Networks, Signals and Systems
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Lecture- 17
Tutorial

Okay good after noon, we will continue with a few more tutorial problems okay.

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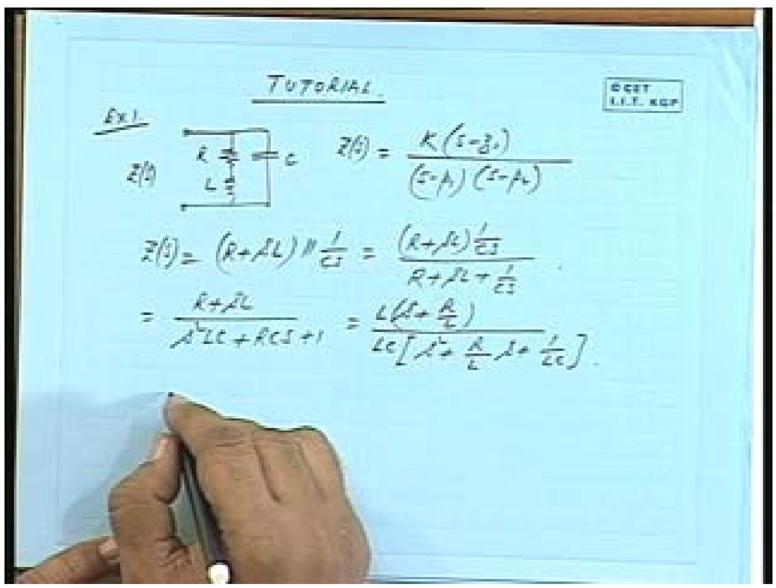

We will take up an example on pole zero distribution and competition of network values from the pole zero distribution network parameter values. First question is you are given a circuit like this show that this impedance function z (s) has distribution of poles and 0s in this form. This function has the form like this, so what would be z (s) from here r plus s l in parallel with 1 by c s okay. So that gives me r plus s l into 1 by c s, r plus s l plus one by c s if I multiply throughout by c s that gives me r plus s l by s square l c plus r c s plus1. So this can be taken taken as s plus r by l into l divided by l c into s squared plus r by l s plus 1 by l c so you can see this is 1 by c into s plus r by l which will be of this form s minus z_1 okay z_1 is minus r by l. Similarly, p_1 p_2 will be the corresponding roots here. The next part of the question is for the given distribution therefore we have a pole here a pole here they are not to the scale this is root over of 111 by 2 j, this is minus root 111 by 2 j and the real part is 5, 1.5, 3 by 2 and the 0 is at minus 3, so compute compute the values of r l and c, r l and c.

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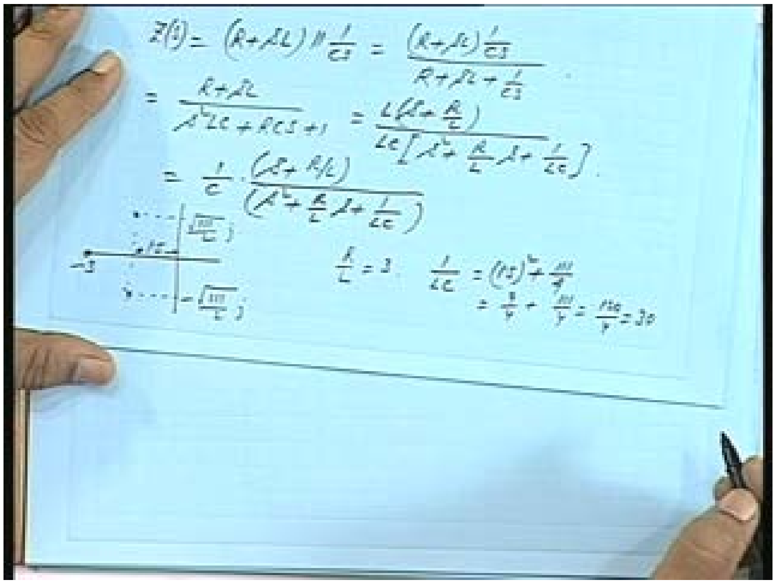

So could you please tell me from here what will be the values? So I can write r by l is equal to minus 3 should it be minus 3, r by l is equal to 3 okay. Next s square plus r by l plus 1 by l c, 1 by c that is a constant okay that is separate, so 1 by l c corresponds to what? See it is product of p_1 and p_2 here the roots are complex. So it minus alpha plus j beta minus alpha minus j beta so what will be the product alpha squared plus beta squared.

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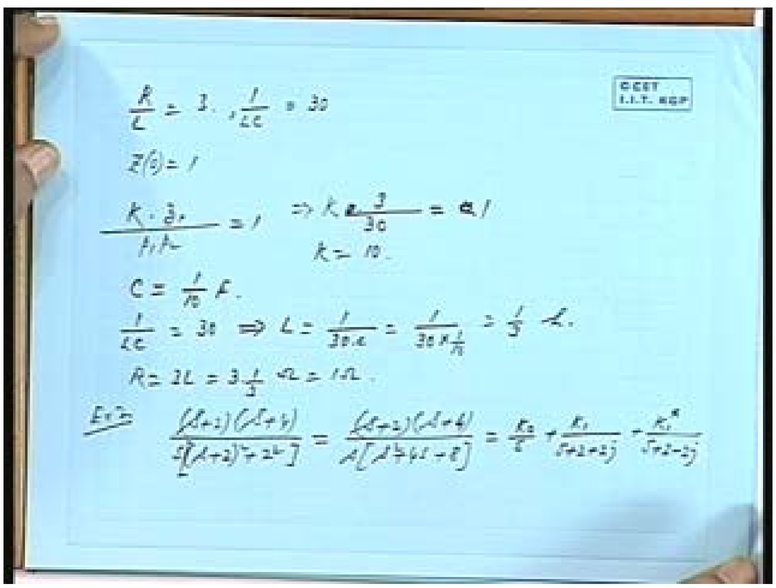

So 1.5 squared plus this squared is it not? So 1 by l c is that all right will be equal to 1.5 squared plus this squared is 111 by 4, so this is 9 by 4 plus 111 by 4 so that is 120 by 4 that is equal to 30 okay. Anything else can you find out, anything else what about the addition of these 2 you want plus p_2 minus alpha plus j beta and minus j beta will get cancel, so 2 times 1.5 so 3 that will be equal to r by l. So r by l is equal to 3 that we have already got r by l from here okay can you find out any thing else? Can you find out see I have got 2 relations r by l is equal to 3, 1 by l c is equal to 30, can you find out the third one? How many poles and 0s are there 3, is it not I have got only 3 values. So 3 values can give you r l c 3 different quantities can they can they give you let us see this is 1.5, this is root 111 by 2.

So what should be c there is some redundancy, can you find that there is no uniqueness in the impedance functions. So far is the poles and zeros are concerned because this k is not reflected in the pole zero distribution all right. So if I double the resistance, double the inductance and half the capacitance then also I will get the same pole zero distribution. So there is no unique solution so r and l can be expressed in terms of some c out of the 3 2 of them can be expressed in terms of the third one you cannot have a unique solution is it not.

So poles and zeros will give you only the roots relating r l and c but not the exact values unless you are given something else. Now the third quantity is z_0 is 1, z_0 is 1 now can you find out z_0 means what s equal to 0, s equal to 0, so k into this one I will get k into z_1 by p_1 p_2 equal to 1 all right. So how much is k, z_1 is 3 minus z_1, k into 3 divided by p_1 p_2 product is, sir j into minus 1 okay if I take here, if I put s equal to 0 will that be all right, could put that. So how much is z if I put s equal to 0 r by l c divided by r by l is 3.

So find out k into 3 divided by see this is my k into s plus r by l this is one root divided by s square plus r by l into s plus 1by l c, is it not. So if I put s equal to 0 k into 3 divided by 1by l c that is 30, 1 by l c came out to be 30 okay, so how much is k that is equal to 1, so k equal to 10. So if k is equal to 10 you have got c that is equal to 1 by c, so c is equal to 1by 1 by 10 farads. So once you have got this so what is l, 1 by l c equal to 30 so l is equal to 1 by 30 into c equal to 1 by 30 into 110, is it not, 1by 3 Henry. So r is equal to 3 into l that is 3 into 1 third ohms 1is that all right.

Next we take up a problem on pole zero distribution and from the pole zero distribution graphically can you compute the residues. Suppose you are given a function s plus 2 into s plus 4 divided by s into s plus 2 whole square plus 2 squared okay. Suppose this is a function you may write this as s plus 2 into s plus 4 divided by s into s square plus 4 s plus 8 basically this is a function, what will be the pole zero distribution and suppose I want to break it up I want to break it up as some k_0 by s plus k_1 by s plus 2 plus 2 j plus some k_1 star this is a residue corresponding to complex root. So the residue corresponding residue for the other complex root will be complex conjugate of k, we will see that so how do you evaluate k_0 k_1 and k_1 star graphically.

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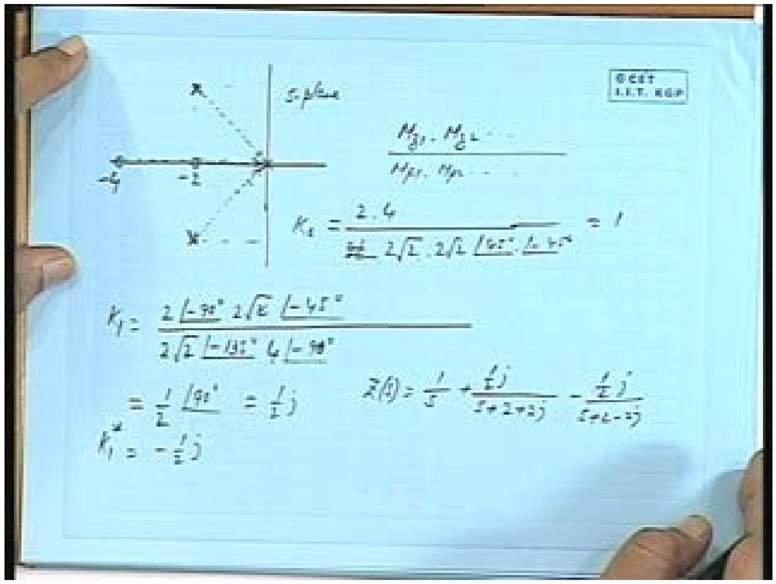

So let us draw the pole zero distribution 2 and 4, 2 and 4 these are the 2 zeros okay and then minus 2 plus 2 j minus 2 plus 2 j minus 2 minus 2 j these are the 2 zeros and the third 0 is at the origin all right, third pole sorry and this is the complex s plane. So when we compute k_0 the residue corresponding to this s that is the pole at the origin you draw vectors from other roots that is all the zeros and poles up to that point. Then the distances of the zeros I will write m_{z1} m_{z2} etcetera distances from different zeros up to that point divided by distances from the poles m_{p1} m_{p2} and so on. So how much is it so distance from this one is 2 into 4 and these are at angle 0, this is horizontal going from negative to positive. So this is all having an angle 0 and how much is this root 2 root 2 sorry 2 and 2, 2 root 2, 2 root 2, root 2 sorry 2 root 2, okay what is the angle 45 degrees, what is this angle minus 45 degrees. So 2 root 2, 2 root 2, 45 degrees minus 45 degrees so that gives me 4 into 2, 4 into 2, 1 okay. Similarly, so this is k_0, this is k_0 can you tell me what will be k_1 so the corresponding root is minus 2 minus 2j.

So minus 2 minus 2 j is this one, so draw all these vectors coming towards this point and then what are the distances of the zeros this is 2 and angle of minus 90 degree is that all right, this 1 is 2 root 2 because this is 2, this is 2, 2 root 2 angle minus 45 degrees is that all right and then this distances, this is 2 root 2 angle 135 degrees minus so 2 root 2 angle 135 degrees and then this one is 2 plus 2, 4 know 4 angle minus 90 degree is that all right when I come towards this it will be 4 minus 90 degrees. So how much is it, 2 2's are 4 root 2 goes so half then minus 90, minus 90 will get cancelled minus 135 minus 1 minus 45 so this will become plus 90 okay. So that is half j you can write either way so k_1 star similarly will become minus half j okay if I am asked to calculate the response this is for realizing the network we might be interested in this okay. So this becomes 1by s plus half j by s plus 2 plus 2 j minus half j by s plus 2 minus 2 j, is that all right. If you are interested in knowing the impulse response, if I excite it by an impulse of current what will be the voltage like it will be I into z (s) impulse I will give you 1, so z (s) itself will be v (s) so what will be corresponding v (t) just Laplace inverse of this.

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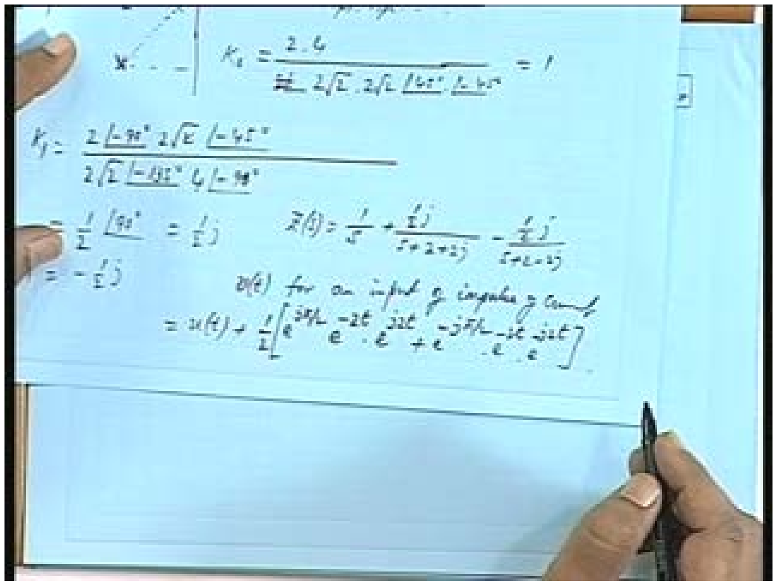

So if I am permitted to write v (t) for an input of impulse of current this one will be 1by s so u (t) plus how much will be this this can be written as half of j can be written as e to the power phi by 2 what about this e to the power minus 2 t alpha plus j beta then e to the power j_2 (t) okay minus half is common e to the power okay I will write plus e to the power minus j phi by 2, e to the power minus 2 t, e to the power j minus j_2 (t) okay. So what will be the net value u (t) plus e to the power minus 2 t can taken out e to the power minus 2 t can be taken out. You have got e to the power j_2 (t) plus phi by 2 plus e to the power minus j_2 (t) plus phi by 2, is it not divided by 2. So that gives me what u (t) plus any mistake, e to the power minus 2 t cosine 2 t plus phi by 2 and how much is that cos theta plus phi by 2 sine sine, is that all right.

So you can plot for many such functions the tutorial problems are there a large number of them we can try at home. Let us now go to a synthesis problem okay there is a very interesting problem a network is given this is z_1 (s) values are 1 ohm, 2 farads 1 forth and 2 farads okay. Here asked to see the possibility of having another network such that if that other network z_2 (s) is put in series with this or in parallel with this whether I can get a net value of 1 ohm which one is possible and what will be the structure is it possible to find out another network z_2 (s) such that if I put this z_2 (s) in series with this. I should get 1 ohm it may or may not be possible or in parallel with this I should get 1ohm.

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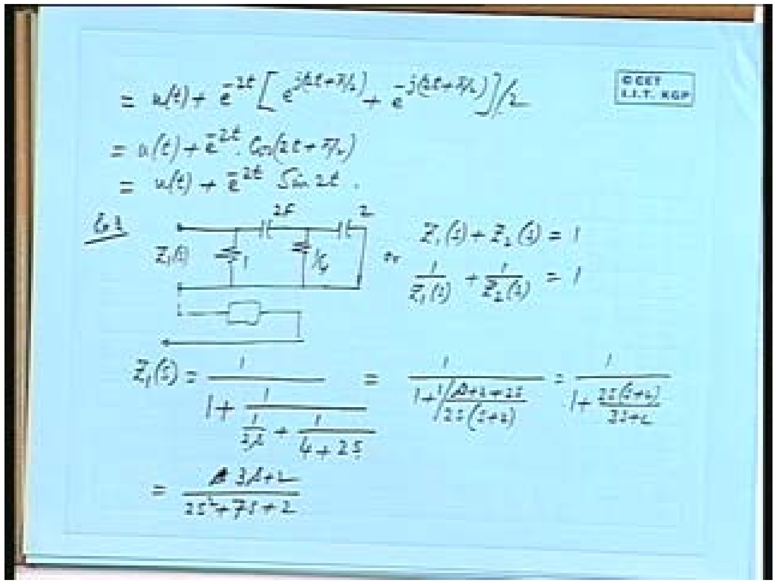

So first of all let us find out z_1 (s) okay what will be z_1 (s) like could you tell me, cauer ladder synthesis you have seen can you try from there it will be 1 by if this is z (s) then what is admittance 1 by z (s) it starts with 1 mho 1 plus again impedance this will be an impedance if I take out 1 mho then again whatever is the balance if I write in the impedance form then it will be 1 by 2 farads means 1 by 2 s plus 1 by 4 mho plus 1 by 2, 1 by, so this plus this now this is the last element what is the admittance of this these are basically two admittances so 2 s, 1 by 1 by 2 s that means 2s. So this is another way of calculating z_1(s) because you know how to do it so it will be 1 by 1 plus 1 by 2 s plus 1 by 4 plus 2 s.

So 2 s into s plus 2 okay, I can otherwise write 2 into s plus 2 so 2 s into s plus 2 and then the numerator will be 1 by 2 s. So s plus 2 plus 2 s check if I am all right, is this okay. This should be 1 by so 1 plus so I will invert it 2 s into s plus 2 divided by 3 s plus 2 okay so 1 by 3 s plus 2 plus this. So numerator will be 3 s plus 2 and the denominator will be 3 s plus 2 plus so 2 s squared plus 4 s plus 3 s, 7s plus 2, is that all right, 3 s plus 2 divided by 2squared plus 7 s plus 2. Now what would be z_2 (s) let us see the first possibility z_1 (s) plus z_2 (s) should be equal to 1 then what should be z_2 (s) 1 minus, case 1 if z_1 (s) plus z_2 (s) is possible the n z_2 (s) will be 1 minus z_1 (s) and what is that 3 s plus 2 divided by 2s squared plus 7s plus 2 that is equal to 2s squared plus 7s plus 2, 2s squared plus 7s minus 3s, so 2s squared plus 4 s okay. So if I take 2s out it will be s plus 4, 2s plus 2, s plus 2 divided by 2s squared plus 7s plus 2. So there are 2 real roots can you compute them if I take 2 outside s squared plus 7 by 2s plus 1 b squared is greater than the 4 a c, so the roots are minus 7 by 2 plus minus so 49 by 4 minus 4 divided by 2.

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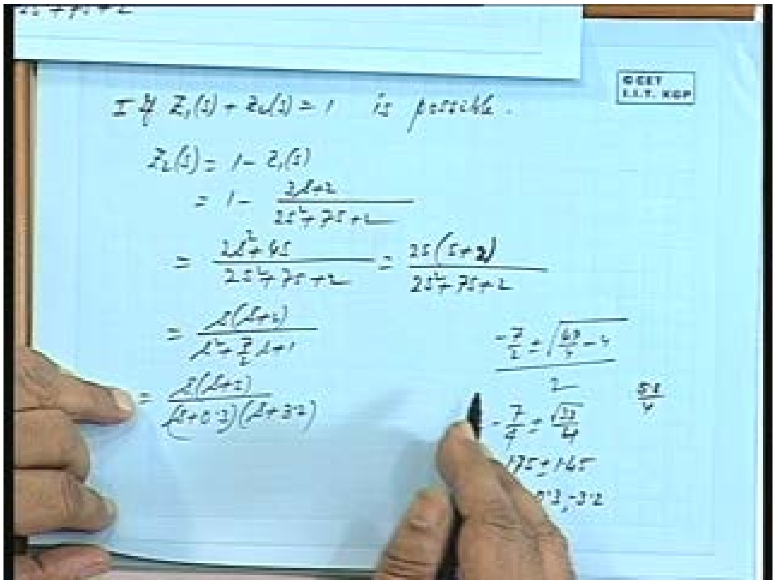

So minus 7 by 4 plus minus 49 minus 16, so root 33 by 2 okay. So by 2 and 2,4 thank you very much ,so minus 7.4 means minus 1.75 plus minus root 33 you can take this to be 5.8 divided by 4 okay, so 1.45 okay. Now you see the distribution of roots minus 1.75 plus 1.45 so that will be minus .4, .3 the other root is 3.2 so this will be approximately s into s plus 2 divided by s plus 0.3 into s plus 3.2 does it sound like any of your known r l or r c network function, does this appear to be any of those r c or r l networks of the poles and zeros alternately coming yes, yes, 0, pole, 0, pole. So it will be either r l or r c now 0 is the closest to the origin so it will be r l network r l network intuitively you can guess it has to be r l because it is an r c network and finally it will be only real, so c part has to be compensated by l, is it not r c network will be all ways giving you some r minus j x form then the other 1 has to be r plus j x form any r l circuit you take series parallel all that for any frequency omega its net value will be r plus j x so if this is r minus j x form this 1 has to be r plus j x so this is possible now you can realize this okay I will leave it to you as a simple exercise you can find a foster, cauer, foster 1, foster 2, cauer 1, cauer 2 any of them to realize this.

Okay let us see the other possibility if 1 by z_1 (s) plus 1 by z_2 (s) equal to 1 is possible then 1 by z_2 (s) will be 1 minus 1 by z_1 (s) okay that is 1 minus z_1 (s) was 2 s squared plus 7 s plus 2 divided by 3 s plus 2. So that will give me 3 s plus 2 minus this so obviously this is minus 2 s squared okay minus 4s divided by 3s plus 2. You cannot realize this a negative transfer function so this is ruled out this is not a p r f, so this cannot be realized another question we will take up another interesting problem can you check whether s plus 2 into s plus 3 by s plus 1 into s plus 4 is positive real or not. I can write this as s squared plus 5 s plus 6 divided s squared plus 5 s plus 4, is that all right. So even z (s) only the numerator part will give me s squared plus 6 into s squared plus 4 minus 5 5's are 25 s squared. So that will give me s to the power 4 plus 4 plus 6, 10 minus okay 10 s squared plus 24 minus 15 s squared.

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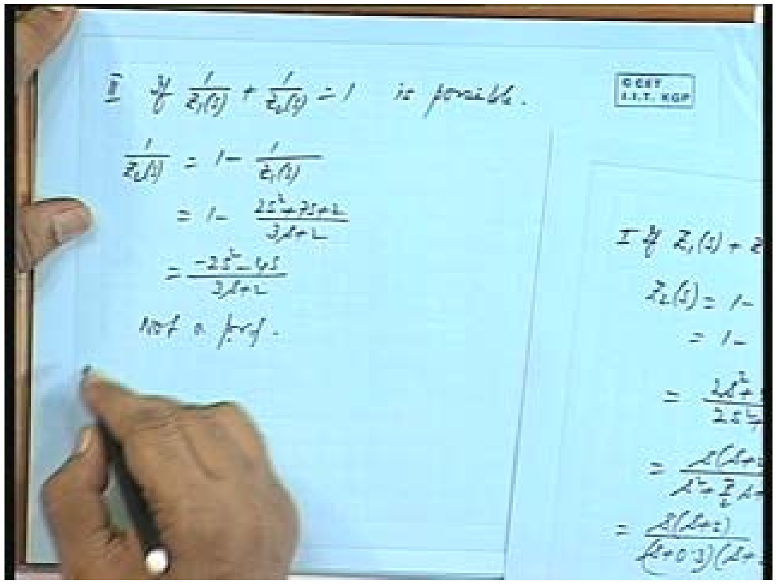

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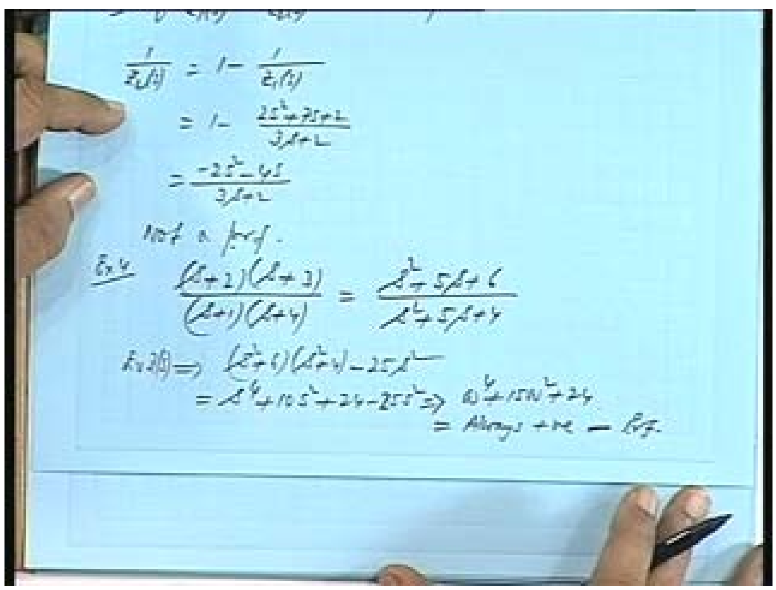

So if I put s equal to j omega that will give me omega to the power 4 minus 15 s squared will become plus 15 omega squared plus 24 and this is always positive for all value of omega minus 15 s squared sorry minus 25 minus 25 so total is minus 15 so that has given me 15 omega squared plus 24, this is always positive. So it is realizable okay so it is a p r f any p r f function will be realizable but will it be realized by r l or r c, see 0,0, pole so poles and zeros are not interlacing so it will not be possible to realize by r l or r c, it has to be by r l c all right. Whenever it is a positive real function but the poles and zeros are not interlacing then it is not possible to get the realization by r l or r c, 2 element synthesis is not possible there is another interesting question y (s) is equal to s plus 2 whole squared by s squared plus 4, is it positive real?

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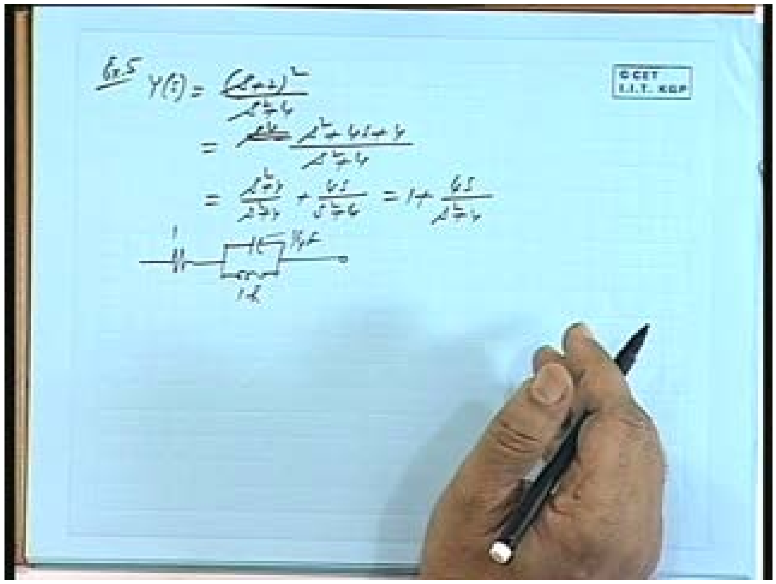

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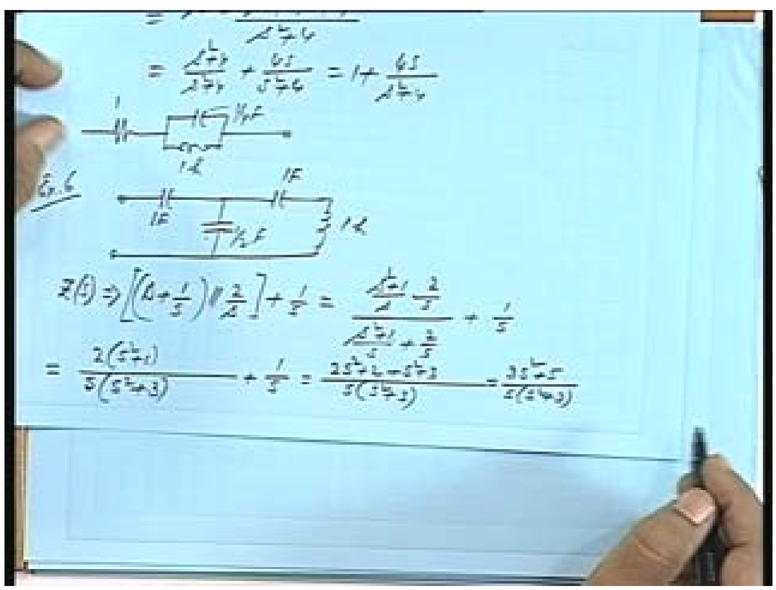

Now let us expand it, s to the power four sorry s squared plus 4 s plus 4 divided by s squared plus 4. Now s squared plus four if I take out if this s squared plus 4 plus 4 s by s squared plus 4 every time I need not go for that m_1, m_2 minus n_1, n_2 if there are simple short cuts I will add up that. So this is a 1 ohm resistance this is an l c combination it is realized also it will be 1 ohm resistance and how much is this make s tending to 0. So it will be 4 s by 4 that is 1 Henry make s tending to infinity it will be 4 s by s squared, so 1by 4 farad. It is a p r f anything that is in a realizable form with pure r l c elements is a positive real function, is that all right there is a very interesting problem on l c network.

Okay that is a network 1farad, half a farad, 1 farad and 1 Henry you see the number of elements here are 4, 1 inductance and 3 capacitances will it be a canonical form, 3 capacitances and 1 inductance its not a canonical form why because in a canonical form the difference in the type of elements will be restricted to 1, either 1 inductance 2 capacitances or 1 capacitance all right. So what would be the canonical form for this, can you find out a canonical form for this.

So let us calculate z (s) what will be z (s) it is 1 Henry plus 1 farad first one is 1 Henry, so s plus 1 by s that is in parallel with 1 by 2 farad. So 2 by s okay this combination is in series with 1 by s, is it not. So let us simplify this this is s squared plus 1 by s so s square plus 1 by s into 2 by s divided by s squared plus 1 by s plus 2 by s okay whole thing is finally added with 1 by s is that okay.

So multiply throughout by s I will get in the numerator 2 into s square plus 1 divided by 1 s will go so s into s square plus 1 plus 2 so square plus 3 plus 1 by s okay. So how much is this s into s squared plus 3 okay 2 s squared this is 2 s squared plus 2 plus s squared plus 3. So that gives me three s squared plus 5 divided by s into s squared plus 3 okay so what will be the canonic form.

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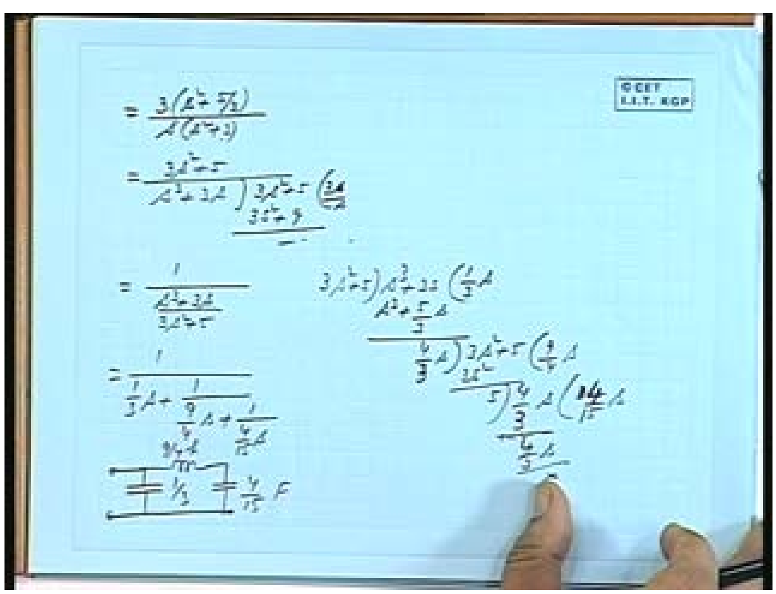

I can write 3 into s squared plus 5 by 3 divide by s into s squared plus 3, now you see how many elements are there 1, 2, 3 you can realize if you want to realize by the same type of network all right that is a cauer synthesis if you make, so I will write 3 s squared plus 5 divided by s cubed plus 3 s can I carry out the division. So it will give negative values 3 by s 3 by s, so 3 s squared plus 9 see if I subtract I will get negative. So this division will not be proper so I will have to invert it 1 by s cubed plus 3 s divided by 3 s squared plus 5 okay, 3s squared plus 5 s cubed plus 3 s okay.

So 1 by 3 s plus 5 by 3 s if I subtract 4 by 3 s 3s squared plus 5 okay, so 9 by 4 s will give me 3 s squared and then 5, 4 by 3 s so 3 how much is it 12 by 5 s no, sorry how much is it 15, 4 by 15, 4 by 15 s so that will give me 4 by 3 s. So this can be written as 1 by 1 third s plus 1 by 9 by 4 s plus 1 by 4 by 15 s okay. So z (s) is this that means this is admittance all right so 1by if 1 third is the admittance what is this? 1 third s is the admittance so it is 1 third farad c s then 9 by 4 s is the impedance, so 9 by 4 inductance and then 4 by 15 farads is that all right. So this is the canonic form for this network all right.

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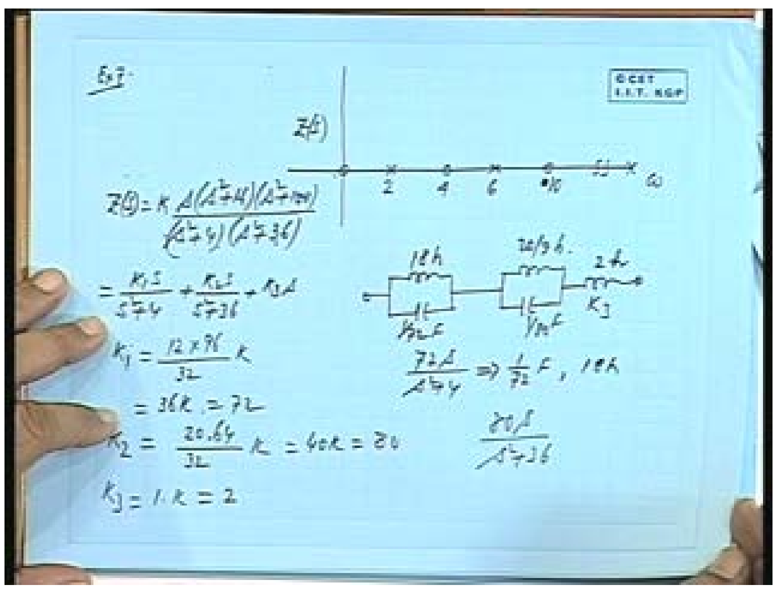

If you are given a function with poles and 0s okay the location of poles and 0s are given say next problem this is a 0, a pole, a 0, a pole, a 0 and a pole okay. So this is 2, this is 4 this is 6, this is 10, can you write the network function. Suppose this is for z (s) and l c function is having poles and zeros at this frequencies what will be z (s) like some k into s, s minus s minus 4 if it is on the imaginary axis basically omega means j omega axis. So the value of x against omega we represent by poles and zeros. So it will be s squared plus 16, 4squared no and s squared plus 100 divided by s squared plus 4 s squared plus 36.

So 0, pole, 0, pole, 0, okay and if I ask you to get foster one synthesis you can do this all right. So what will the foster 1 structure be like s squared plus 4 will be 1 pole so that will give me something like k_1 (s) by s squared plus 4 okay plus k_2 (s) by s squared plus 36 will there be k_3 (s) yes, yes because s square, s squared s s to the power 5 and that is there will there be k_4 by s no because there is nothing like s in the denominator. So this is all so what will be the structure like corresponding to each of these factors s by s squared plus alpha or some omega squared there is an l c parallel combination, is it not?

Similarly there will be another one for this and thirdly it will be k_3 Henry okay. Now suppose we give you in the foster structure k_3 equal to 2 Henrys what are the values of these what are the values of these, let us work out for this problem it self how much is k_1 in terms of k, k is not known k_2 will also come in terms of k. So will be k_3 and there we will evaluate the value of k because k_3 is known 2 Henry that is given all right. A network function whose poles and zeros are given like this and which is having a structure like this with 2 Henry inductor in series you are asked to calculate the other element values all right.

So k_1 is how much is k_1, s squared plus 4 divide by s s will go make s squared plus 4 equal to 0 so this will be 12 into 96 divided by 32 correct me, if I am wrong is that all right into k. Okay so this is 3 into 12, 36 k, so k is known 2 Henrys a sorry k_3 let me calculate k_2, k_2 similarly s squared plus 36 equal to 0 so this will be 20 into 64 divided by 32 into k and how much is that 40, 40 k, how much is k_3, k_3 divide by s if I divide by s so they are all free from s and then make s tending to infinity. So that this will become 0, this will become 0 denominator becomes very heavy. So here infinity means s to the power 4 by s to the power four this s has all ready been eliminated dividing by s so that is 1 into k so k_3 is equal to k it self and that is equal to 2, 2 Henrys is it not therefore k_2 is equal to fort 80 and this one is 72.

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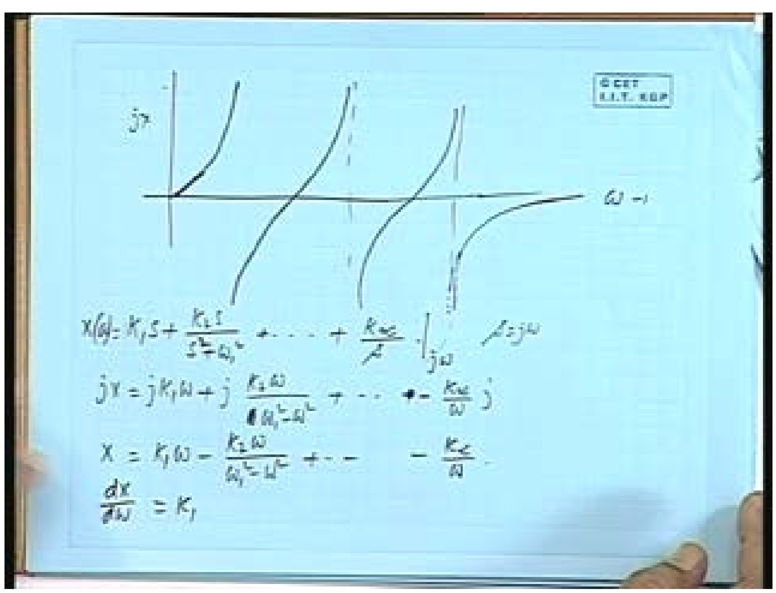

So 72 s by s squared plus 4 will be the factor corresponding to the first set this one, so what are the values s tending to infinity, s tending to infinity will be giving me 72 by s so 1 by 72 farads okay 1 by 72 farads and s tending to 0 will give me 72 by 4. So 18 Henry, 18 Henry. Similarly you can calculate the element for this element values for this okay I hope all of you can do that 80s by s square plus 36. So that will give me 1 by 80 farad and 80 by 36 okay that is 20 by 9 Henry, is that okay.

You may be given sometimes certain other conditions a slope of a function z (s) if you remember z (s) was something like this or y (s) for an l c network z (s), y (s) this were like this, is it not, it may be like this. So if I am permitted to write the general term k_1 (s) plus k_2 (s) by s squared plus omega 1 square and so on plus k infinity by s you may write like this what will be the slope at a particular frequency say at the origin when s tends to 0 what would be the slope like? For this type of a general type of function how do I calculate that so each one of them you take the derivative and then put s tending to 0 or s tending to infinity, then s tending to infinity you will get this slope okay or this slope s equal to 0, this is a slope all right.

So like that you can find out we will not make s tending to infinity s equal to j omega after taking the derivative will put s equal to j omega because this is against omega and then evaluate the slopes okay that will all ways come out to be positive at any point. You can take the derivative and you can show this this evaluated at j omega see if I put s equal to j omega so how much is j x this side is j x, so how much is j x is k_1 j omega plus j times k_2 omega by omega 1 squared minus omega square because s squared is minus omega squared and similar terms plus k infinity by j omega.

So it will become minus k infinity by omega into j now j will be cancelled with this. So x is given by k_1 omega minus k_2 omega by omega 1 squared minus omega squared plus similar terms k_2 k_3 etcetera and minus k infinity by omega. If I take the derivative with respect to omega d x by d omega this one will be k_1 if you take the derivative of these terms it will be all positive you can see for your self and minus k infinity by omega this will also be positive plus k infinity by omega square. So all of them will give you positive terms and hence the slope is all ways positive with respect to omega okay. So we will stop here for today, we will take up a new topic in the next class. Thank you very much.

Preview

Lecture - 18

Graph Theory

Good after noon friends, today we shall be taking up graph theory that is very much useful in networks problems. Now let us first of all define some of the basic terminologies and and then we will see how to use these terminologies and how to use the basic network loss in a very compact form especially when you have multi node so 100 of nodes in a network the solution of the network problems with different conditions say voltages at different nodes may be given some may be, may be may not be given and there were different types of elements, four terminal, there is two port network elements. So it is a complex mixture of different kinds of elements passive and active, so solution of such networks become very involved we resolved to graph theory in such cases. Topology is a branch of mathematics; it is a branch of mathematics concerned with selected properties of collections of related physical and abstract elements all right. In network topology we include the particular configuration of nodes and branches, the configuration of nodes and branches without trigger to what elements exist in those branches.

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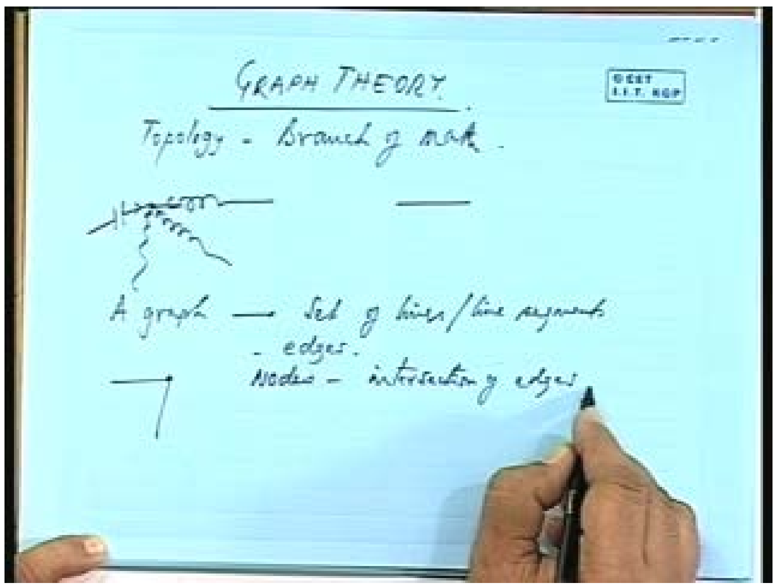

Suppose you are having an inductor, a resistor, a capacitor, for example when we apply kirchoff's law currents flowing in these elements will sum up to 0 irrespective of the type of elements that you have the net in the network, is it not. So this can be used, this network topology can be used for different types of networks like water and gas mains, mechanical network, traffic flow etcetera. See network theory this kirchoff's current law we can also apply to traffic flow, is it not number of vehicles moving towards a particular junction and number of vehicles going out at an instant that will be summing up to 0. A graph is a set of lines, we call lines or line segments known as edges these are called edges. So each element in a network will be represented by a line or an element okay these are called edges. Then intersection of edges will be known as nodes or vertices intersection of edges.