Networks Signals and Systems

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Lecture -16

Tutorial

Okay good morning friends, Today we will have a tutorial session will be working out a few problems on bode plot and positive real functions, if possible something on synthesis, 2 element synthesis of driving point impedance functions. The first problem is given a bode plot like this this is 1 by root 3 t, this is 1 by t okay, this is 12 db per octave and this is minus 6 db per octave, this value is 3 db. Estimate these frequencies omega c and omega d this frequency, omega c is how much, omega d is how much and then determines g (s) okay.

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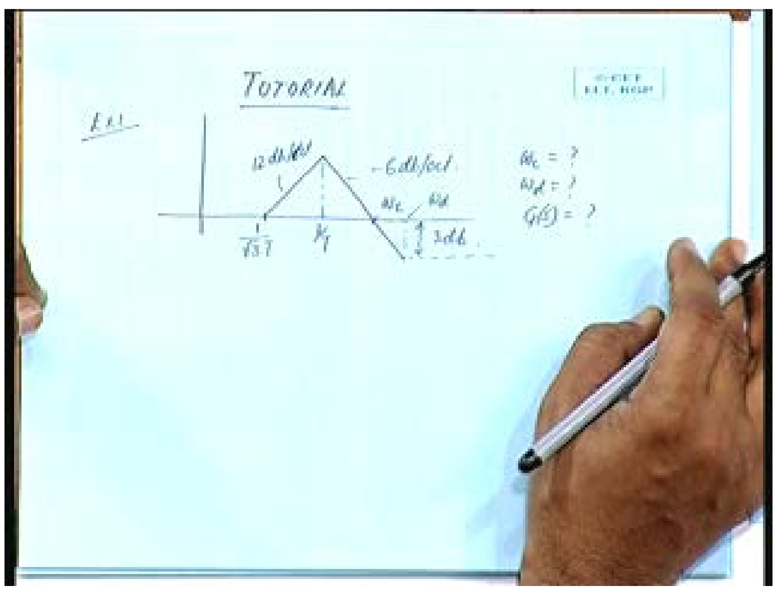

So for this problem this is the db gain, this is the 0 db line. So could you suggest the function like what g (s) what it will be like g (s) will be some constant times it is a slope going up at this frequency okay. So 1 by root 3 root 3 t, let me call it omega 0 okay omega 0 then it will be 1 plus s by omega 0, s by omega 0 whole squared, whole squared this is 12 db per octave whole squared alright then okay 1 plus s by omega okay 1 by t will be how much then root 3 omega, root 3 omega 0. So if I call this 1 by t is equal to root 3 omega 0 so s by root 3 omega 0.

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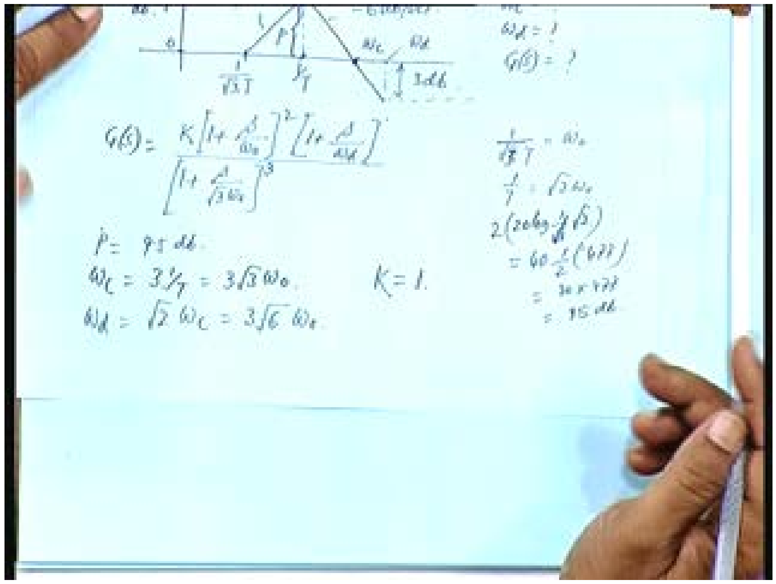

Then any other answer squared, cubed so it was going up by 12 db and it is falling by 6 db so it will be 3 times 6 db, so cubed okay then at what should I write s by omega d again it is from 6 db it is coming to why should it be squared? Why should it be cubed? Now it is being raised by only 6 db or octave okay so you are beaten, now what would be omega d and omega c, what is this gain this is to be calculated alright that is you call it some p, what is p?

So this is also to be determined, how much is p? Could someone tell me 12 db per degree, per octave? Now you are going to a multiple of root 3, is it not? So root 3 what would be this gain? Yes, what would be this gain it is at a slope of 12 db per decade 12 by, no at 3, 3 times omega say at root 3 times omega naught that means the frequency is increasing by root 3. So how much is 20 log of 1 by root 3, 20 log of root 3 sorry 20 log of root 3, 12 db per decade is basically 40 db where 12 db per octave means 40 db per decade alright?

So 20 log of root 3 into 2, n is equal to 2 is it not so how much is it 40 log of root 3 is 1 by 2 log 3 is .477, so 20 in to .477 about 9.5 d b is that okay, at 2 times this it would had been 12 db, it is a little less because root 3 is less than 21. 732. So at that frequency it will be less than 12 db it is approximately 9.5 db okay. Now when does it become 0 how much is this fall 9.5 db, so at 9.5 db with a rate of fall of 6 db, how much is this frequency? How much is this frequency? 9.5 db had it fallen at the rate of 12 db per octave it would have been that much time that is root 3 times further but it is root 3 into root 3, 3 times 1 by t, is it not? If this is raising from root 3 to some height say 1 by root 3 t to some height with 12 db per decade per octave slope with 6 db that means with a slower rate just half the rate of fall it will be twice that multiple, is it not? So this is root 3 times, so it will be root 3 multiplied by root 3 is that alright.

So this will be 3 times omega c is 3 times 1 by t or 3 times 3 root 3 omega naught, is that okay. Then you are having a fall of 3 db with 6 db with 6 db fall I mean rate of fall of 6 db, 3 db is obtained at what value omega d is how many times omega c? Had it fallen to 6 d b then omega d would have been twice omega c. So at 3 db it will be root 2, root 2 so omega d is root 2 times omega c that means 3 root 6 omega naught is that alright so once you have identified this frequencies just put the values is that okay. How do you determine k after doing all that how do you determine k? 20 log k is equal to 20 log k is 20 log of k is, 20 log of k, no at a very very small frequency all these will be knocked off 20 log of k is 0. So k is 1, is it not, is this alright?

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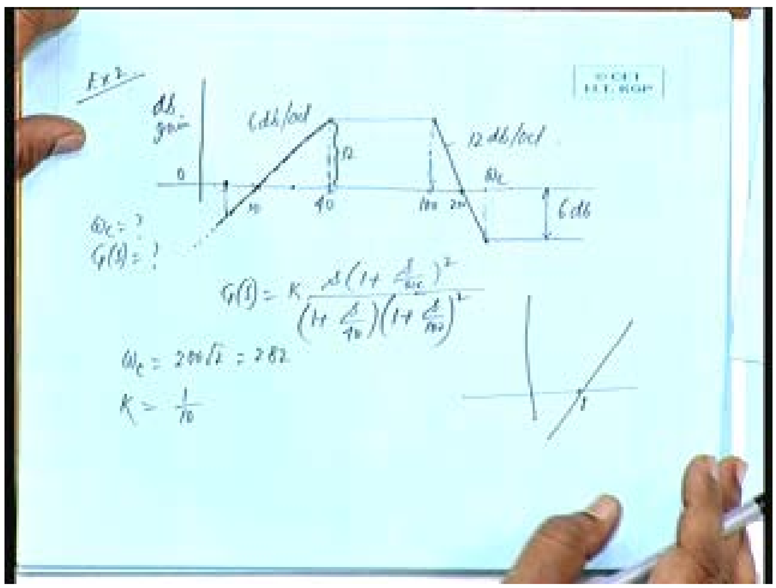

I may may be wrong correct me, if there is any doubt. This another problem, this is at 10 this is at 40, this is at 100, this is 6 db minus this is 0 db line and this is minus 12 db per octave, this is 6 db per octave almost a similar problem. This is db gain, what would be once again, what would be g (s) and this frequency omega c, omega c and g (s). So this is going at a slope of 6 db per octave. So what could be the structure k into s in the numerator then 1 plus s by 40?

Okay then s by 100, what squared is that alright and then this will be 1 plus s by omega c squared, is that okay from minus 12 db it becomes 0 db now what is omega c from 6 db, 6 db per octave if that is a slope so from 10 to 40 how much will be this height 10 to 20, 20 to 40. So there are 2 octaves, so it will be 6 plus 6, 12 is it not, 6 db per octave. If you double it you get 6 db increase, so 10 to 20 it will be 6, 20 to 40 it will be 12, another 6, so this is 12 db okay. So from 12 db per octave if it is falling so how much is this it is falling at the rate of 12 db per octave and it has fallen by 12 db. So how much is this? This frequency 200, 200, 100 to 200, this is the octave and 200 to omega c it is 6 db, so how much is this? Omega c, how much is omega c 200 instead of going to 400, 3 it is not an arithmetic mean root 2, instead of 2 it will be 2 to the power half it is the power so omega c is 200 root 2 alright, 282 approximately, is that alright? So substitute that omega c you get the form what about k, what about k how much would be k, 6 db per octave line other contributions are not there at a low frequency these are all insignificant, what is the contribution of s had there been no k s would have been a line through 1, s is a plot which is passing through omega equal to 1, is it not?

Now instead of passing through omega equal to 1, it is passing through 10 that means this is brought down by how much? It is 1 decade, 1 to 10 is 1 decade 1 decade corresponds to 6 db per octave, 6 db per octave is equivalent to 20 db per decade. So in 1 decade it is 20 db. So you have given a boost in the negative direction by 20 db, so how much is k how much is k 20 log of k is minus 20, so k equal to 1 by 10 log of 1 is a log of 10 is 1. So minus 1 means 1 by 10, so k is 1 by 10 is that okay so this will be the transfer function.

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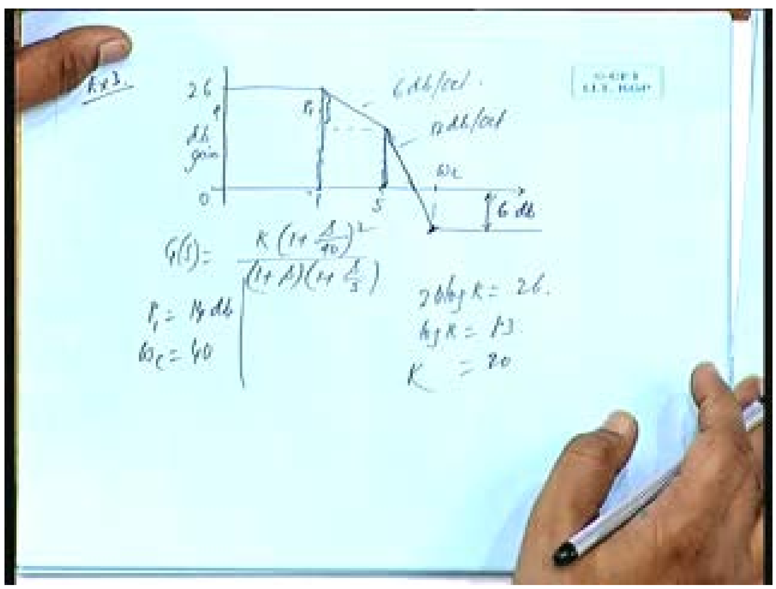

Now another problem on bode plot this is 1, this is 5, this is omega c. Okay this is 12 db per octave, this is 6 db per octave and this is 26, 26 db is db gain okay this is 0 db line. So what would be g (s) like first of all what would be this height, this is 26, 6 db per decade per octave. So from 1 to 5 from 1 to 5 how much is the fall from 1 to 5 how much is the fall? 6 db per decade so how much is this fall? How much is this 6 db per octave. So 1 to 2, 6 db, 2 to 4 another 6 db, so 12 db but then 4 to 5 if you take 20 log of 5, log of 5 is .7, .7 into 20, it is 5 times this frequency 14 db is that alright. So I call it p_1 this small height, so p_1 is 14 db so how much is this? This is 5, how much is this 26 minus 14, 12 is that alright. So 12 plus 6 this is given as sorry this is given as 6 db again this is also given as 6 db. So 12 plus 6, 18 db fall is taking place. so how much is this omega c if it is a multiple of 6 then you take the octave 3 fold 5 to 10, 1 octave 10 to 20 another octave, 20 to 40 another octave.

So every time per octave you are having an increase of 6 db, 6 db, 6 db like that. So for an 18 db there is any multiples of 6 the calculation is very simple is it not 5, 10, so 5, 10 20, 40 okay. So omega c is 40 so once you know this g (s) is very simple k into 1 plus first 1 is taking place at 1. So s, s by 1 into 1 plus s by 5, is it not and then in the numerator it will be 1 plus s by 40 whole squared from 12 db it is coming to 0 db, how much is k? When s is very very small 20 log of k is 26. So log of k is 1.3, 1.3 is .3 is corresponding to log 2. So 20 is it not so k equal to 20, so k is 20 and this is the function is that alright.

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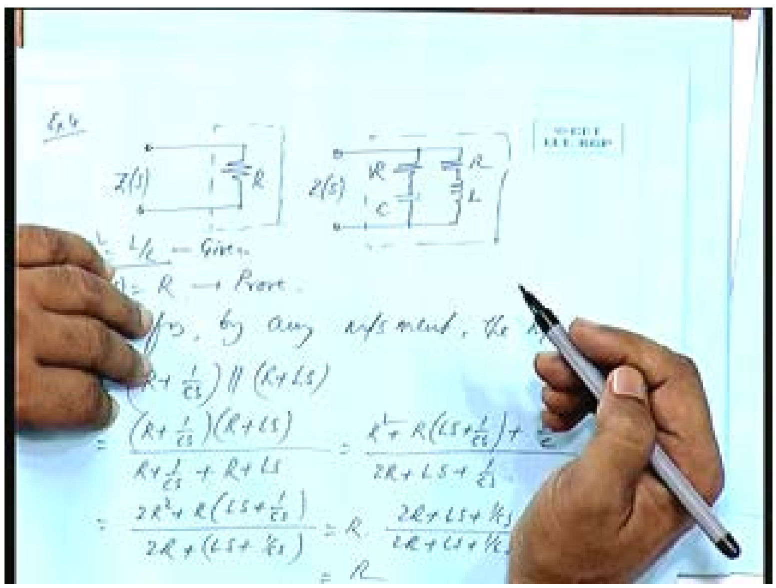

Let us take an interesting problem there are 2 black boxes, this is l Henry, r resistance resistance, c farads, the condition is r squared is equal to l by c, z (s) show that z (s) in both the cases are same that means this z (s) is having a value r given this, prove this okay. The second part of the question is this is given we have to prove this, this is part 1, part 2 by any measurement, any external measurement can you identify this one from this one, can you identify that this is a pure resistance and this is not? Okay will take up, okay identify by any measurement that means any external measurement you carry out any experiment that you want make any measurement and identify the networks.

Okay so let us computer the impedance function z (s) is that alright. Okay thank you how much is z (s) it is r c combination r plus 1 by c s in parallel with r plus l s okay. So that gives me r plus 1 by c s into r plus l s divided by r plus 1 by c s plus r plus l s okay. How much is this r squared plus r into l s plus 1 by c s plus l by c, is that okay divided by 2 r plus l s plus 1 by c s numerator is l by c is r squared so r squared plus r squared that gives me 2 r squared plus r into l s plus 1 by c s divided by 2 r plus l s plus 1 by c s. Obviously r can be taken common that means for any frequency I measure the impedance it is all ways r.

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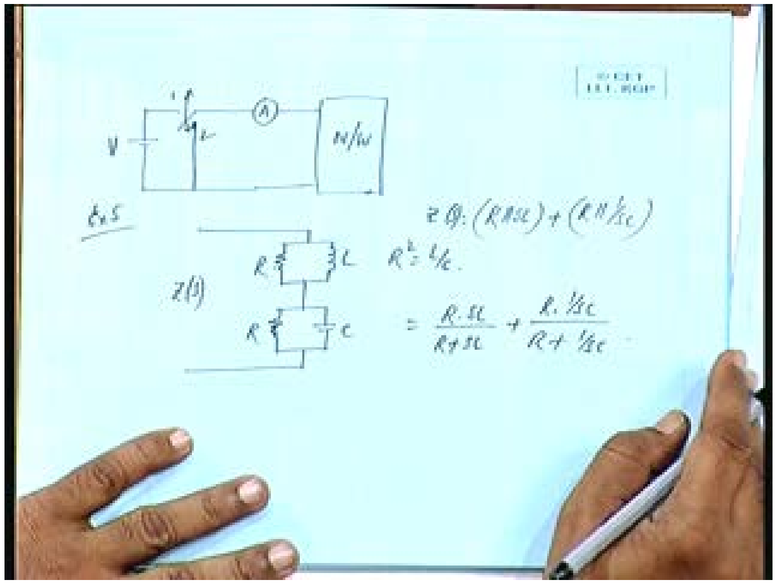

So how do I distinguish, I have chosen say 1 ohm, 1 farad, 1 ohm, 1 Henry, then effective value will be all ways 1 ohm irrespective of the frequency s can be anything okay. So you have to suggest a method to identify this, could you suggest anything? Bode plot, bode plot will not give you bode plot will give you a constant value r any plot see function value g s is r it is a constant so by that you cannot identify you have to device an experiment by measuring something, by observing something you may be able to tell that this is resistive and this is not. Let us have a network like this is a black box okay, if I have a switch initially kept in position 1 what do I observe in the emitter for this circuit v by r will be the current in the ammeter, is it not? In this circuit this will get charged up to v volts and there will not be any drop across the inductor because current will be stabilizing no di by dt. So the entire current will flow through r this r is that alright, so there will be a current through this is it not? These are 2 energy storing devices alright.

Now if I put this on suddenly put this switch on to position 2, what will happen to this? There is a voltage v that is appearing across this there is no current through this and there is a current that is v by r that is flowing through this inductor. Now because there is a capacitance which is fully charged alright, this is charged to plus and minus. So this current can it flow through thism, this current it will try to maintain the same current had there been no connection here it would like to maintain that current there can be a current but then there is a short circuit path, there is some kind of a resistance because this resistance is present, this resistance is present whereas so far as these 2 terminals are concerned there is a short circuited path. So initially this will try to maintain a current through this, is it not? This v by r will be initially flowing through this some part will gradually flow through this also because this will try to discharge in this manner. So both of them will be additive there will be a current but this is a shorter path.

So there will be a large current flowing through this emitter, what about this if I short it no current so long as there is no source resistance cannot withhold any charge or flux. So there is no energy storing device so this will not show any current whereas this will show is that all right. So this is a simple experiment you need an ammeter and a voltmeter voltage supply and you can see this will be a good indicator there is another experiment an extension of this, we have r and c, r and l once again r squared is equal to l by c alright, What would be z (s) prove that z (s) is equal to r. So once again it is r l parallel combination r parallel s l series combination with r paralled s c, 1 by s c. So it is equal to this is z (s) equal to r in to s l by r plus s l plus r into 1 by s c by r plus 1 by s c okay.

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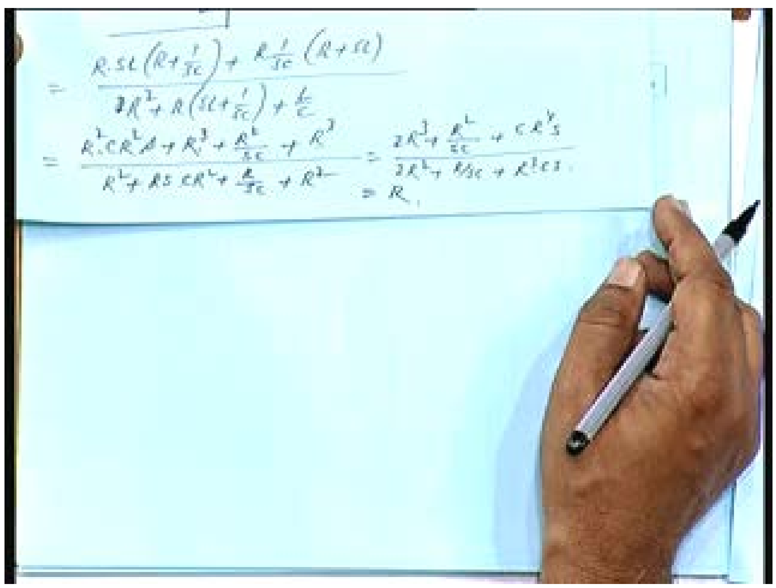

Once again if I multiply r plus 1 by s c plus r into 1 by s c into r plus s l divided by r plus s l. So r squared plus r into s l plus 1 by s c plus l by c okay. So l is equal to I will c r squared wherever there is l, I will put c r square. So I get r into r squared into l is c r square r square s l into s plus r into l by c which is r squared, so total r cubed plus r square by s c plus r into l by c that is r cubed, correct me if I am wrong is that alright? Again r squared plus r s into l is c r squared plus r by s c plus r squared okay. So that gives me 2 r cubed, r cube plus r cube 2 r cubed plus r squared by s c plus c r^4 s divided by r square 2 r squared alright plus r by s c plus r cubed c s. So you can see the ratio if I take r common from the numerator numerator and denominator will get cancelled so that is equal to r, is it alright? Now the second part I leave it to you as an exercise how do you find out this network?

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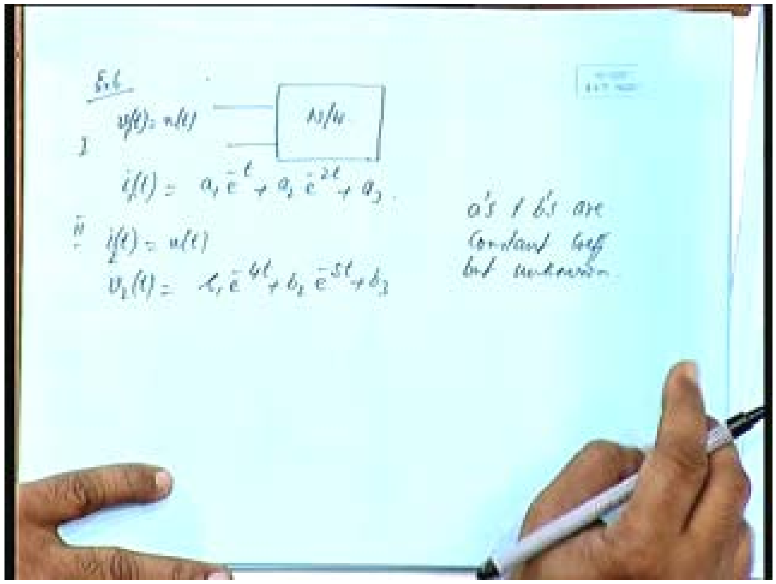

Then another problem, we have a network which is to be identified if you apply a unit step of voltage, if you apply a unit step of voltage then the current is a_1 e to the power minus t plus a_2 e to the power minus 2 t plus a_3. If you apply a unit step of current this is case 1 if unit step of current is applied I will call it v_1 I_1 similarly, I_2 then I get a voltage across the terminals as b_1 e to the power minus 4 t plus b_2 e to the power minus 5 t plus b_3 all these are constants but not known okay a's and b's are constant coefficients but unknown if I apply a steady dc for a steady dc, steady current of 1 ampere you we get for a dc voltage of 50 volts what be z (s) question 1 what would be z (s) and next question is is it a p r f, is it positive real.

So let us see the first condition I_1 (t) is this so much will be I_1 s I_1 s is a_1 by s plus 1 plus a 2 by plus a_3 by s so that will be some p (s) by s into s plus 1 into s plus 2 okay what is the corresponding v_1 (s) input is unit step is it not, so 1 by s, so what could be z (s) like v_1 (s) by I_1 (s) okay. So this divided by this how much is it s plus 1 into s plus 2 divided by some p (s) not known as here but the numerators factors will be s plus 1 and s plus 2 okay.

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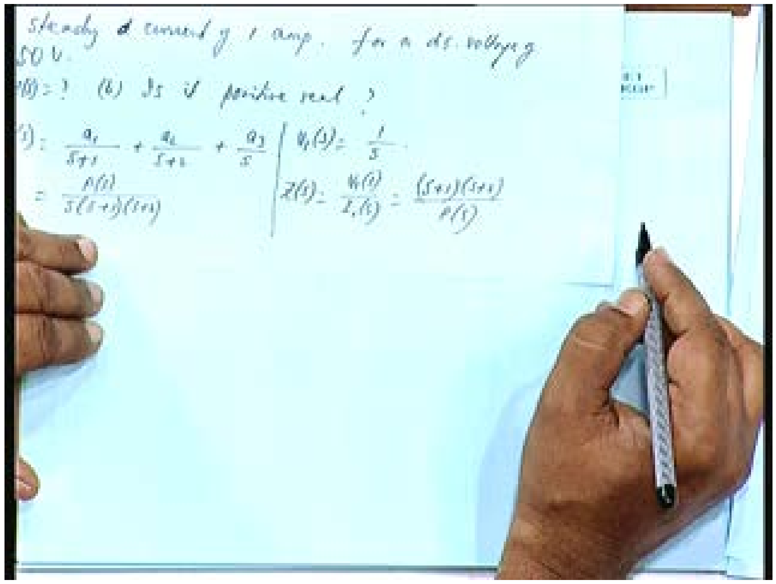

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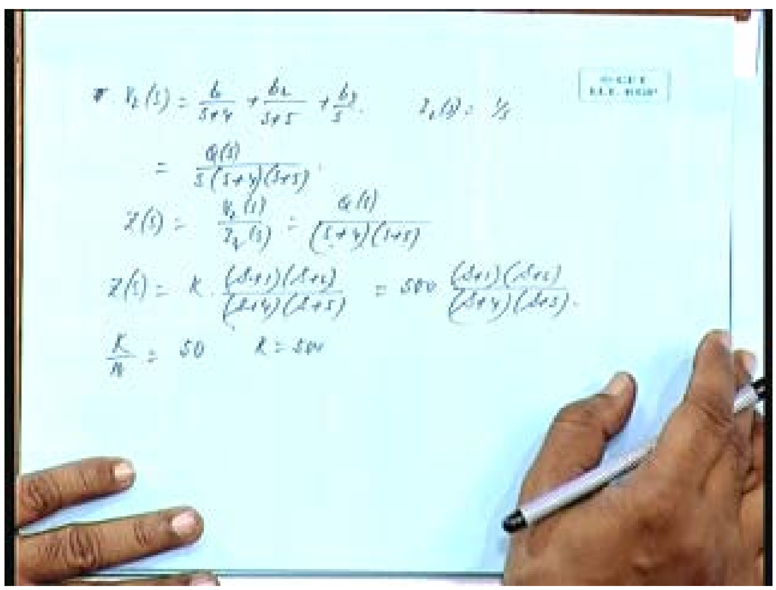

Next you apply I sorry v_2 (s) is how much you apply I_2 (s) as 1 by s is it not second time you are applying a unit step of current so I_2 (s) is 1 by s what is the corresponding v_2 (s) b_1 by s plus 4 plus b_2 by s plus 5 plus b_3 by s. So it will be some q (s) by s into s plus 4 into s plus 5 okay. So how much is z (s) v_2 (s) by I_2 (s) and that will give me some q (s) by s plus 4 into s plus 5 okay. So z (s) from these 2 you can see in 1 case I am getting the 0s in the other case I am getting the poles. So I can write this as some k times s plus 1 into s plus 2 by s plus 4 into s plus 5 is that okay. Sir could be just rewind this, okay you are given a network function, network to be identified you are applying an input voltage of step function u (t) input is step, unit step so how much is its Laplace transform 1 by s agreed yes, current is in this form this a_1, a_2, a_3's are not known.

So what it could be what could be the current transform I_1 (s) a_1 by s plus 1 a_2 by s plus 2 plus a_3 by s so there that will be a common denominator s into s plus 1, s plus 2 numerator after all this additions let it be p (s). So I have identified this, so v by I, v_1(s) by I_1 s that will give me z (s) so that will give me s plus 1 into s plus 2 in the numerator. So z(s) has a numerator which is this that means that these are the 0s denominator I do not know from the second case I am getting similarly the denominator z (s) equal to v_2 (s) by I_2 (s) that is s plus 4 into s plus 5. So by the 2 experiments I am able to identify the poles and 0s so what could be z (s) it will be some k times s plus 1 s plus 2 divided by s plus 4 s plus 5 okay and then for a dc voltage of 50 volts the steady current is 1 ampere. So what is the voltage by current ratio 50 by 1, 50 ohms and how much is that in this what do I put for a dc, what is the dc impedance of this put s equal to 0 so that is 1 into 2 by 4 into 5, 1 by 10, 1 by 10.

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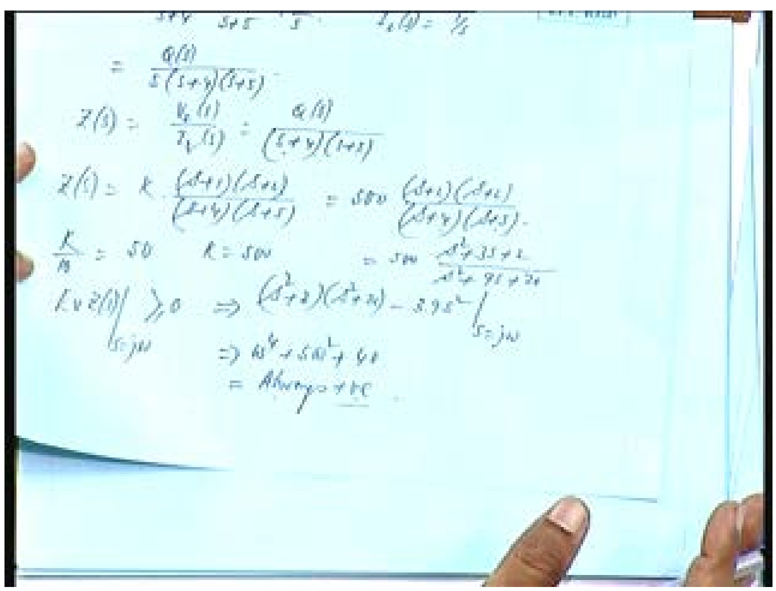

So k by 10 is equal to 50 ohms is it not, 50 volts 1 ampere so k equal to 500, is that okay okay. So 500 into s plus 1 by s plus 2 by s plus 4 into s plus 5 is that okay, is it a positive real function second part is is it a positive real function, roots are in the left half side, left half plane then the degree difference is restricted to 1 all those conditions are satisfied. So no no, that is for r l or r c it can be an r l c synthesis that is possible so we are not concerned about that the test that you have to make is even z (s) if I put s equal to j omega should be greater than equal to 0 for all omegas is it not.

So let us find out the even z (s) this is s squared or can write this as 500 into s squared plus 3 s plus 2 divided by s squared plus 9 s plus 20, is it not. So even part is s squared plus 2 into the even part of the denominator is s squared plus 20 minus 3 into 9 s square20 s squared plus 2 s squared, 22 s squared minus 27, so minus 5 s squared.

So that will become plus 5 omega square if I put s equal to j omega plus 40 this is all ways positive. So it is a positive real function, can you realize it by r l or r c no because the poles and 0s are not interlacing 0, 0, pole, pole they are coming in this order. So it will not be possible to realize this by r l or r c we will see later on how this can be realized by r l c that will be taken up later but we know it can be realized because this is always positive, is that okay.

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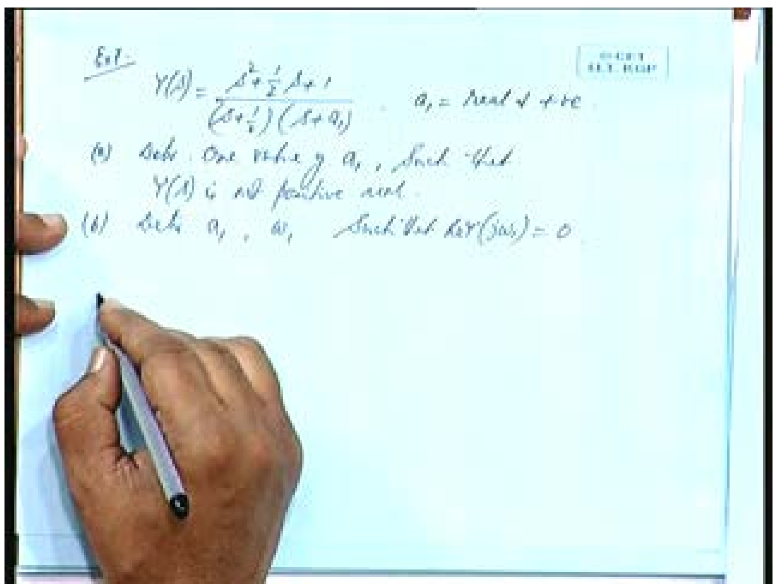

There is another problem consider the admittance function y (s) s squared plus half s plus 1 divided by s plus half into s plus a_1 where a_1 is a real and positive question is question is determine 1 value of a_1 determine 1 value of a_1 such that y (s) is not positive real, is not positive real, second condition is determine a_1 and also the frequency omega 1 such that at that frequency y j omega 1 the real part of this is equal to 0.

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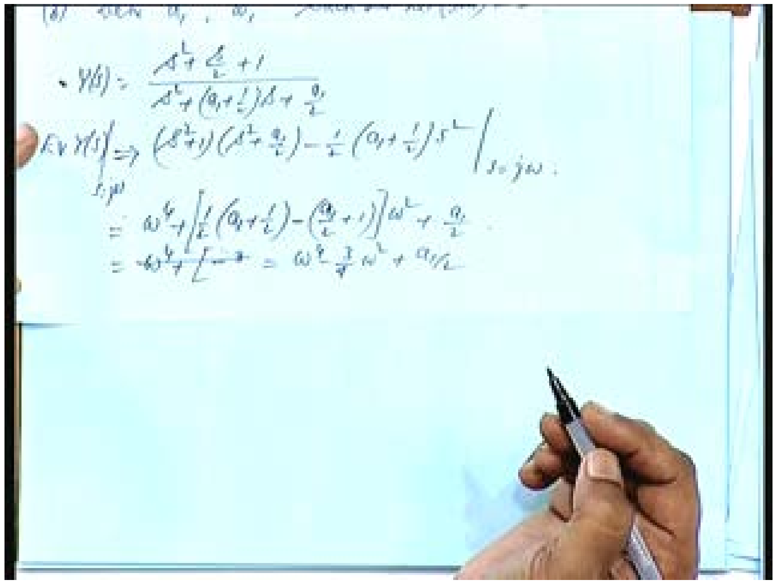

Okay so let us once again find out the condition for positive realness y (s) if I break it up it will be s squared plus s by 2 plus 1 divided by s squared plus a_1 plus half s plus a_1 by 2 okay. So even z (s) or even y (s) what is the even part s squared plus 1 mind you earlier also when we are considering even part this m_1 m_2 minus n_1 n_2 that we derived divided by m_2 squared minus n_2 squared that denominator was all ways positive. So we are not considering the denominator this is to be divided by s squared plus 20 whole squared minus 81 s squared but that is all ways positive, is it not.

So here also we will consider only the numerator of that even z (s) so s squared plus 1 into s squared plus a_1 by 2 minus this into this half a_1 plus half s squared this is to evaluated that the s equal to j omega though I am writing even y (s) but I am not writing the denominator which is a positive quantity I should better put arrow. Now how much is this if I put s equal to j omega omega to the power 4 okay plus if I put s equal to j omega this minus sign will become plus half of a_1 plus half minus a_1 by 2, a_1 by 2 into s squared and 1 into s squared plus 1 this whole thing into omega squared, correct me if I am wrong half into a_1 plus half this s squared this is continuing s squared terms s squared terms will be all ways generating a minus omega squared.

So I have taken the minus 1 as positive and this one as negative the coefficients coming out from here is a_1 by 2 and 1 okay plus a_1 by 2 okay. So this is omega to the power 4 plus if I simplify this a_1 by 2, a_1 by 2 will get cancelled and 1 forth and minus 1 so minus 3 by 4 so omega 4 minus 3 by 4 omega squared plus a_1 by 2 okay.

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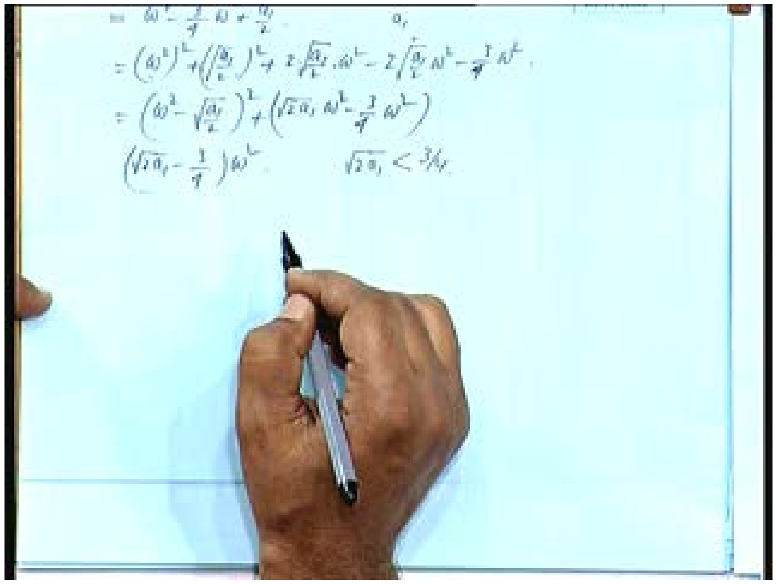

So omega 4 minus 3 by 4 omega squared plus a_1 by 2, is it negative for any value of a_1 any value of a_1, is it negative sometimes when when is it negative suggest any value of a 1. So that it is negative sometimes, would you like to put some values keep on trying it yes, less than g how much is it for what value of a_1 will be will this be less than 0 at least for some omega, no a_1 is positive, a_1 is positive, 1 you suggest 1 if I put 1 then this is basically omega squared squared plus plus a_1 by 2 whole squared plus 2 into root over of a_1 by 2 into omega squared if I add this and subtract it okay so that I can choose what should I choose this plus this plus this or minus this either way okay.

Let us see it is better to see it how it works. Suppose you put minus this one you choose to include in this then it will be root 2 a_1 root 2 into that gives me this omega square minus 3 by 4 omega square okay, this is all ways positive okay whatever be the value of omega a_1 this is all ways positive, this can see root 2 a_1 minus 3 by 4 into omega squared this can become negative if 3 by 4 is greater than these terms that is if root 2 a_1 is a less than 3 by 4 this may be negative, is it not. So it may become negative sometimes if this is a condition so a_1 is less than 3 by 4 root 2 if you take the positive term on the other hand it will be omega squared plus root a_1 by 2 whole squared minus 3 by 4 omega squared minus root 2 a_1 omega squared okay one may write this squared term minus omega square into 3 by 4 plus root 2 a_1, sir a_1 less than 9 by 4 root that root a_1 root a_1 sorry, so a_1 will be a_1 will be square square okay. Thank you very much.

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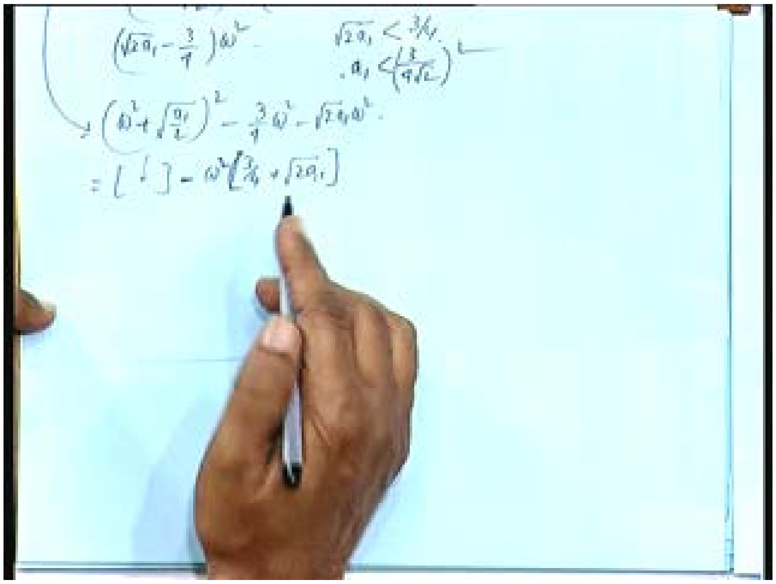

So here is 1 condition does it mean for any value of a_1 it will be negative though there is a negative term but this value is generating more values I mean this whole thing is giving me sufficient margin may be. So for any value of a_1 this cannot be tested so I should all ways put the minus sign here and then plus something and then try to see if in that plus term there is any possibility of getting a negative value alright. Otherwise, this will not be able to identify the actual range of a_1 you may be mislead then the second part is I think the time is over, I will continue with this in the next class okay.

Preview

Lecture - 17

Tutorial

Okay good after noon friends, we will continue with a few more tutorial problems okay. We will take up an example on pole 0 distribution and competition of network values from the pole 0 distribution network parameter values. First question is you are given a circuit like this show that this impedance function z (s) has distribution of poles and 0s in this form. This function has the form like this so what would be z (s) from here r plus s l in parallel with 1 by c s okay, so that gives me r plus s l into 1 by c s, r plus s l plus 1 by c s if I multiply throughout by c s that gives me r plus s l by s square l c plus r c s plus 1. So this can be taken taken as s plus r by l into l divided by l c into s squared plus r by l s plus 1 by l c.

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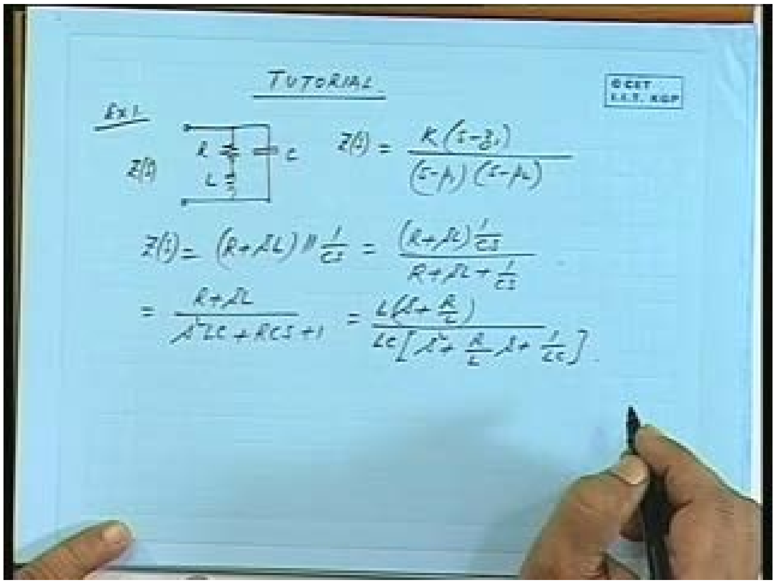

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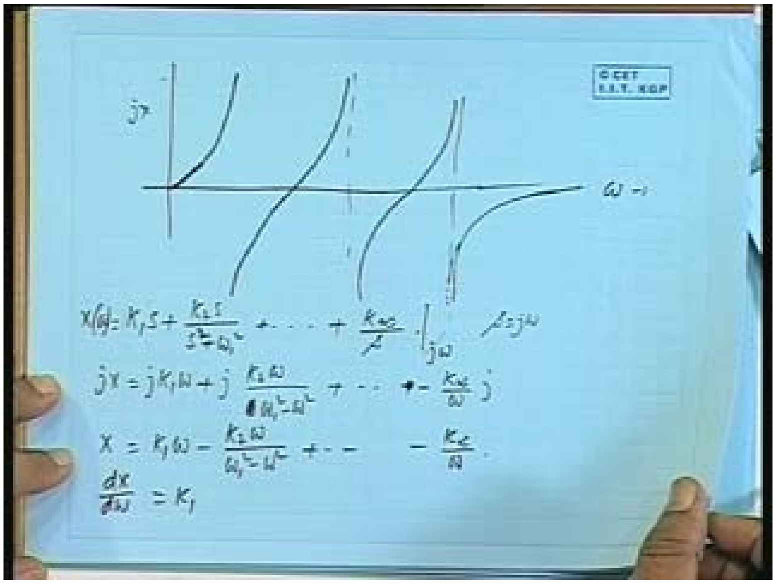

Okay that will always come out to be positive at any point, you can take the derivative and you can show this. This evaluated at j omega see if I put s equal to j omega so how much is j x, this side is j x. So how much is j x is k_1 j omega plus j times k_2 omega by omega 1 squared minus omega square because s squared is minus omega squared and similar terms plus k infinity by j omega. So it will become minus k infinity by omega into j.

Now j will be cancelled by this, so x is given by k_1 omega minus k_2 omega by omega 1 squared minus omega square plus similar terms k_2, k_3 etcetera and minus k infinity by omega if I take the derivative with respect to omega dx by d omega. This one will be k_1 if you take the derivative of these terms it will be all positive you can see for your self and minus k infinity by omega this will also be positive plus k infinity by omega squared so all of them will give you positive terms and hence the slope is all ways positive with respect to omega okay. So we will stop here for today, we will take up a new topic in the next class. Thank you very much.