Network Signals and Systems

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Lecture - 15

2- Element Synthesis (contd…)

Okay good morning, last time we are discussing about foster two realization of network functions.

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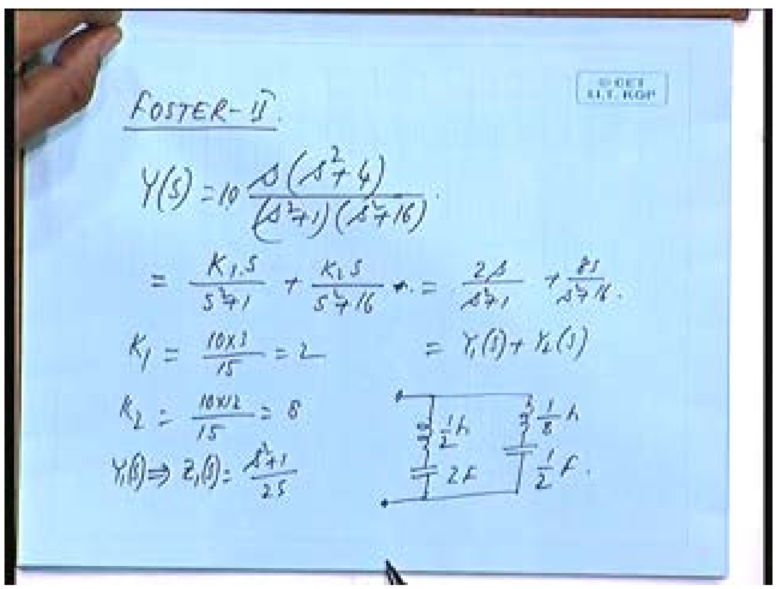

Suppose you are given z (s) and you have obtained y (s), y(s) is a s into s squared plus 4 divided by s squared plus 1 into s squared plus 16 okay. Then I can write this as k_1 by k_1 (s) by s squared plus 1 plus k_2 (s) by s squared plus 16, how many elements will be there? Can you guess how many elements will be there? How many informations are given to you I could have add a constant here 10 or so so this is one information 1, 2, 3, 4. So you can have only a 4 element synthesis.

Now each of these factors will take away 2 elements so 2 plus 2, 4 elements are already gone all right so blindly you can write there is no other term k_4 (s) or k_5 by s okay. So how much is k_1 multiply by s squared plus one divided by s make s square plus 1 equal to 0. So 10 into 3 by 15 so that is equal to 2, k_2 so 10 into 12 by 15 make a square plus 16 equal to 0. So this will be 12 into 10 by 15, 8.

So I can write this as 2 s by s squared plus 1 plus 8 s by s squared plus 16 okay, so this is y_1 (s) plus y_2 (s) so how much is y one s what are the values of l and c corresponding z_1 (s) y_1 (s) gives me z_1 (s) which is just inversion of this s squared plus 1 by 2 s. So that is s by 2 half Henry 1 by 2 s, so 2 farads is that okay another combination y_2 (s) how much will be the element values of y_2 (s) invert it s square by 8 s that is s by 8, 1 by 8 Henry and 16 by 8 s. So 2 by s that will give me half a farad, is that all right.

Now by realizing the network in foster 1 and foster 2, we try to see whether these values are within the working range of the available element the components in the markets. So depending on that we keep on try different synthesis methods it is not necessary that we stick to either foster 1 or foster 2 you can have a mixture of foster 1 and foster 2 that is also possible you realize partly by foster 1 and rest of the network you realize by foster 2 okay. If it is a very big z (s) function we can realize like say terms like this k_1 (s) by s square plus 1 rest of it, it will not realize.

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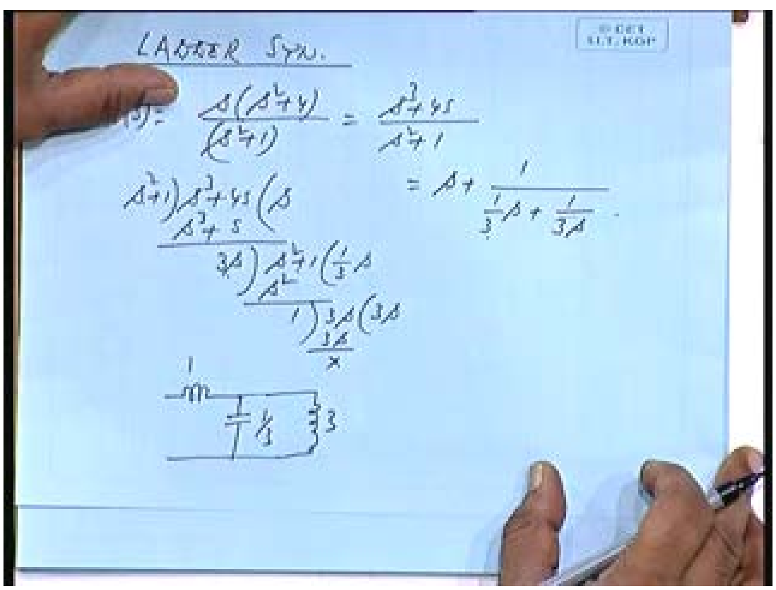

So subtract it from here whatever is the balance you switch over to foster 2, suppose you have started off with foster 1 realize only one pole by foster 1 rest of it again you combine and then you realize by foster 2 that is also possible any combination is possible there is another, another technique realizing the network function by ladder synthesis.

Suppose z (s) is written as, say s I will take a simple function for convenience here s into s squared plus 4 by s squared plus 1okay, lets us take a simple one to start with.

So I can write this as s to the power 3 plus 4s by s to the power 2 plus 1, s squared plus 1, s cube plus 4 s if I divide this I will get s cubed plus s subtract 3 s again divide this s squared plus 1. So one third s, s squared you get 1,3 s okay. So this function can written as s plus 1 by 1 third s plus 1 by 3 s okay. As you have done in high school mathematics vulgar fraction so you go down like this or in HCF, you have done is it not LCM, HCF you keep on dividing this becomes HCF.

So the similar method of division you carry out here. So that the quotients appear as functions of s some multiple of s then we have started of with z (s). So this is an impedance, this is an impedance of 1 Henry okay, z (s) is equal to this much, this is one Henry then it is 1 by this. So what is this an admittance so then it is some admittance part of which I have realize as 1 third s, 1 third s is the admittance of 1 third farad sorry.

First term is directly written s, yes second and third term then I have taken s squared plus ,1well 1by 3 s if I multiply I will get s square yes first term is written directly second and third term are written okay okay okay I will explain this z (s) is what?

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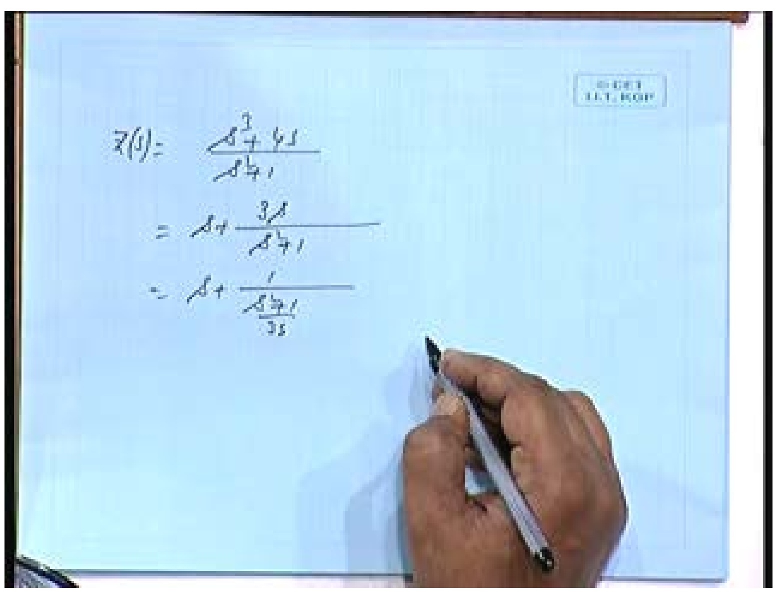

I have written s cubed plus 4 by s squared plus 1, is it not I have written this as s plus what was the residue part of one , by 3 s plus by 1 by 3s by s squared plus 1 is that all right if I divide by 3s this is precisely I have done so is that alright. So it will be like this, so this is an impedance, so this whole thing is an admittance 1by n impedance, is it not? So this is an admittance the first element I have realized by 1 third s, 1 third s is the admittance of how much 1third farad, c s is the admittance so 1third farad and then this is also an admittance, the balance is also an admittance one over that is an impedance.

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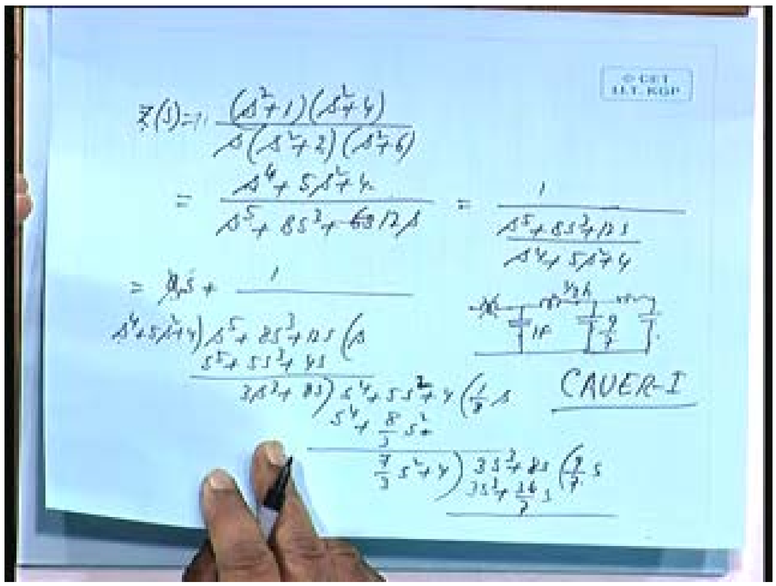

So this is an impedance 3 s is the impedance. So that admittance is appearing here as an impedance of 3Henry and its stops here. Let us take another example a little bigger in size suppose z (s) is given as s, s squared plus 1 into s squared plus 4 divided by s into s squared plus 2into s squared plus 6 okay. So how much is the numerator s to the power 4 plus 5s squared plus 4 divided by s to the power 5 plus 8 s to the power 3plus 6 s is that okay sorry, 12, 12 thank you, 12 s okay.

Now in the first division I want to extract some a_1 (s) plus whatever is the residue divided by this numerator is it not now obviously this is s to the power 4 this is s to the power 5 this will not be present. So I will go to s to the power 5 plus 8 s cubed plus 12 s divided by s to the power 4 plus 5 s squared plus 4. Okay so carry out the division let us try that s to the power 4 plus 5 s squared plus 4 s to the power 5 plus 8 s cube plus 12s. So this will be s s to the power 5 plus 5s cubed plus 4 s if I subtract I will get 3 s cubed plus 8s, s to the power 4 plus 5 s squared plus 4.

So this will be 1 third s, do you agree s to the power 4 plus 8 by 3 s squared that is all, so 5 minus 8 by 3, 7 by 3s squared plus 4 this side it is 3s cubed plus 8s. Okay if I carry out further this will be 9 by 7 s all right that will be may 3s cubed plus 36 by 7 s alright. You keep on dividing and finally you will be getting a 0 residue all right and these are the quotients okay. So 1 s is the admittance first element was a_1 (s) that is 0, first one if it is z (s) if you are starting from z (s), first one is a 1s means a_1 Henry, next one it will be 1farad. Now this a_1 is, in this particular example is 0, next this one is an inductance of one third Henry, straight away these quotients with s, the term associated with s this gives you the element values 1 third Henry, then 9 by 7 farads and so on. How many elements are there 1, 2, 3, 4 and a constant 5. So that will be 1, 2, 3, 4, 5 it will end up like this.

Okay so if you go from z (s) with highest power of s, z (s) or it may start with y (s) for example here the because the numerator is of a lesser order. So it started with a y (s) the first element is not present but as started with s to the power 4 polynomial in this order and s to the power 5, 8 s cubed and so on. So if I go from the highest power of s that is the polynomial is written in descending order of s and then carry out the division I get a structure like this, starting with an inductor in series capacitor in short inductor, capacitor inductor, capacitor and so on this is known as cauer one synthesis.

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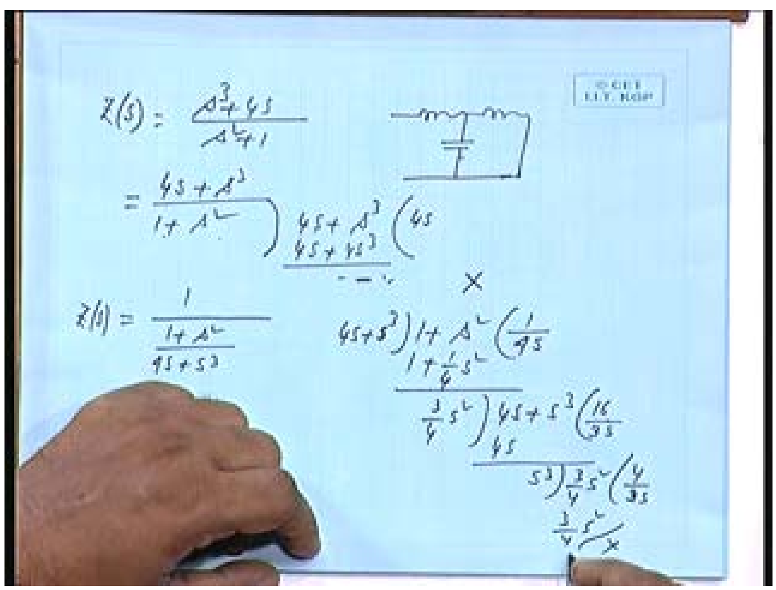

If instead we will started with say earlier example I will take that would be better say z (s) we had s cubed plus 4s by s squared plus 1, is it not that was our first example we got the inductance, capacitance, inductance. So series element was inductance, shunt was capacitance if we reverse the order of the polynomial that is if we start in ascending order of s and carry out the division, what we get? Yes, let us see 4 s plus s cubed 1 plus s square 4 s plus s cubed. So this one will be 4 s then 4 s cubed.

You will get negative sign, so this division fails. So put it below this you get negative term so it is truncated, so I rule out this possibility so I can write z (s) as 1 by 1 plus s squared by 4s plus s cubed that is possible I start with y (s) basically. So if I have 4s plus s cubed 1 plus s squared. Now if you carry out the division it will be 1by 4s, so I will get 1 plus 1 by 4 s squared alright, if you divide 3 by 4 s squared 4 s plus s cubed, so what do? We get here 16 by 3 s, do you get do you get it so that will give me 4 s s cubed and 3 by 4 s squared. So it will be 4 by 3 s so that elements we get are z (s) equal to 1 by this.

So this is the admittance say it starts with an admittance of element 1 by 4 s, what is the admittance of 1 by 4 s, what is that corresponding element 1 by 4 s, is the 4s is the impedance so 4 Henry okay, then 16 by 3 s is the impedance alternately it will be impedance, admittance, impedance, admittance, is it not? So 16 by 3 s is the impedance what is this 3 by 16 farads okay and then 4 by 3 s is the admittance. So 3 by 4 Henry inductance is it all right that means now I have another network if I reverse the order of the polynomials put them in descending order, in ascending order and then carry out the division such that the quotients are all ways one by s, multiples of 1 by s older it was coming as multiples of s. So alternately it was inductance, capacitance, inductance, capacitance and so on. Now this is something divided by s all right and one over that coefficient is the element value 1 by 4.

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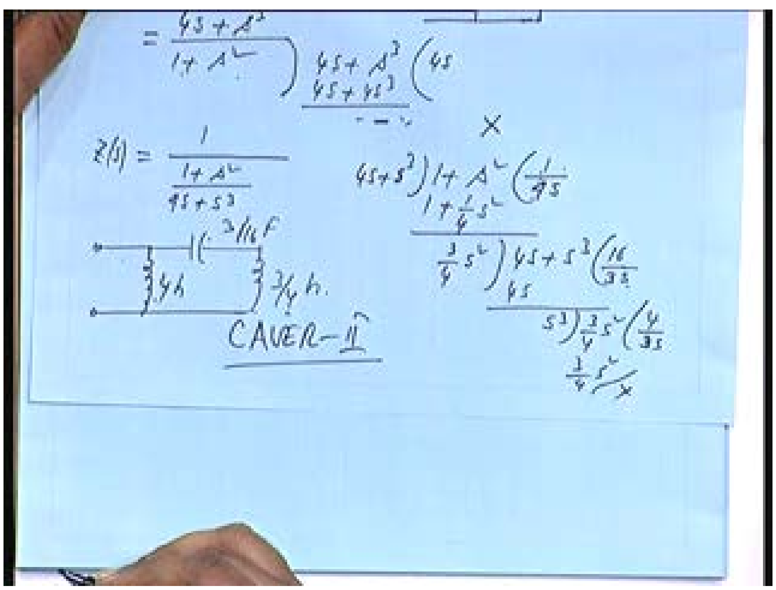

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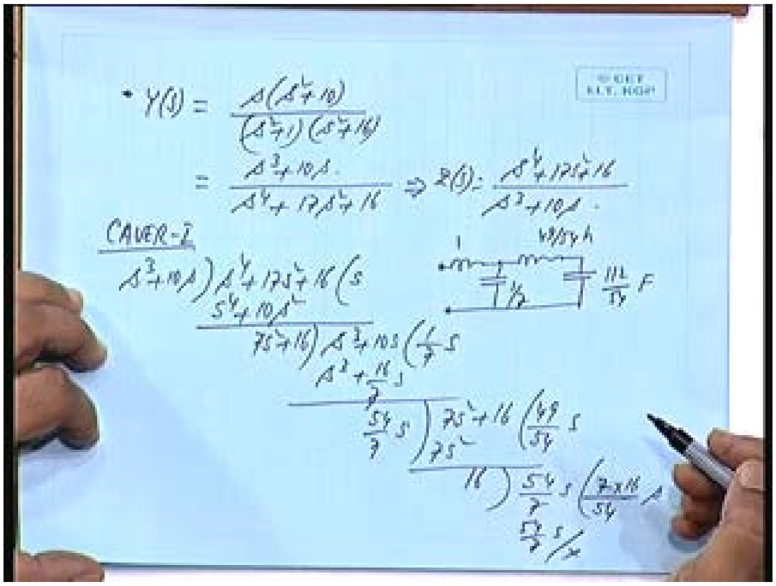

So invert it 4, 16 by 3 invert it 3 by 16, 4 by 3 invert it 3 by 4 so now the element values adjust inwards of those coefficients of 1 by s all right and shunt elements are inductances, series element elements are capacitances. So this is known as cauer 2 realization, okay this is cauer 2 realization. So if we have say y (s) lets us take an example y (s) is s into s squared plus 10 by s squared plus 1 into s squared plus 16 if you ask it to realize it by cauer synthesis, what will be the cauer one synthesis let us write it s to the power 3 plus 10 s divided by s to the power 4 plus 17 s squared plus 16, is it all right? So they are all in descending order so in cauer one this is y (s) I should try to get s to the power higher power s to the power 4 in the numerator then only I can divide.

So this can be written as z (s) equal to s to the power 4 plus 17 s squared plus 16 divided by s to the power 3 plus 10 s okay and then carry out the division s to the power 3 plus 10s s to the power 4 plus 17s squared plus 16. So s s to the power 4 plus 10 s squared 7 s squared plus 16 s to the power 3 plus 10s, 1 by 7 s 16by 7 s okay is that all right. So 70 minus 16, 54 by 7s and here you have got 7s squared plus 16.

So 49 by 54 s that will be my 7 s square and then 16, 54 by 7 s okay so this will be how much 7 by 54 into 16. Okay so you get 1 Henry inductor, 1 by 7 farad capacitor, 49 by 54 Henry inductor and 7 into 16 by 54 so 112 by 54 farad capacitor, 49 by 54 Henry inductor. So this will be the realization okay if you reverse the order and start the division cauer 2. If I am asked to realize this by cauer 2 y (s) was 10s plus s cubed divided by 16 plus 17 s squared plus s to the power 4. Now when you are carrying out cauer 2 division you must see the first 2 terms in the division if I divide 10s by 16 I will get in the quotient s I should get 1 by s. So obviously this will fail so I will write this as 1 by 16 plus 17s squared plus s to the power 4 divided by 10 s plus s cubed and now carry out the division with quotient as 1 by s and its multiple, is that okay.

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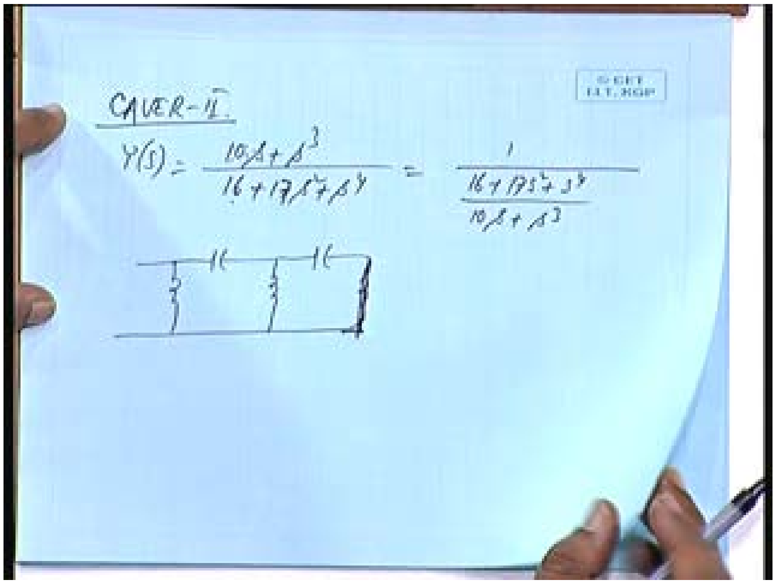

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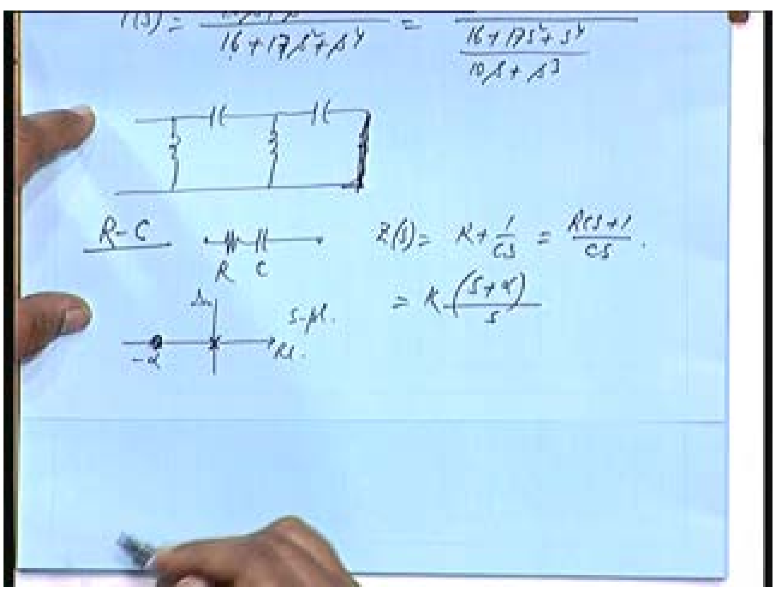

So you will get cauer 2, cauer 2 elements will appear as inductor in shunt capacitor in Series, you had two inductors and two capacitors here. So here also you will have 2 inductors and 2 capacitors this is a original form. So for this example it will be like this okay so far we have studied two element synthesis with l and c, what will be the network function for r c elements if I have only r and c what will be z (s) r plus one by c s okay so that is r c s plus 1 by c s so it is k into s plus alpha by s in this form. So the roots are 0 at minus alpha this is s plane it is real and pole sorry 0 at minus alpha and pole at the origin. Okay let us take r and c parallel combination what will be the values r c s sorry, r by r c s plus 1 okay, is it not? So that is k by s plus alpha so pole is at minus alpha and 0 is at when is it 0 when s tends to infinity, so when s is infinity I will show you with a break it is a 0 okay. This is a dl imaginary function is that okay, when we have a function like this say 1 ohm resistance, 1 farad I am just taking one one all ones what would be the pole 0 configuration.

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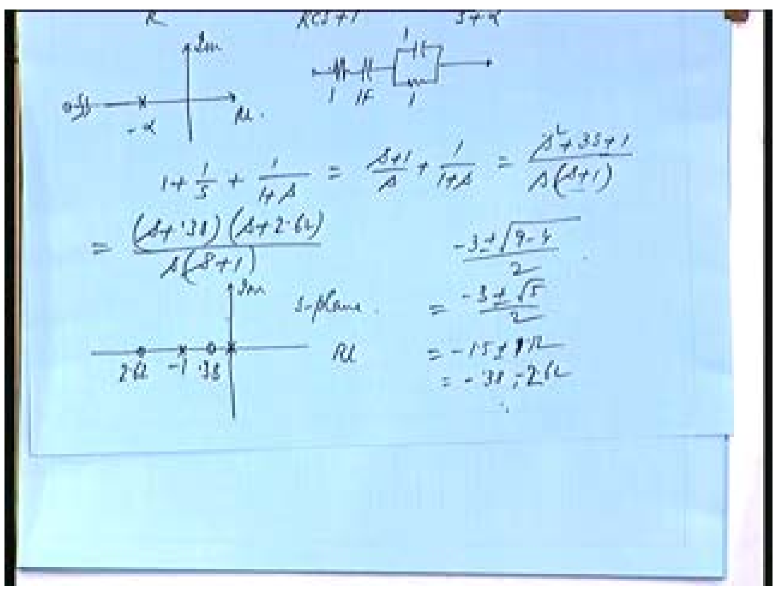

Let us simplify this 1 plus 1 by s plus 1 by 1 plus s this is 1 and 1 by s parallel combination that will be the 1 by 1 plus s. So that gives me s plus 1 by s plus 1 by 1 plus s, so 1 plus s whole squared plus s s squared 2 s plus s 3s alright. You can find out the roots, what is the roots s squared plus 3 s plus 1 minus 3 plus minus 9 minus 4 by 2 okay. So how much is it minus 3 plus minus root 5 by 2 sometime back we made a slip I do not recollect in the earlier lecture whether we got these values minus 1.5 plus minus 2.2, 1.12 so .38 and 1.62 did you get the same values those are slip there minus 3 by 2 the same condition minus 3 plus minus root 5 by 2, did I get minus 3 plus minus root 5 by 2. So there is 1.5 and 2.24 by 2, 1.2.

So this is 2.62 and .38 is that alright is .3 union to multiples okay fine. So it will be s plus .38 into s plus 2.62 divided by s plus 1 so where at the poles and zeros located for this, this is a real and imaginary axis okay s plane okay so at minus 1 it is a pole, at .38 it is a 0 minus s into s plus 1 thank you very much. So s is a pole okay s is a pole .38 is a 0 again minus 1 is a pole and 6.62 is a 0, what about infinity at s equal to infinity it is s square by s square 1.

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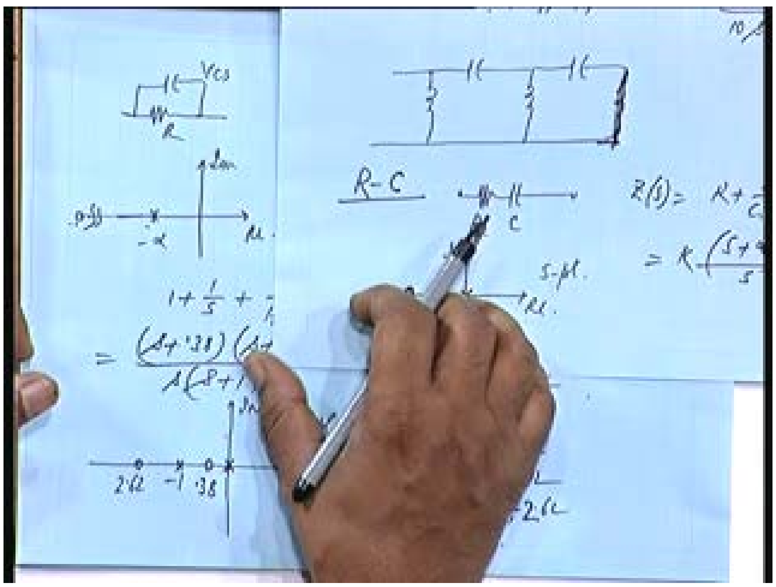

So it is neither a pole nor a zero okay so if you have observed the pole zero configuration for these 3, you can take a few more examples and then see for yourself, either it may be starting from the origin or it may not start from the origin but the first one is a pole, its starts with a pole as you go along the negative real axis, first of all poles and zeros are all on the negative real axis r c circuit there is no scope for oscillation.

So poles and 0s are all on the negative real axis and the first one closes to the origin is a pole, first one starting one is a pole again poles and zeros are coming alternately, pole followed by a 0, infinity can be a 0 or a pole but it is not necessary to be either of them okay it will start with a pole and then alternately poles and zeros will be appearing okay. So if you are having a function if you are having a function z (s), z (s) equal to s plus 1, s plus 4 into s plus 10 into s plus 16, is it an r c network, it is not an l c network is it possible to realize these by r c elements poles and zeros are coming alternately minus 1, 4 minus 10,16. So poles and zeros are interlaced okay first one is a pole then it will be possible to realize it by r c combination, if in the impedance function first one is a pole obviously in the admittance function, first one will be a 0 and there should be poles and zeros should be appearing alternately okay.

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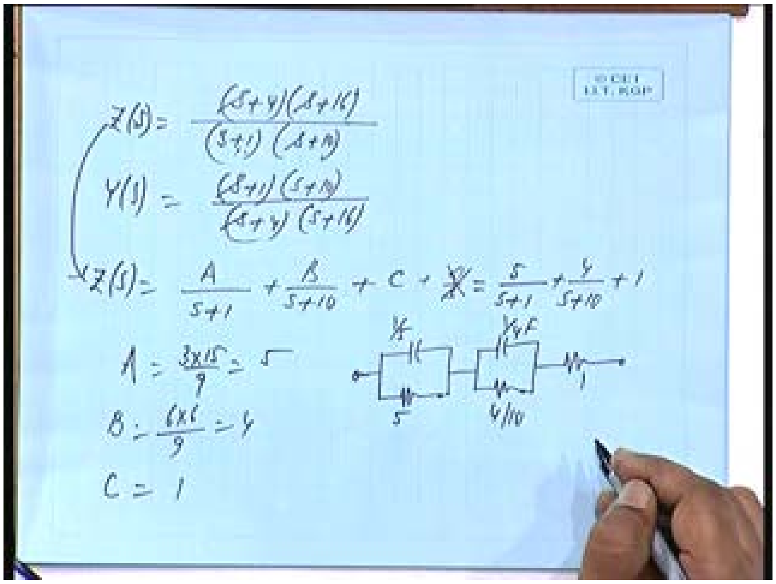

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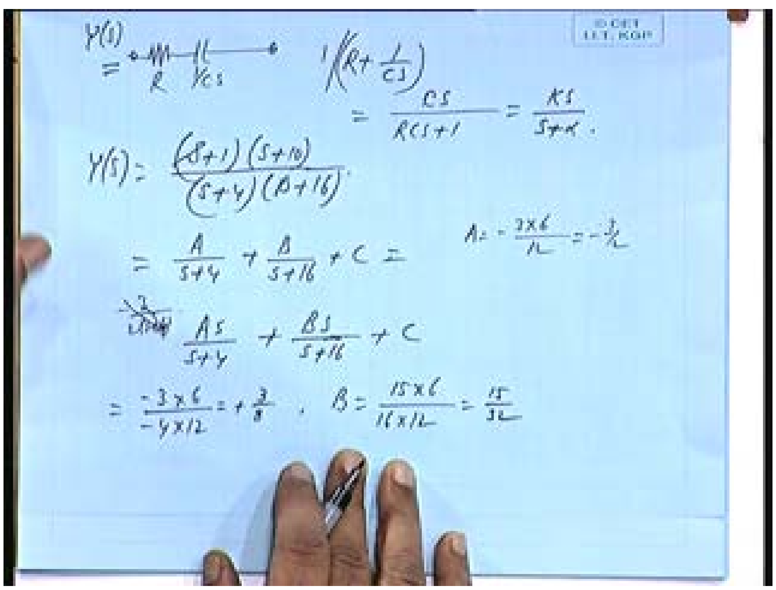

Then this is an r c network if I have z (s) given like this say this particular example I can make partial partial fractions a by s plus 1 plus b by s plus 10 okay plus some constant c alright d by s is ruled out because s is not there as a pole in this particular example alright. Otherwise, this will be the general form in this case d is not there is it possible to have c yes, it is s squared numerator is also s squared. So let us cal calculate the values of a, b, c what will be a multiply by s plus 1, we are considering z (s) and then make s plus 1equal to 0 it will be 3 into 15 by 9 that is equal to 5 b. Similarly, if I multiply by s plus 10 make s plus 10 equal to 0, this will be minus 6 plus 6 and minus 9.

So 6 into 6 by 9, 4 then if I make s tending to infinity this will vanish, this will vanish will be left with c so s tending to infinity means s squared plus s squared1 okay. So this can be written as 5 by s plus 1 plus 4 by s plus 10 plus 1 okay 5 by s plus 1 is what if I make s tending to 0 it is 5 ohms, if I make s tending to infinity 1 by 5 farads. Similarly, here it will be 4 by 10 ohms s tending to infinity it will be 4 by s 1 by 4 farad and then 1 ohm resistance. Okay this will be foster one synthesis for z (s) is that alright foster 1 synthesis for z (s) there is something very interesting here if you try to realize foster 2 what is the form of y (s) function for r c series elements, what is the admittance of this r plus 1by c s is the impedance. So 1 by this, is it not so 1 by r c s plus 1 into c s.

So now the admittance function if you consider admittance function of this element, it is k s by s plus alpha in this form unlike the impedance function which is k by s plus alpha admittance function for an r c network is k s by s plus alpha. So let us realize the same network by foster 2 that is starting with admittance remind you foster 1 starts with impedance foster 2 starts with admittance cauer 1starts with a ladder structure with a descending order of s s to the power 4, s to the power 3 and so on in case of cauer 2, it is in the reverse mode.

So let us see this particular example y (s), s plus 1 s plus 10 by s plus 4 into s plus 16, if you do not take note of this if you try to find out a by s plus 4 plus b by s plus 16 plus c you will land up in trouble. Let us see what you get what will be a if I multiply by s plus 4 make s plus 4 equal to 0 this will be minus 3 into 6 divided by 12. So you get a negative value you cannot realize minus this is 3 by 2 minus 3 by 2 s plus 4 that is not possible, is it not minus 3 by 2 into s plus 4 this is not the admittance of any r c network. So you land up in this problem suppose now we put a s by s plus 4 plus bs by s plus 16 plus c. Now let us see what will be the value of a multiply by s plus 4 make s plus 4 equal to 0 s is equal to minus 4.

So on the left hand side or multiplied by s plus 4 divide by s and then make s plus 4 equal to 0. So this will be 3 minus 3 into 6 okay divided by minus 4 into 12 is that alright. So minus minus plus 3 by 2 is that alright 3 by 8, 3 by 8 sorry 3 by 8 how much is b similarly, multiply by s plus 16 divide by s and then make s plus 16 equal to 0. So that gives me 15 into 6 divided by 16 into 12 alright, so 15 by 32 is that okay.

So we get this as 3 by 8 s and here s plus 4 plus 15 by 32 s plus 16 and see whether same logic if I make s_{10} into infinity it will not work here because here s by s that will be again giving me a, so make s equal to 0 this will vanish, this will vanish, this will be 10 by 64 is that alright 10 by 64. So what are the element values I have got y_1 plus y_2 plus y_3 I can write this as y_1, y_2, y_3. So how much is y_1 it is 3 s by 8 s plus 4 if I invert it corresponding z (s) will be z_1 (s) will be 8 s plus 32 by 3 s that is 8 by 3 plus 32 by 3 s. So one resistance, one capacitance this is 8 by 3 ohms and this is 3 by 32 farads okay.

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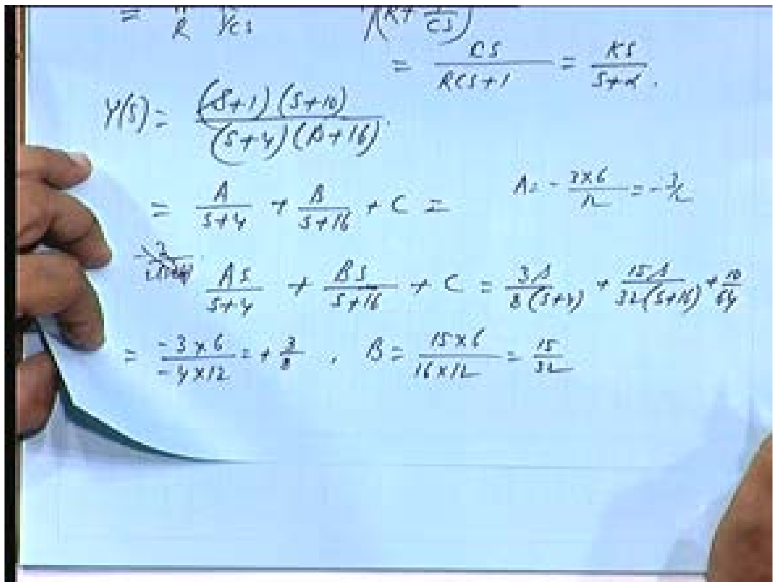

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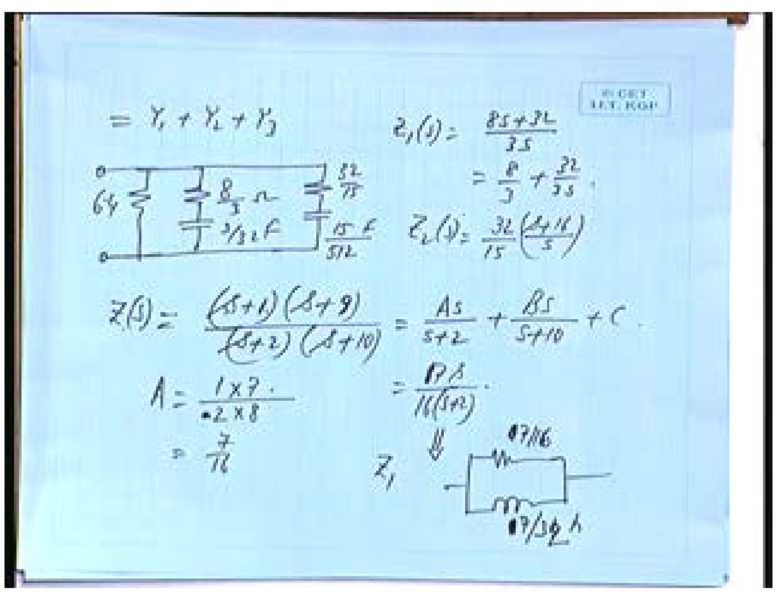

Similarly y_2 will be another r c combination, how much is that? 32 by 15, 32 by 15 by 32 so z_1 z_2 (s) equal to 32 by 15 s plus 16 by s. So 32 by 15 ohms resistance okay and 32 into 16 by 15 so 15 by 500 and 12 farads is that all right then I am left with another admittance of y_3 that is 10 by 64 more. So 64 by 10 ohms I will put on this side y_3 that is 6.4 ohms so this will be the realization by foster 2 is that alright.

So whenever you are given a network function z (s) network function z (s) where poles and zeros are appearing alternately in this form s plus 1 into s plus okay, guess whether this will be an r c network s plus 2 into s plus 1, 2, 9, 10 will be, will it be an r c network, will it be an r c network why? 0 is coming poles and zeros are alternately coming, 0 is close to the origin, 0 is close to the origin so this is not realizable by r c we have observed for rc network poles will be closest to the origin. The first one will be a pole if you calculate these roots is that alright so what could be this function like let us see can I can I break it up into some AS by s plus 2 but if I have y s in this form then it is an r c network.

Suppose I have this given as an y (s) admittance function then it is an r c network, is it not for z (s) the closest one, closest to the origin should be a pole that means for y (s) closest to the origin should be a 0 and we have got factors for that suppose z (s) is given in a similar fashion for which we have all ready factorized in this form like the previous one. Okay let us calculate just one of them, what is the value of a? If I multiply by s plus two divide by s make s plus 2 equal to 0 it will be 1 into 7 divided by minus 2 okay I will not put minus 2 because minus 1 is also there 2 into 8, so 7 by 16.

So this will be 7 by 16 into s plus 2 into s this is an admittance function is it not for r c network suppose this is an impedance function what would be the impedances elements suppose this is an impedance function we start with this as z_1, if I make s tending to infinity if I make s tending to infinity this will go 17 by 16 s will get cancelled 17 by 16 ohms resistance, is it not and if I make s tending to 0 s tending to 0 then this will go 17 s by 16 into 2, 17 by 34 what? 17 by 34 into s is the impedance of what ? 17 by 34 into s is the impedance of what? It will be an inductance Henry 17 by 34 s means 17 by 34 Henry.

So it is basically a parallel combination of r l elements, is that alright 7 by 16, 17 by 32, 16 into 2, 32, a is 7 by 16 7 not 17 sorry it is not 17, thank you very much. It is 77 by 16 ohms resistances and 7 by 32 Henry inductance. So if you are having in the impedance function the first one first factor as a 0 then it will be an rl network that means admittance function of r c network and impedance function of r l networks are identical, conversely impedance of r c networks and admittance of r l network are identical poles and zeros, thank you, poles and zeros will be alternately coming if in the impedance function it starts with a pole it is an r c network, if it is in the if it starts with a 0 it will be an r l network and then the method of partial fraction is identical alright the partial fractions that you have made for admittance of r c network will be similar to impedance of r l network, is it not. So in the next class we will be taking up some tutorial exercise on synthesis by different techniques of different r l, r c and r c, l c networks, Thank you very much.

Preview of the next lecture

Lecture -16

Tutorial

Okay good morning friends, today we will have a tutorial session.

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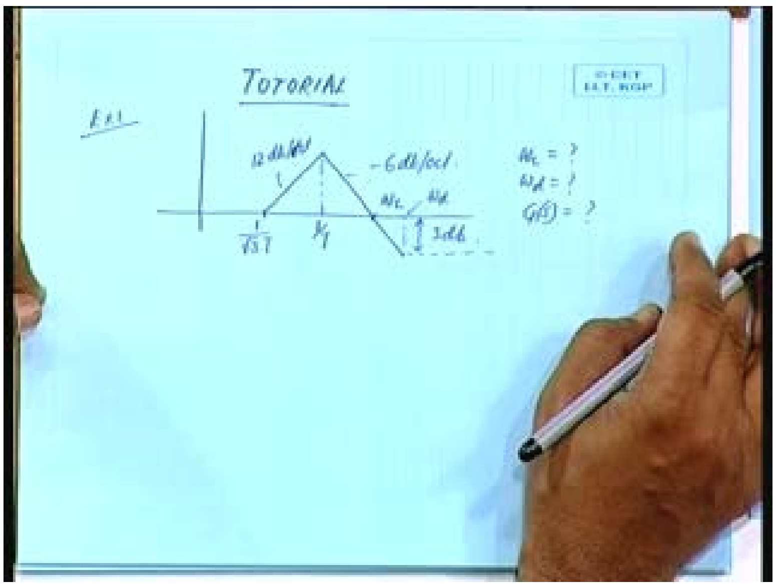

Will be working out a few problems on bode plot and positive real functions if possible something on synthesis, 2 element synthesis of driving point impedance functions. The first problem is given a bode plot like this, this is 1 by root 3 t, this is 1 by t okay this is 12db per octave and this is minus 6 db per octave, this value is 3 db. Estimate these frequencies omega c and omega d, this frequency omega c is how much, omega d is how much and then determine g (s) okay.

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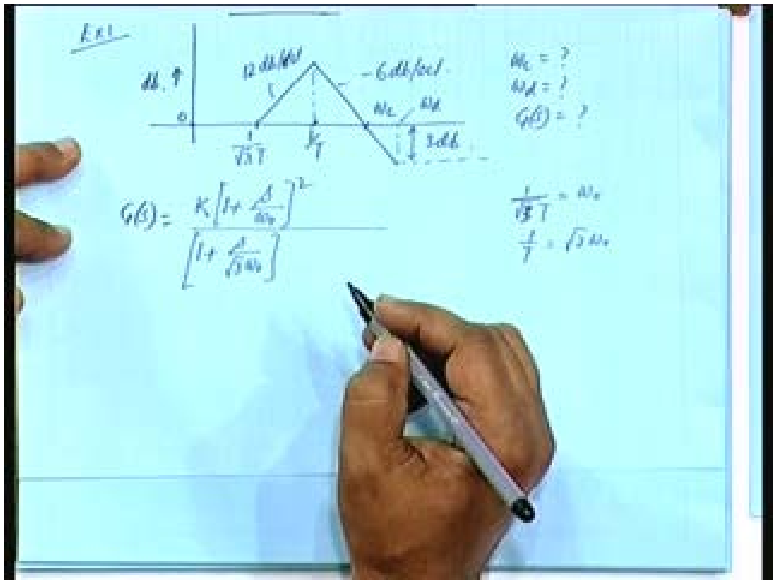

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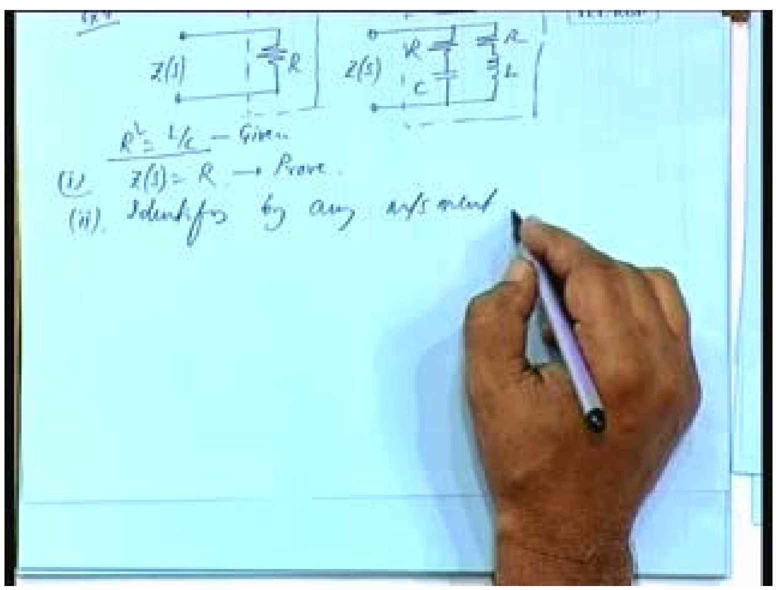

So for this problem this is the db gain, this is the 0 db line. So could you suggest the function like what g (s) what it would be like g (s) will be some constant times it is a slope going up at this frequency okay. So 1by root 3 root 3 t, let me call it omega 0 okay omega 0 then it will be 1 plus s by omega 0 s by omega whole squared whole squared, this is 12 db per octave whole squared all right then. Okay 1 plus s, s by omega okay 1 by t will be how much then root 3 omega 0 root 3 omega 0 yes sir. So if I call this 1 by t is equal to root 3 omega 0, so s by root 3 omega 0 then any other answer there are 2 black boxes this is l Henry, r resistance, resistance, c farads. The condition is r square is equal to l by c, z (s) show that z (s) in both the cases are same that means this z (s) is having a value r, given this, prove this okay.

The second part of the question is this is given we have to prove this this is part 1, part 2 by any measurement, any external measurement can you identify this one from this one can you identify that this is a pure resistance and this is not okay, will take up okay identify by any measurement that means any external measurement you carry out any experiment that you want make any measurement and identify the networks.

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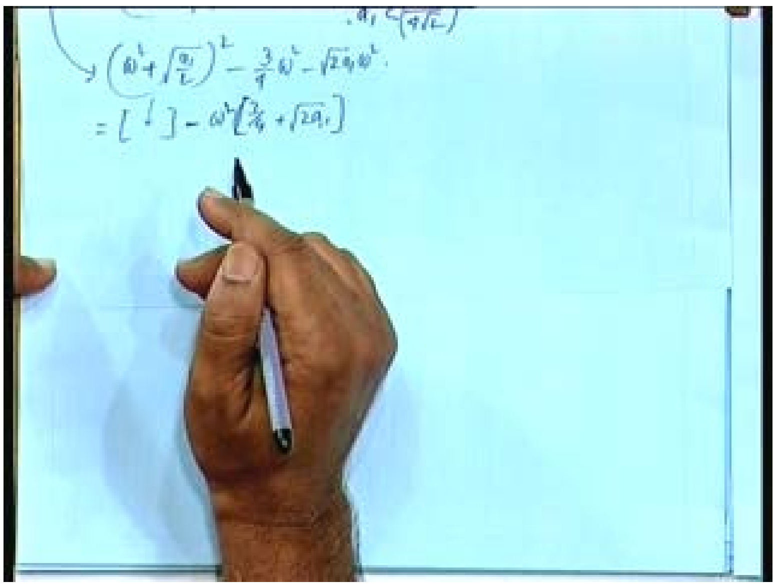

S squared minus root 2 a_1 omega squared okay one may write this square term minus omega square into 3 by 4 plus root 2 a_1 sir, a_1 less than 9 by 4 root that root a_1 root a_1 sorry. So a_1 will be a_1 will be square square okay, thank you very much. So here is one condition does it mean for any value of a_1 it will be negative though there is a negative term but this value is generating more values no I mean this whole thing is giving me sufficient margin may be.

So for any value of a_1 this cannot be tested so I should all ways put the minus sign here and then plus something and then try to see if in that plus term there is any possibility of getting a negative value alright. Otherwise, this will not be able to identify the actual range of a_1 you may be mislead then the second part is I think the time is over I will continue with this in the next class.