Networks, Signals and Systems

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Lecture - 12

Bode Plot (contd…) - Poles & Zeros

Good morning friends, we will be continuing our discussions on bode plot. Before we start I would like to just bring to your notice some small slip that I have made yesterday. I was mentioning about 2 frequencies omega n_1 and omega n_2 remember somebody pointed out later on this one.

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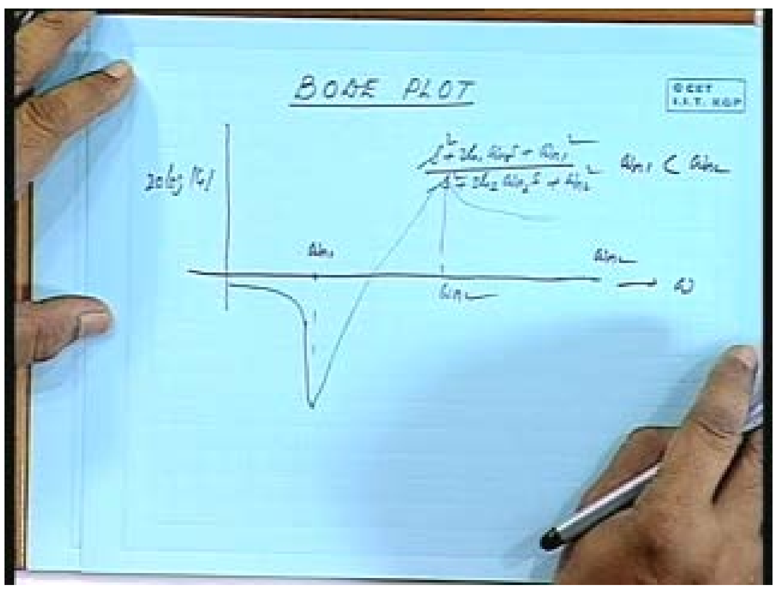

Anyway it was like this, omega $n_1$was a frequency omega n_2. So what I meant was omega n_1 is less than omega n_2 and it was in the numerator s squared plus twice zeta 1 omega n_1 s plus omega n_1 squared divided by s squared plus twice zeta 2, omega n_2s plus omega n_2 square. So the curve was somewhat like this. Okay this was omega n_2, this was omega n_1 okay. In the phase plot this was 20 log of G, in the phase plot we discussed for a function like 1 by s plus omega c approximately 110th of omega c approximately 110th of omega c the angle was about 6 degrees minus 6 degrees and at 10 times this it is 84 degrees all right. So it is close to 0 degree and this is close to 90 degrees, so almost in 2 decades you are having a fall of 90 degrees approximately. So if one draws a line 45 degrees per decade, in many books they write 45 degrees per decade straight line and this will be slightly over this and slightly below this okay, its one decade around omega c okay. So this is again another approximate sketch in the phase. Today will be taking up the quadratic form in a little more detail.

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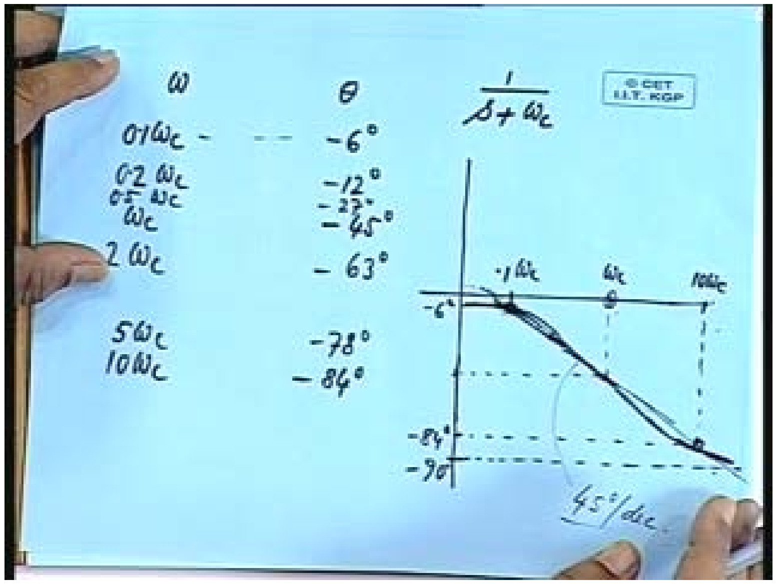

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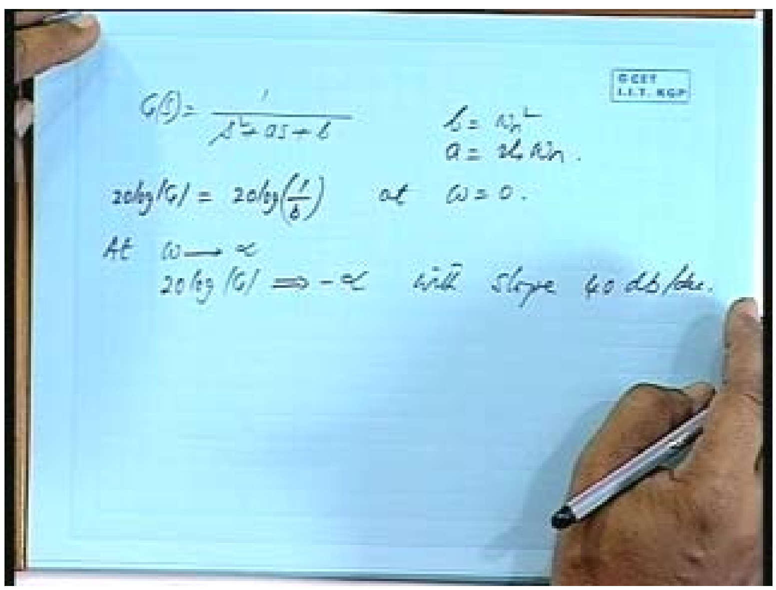

Suppose G (s) equal to 1 by s square plus a s plus b, b is same as omega n square is just we write twice zeta omega n okay, what will be 20 log of G at s equal to 0, tending to 0 will be 20 log of 1 by b at omega equal to 0 that means b c value okay, at omega very very high omega tending to infinity it will be 1 by s squared, 1 by omega square. So it will be tending to infinity with a slope of 40 db per decade, so at omega tending to infinity 20 log of G will be tending to what minus infinity, minus infinity with a slope of slope 40 db per decade because it is proportional to 1 by s square it is approximating 1 by s square 40 db per decade when is it maximum, suppose a reasonably small, when is it maximum. Let us compute that.

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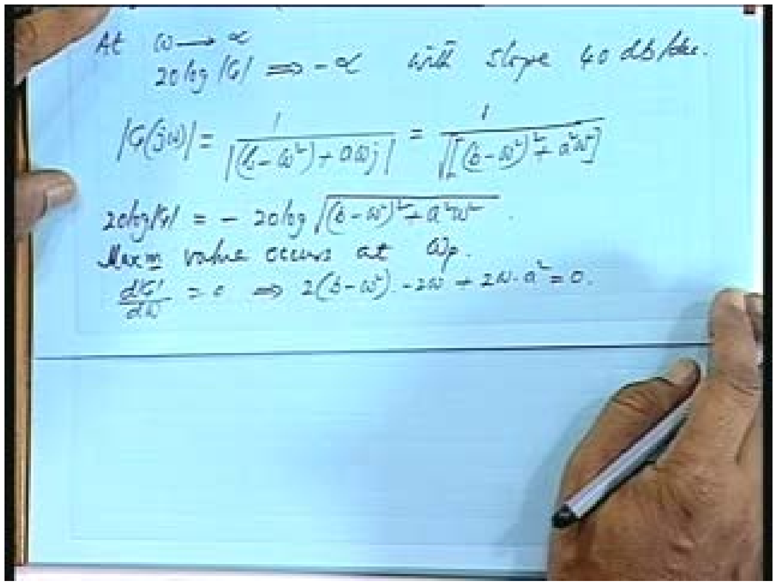

So G j omega magnitude is 1 by b minus omega squared plus a omega j magnitude of this, is it not? I am just putting s is equal to j omega equal to 1 by root of b minus omega squared whole square plus s squared omega square at the, so 20 log of G now when is this maximum this quantity 20 log of G is equal to minus of 20 log of this whole quantity b minus omega squared whole squared plus s square omega square, is it not and to get the maximum value occurs at say omega p, what is this omega p, we differentiate this with respect to omega and then equate to 0.

So if I differentiate the quantity inside the bracket what do I get so d g by d omega equal to 0 that gives me 2 in to b minus omega square in to minus 2 omega plus twice omega in to s square equal to 0. So 2 omega gets cancelled that gives me, if you allow me to write b minus omega squared is equal to a squared by 2, is that all right or omega equal to root over of b minus a square by 2. We call it omega p when a is small this is approximately equal to root b if a squared by 2 is much less than b that means for small value of zeta it is very close to root b that means omega n basically root b means omega n, b is equal to omega n square is it not natural frequency square. So the plot will be this is 40 db per decade and this is omega p pretty close to omega n.

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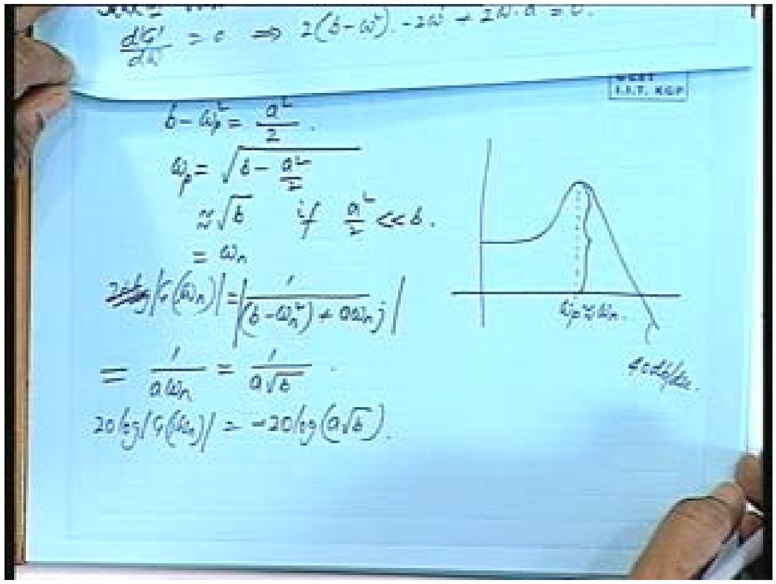

At this value omega n how much is this height, so what is the value of that 20 log of G at omega n, omega p is close to omega n. So if I put just omega n how much is that at omega n, omega n squared minus omega n square and b they will get cancelled. So it will be just 1 by a omega n is it not 1 by b minus omega n squared which will be 0 plus a omega n j magnitude of this which should be equal to 1 by a omega n and omega n is nothing but root b. So a root b okay a root b sorry this is 20 log of G therefore at omega n will be minus because it is in the denominator 20 log of a root b is that all right.

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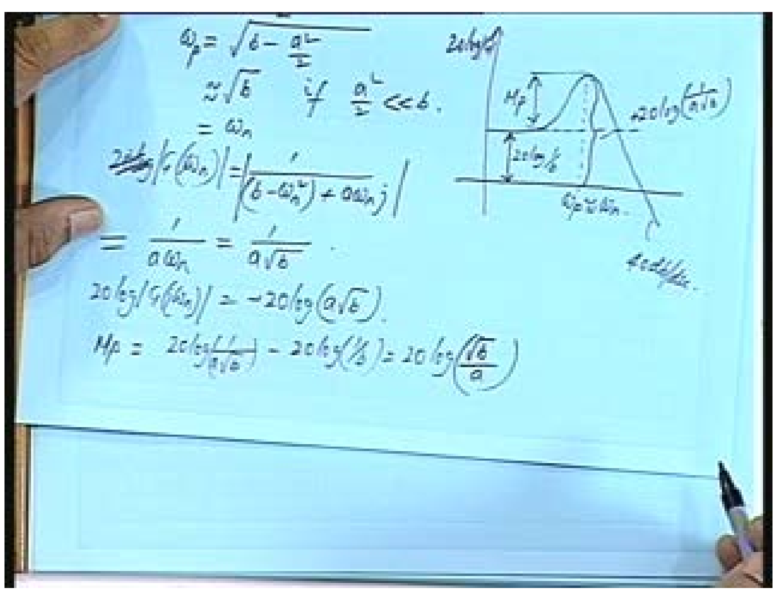

What about this starting value for omega tending to 0? This was 20 log of 1 by b all right. So how much is this p this is 20 log of 1 by b, this one and this one we have computed is 20 log of a root b minus that means basically if I put plus then it is 1 by a root b, is that all right? So if I subtract from this this side I will get this peak okay so how much is this I would like to compute? So if I call it m p something like your over shoot in a transient process you measure above the reference that is the steady state value, is it not? so something like over shoot if I want to measure m p how much is it this is 20 log of G m p will be 20 log of 1 by a root b minus 20 log of 1by b. So that will be 20 log of this divided by b so that will be root b by a is that all right this very important 20 log of root b by a okay. Let us work out one or two simple examples.

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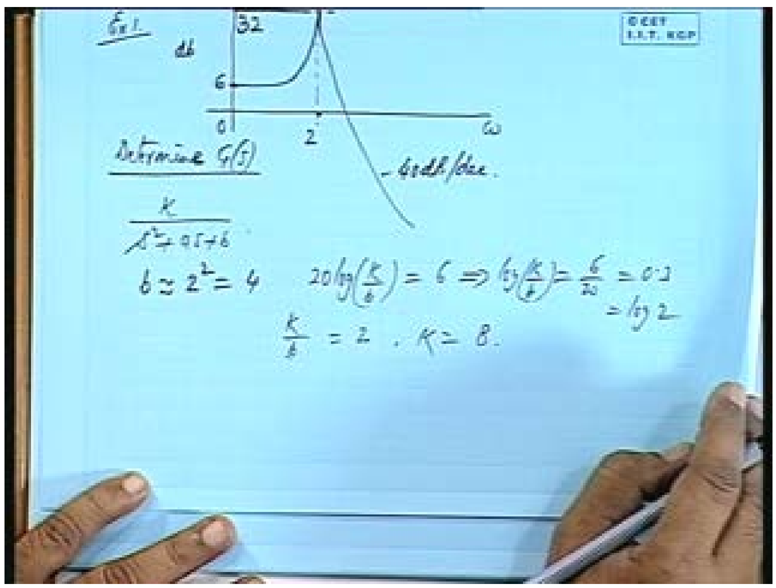

There is a question that is asked we are given the db gain 20 db, 20 log and G as this is 40 db per decade and this is given as 2, this is 0, this is 6db and this is given as 32 db, 32 db okay determine g (s) mind you we are making approximate estimates of such functions g (s). So let g (s) be it is a function of this kind k by s squared plus a, s plus b is that all right. Let this be g (s) could you suggest from these values 2 omega equal to 2 at d c that is omega very very small this is 6 and at the peak it is 32 can you estimate this a, k, a and b yes, how much is b approximately we know no b is corresponding to this frequency omega n is omega p, so b will be 2 squared b is equal to omega n squared 4 agree and then at s very very small it is k by b, 20 log of b is 620 log of k by b is 6, so 6 by 20 is .3.

So k by b is anti log point 3 is 2, is it not? Say log of k by b is 6 by 20, 0.3 that is log of 2. So k by b is 2, so how much is k_4 in to 2 b is 4 we have already computed so k is 8, is that all right. Then what you do how much is this, just now we computed how much is a peak this value 20 log of 1 by a root b, you can calculate from here or you can calculate from here either way. So 20 log of 1 by a root b is 32 is that all right.

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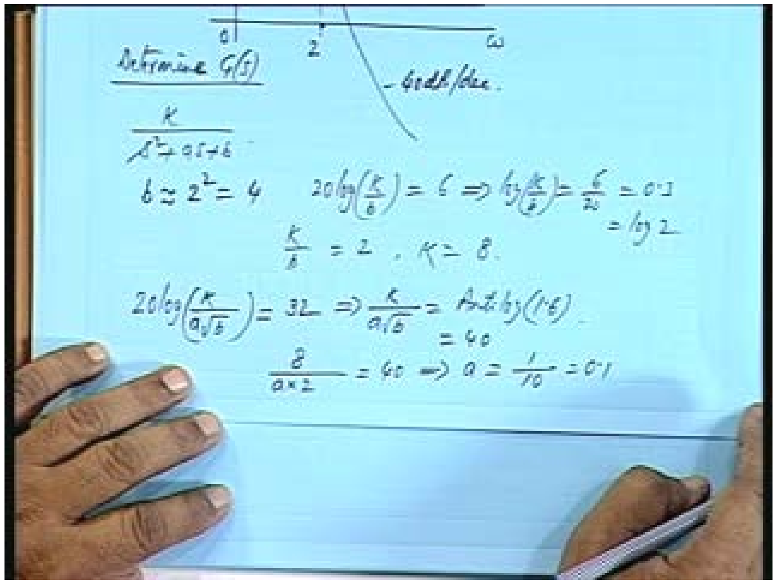

So 20 log of pardon k is eight already calculated 20 log of no that time we are taking 1 by see if there is a k_{20} log of k will be added is that all right. So okay, so 20 log of k by a root b instead of it was 1 and the maximum was obtained as 20 log of a by sorry g (j) omega n this maximum was 1 by a root b.

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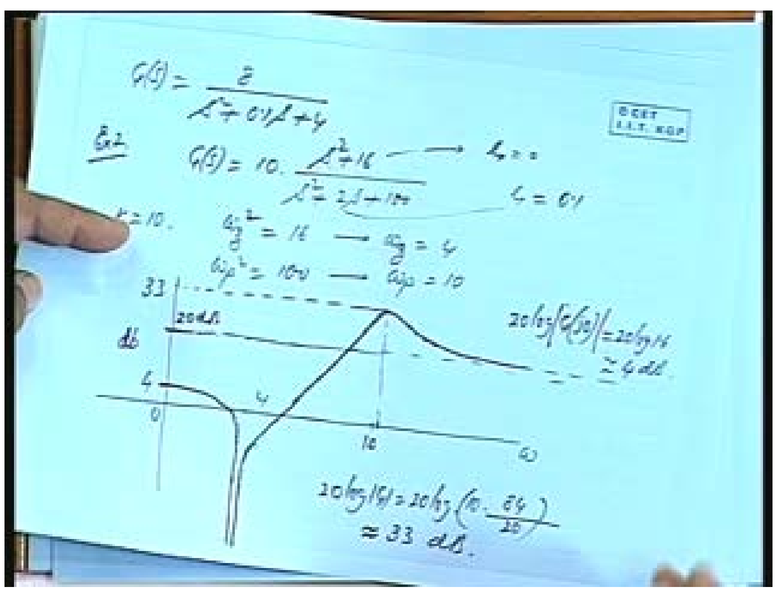

So if there is ak it will be k by a root b okay instead of 1if I had k here then this should have been k by a root b okay so 20 log of k by a root b is equal to 32 which means 32 by 20, so k by a root b is antilog 32 by 20 is 1.6 okay. If you remove that 1 so anti log of .6 is 4 so 40, so k is 8, a is not known root b is 2 is equal to 40. So how much is a, 1 by 10 1 by 10, 0.1 is that all right. So what will be the function g (s), therefore g (s) is 8 by s squared plus 0.1 s plus 4 as in pleasure any question? Let us take another example here you are ask to sketch that is pretty simple 10 s square plus 16 divided by s squared plus twice s plus 100 sketch the gain function okay so our k is 10. Now in the numerator if I now will be talking in terms of poles and zeros. So if I call this as omega z square that is that omega n that we are referring to natural frequency in the numerator will put a subscript z okay, z means corresponding to 0s or numerator it is corresponding to 0 natural frequency and this will be the natural frequency in the pole.

So omega z squared is 16, omega z is 4, omega p squared is similarly, 100 omega p is 10 is that all right and here what is zeta for the numerator zeta in the numerator function is 0, there is no middle term zeta here you can calculate 2 in to zeta in to 10 is 2 so zeta is .1 okay. So the second one so we can sketch it like this at 4 it will be going to infinity from there again it will come up at 10 will be settle somewhere. So you have to specify this value this value and this value where it settles okay so what will be the starting value this will be the sketch how much is this 20 log when I put s equal to 0, 160 by 100 is it not. So this starting value 20 log of G at j tending to 0 very very small. So j_0 we call it the dc gain is 16 by 100 and multiplied by 10, so 1.6, 20 log 1.6 that is approximately 4 db.

So this is 4 all right what about this maximum value when omega equal to 10. Let us compute here the exact value you can compute omega equal to 10 means, 10 square minus 10 square plus 16. So minus 84, so 20 log of G will be 20 log of 10 in to 84 and denominator it will be 100 and this one will get cancelled. So 2 in to 10, 20 is that all right. So that gives me approximately 20 log of 10 is 20 and 20 log of 42, log of 4 is .6 all right.

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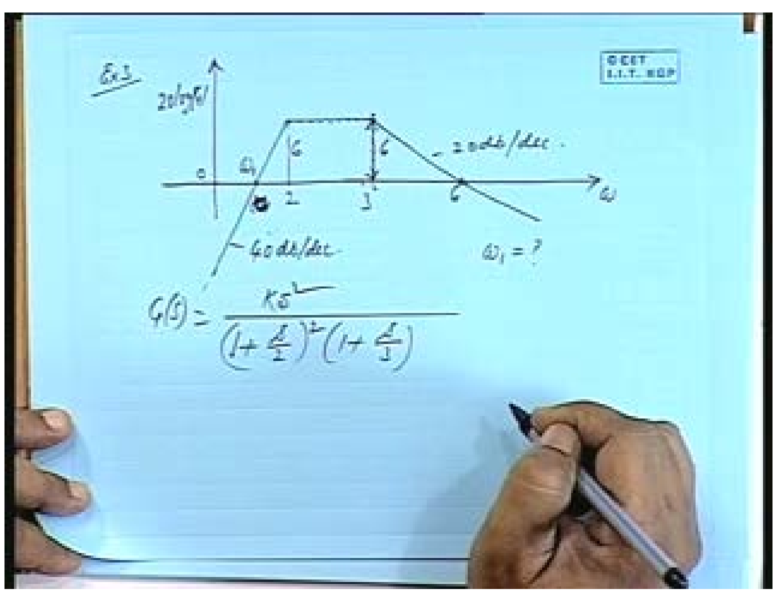

So it will be slightly more than that, so approximately 20 in to .6 about 12 so approximately 33 so this is all this calculations are somewhat approximate 33, where will it settle down, where will it settle down s tending to infinity will make it is square by s square 1. So only 10, 20 log of 10 is 20, so this one will be 20 db where it will finally settle down, is that all right? Let us take another example know I hope this will be challenging one I am expecting all of you to participate effectively in this problem, tell me at every stage you must guide me. This is 1.4, this is 2, this is 3, this is 6, this is 20 db per decade, this is 40 db per decade, this is 0, is a 0 db line. Now tell me what to be the functioning let us estimate g (s), what are the frequencies they will be some gain k there is a line that is starting with a slope of 40 db per decade. So what will be the nature of the function will it be k by s or k (s) or k by s squared as you said k by s squared k (s) squared it is going up. So k(s) squared, thank you very much and then it was going up like this and then it takes a bend at 2 is equal to j_2 then 1 plus s by 2 whole square there was a rise of 40 db per decade and you are bringing it to 0 that that means it will be whole squared is it not then 1 plus s by 3.

Here you are having a further fall of 20 db per decade than anything else, nothing else it continues okay then how do identify k. So you have to identify this height, how do you do that actually there are 2 information is which are redundant I could have ask for this frequency, I could have ask for this frequency or I could have ask for this frequency, the question that was given actually I had given one excessive information the same information. So you can calculate either from this side or from this side I given you the hints.

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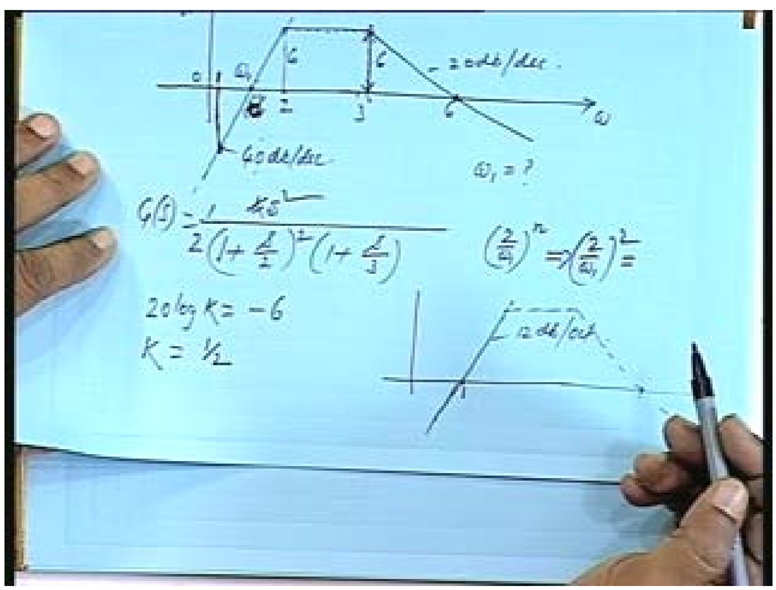

Now tell me see 6 divided by 2 is 3, so basically you are going up like this so what is the increase in this in octave 20 db per decade means 6 db. So from 0 it will rise by 6 db is it not or from 3 if I double the frequency 3 to 6 it is having a fall of 6 db, so this height is 6. So is this because this are horizontal line I could have asked you what would be the frequency here okay suppose, this is omega 1, what is omega 1? The question is what is omega 1? What is omega 1? So 2 by omega 1 is a fall of 6 db in a line of 40 db per decade that means 12 db per decade all right.

So 2 by omega 1 to the power n should correspond to how much 40 db per decade is corresponding to n is equal to 2, is it not? So what would be omega 1, n is 2, is it not. So 2 by omega 1 to the power 2 is equal to 20 log, if you take is equal to 12 db per octave, db per octave see per octave means 2 to 1, if I go from 2 to 1 there is a jump of 12 db all right. So 2 to what value if I go high level jump of 6 is a geometric mean of 2 and 1 is it not.

So you are in the middle, in the logarithmic scale equal distance means basically it is a ratio instead geometric mean. So it will be root 2 times less that means 1 by root 2 of 2, so it will be root 2, 1.41 okay. So how much is k now we know this is 6 so 20 log of k, 20 log of k, so at 1 it is how much at omega equal to 1 how much is this minus 6 because it is 12 db per decade, 40 is sorry 12 db per octave, 40 db per decade means 12 db per octave that means 2 to half of this frequency if I go 2 by 2 is 1 at 1 it will be 6 db minus then only it will be minus 6 plus 6, this total height is 12 db. So at omega equal to 1 it is minus 6 db all right normally when we compute these these are all asymptotes so at a very small value of s this does not figure, this does not figure so it is k s squared has it been only s squared it would have started like this at 1 it would be 12db per octave or 40 db per decade, is it not? Now this as gone down by 6, it is here at omega equal to 1 it is 6 actually otherwise it would have been much above this, this would have taken place much later not at 6 but this has been push down by 6 db and instead of hitting at omega equal to 1, it is hitting 0 at 1.4, it has been push down by how much 6 db. So 20 log of k is minus 6. So how much is k half, so that z function is half of k equal to half so you write half of s squared in to 1 plus s by 2 whole square in to 1 plus s by 3. So from the sketch of the asymptotes you are ask to find out identify g (s).

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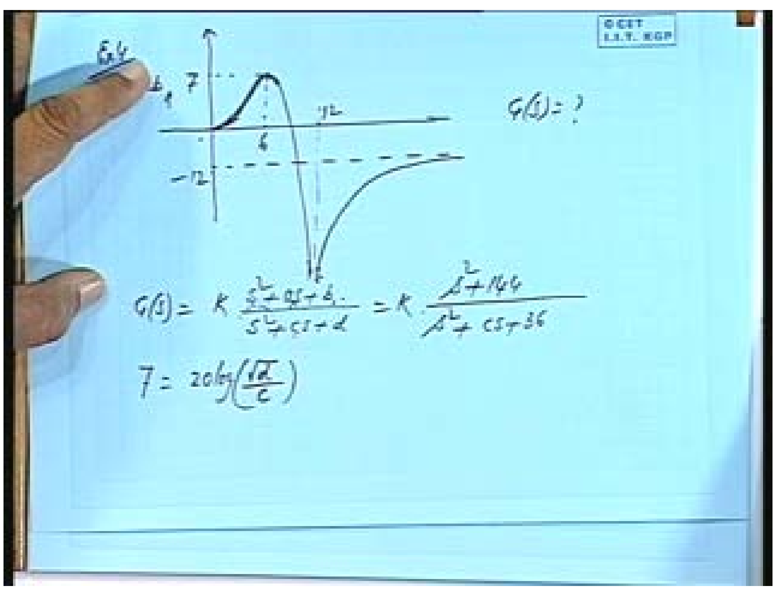

Let us take up another example. I am asked to compute g (s), this is given as 12, this is given as 6, this is given as 7 db, this is settling at minus 12 okay estimate g (s). So you can write g (s) as some k times say s squared plus a, s plus b by s squared plus c, s plus d okay, this is if you remember in the last class we have discuss omega n_1 and omega n_2. So similarly, they are 2 such peaks 1 in the numerator, 1 in the in the denominator will come, so s squared plus a s plus b.

Now tell me in the numerator function s square plus a, s plus b here it is 10 in to infinity, so how much is a the numerator will correspond to the one where the peak goes in the negative direction this is corresponding to the denominator function. So for this function if it is 10 in to infinity the middle term will be a will be 0, zeta is 0, is it not. So it will be s squared plus this is not that b and how much is b, 12 squared 12 squared 144, this frequency square divided by s squared plus c s plus how much is d approximately 6, 36, 6 square okay, how much is this height about the reference from the earlier value, how much is the peak 7 db is equal to how much 20 log. We calculated root d by c, is it not? This is what we saw some time back root b by c remember root b by a, 20 log root b by a was this peak m p, where it was in the denominator.

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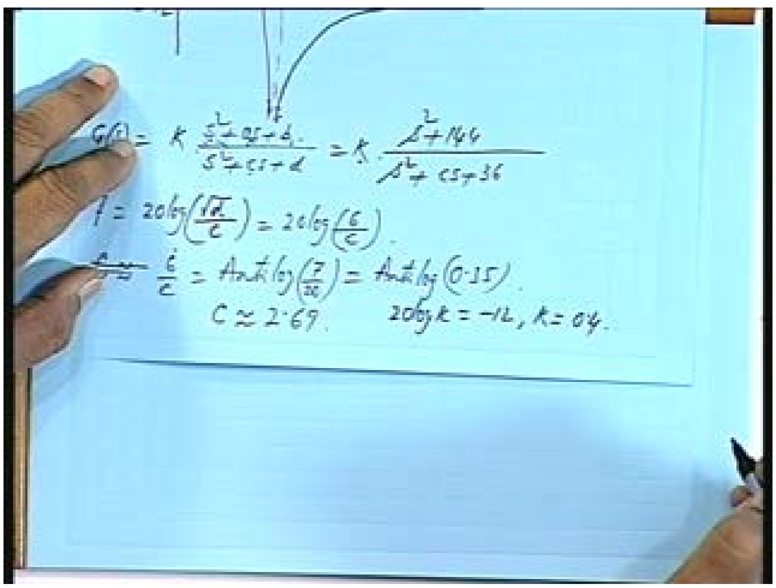

So 20 log root d by c, how much is that 20 log of root over root over of d means 6 by c. So from their c if you compute 7 by 20, so it will be approximately .35, anti log .35 okay 6 by c is antilog 7 by 20, so .35, anti log .35. Hence you can calculate c, I have calculated c approximately equal to 2.69 or so it may be wrong I mean it is somewhat some gross approximations are made. So once you know c now what is to be calculated these k when s tends to infinity this will be 1, so 20 log of k and that is equal to minus 20, is it not? As as s tends to infinity so 20 log of k is minus 12, so k is approximately .4 is that all right. So you know all the values.

So so far we have studied the variation of the gain with frequency in terms of the asymptotes, this will be useful for estimating the frequency response for a given function. Now we will introduce the concept of poles and 0s of a network function.

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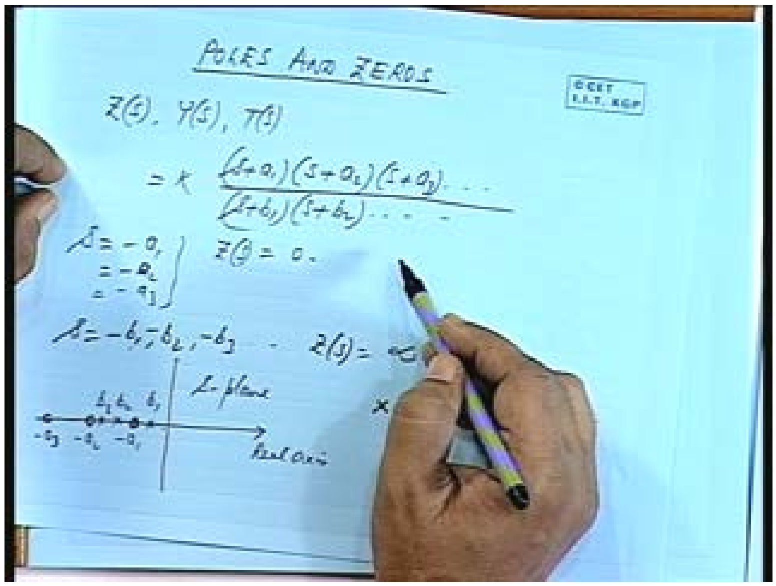

Any function say it may be z (s) it may be y (s) okay or may be transfer function V_2 by V_s V_2 by V_1 and so on or transfer impedance transfer admittance Y_{12}, Z_{12} and so on. It can be any function we can write as some constant in to say s plus a_1 in to s plus a_2 in to s plus a_3 and so on divided by s plus b_1 in to s plus b_2 and so on okay, some finite number of factors in the numerator finite number of factors in the denominator if you can write like this then at s equal to minus a_1 the function is say z (s) equal to 0 similarly at minus b a a_2 minus a_3, z (s) will be always 0 if you take s equal to minus b_1 or minus b2 or minus b_3 then z (s) will be 10 in to infinity. So when any function grows up when it tends to infinity the corresponding values of s we call them poles when the function tends to 0 we call those roots as 0s of the function they may not be so explicitly given it may be any function but wherever the function becomes infinity we call them poles, wherever the function becomes 0 we call them 0s.

So in the complex s plane we may have minus a_1, a_2, a_3 and so on. We will show the 0s in the s plane, this is the real axis if a_1, a_2, a_3 these are positive quantities then at minus a_1 minus a_2 minus a_3 they will be represented in the negative side and negative real axis, the function becomes 0. So we show them by a small circle or a_0 okay similarly at minus b_1, b_2, b_3, b_1 may be here, it may be here, it may be here. So b_1, b_2, b_3 and so on the function becomes infinity, so when the function becomes infinity the corresponding roots we call them poles will be denoted by a cross. So for a function like this it will be denoted by 0s and cross like this, it is not necessary that the numerator and the denominator will be in such simple factors all right there can be quadratic forms where the roots may be complex.

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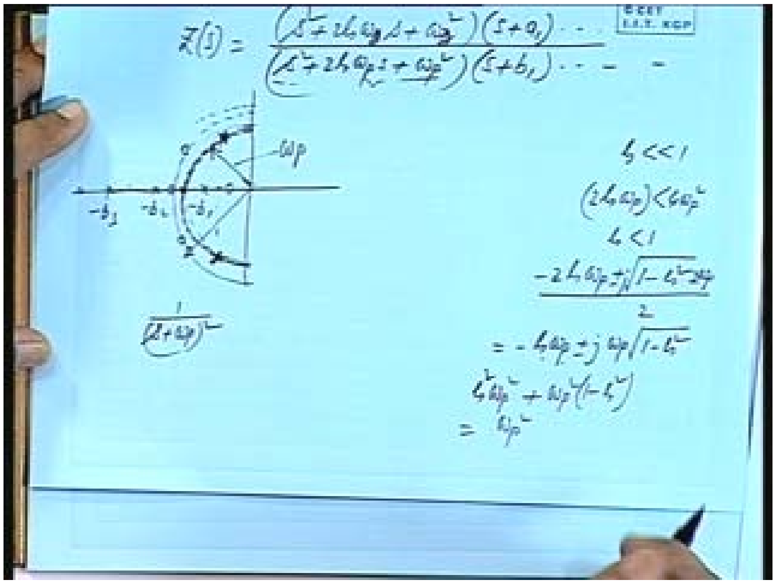

So if you have say in this a function like s squared plus twice zeta omega n s plus omega n squared then s plus a_1 and so on. Similarly, you may have its not necessary that you will have only one quadratic you can have multiple number of quad quadratics similarly, twice zeta I will write the numerator as omega z, so omega z corresponding to 0s that means numerator corresponds to 0s omega p s plus omega p squared in to s plus b_1 and so on. Then the roots will be distributed like this there can be poles here at minus b_1 minus b_2 minus b_3 and so on there will be also poles corresponding to this quadratic and if zeta is very small okay much less than 1 such that zeta omega p whole squared is less than 4 a c that is 4 in to omega p square that means zeta is less than 1, if zeta less than 1 then it will be the roots will be always complex conjugate okay, what are the roots what are the roots minus b minus twice zeta omega p plus minus root over twice zeta omega p square minus 4 omega p square. So how much is it 1 minus zeta squared in to omega p in to 2 outside okay in to j divided by 2. So that will be minus zeta omega p plus minus j omega p in to 1 minus zeta square that means omega p minus zeta in to omega p will have some real value and plus minus say here.

The roots will be complex conjugates, now you see the real parts squared and the imaginary part square if I take that is constant real part squared is zeta square omega p square if you take the imaginary part square will be omega p squared in to 1 minus zeta square and that is equal to omega p square, this minus term will get cancelled that means with omega p as the radius if I draw semicircle then for different values of zeta, for different values of zeta the roots will be lying on the unit circuit, on this semi circle okay. So if I keep on changing zeta only returning omega z or omega p corresponding poles are 0s will be shifting along this okay, when zeta is made equal to 1 this term will be 0 so the their will be no imaginary part only 2 roots will be both the roots will be real. So the roots will be here there will be 2 roots here is it not, so that time will get 1 by s plus omega p whole squared okay zeta equal to 1 it will be twice omega p s plus omega p square similar, will be the behavior for the 0s. So 0s may be located anywhere here may be may be here like this and 0s also may fall on a circle of radius omega z this was of radius omega p, 0s, complex 0s may occur here and here on a semicircle of radius omega z okay.

So depending on the quadratics that will be given to you the corresponding natural frequencies you choose the semi circles and the roots the poles and 0s for that particular quadratic will be lying on that semi circle and they will be appearing in complex conjugate form. There are some situations where we have to investigate the nature of this poles and 0s their locations and other behavior okay while identifying network functions specially for studying the realisability of a network function, whether a function can be realized by a network element set of network elements if you want to study that then you have to study the behavior of the poles and 0s that is the distribution of the roots.

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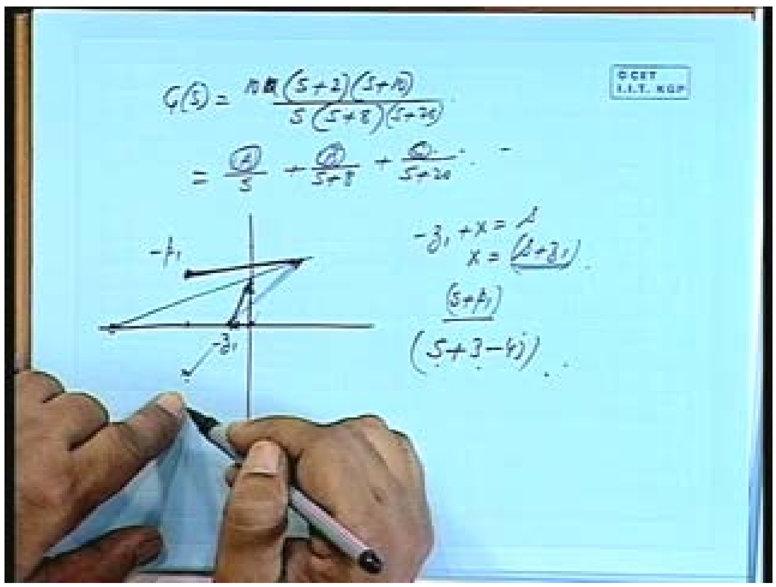

Sometimes we are interested in knowing the transient response how do you compute the transient response of a system if you are given say G (s), g (s) is a very general function as k in to okay k may be say 10 in to s plus 2 in to s plus 10 divided by s in to s plus 8 or in to s plus 20. Now this one standard technique that you already know is to calculate A, B, C by partial fraction analysis these residues you calculate by simple algebric method multiply this function by s make s equal to 0 you get s similarly multiply by s plus 8 make s plus 8 equal to 0 and so on, is there any other approach given the pole 0 distribution. Suppose we have, poles 0s like this is it possible to find out, is it possible to find out this A, B, C etcetera, what is A after 1, what is A or what is Bm these are the residues that you are computing for for this function. If I take any value of s here what does this vector show. Suppose this is at some z_1 this is at minus z_1, is it not? So this is minus $z_1$this is s so what is this factor? what is this vector? No, there is no pole I am taking any value of s, s I am varying along this imaginary axis all right then what does it show me, if I vary s along this what does it show? Now for that method any where if I take what would be this vector line this is s, so minus z plus this unknown vector x minus z_1 plus unknown vex vector x is equal to s, so how much is that unknown vector s plus z_1 all right.

So if I take any s it need not be of this line any s then a distance this vector connecting minus z one and this will give me s plus z_1 vector all right similarly, for a pole p_1 if I draw this what will be this representing this p_1 is basically minus p_1, is it not? Suppose it is at minus p_1, p_1 is a complex quantity all right then it will be representing s plus p_1 vector if s plus p_1 is a factor then s equal to minus p_1 is the root all right. So if this is minus 3 plus 4 j then the factor corresponding to this vector will be giving me s plus 3 minus 4 j, if minus 3 plus 4 j is this point then this vector is s plus 3 minus 4 j okay. Similarly, for this 1 this 1 and so on okay. So in the next class we shall discuss about how to compute these A, B, C these constants with the help of these phases or these vectors, these vectors okay. So we know the location of poles and 0s from the factors and then we will find out how to compute these residues geometrically okay. Thank you very much.

Preview of Next Lecture

Lecture-13

Driving Point Immittance Functions-Realisability Conditions

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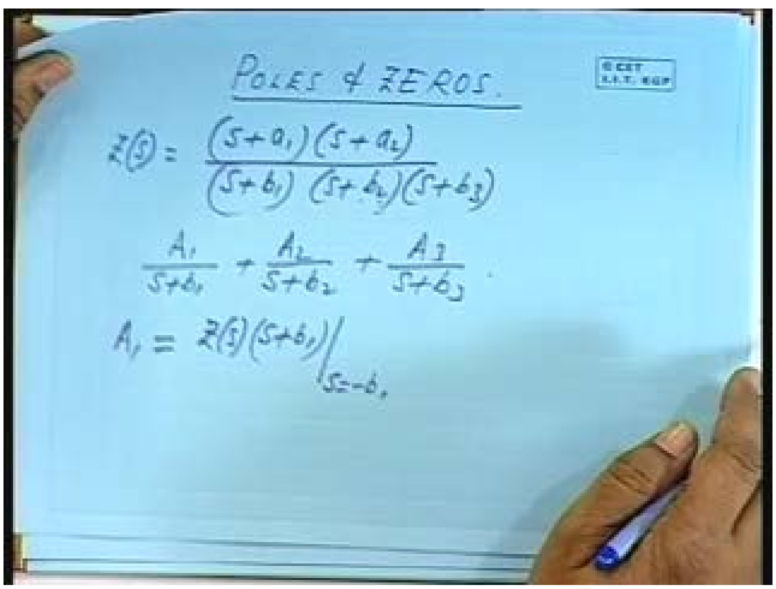

Okay good afternoon friends we are discussing about poles and 0s, location of poles and 0s and determination of residues. Suppose we have function z (s) s plus a_1 in to s plus a_2 divided by s plus b_1 in to s plus b_2. So this type of function or may be s plus b_3 also when we make partial fraction of such functions it may be just the response function or when z (s) is there if you give an impulse input then this itself will give you the output function. So any function of this kind when we want to measure there when we want to calculate the residues A_1, A_2 etcetera. We computed say A_1, we computed like this this function z (s) multiplied by s plus b_1 and then evaluate this at s equal to minus b_1, is it not. Now this b_1, b_2, b_3 they can be complex conjugate also if they are complex conjugate they should be in pair conjugate pairs okay.

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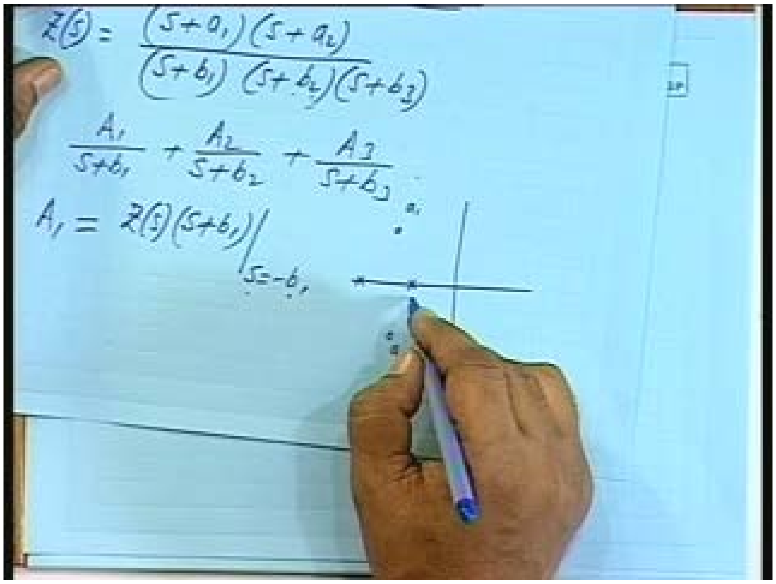

So z (s) in to s plus b_1 s evaluated minus b_1, I mean by putting s equal to minus b_1 evaluate $a_1$what does it mean if I multiply by s plus b_1 this will go z (s) in to s plus b_1 means this we are evaluating at s equal to minus b_1. So suppose you are having b_1 is here b_2 may be here okay, a_1 may be here, a_2 may be its complex conjugate a_1, a_2 both could have been real also similarly, there can be a_3 a_4 and so on. So z (s) in to s plus b_1 when we are evaluating at this point that means I am placing s at m_1 m_2 sorry m_1 n_1 minus m_2 n_2 will become s squared plus 10 in to s squared plus 6 minus 11 in to 5, 55 s squared okay.

So that is s to the power 4 plus 10 plus 6, 16 s squared minus 55 s squared plus 60, so that is s to the power 4 is that all right minus 39 s square plus 60. Now if you evaluate at s equal to j omega what is this give you if i evaluate this at s equal to j omega, omega to the power four plus 39 omega squared plus 60, is it always positive for all values of omega? Yes, so it is realizable because power dissipated if it is negative what is it physically mean if the real part is negative that means if I pass a current instead of getting heated up, it will be cooling down is it possible I square R is negative means what it is cooling down that is not physically possible so the real part of the impedance function z s the impedance function z (s) must be always positive. So even though we have seen earlier, so this example the residues are coming negative.

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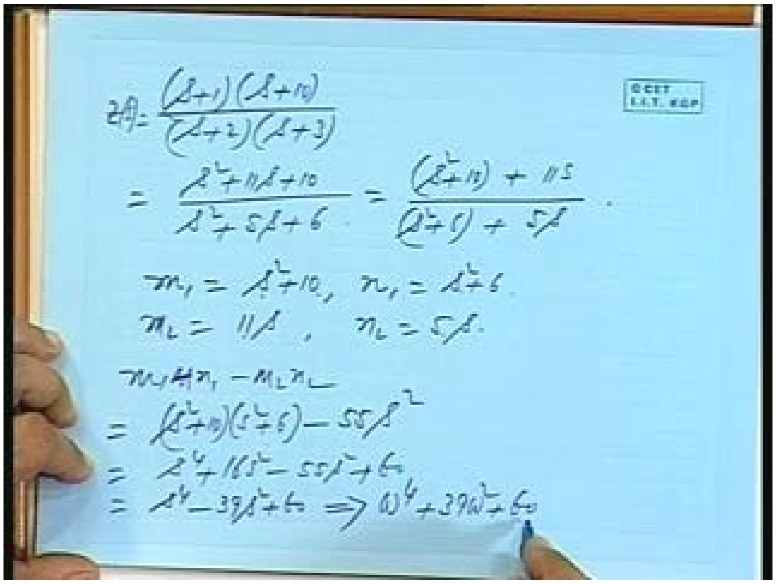

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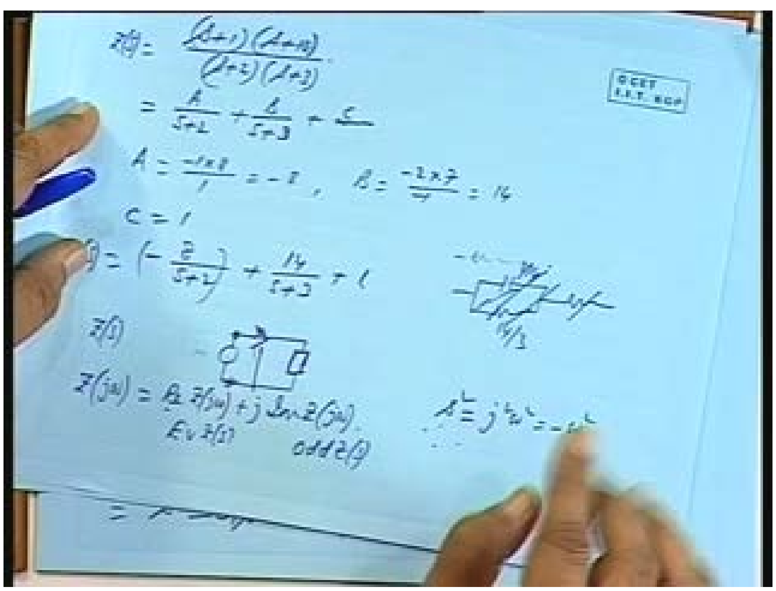

It may not be possible to realize by RC combinations, you may try with LC it may fail it may be possibly the RLC all right. So but it is a realizable function so if you find that this test fails it is not positive throughout for all values of omega somewhere it is becoming negative then you can say it is not realizable by passive elements this is not an network function, is that all right? So we will stop here for today will continue with this and a few simple realizations will see with LC RC and so on.