Networks, Signals and Systems
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Lecture - 18
Graph Theory

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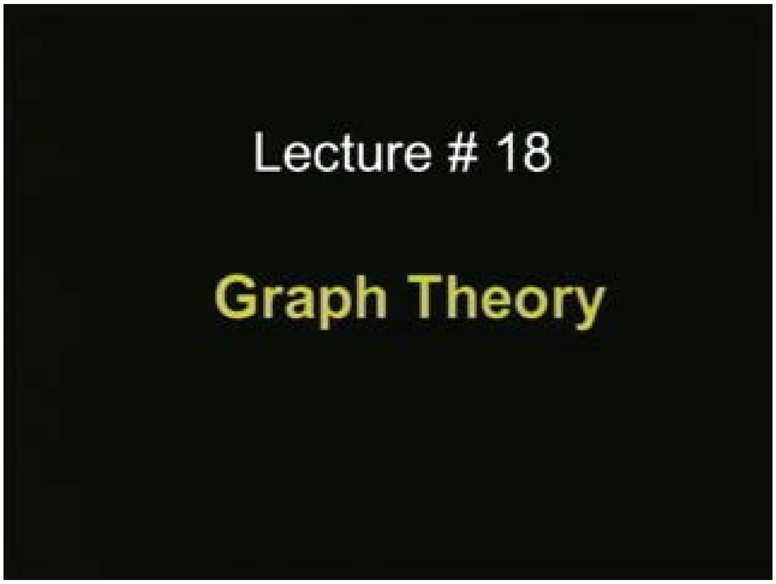

Good after noon friends, today we shall be taking up graph theory that is very much useful in networks problems.

Now let us first of all define some of the basic terminologies and and then we will see how to use these terminologies and how to use the basic network loss in a very compact form especially when you have a multi node say 100 of nodes in a network the solution of the network problems with different conditions, say voltages at different nodes may be given some may be may be may not be given and there are different types of elements, 4 terminal that is 2 port network elements. So it is a complex mixture of different kinds of elements passive and active. So solution of such networks become very involved we reserve to graph theory in such cases topology it is a branch of mathematics, it is a branch of mathematics concerned with selected properties of collections of related physical and abstract elements all right. In network topology we include the particular configuration of nodes and branches, the configuration of nodes and branches without trigger to what elements exist in those branches.

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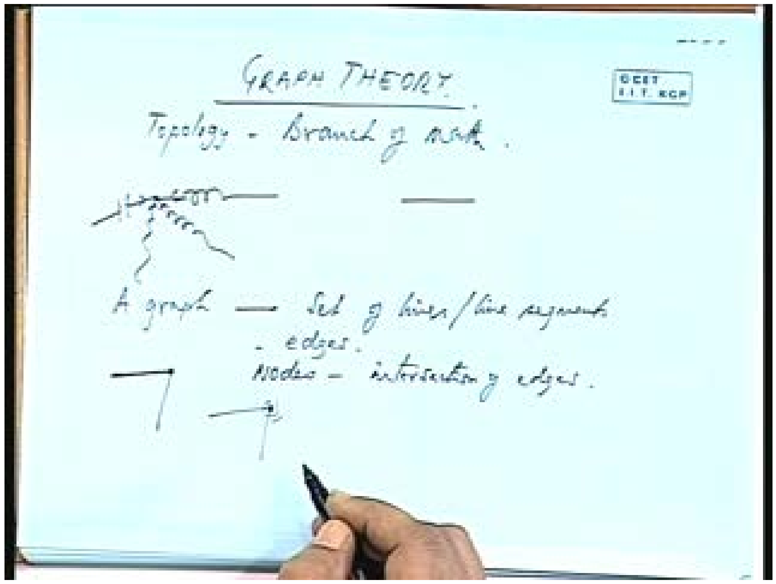

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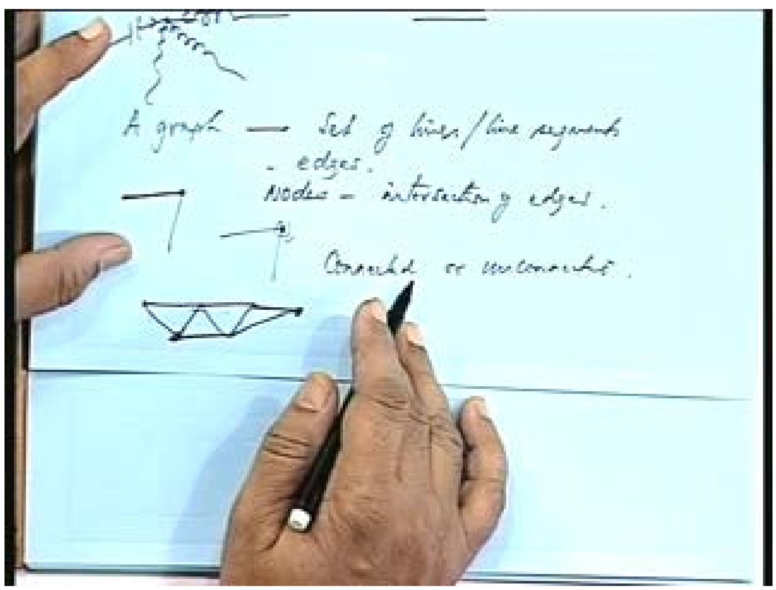

Suppose you are having an inductor, a resistor, a capacitor, for example when we apply Kirchhoff's law currents flowing in these elements will sum up to 0 irrespective of the type of elements that you have the net in the network, is it not. So this can be used this network topology can be used for different types of networks like water and gas mains, mechanical network, traffic flow etcetera. See network theory this Kirchhoff's current law we can also apply to traffic flow is it not number of vehicles moving towards a particular junction and number of vehicles going out at an instant that will be summing upto 0. A graph is a set of lines, we call lines or line segments known as edges these are called edges so each element in a network will be represented by a line or an element okay, these are called edges then intersection of edges will be known as nodes or vertices, intersection of edges, edges are incident only with vertices, they cannot hang, they will be always coming to a vertex okay. The degree of a vertex is a number of elements falling on to that vertex okay a graph may be connected or unconnected that means between any two vertices if there is a path existing all right. You may have between this point and this point I can always find some path okay. So this is connected, this is a connected graph.

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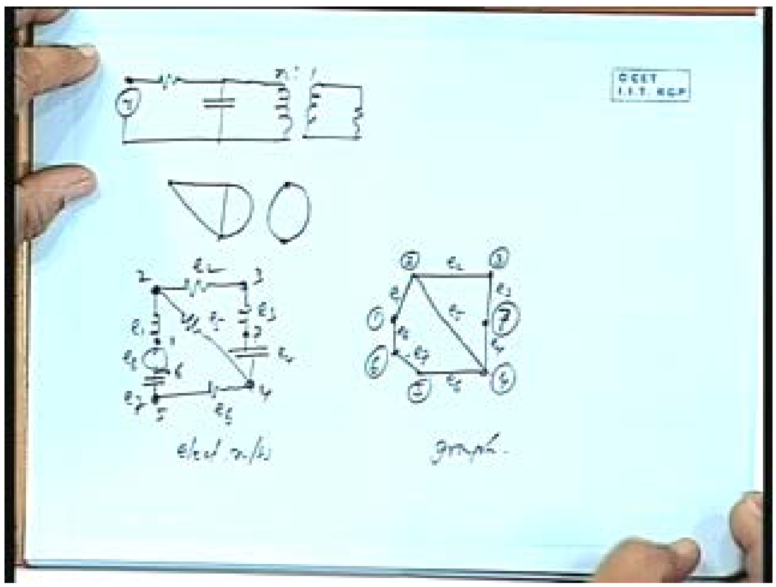

Now consider a situation you are having a transfer winding okay n is to 1. Here I can put all these elements generator as well as the resistance here as a single edge or if you want generator can be shown separately so I can have another edge for this each element will be represented by an edge. Similarly, a capacitor by this okay then there will be some leakage reactance here, resistance here in the winding, so that will be represented by another edge this side, there is some leakage inductance here, mutual inductance and on this side there is a resistance. So these two are connected like this.

So this is unconnected there are 2 graphs unconnected, you may have a circuit like this is source there is a there may be a capacitor here, there may be a capacitor here okay. So I can number the elements e_1, e_2, e_3, e_4, e_5, e_6, e_7 like that and then I can number the nodes also 1, 2, 3, 4, 5 and I put one node here 6, okay then 1 to 2 I will represent by a by an edge here 2 to 3 like this 3 to 4, e_3 and e_4 there may be combined or I can put another node here after this element. So that may be 1, 2, 3, 4, 5, 6, 7, so 3 to 7, 7 to 4, 4 to 5, 5 to 6, 6 to 1 and 2 to 4 okay. We can also write the element numbers against each of these edges this is e_1, this is e_2, this is e_3, this is e_5, this is e_4, this is e_6, this is e_7 and this one if you want e_8, it will between 1 and 8. So this will be the network graph for this circuit okay, so this is the electrical network and this will be its graph. So these are the nodes and these are the edges okay. A degree of the node 4 will be how much 1, 2, 3 okay now we define some more terms tree, co tree, loops, branches rather twigs and links.

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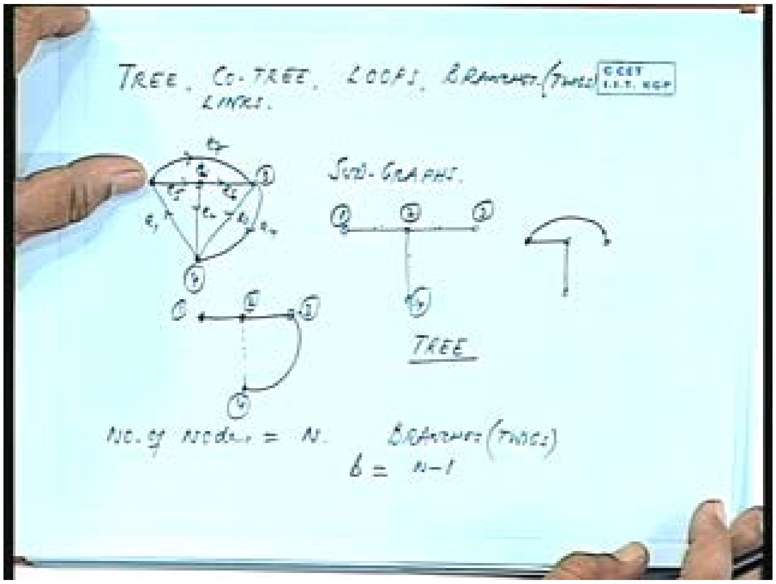

Suppose you have a network graph like this okay, we choose a particular orientation of the elements we can choose the direction of currents arbitrarily in any network, you can select some current directions okay. So we put those directions on these edges okay then this is known as a oriented graph, this is an oriented graph. Once we give an orientation to the edges it is an oriented graph, so we can give some arbitrary orientation to these elements all right. Let this be 1, 2, 3 and 4, this nodes be 1, 2, 3, 4 and these are the orientations. We can number the elements also the edges e_1, e_2, e_3, e_4, e_5, e_6, e_7.

Okay now a tree is a sub graph, a sub graph which contains all the nodes but there is no loop present, this can be a possible tree all right. There can be a tree like this, this is another possible tree, this is another possible tree including e_4, e_6, e_5 okay. You can number the nodes every time elements also you can show, so there can be so many possible trees okay you put any other element there will be a loop suppose we now choose e_2 then there is a loop, is it not. So a tree means there is no loop but all the nodes are connected all the nodes are connected from any node you can go to any other node there will be a path a variable, so this is known as a tree okay how many elements are there in a tree? Suppose the number of nodes is equal to capital N say n minus 1, n minus 1 to connect n number of points you need n minus 1 number of elements all right. So we will call it twigs.

Okay or many books write branches of a tree most specifically it should be termed as twigs because in many other books they term all the elements as branch, branch of a network. So there is confusion, so it is better to mention branch of a tree or better to say twigs, twigs means branches of a tree. So number of branches for a tree will be n minus 1, so number of branches in a tree will be n minus 1. If from the network, if I take out the tree what is left is co tree, so co tree is the sub graph left after the removal of a tree, of a tree from the main graph okay will the co tree have interconnect a all the nodes connected may not be not necessarily.

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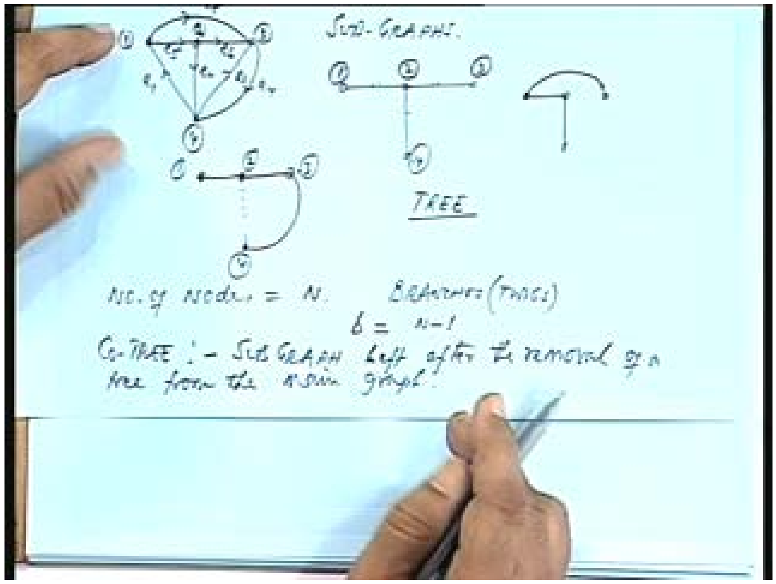

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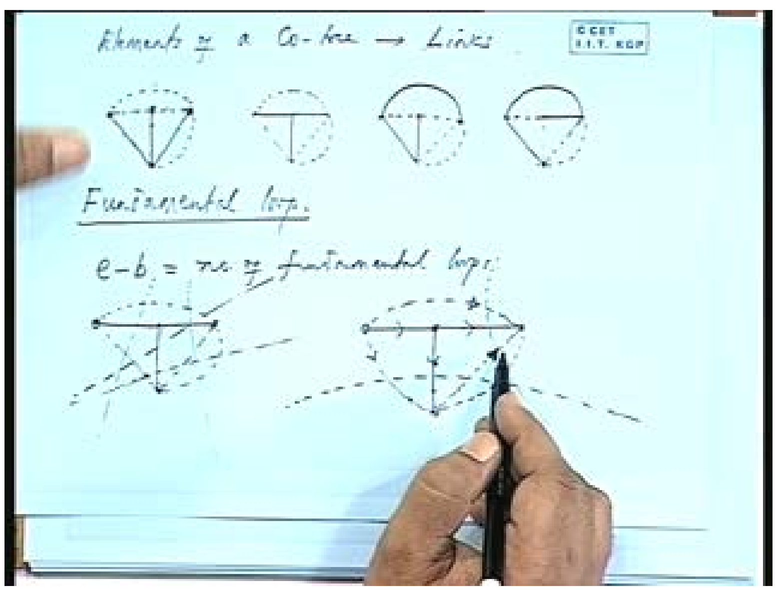

So co tree can be an unconnected set of sub graph whereas a tree is a connected sub graph where there is no loop but all the nodes are connected. So the elements of a co tree are known as links okay, so co tree is basically complementary to tree to a tree okay or the vice versa. Now suppose we have a tree like this with the previous example then what will be the co tree co tree will consist of these dotted lines okay if I choose this as the tree then the co tree will be consisting of these dotted lines. If I choose this as the tree then co tree will be this dotted lines and so on the tree can be this one also then co tree is like this okay, is this clear. Fundamental loop by fundamental loop we mean once you select a tree, once you select a tree you put one link at a time and the loops formed by replacement of such links will be known as fundamental loop okay. So this is if I have selected this as the tree then this is a fundamental loop, this is one fundamental loop, this is one fundamental loop and this is one fundamental loop okay. The loops are consisting of that particular single element and the other necessary edges of a tree.

Okay to form that loop that means the particular link is closing a loop and that is a fundamental loop, this will not be a fundamental loop that means a fundamental loop will consist of only one link you may not this may be a loop but this is not a fundamental loop because there are two links, you take one link at a time whatever loop is found that is a fundamental loop okay. Why do we need fundamental loops? When you are having a very large network you keep on taking while writing loop equations you keep on writing loop equations you never know what will be the order how many loops are minimum required to solve the network problem? How do you really select the number of equations, number of loops? When especially you are having so many variables so select a tree whatever is left over will be the links that is elements of the co tree take one link at a time that will be forming the loops okay.

So number of equations loop equations, number of minimum number of loop equations required to solve the problem will be number of links okay. So total number of edges you say e then what will be the number of links e minus b the total number of links will be total number of edges here 1, 2, 3, 4, 5, 6, 7 and how many are required to form the tree, tree. So there will be 4 fundamental loops because there are 4 links okay e minus b will be number of fundamental loops. The minimal number of edges which if cut separates the graph into 2 unconnected sub graphs, we known as cut sets, cut set means it is a set along a line of cut. So once again I will take an for the same network graph one particular tree like this okay and this are the elements of the co tree that is a links. Now you cut one branch of the tree at a time and separate the graph into 2 sub network, sub graph. So I can cut along this this is a cut set okay fundamental cut set then I can cut along this this is another fundamental cut set.

I can cut along this, this is the third fundamental cut set that is I am cutting this edge of the tree, I am cutting this edge of the tree, I am cutting this edge of the tree okay. So you segregate the 2 parts by cutting one branch of the tree at a time okay there can be many other cuts possible, I can have a cut like this I can cut 2 branches that is also a cut set but that will not be a fundamental cut set okay. We choose another tree then this particular cut may be a fundamental cut set it all depends on the tree you choose okay like in the previous case, the fundamental loops will be dependent on the type of tree that you have selected and hence the type of links that you selected, the elements that you have chosen as the link, is that clear. So in a cut set suppose these are the 3 directions of the tree elements okay when I have a cut set here, this is one cut, so the links here, here, here, here, you will find these 3 links are getting cut along with this okay, this particular twig. We shall be associating a positive direction to the cut set particular cut set according to the direction of the tree element.

So this particular tree element is having this direction, so this direction will be taken as positive for the cut set okay, therefore if the direction of this element initially was given was chosen like this this was the direction chosen then this will be negative and this will be positive if this element was having a direction like this then this will also be positive because the line of cut is taken as positive when the elements are going inside. I have selected the direction of the tree element as the reference okay so all other elements going inward for this cut set will be positive all other elements will be negative there will be given a sign negative, will come to cut set matrix later on all right. Similarly, if I have a cut here then this is the direction which is taken as positive so this element will also have a positive value this element depending on the direction if this is the direction then this will also be positive. So this is the direction of the elements and hence their sign for different cut sets these how it is selected.

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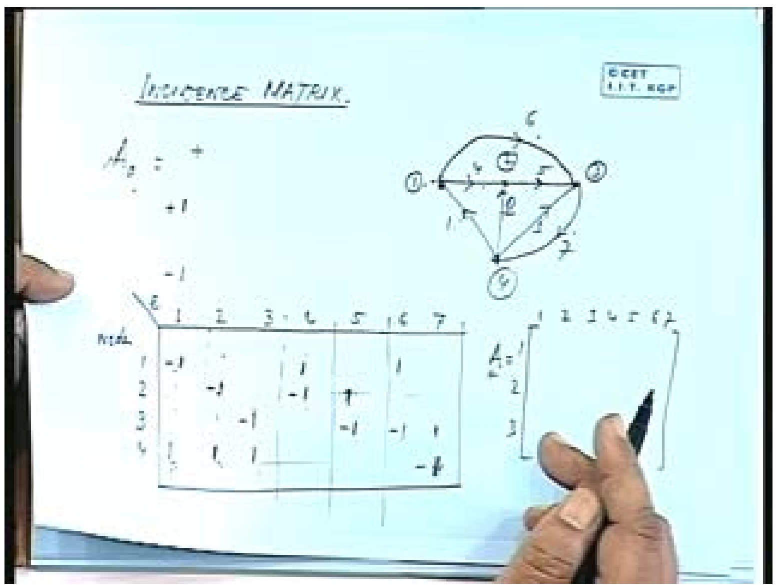

Now we define a matrix, incidence matrix complete in complete incidence matrix A_a, we select say a direction fall the elements suppose this is 1, 2, 3, 4, this is element 1, 2 suppose this is the direction okay I put a circle on the node numbers and these are the edge numbers 1, 2, 3, 4, 5, this may be 6, this may be 7, these are the edge numbers we give a sign plus 1, when it is going away from elements in the a matrix we will put plus 1, when it is going from node 1 to 2, if it is going away from 1 to 2 then against the node 1 will put plus 1 sign and against node 2 we will put minus 1 sign where it is entering. So going away means plus 1 entering a node is minus 1 going away from a node, it will be given plus 1 going into a node it will be given a minus 1 sign and it is not present anywhere else so it will be 0.

So for this we write the matrix like this elements or edges there are 7 elements 1, 2, 3, 1, 2, 3, 4, 5, 6, 7 okay and nodes 1, 2, 3, 4. So the first element is going from 4 towards 1, so at node 1 it is minus 1 and at node 4, it is plus 1 okay, element number 2 similarly at 4 it is plus 1,at node 2 it is minus 1, so this element will be minus 1 here plus 1 here, is that okay rest are all 0 so I am not writing them element number 3, could you please tell me, plus one and at node 3 minus 1 minus one then element number 4, element number 4 is plus 1 at 1 minus 1 and minus 1 at 2 okay element number 5 is plus 1 at 2, minus 1 at 3, is that okay and element number 6 is plus 1 at 1and and at 3 is minus 1, at 3 at 3 it is minus 1 very good add element number 7 it is plus 1 at 3 and minus 1 at 4, is that okay.

Now you see if you add the columns they are all zeros, is it not, 1 minus 1 sorry. So they will be summing up to 0 this, this, this, this, so the equations that you write here if I write 4 equations involving this element connections one of them is redundant because if you have 3 equations forth equation can be generated from the first equation, first equations can give rise to the forth equation When you have addition of these elements equal to 0 okay then this is not an independent equation without that I can fill it up wherever there is minus 1 rest are all 0.

So it has to be plus one because total sum has to be 0, so this is minus 1, 00 then this will be 0 then this will be plus 1 so any one of them you say remove row 2 if I give you 1, 3, 4 you can all ways generate row 2 is it not total sum has to be 0. So minus 1 and plus 1 is already 0 so there wont be any thing it will be 0 here it is only 100 so this will remain as 1 so I can all ways generate row 2 even if it is not given so information of row 2 is contained in 1, 3 and 4, hence we define another matrix a where any of the nodes is taken out as the reference.

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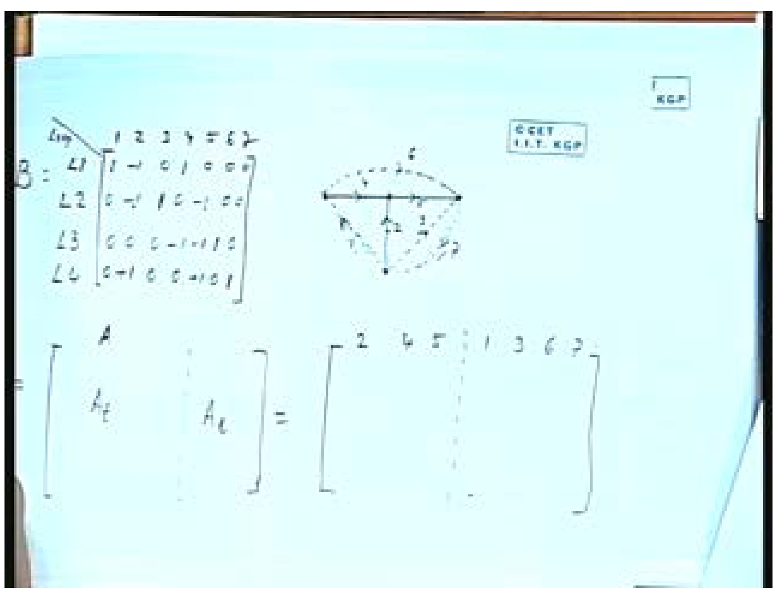

I can choose number 4 as the reference or number 1 as the reference any one of them. So if I remove that then this matrix will become it will be 1, 2, 3, 4, 5, 6, 7 but on this side you will have 1, 2 only 3 nodes that means number of nodes will be equal to 1 less than the given number of nodes in the network, original network, original graph okay. So you can fill it up so delete any of the rows of a matrix you will get the matrix a all right. So this is the commonly used admi incidence matrix, you call it a matrix. Now we will be going into some of the properties of these matrices we similarly define loop matrix b, you chose a tree, suppose this is a tree okay. These are the elements 1, 2, 3, 4, 5, 6, 7 you take one element at a time. So let us take this let me see the direction that we choose choose, this was the direction, this was element number 425, these are the directions 376 okay. The direction of the loop is decided by the direction of the link, so the first loop, loop 1, I will call it loop 1 is with this element 1okay link 1, now that completes this loop with loop, is it not.

So you go along this loop taking this to be positive all right you take the reference of the direction of the loop as the direction provided by the link okay, link direction is taken as the direction of the loop. So if it is in the opposite direction I will put a minus 1 sign, if it is in the same direction I will put a plus 1 sign, so here it is 1, element number 1 it will be 1, element number 4 it will be 1, element number 2 it is minus 1, rest are all 0, no other element will be connected with this is that okay, then loop 2 I can choose the second link as this one then this will be the direction because the direction of element 3 is this.

So element number three will be plus 1, element number 5 is minus 1, element number 2 is also in the opposite direction, so 5 and 2 will be minus 1 rest are all 0 okay. Then loop 3 I can choose this as the loop 3, third loop where direction of 6 is plus 1, 4 and 5 both are negative okay, loop 4 I can choose element number 7, so element number 7 is plus 1, what are the other elements 3, 3 is a link, 2 links will not form a loop 2, 5 in a loop only one link will be present. So 2 and 5 will be minus 1, minus 1 okay what was loop 2 plus 1 plus 1, what was loop 2 2, 5, 3 minus 1 minus 1 and plus 1 okay.

So this will be the elements in the loop matrix, b matrix, is that okay, matrix a I can re write matrix a once you have selected the tree, once you have selected the tree you can put the tree elements in one side and the link elements on the other side okay. We can put it as at partition, a link okay, so what would be a t like I will put the column numbers as element 2, 4, 5 this is my tree so 2, 4, 5 I will partition it then 1, 3, 6, 7, 1, 3, 6, 7 I may fill up these two sub matrices with ones plus minus 1 and zeros from here okay I know element number 1 is between 14 is eliminated so I know the elements here, so element number 1 will be having those elements here okay.

So I will get an get 1 A_t matrix and A_l matrix, 2 partition sub matrices for matrix a similarly, we can also partition b if we take b transpose it will be like 1, 2, 3, 4, 5 and this will be loop 1, loop 2, loop 3, loop 4 and so on okay. We will come back to this, we will come back to this arrangement of b matrix also in a partitioned form before that we will show a very interesting relationship between a and b matrix, what is it matrix a into b transpose will look like? If we we will have matrix a 1, 2, 3, 4, 7 elements here and b transpose will have 7 elements here okay and this side will be loop 1, loop 2 and so on, this side you will have the nodes all right. Now if you multiply elements of this and elements of this and then add them together that will be ab matrix, now you see elements of these and elements of these will be summing up to 0, can you tell me why? B transpose, no but why should that that be 0, because plus minus will take you plus minus.

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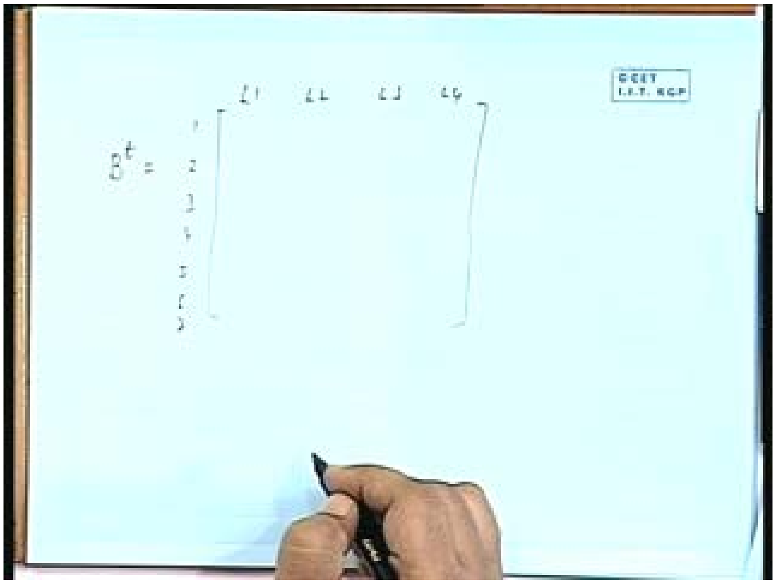

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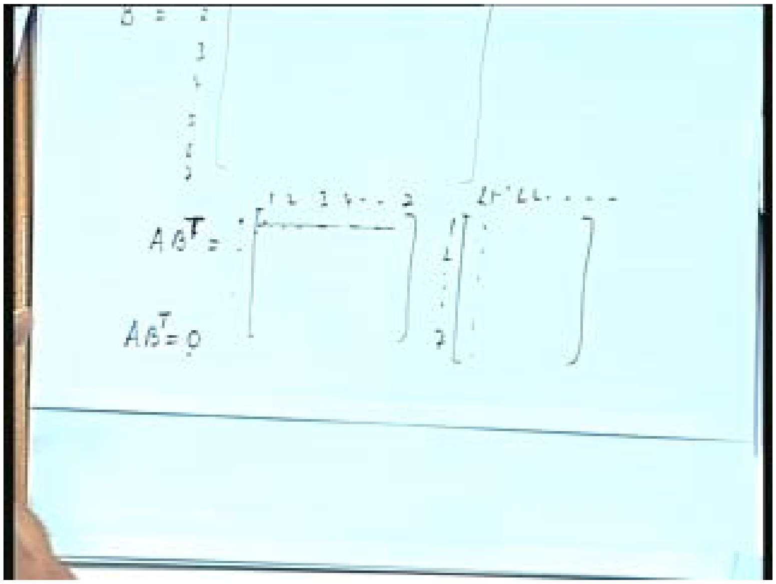

Let us see, let us see yes, these are the nodes now a particular node, so particular node is say this node, node 2 is connected with element number 5, element number 2, element number 4 okay node 2 is having connection with 2, 4 and 5, is that all right. So these elements 2, 4, 5 in b matrix how do they appear 2, 4, 5 now out of these 3 elements, there can be 4, 5 connections all right. So far as the a matrix is concerned in the row there can be in this case there are 3 elements non 0 elements there can be 4, 5 non zero elements depends on the interconnection of that particular node is that all right. Now out of this only 2 elements will be involved in the b matrix, b matrix forms a loop is that all right. So from here though there are 3 elements you take this loop then only 2 and 5 will be involved 4 cannot be involved 3elements of a particular node cannot be involved in a single loop.

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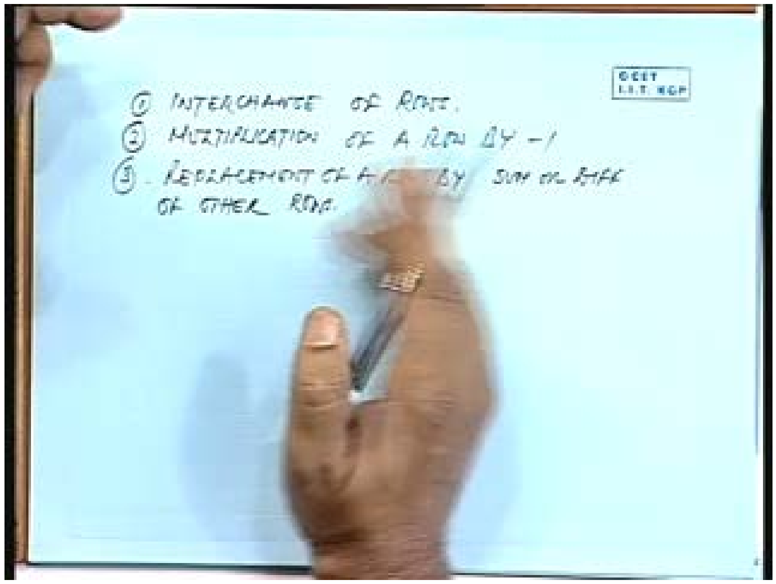

I can have a loop going out like this and which will be involving 2, so either 2 and 4 or I can have a link with 4 and 5 or 5 and 2 but not all the 3, so only 2 elements of a in a row will be involved in elements of b. Now what will be the nature of those elements for a, suppose they form all plus signs in the loop means there in the same direction in a particular loop but then so far is the elements of a are concerned one is going out the other one is coming in then only it can form a loop. So elements will a plus 1 and minus 1 sign, so whereas in b it will be corresponding elements will be plus 1 and plus 1 so if we take the products 1 into 1 plus minus 1 into 1 so it will be always 0. So a into b transpose is all ways 0 you take any row and corresponding columns you take the products it will be always 0, need not be a fundamental loop or any other this thing over that a matrix after taking out the reference it can be even with the entire a matrix, original a matrix because for each row that relationship will be satisfied. So for each row and corresponding column of b you take the products it will be 0.

So this is a very important relation a into b transpose is 0 okay. Now what would be the procedure for developing a tree I can choose n number of trees all right, what is a possible algorithm for getting a tree whereas if you keep keep on connecting the elements into the network you may, may not have a tree because you may connect some elements on this side some element on the other side they may not be connected.

So I have to connect all the nodes and there there can be formation of loops also I have to avoid loops so we will discuss about formation of a tree an algorithm, for formation of a possible tree. For this there are 3 elementary operations that we shall resort to that is interchange of rows, multiplication of a row by minus 1 and then replacement of a row by sum or difference of other rows, by sum or difference of other rows sorry, okay. This is precisely the procedure that you follow in the determination, calculation calculation of determinants of any matrix, is it not. Similar operations you use so we shall be using these okay these are known as elementary operations we shall be using for the formation of a tree. We will stop here for today, we will continue with this in the next class, thank you.