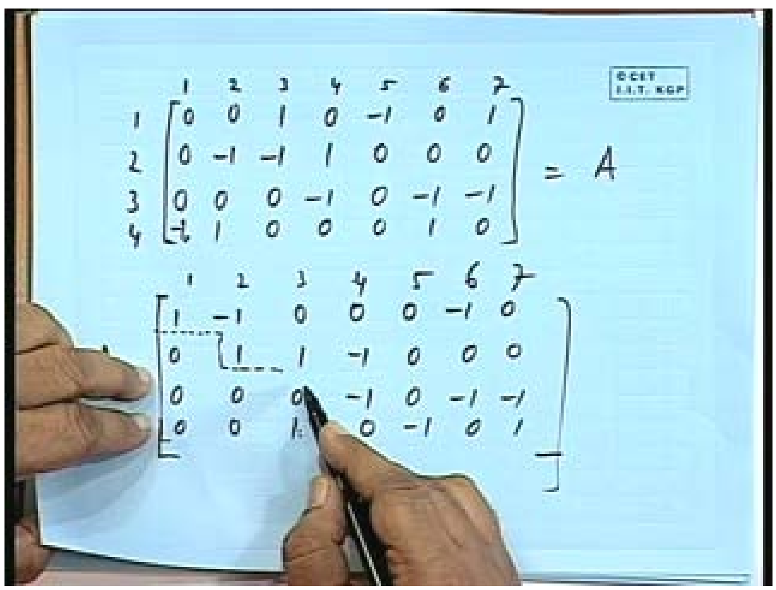

So we are going to identify those nodes where this one starts okay below which the elements will be all zeros this information is no, you have to create one and then you will know what are the nodes to be selected for the possible tree if you connect, what are the element 1, 2, 5, 6 they will form a possible tree. So this is this is not unique so somebody make get 1, 2, 3, 7 that also can be another possible tree then element number 1, 2, 3, 7. So let us take up an example and see what results we get, suppose you are given 1, 2, 3, 4, 5, 6, 7, 1, 2, 3, 4 the values are 0, 0, 1, 0 minus 1, 0, 1, 0 minus 1, minus 1, 1, 0, 0, 0, 0, 0 this is my a matrix 0, 0, 0 minus 1, 0, minus 1, minus 1 then forth one is minus 1, 1, 0, 0, 0, 1, 0 sorry by the way can you draw the, can you draw the graph, can you draw the graph for this.

So reference is the fifth that is minus 1, plus 1, so let us see what the connection is like. So from the a matrix can you re draw the graph I am not using this space because I will be using for the conversion of matrix a. So I will just show you how the graph will look like okay what will be the graph like element number one is connected between minus 4, 4 and the fifth one.

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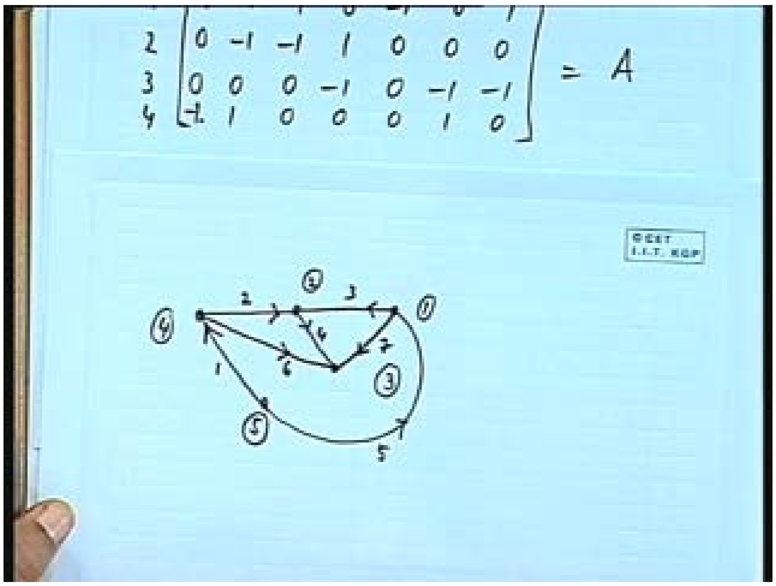

Suppose this is 5, this is 4, so this is element number 1 okay element number 2 is between 2 and 4 so suppose this is 2, 2 and 4. So from 4 it is going towards 2 then 3 is between 1 and 2 suppose this is 1 then 3 is between 1 and 2, 1 is positive, 2 is negative okay 4 is between 2 and 3, suppose 3 is some where here okay between 2 and 3 all right this was element number 1, this was element number 2, element number 3, this is element number 4, okay.

So from 2 it is positive 3, this is node 3 then element number 5 is between 1 and 5 so between 1 and 5 this is element number 5 okay 6 is between 3 and 4. So node 3 and 4 is that all right, so this is 6 and 7 is between 1 and 3, so between 1 and 3, 1 to 3, this is element number 7. So this is the type of graph is that all right so from a given matrix from a give matrix you an always draw the graph now what is the possible tree for this there can be many possible trees. So we are trying to identify one of them I hope this is clear. Now what do we do the first operation I want a matrix as far as possible, first one will be there if it is possible all right we will start with one there so put 4 interchange row 4 with 1 and then change the sign of this.

So next I will call it a one sir a one is root transformation already there allowed or only we call and assume transform no you see elements they you cannot keep on changing both then what will be the location. So these are the nodes that will be selecting or these are the elements that will be selecting for the tree so this will be retained 1, 2, 3, 4 etcetera this side we have changed interchanged 4 with 1 and then change the sign.

So it will be one minus 1, 0, 0, 0 minus 1, 0 okay then 0 if you permit me I will skip some steps I will also change the sign of row 2 in 1 go so it will be 0, 1, 1 minus 1, 0, 0, 0 okay third one I will retain 0, 0, 0 minus 1, 0, minus 1 minus 1 and this 1 will be 0, 0, 1, 0, minus 1, 0, 1, correct me if I am wrong is that all right. So I have already identified you see this is below this step it is 0, now if I interchange these 2 then this one will come here.

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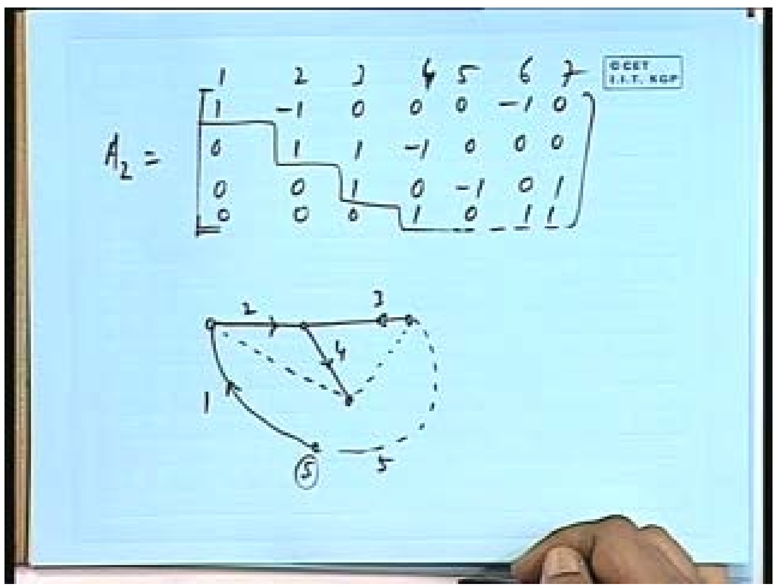

So we can change the sign of then a_2 if I interchange 3 and 4, this is 1, 2, 3, 4 new row numbers so 3 and 4 if I interchange then this will come here and 4 will go there I mean this minus 1 will go there okay and then change the sign. So you will get 1, minus 1, 0, 0, 0, minus 1, 0 then 0, 1, 1, 1, minus 1, 0, 0, 0 then we get 0, 0, 1, 0, minus 1, 0, 1 and then 0, 0, 0, 1, I will change the sign okay I will change the sign 0, 1, 1, 0, 1. So element number 1, 2, 3, 4 may form a possible tree these are very simple case in this particular example that we have taken it came very easily so 1, 2, 3, 4.

Let us see the graph here that we got element number 1, 2, 3, 4, let us see the tree how it looks like, this was 2, this was 4, this was 1, this is node 5 and then 3, this was 3. Now does it look like a tree 5, 6, 7, this is element number 5, you see so this is a possible tree 1, 2, 3, 4 and 5, 6, 7 they form the links, is it not.

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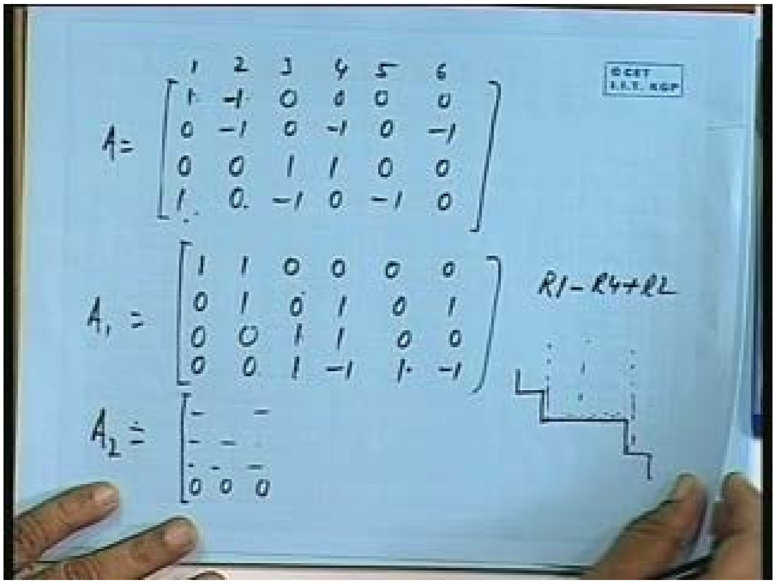

Another example, we try suppose you are given matrix a, 1, 2, 3, 4, 5, 6 1, 1, 0, 0, 0, 0 then 0, minus 1, 0, minus 1, 0, minus 1 then you have 0, 0, 1, 1, 0, 0 and then 1, 0, minus 1, 0, minus 1, 0 this is the matrix a. All of you can reconstruct the graph from the given mat matrix a, I told you just now how to make it. So a_1 what could be a possible transformation of this I can keep 1 there and then 2 here, second row change the sign okay so I will get 1, 1, 0, 0, 0, 0 then 0, 1, 0, 1, 0, 1 then third one is also possible 0, 1, 1, 0, 0, fourth 1 if I subtract from here if I subtract from here then what do I get 0, 0, minus 1, then you are subtracting one from 4, 2 so it will be 0 minus 1okay.

So how to eliminate that multiply 0, minus 1, I wanted to skip some steps so that I can write in a compact form here. So 0 if I subtract I get 0 if I subtract I get minus 1, 1 or minus 1, I can subtract this one from here, so say plus 1then if I add with this second one then that will create a 0. So subtract this from here so row 1 minus row 2 plus row, a row 1 minus row 4 plus row 2 what what we get okay let us be satisfied with that operation row 1 minus row 4 plus row 2, what do we get this minus this 0 then this minus this plus this so 0, so this minus this plus 1 plus 0, 1 plus 0, 1 then this minus this and plus 1, so minus 1 then this minus this so plus one this plus this and minus 1 okay.

Now subtract from the third row a_2, yes if I subtract from the third row will that be all right so up to first 3 I am not writing anything, it is only the last 1 which is of interest which one to modify okay for 0, 0 if I subtract from a the third one what do we get 0, 0, 0 and then and add the fourth one then what we get then this one will come. So that will not be desirable so if you subtract one from the other I will get a 0 here okay I will get 0, 0, 0 then I will get 2 here then what will you do we can yes, if others are 0 in that row can we get divided by 2 in the row 4 if other element in the row 4 is 0, we can divide it by 2 but for

Otherwise, otherwise we have to multiple another another row by two and then subtract it to be zero, can you try that, can you try that? So this is the question that means 1, 2, 3, 4 will not be a possible tree may be 5 or 6 so the Echelon structure will be like this that means it will be again coming down at some other element okay, rest of the elements here will be plus 1 or minus 1 you have to tolerate that that means here you are not in a position to get a 1 and below this all zeros at this point it may come here it can be 2 zeros it can be 0, 0 then 1, did you get my point so we will take this in the next class since it is already time I deliberately took a very simple example first. So that 1, 2, 3, 4 you can always arrange but here it will be stalemate you cannot get 1, 2, 3, 4 as the tree elements okay. Thank you very much.

Preview of Next Lecture

Lecture - 20

Graph Theory (Contd…)

Good afternoon friends, we shall continue with our discussions on graph theory yesterday, we had taken up a problem the a matrix was given please correct it there is a minus sign here, I forgot to write that 1, minus 1, 0, 0, 0, 0, 0, minus 1 and so on.

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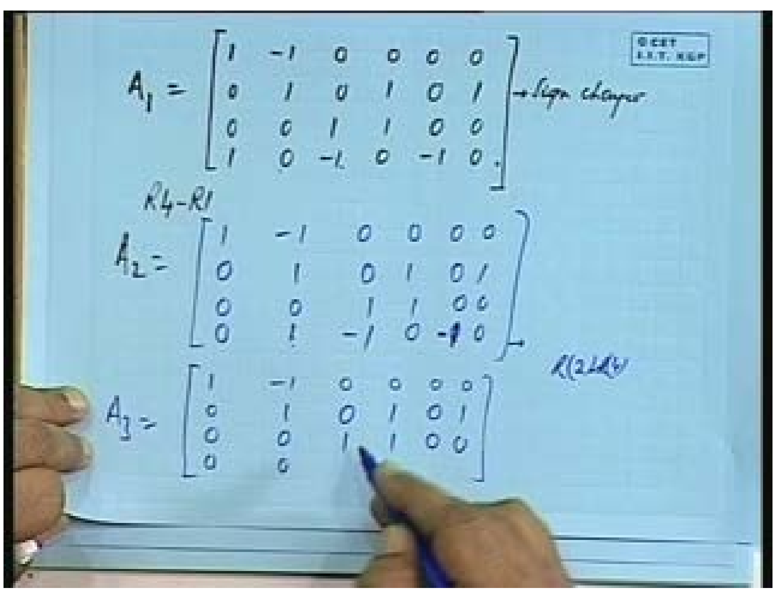

So we shall work it out here, so from a matrix if you apply those transforms what would be the possible next transform I can change a sign of this okay, I can change the sign of the second row, so this will become 1, minus 1, 0, 0, 0, 0 then 0, 1, 0, 1, 0, 1 okay then 0, 0, 1, 1, 0, 0, 1, 0, minus 1, 0, minus 1, 0 okay. Then from here if I subtract this this one was sign changed from the previous matrix, then if I subtract row 1 from row 4, a_2 row 1 you subtract it from row 4, you get 1, minus 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1 then 0, 0, 1, 1, 0, 0 then this minus, this will give me 0, then 1, is that alright then minus 1, minus 1 then 0, 0 sorry, minus 1, 0, is that okay 1, minus 1, 0, minus 1, 0 and then what you do this is having 1 here. See if I subtract, if I subtract this from this 2 minus 4, row 2 minus row 4 what you get? a_3 1, minus 1, 0, 0, 0, 0 then 0, 1, 0, 1, 0, 1 then 0, 0, 1, 1, 0, 0 then 2 minus 4 will give me 0, 0 next, guess please minus 1 .