Networks, Signals and Systems

Prof T. K. Basu

Department of Electrical Engineering

Indian Institute of Technology, Kharagpur

Lecture - 11

Bode Plot (contd…)

Okay friends, we will continue with some more examples on bode plot. Last time we were discussing about the plot for function of this form 1 by S squared plus 5 S plus 100 that is twice zeta omega n S plus omega n squared for different values of omega n omega sorry for different values of zeta we will find this peak value occurs at a shifted value, shifted point here that means the peak gradually shifts to the left at a frequency omega D which is equal to omega n into root over of 1 minus zeta square, omega d equal to omega n into 1 minus zeta squared.

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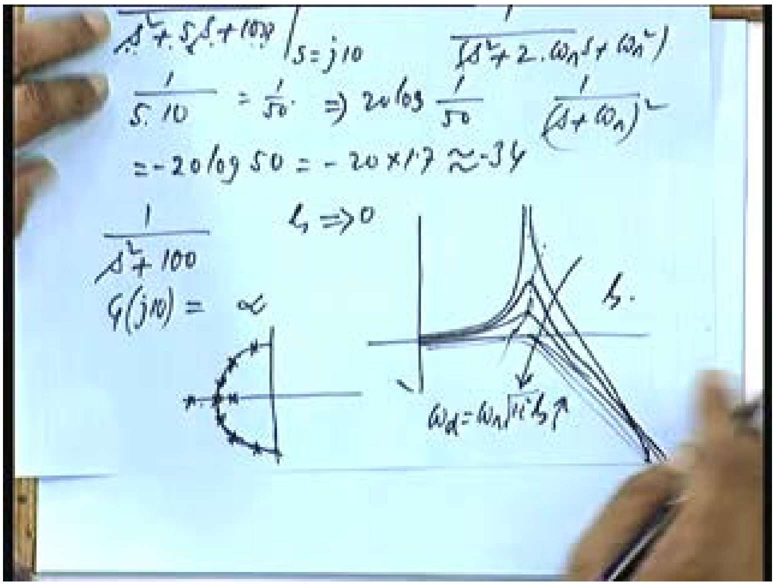

When zeta is small we can approximate this 2 omega n. Now we come to a frequency plot, sorry phase plot. Let us once again compute the phase for each of these factors like say 1 by S plus 10, 1 by S plus 20 and so on if you have a factor like this, what will be the corresponding phase if I put S equal to j omega. So omega equal to 1 omega equal to 10 omega equal to say 5 and so on when it is S plus 10 that is 1 by j omega plus 10 when you take omega equal to 1 omega equal to 1 how much is this 1 by 10 plus j okay, so 10 plus j how much is the angle 10 inverse, tan inverse 1 by 10, 10 inverse 1 by 10 that is 10 inverse of .1 okay.

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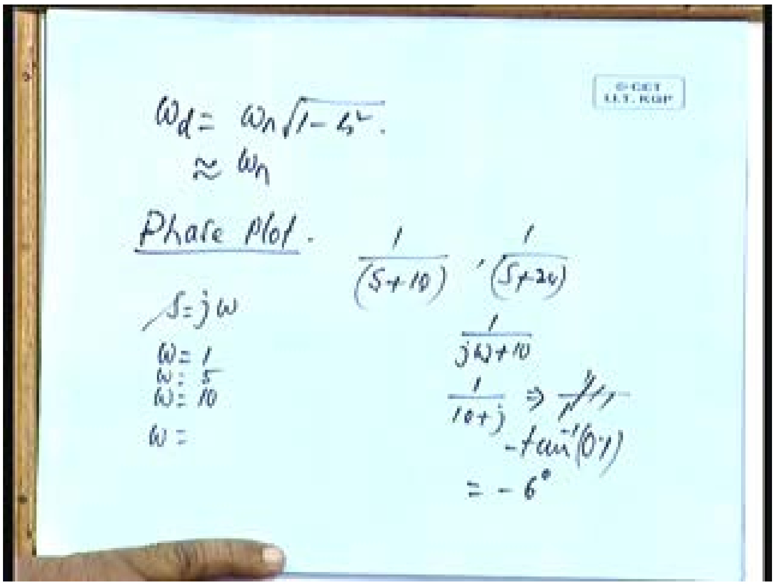

So that gives me tan inverse of .1 minus tan inverse with a negative sin, so that will be minus how much is it 10 inverse .1 is .1 tan theta is equal to theta when theta is small .1 radiant approximately 6 degrees, 57 degrees will become 1 radiant. So .1 radiant means 5.7 degrees approximately 6 degrees okay, if I take 10 times this frequency that is when omega is 100 then it will be 10 plus 100 j all right. So how much is tan inverse 10, tan inverse .1 is .6 so just 1 by .1 that means 90 minus 6 degrees will be 84 degrees okay.

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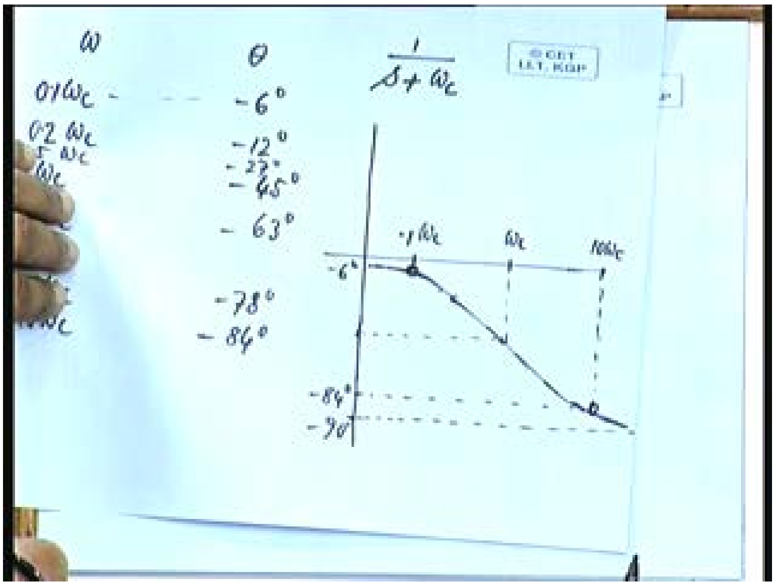

So we will make a table like this omega if the function is like 1 by S plus some omega c, some critical frequency then omega if it is omega c by 10 that is .1 times omega c and 10 times omega c then the corresponding thetas are this should be minus 6 degrees and this one will be 90 minus 6 degree, so minus 84 degrees, you just compliment, is it all right? if it is similarly, if it is 0.2 omega c approximately it will be double this is an approximate calculation. So similarly, at 5.2 so inverse of .2 how much is it 5 times omega c it will be approximately 78 degrees okay.

Now omega c omega c at omega equal to omega c, how much is it minus 45 degrees at 2 times omega c 90 tan inverse 263 degrees 1 by 2, 90 minus 63 degrees is 27 degrees is good enough to make a sketch. Suppose this is .1 omega C, this is omega c this is 10 times omega c then for a factor of this kind it will 10 to 0 as omega tends to 0 okay but it may not go down further at this point I know it is minus 6 degrees, at this point I know it is 10 into minus 84 degrees. Okay, it will asymptotically go up to 90 degree only 1 by when S is very very large compared to omega c it will tend to 1 j omega that is minus 90 degree it cannot go beyond that

So asymptotically it will be approaching minus 90 degree okay and at omega equal to omega c it is 45 degrees okay. You can take some intermediate values like at .2 it is 27 degrees and so on. So approximately we can sketch the curve like this if you take the intermediate values and if the scales are selected properly then you will be getting a curve like this.

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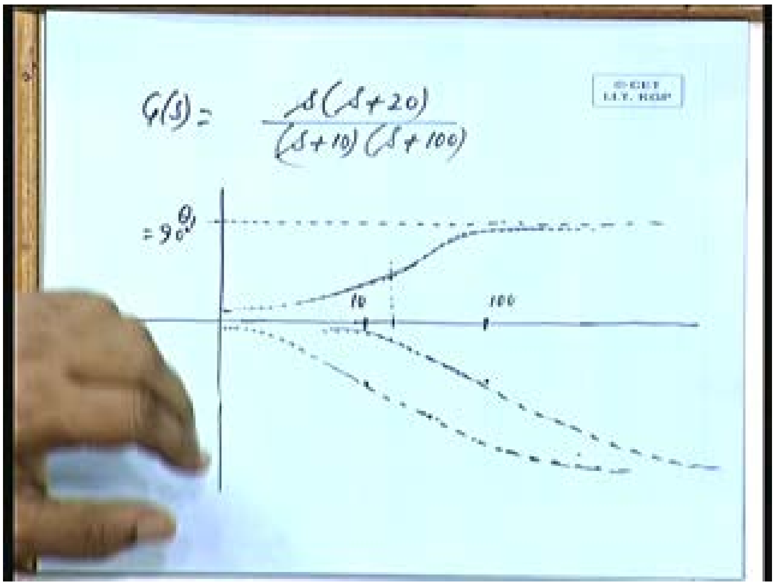

Okay for any factor like this. Now let us see what will be the phase plot for a journal function G (s) say equal to S into S plus 20 by S plus 10 into S plus 100, for S it will be giving me plus G omega, so it will be plus 90 degree a component like this all right angles are all added is it not theta 1 plus theta 2 minus theta 3 minus theta 4 and so on that will be the total angle. So I can like in the gain plot if I take log they become additive. In the phase you do not have to bother they will be always additive, so theta 1plus theta 2 minus theta 3 minus theta 4 if there in the denominator it will negative.

So this will be theta 1 equal to 90 degree, theta 2 if I take this one S plus 20, so suppose this is 10, this is 100 then somewhere here 20 it will have S plus 20 means, this is the critical frequency. So it will be 45 degree plus 620 so at 200 it will be 84 degrees and so on, a curve like this that means this very curve if I take the mirror image this will be reflected here if it is in the numerator all right a similar curve will be there in the numerator function. So this kind of curve will be there then 1 by S plus 10, so it will minus 45 degrees somewhere here and again this will be approaching minus 90 like this and at S equal to 100 there will be again another function like this one thing is very interesting, if I fix omega c then I know at one 10th this frequency it is minus 6 degree at 10 times this frequency it is minus 84 degrees and at various other frequencies I know where it will be some 12 degrees, 27 degrees, 63 degrees and so on. So this curve is fixed the centre point is omega c where the angle is minus 45 degrees.

So I will just take a stencil I will cut a piece of paper say may be sorry a cupboard and then I slide it over this entire scale of frequency fixing omega c at the request point, do you get my point. So this omega c in this particular case is fixed here then it is sliding down this and that at 100 omega is equal to 100 I again draw this curve, the same curve all right and then I take its minor image just invert this and then take the curve at omega c is equal to 20 all right. So I can prepare one slide, I can prepare one stencil just take a cardboard and then cut it along this all right prepare say take from the market any similar graph sheet so that this these decades are already fixed it can be matched with any graph sheet and okay and then you make a profile like this and then use it the same stencil can be used for drawing all the phase component and then add them together okay.

So here it will be plus 90 then it is gradually falling because S_{10} comes earlier, so it will be falling like this this plus minus so much so it will be slightly less so it will be falling like this and then gradually this will start coming so the rate of fall and rate of increase they may be more or less same so it may remain horizontal and then again a fall starts at around this point it is falling mind you very slowly because here this is increasing here it is this is decreasing here and this is increasing but they are not at the same rate somewhere, here again additional term comes so fall will be much faster at the end this is minus 90, minus 90, minus 180 this is plus 90, plus 90, 180, so finally it will come down to 0 okay and so on.

So phase plot is much simpler what you have to draw is first around omega c draw this curve for one single phase, one factor and then keep on repeating that all right like this and then add them together the network functions sometimes we are interested in knowing say what will be say Y or Z parameters for a journal network 1, 1 dashed 2, 2 dashed and what will be its variation with respect to frequency so you may be given may be something like a transmission lines, what will be V_2 (S) by V_1 (S). I excite it by variable frequency source then R_1, L_1, R_2, L_2, C what will be V_2 by V_1 in the Laplace domain if I keep it open how much is V_2 by V_1. So it will be one by CS divide by R_1 plus L_1 S plus 1 by CS this will give rise to a quadratic see if I multiple throughout by CS, it will be L_1 CS squared plus R_1 CS plus 1.

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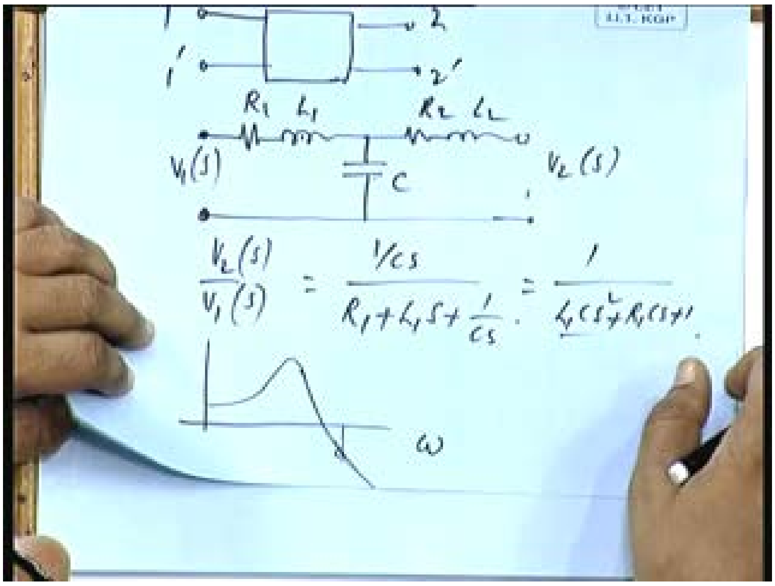

So this is a quadratic depending depending on the value of R_1 L_1 and C we will have different values of zeta, roots may be complex all right or real and we will get V_2 by V_1 for different values of omega something like this, is it not 40 db per decade. So for any frequency, for any frequency you can know what will be the gain all right for any frequency you will be knowing the gain. So if I give an input voltage you can calculate output voltage immediately once you know this function we may be interested in Y or Z parameters that is short circuit or open circuit parameters A, B, C, D parameters and so on.

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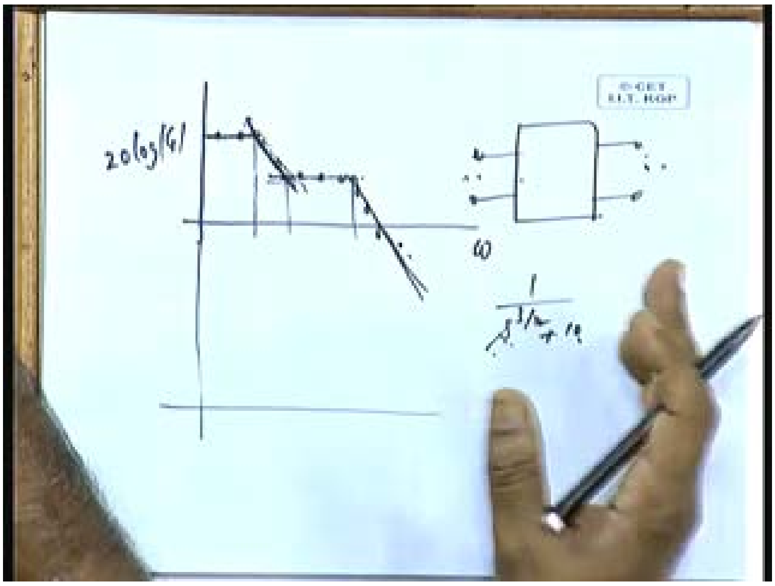

So all this can be computed in terms of S we will get a function like this and then from the bode plot you can get the response at different frequencies. So let us work out 1 or 2 problems. Now before we go to numerical problems, let us discuss about the inverse problem that is an identification problem given and experimental data. Can you estimate the transfer function? quiet often we do experiments on network system which is may be in the given which may be given in the form of a black box all right I may be interested in the transfer function V_2 by V_1.

So we apply a voltage of frequency varying from say 1 hertz to 10 kilo hertz all right. You measure the ratio of the voltage V_2 by V_1 at every frequency. Suppose the experimental points V_2 by V_1 will give you gain V_2 by V_1 will be giving you gain against frequency. For different frequencies you measure the ratio and then take 20 log of that ratio V_2 by V_1, you may be getting some experimental points like this they will be scatted around some approximate straight lines. You are approximating them mind you by the best fit. Suppose we get some lines like this you are you should attempt to fit this points along either a horizontal line or a 20 db per decade slope or a 40 db per decade slope as far as possible because you cannot have a system function why 20 db per decade slope. Can I have 30 db per decade slope, can I have 30 db per decade slope, can we have the function S to the power 3 by 2 plus 10 no we can have 1 by S plus 10 1 by S plus ten the whole square or a quadratic but power of S cannot be a fraction and from all this we know the asymptotes will have to be either 20 db per decade or 40 db per decade or a horizontal line it cannot be anything other than that.

So you have to match this points as far as possible there all experimental points. So there can be errors always best possible fits with these lines once you have identified these you know these frequencies omega 1, omega 2, omega 3 and so on ou can write the transfer function once you have written the transfer you can also obtain the phase all right. From the transfer function you can draw the phase and while doing the experiment you also put a phase meter measure the phase, see whether the experimental phase values and the estimated phase curve they match closely or not if they do not then you again try to fit in another set of 20 db per decade and horizontal lines.

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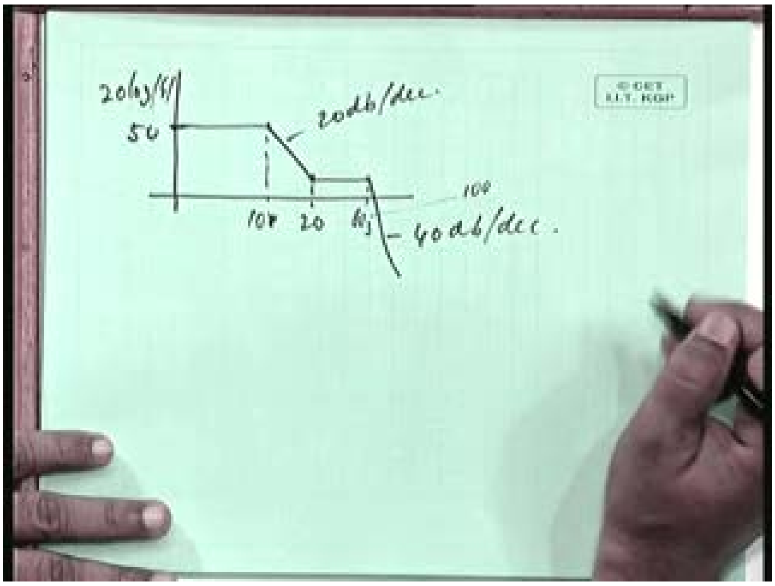

So as to get the best possible combination that means 2 experimental data set will be given to you, you have to make best possible match for the transfer function all right. This is an identification problem. So let us identify 1 or 2 functions okay I have got 50 db this is 20 log of G, this is 40 db, this is 20 db per decade, 40 db per decade, this is omega 1, 10 radiant is per second, this is omega 2 that is 20 and this one is omega 3, omega 3 that is 100 okay, what will be the transfer function, can you suggest?

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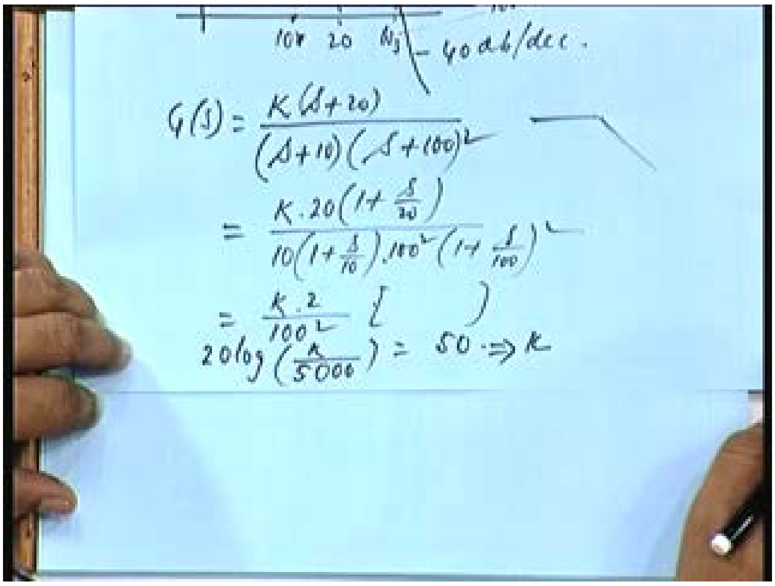

What will be the function like yes, can I write S plus 10, S plus 10 okay s plus 10 that will be this curve is falling at 20 db per decade so S plus 10 then numerator S plus 20 fine then S plus 100, S plus 100 very good, S plus 100 is what squared, squared yes S plus 100 from this point onwards it is 40 degree per decade, 40 degree per decade so squared that is all, there is a gain gain as 50, 50 is equal to 20, 20 log, 20 log equal to 50 is that so no 20 log gain equal to 50, K into 20, S plus 20, can be written as 20 into 1 plus S by 20, S by 20 it is 1 by 1 plus S by 20 which gives me 0 degree line and this okay divided by 10 into 1 plus S by 10 multiplied by 100 squared into 1 plus S by 100 squared okay. So that is equal to K by 20, so 2 by 100 squared into this functions. So it is 20 log of this which is equal to 50, so find out K and you can substitute that value of K here is that all right so 20 log of this quantity, how much is it K by 50000, 5000, 5000 is equal to 50 whatever that may be so that will be giving me the value of K, is it all right?

There is a problem 20 db per decade slope is there, this is omega 2, this is 10000, this is 2, this is 40 db per decade okay and this is 12, this is 20 log of G then you estimate the function can you estimate this function S square, no this is 10000 this is 20 db per decade slope, this is 12 db, how much is omega 2, question is what is the value of omega 2, omega 1 and then G (S) is that all right. Now this is 20 db per decade I can consist this as 6 db per octive.

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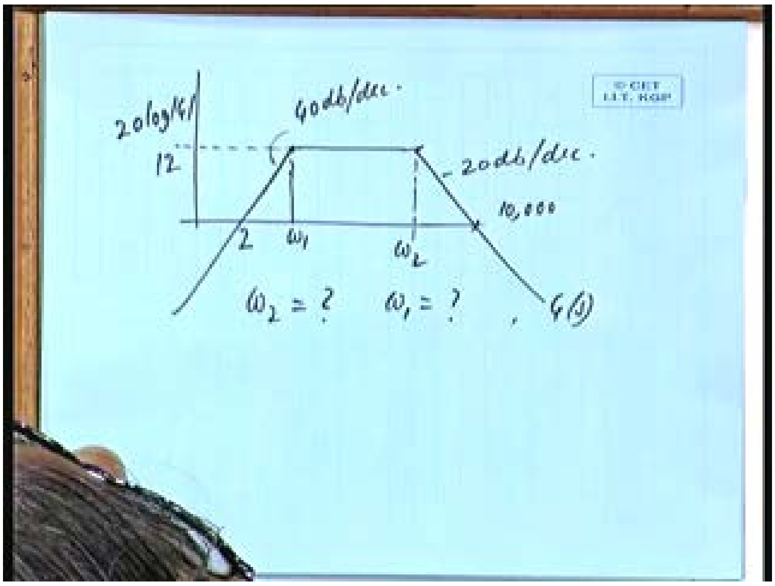

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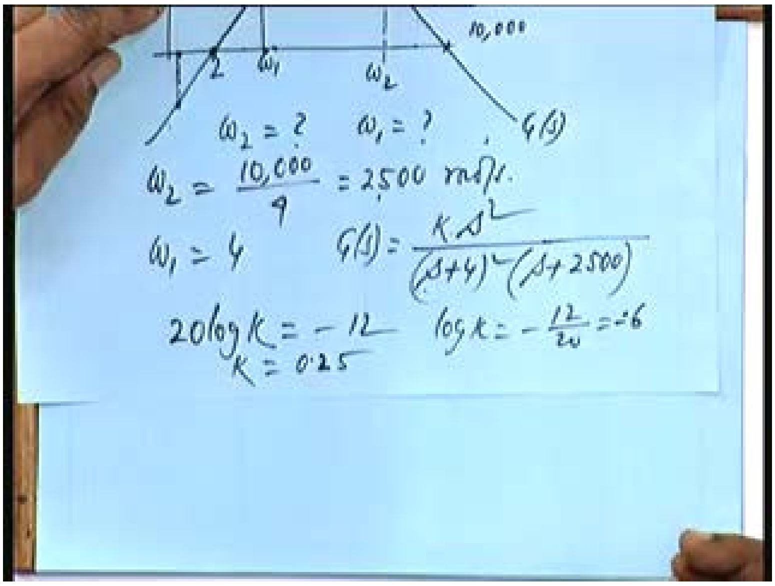

So this is 12db, so when it is raising the 12 db that means it as fallen from here to this point 0 at a rate of 6 db per octive and this fall is 12 db so how many octives are there <a_side> 2 that means what is this frequency omega 2, 10000 minus 2, 10000, 10000 divided by 2, 2 and 2, 4, 4, 4 per octive there are 2 octives had it been divide by 2 that means this raise would have been only 5 6 db so 6 plus 6,12 db okay so 12 db fall or raise if you take it from the reverse side will be at 10000 divided by 4, is it all right?

So omega 2 is 10000 divided by 4, 2500 radiant is per second, what about omega one, what about omega one, this is 12 db, this is 12 db fall and 40 db per decade means 12 db per octaves, 12 db per octaves. So this is if this is 2 how much is omega 1, 1, 4, 4 just next twice this frequency this is slope is double the 20 db per decade, 40 decade per decade means something like S squared okay. So 40 db per decade rise is as good as 12 db per octave so it has gone up by 12 db then this is one octave that means 2 multiplied by 24 omega 1 is 4 is that all right.

So how much will be G (S) therefore some K into what should I write K into S plus 4, S plus 4, no S plus 4 whole square S plus 4 whole square where is 4 coming S square S square in the numerator or the denominator, S squared denominator denominator, s square in the denominator means it will be falling down S square in the numerator, S squared in the numerator and then in the denominator S plus four S plus 4 whole squared whole squared because 40 db per decade will have to be counted. So it also has to be whole squared and then S plus S plus 2500,omega 2, 2500 2500 what only S plus 2500 yes it is 20 db per decade and then how do you evaluate K that gain taking outside that 400, 4 into 4 square 16, 2 by 20 log of the so you can evaluate at any point here okay any other method that 400, we will take common 16 that 400 we will take common 16 into that 400 we will take they are we will take common and this will be a mind you these are all asymptotes so for evaluating that you may get two three different values if you evaluate at 23 different points all right because there is an approximation.

So you have to take the values which are to be neglected you have to take that into account so whenever you are taking a point say somewhere here at omega equal to 1 at omega equal to 1 how much is this, at omega equal to 4, how much is this 12, 12 at omega equal to 2 zero 0 then half of that frequency it will be at half the frequency it will be minus 12, minus 12, no minus 10, this is plus 12 this has to be minus 12, minus 12 there of equal length. So for as the logarithmic scale is concerned this is 2 time this is also 2 times is that all right so this is minus 12 at omega equal to 1, 1 and that time this is neglected this is neglected your considering only asymptotes so at omega equal to 1 it will be 20 log of K and this will b e 0 so 20 log of K is minus 12 calculate K is that all right.

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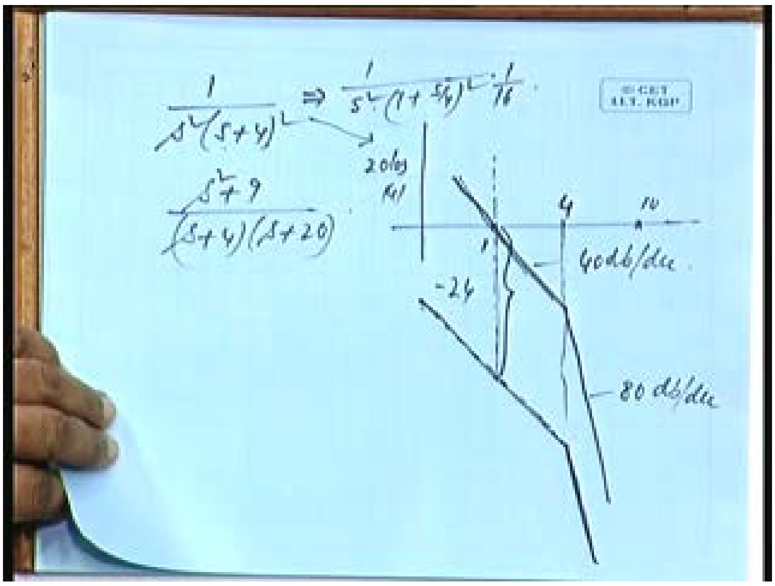

So 20 log of K is minus 12, so log of K is minus 12 by 20 that is .6. So K is how much .6 minus .6, 4 or 1 by 4 log of 4 is .6, so log of antilog of minus .6 will be 1 by 4 that is .25 is that all right, what will be the plot for S square plus S plus 4 whole square. We have got some problems okay or S squared plus 9 divided by S plus 4 into S plus 20. Let us take up these 2 problems 1 by S squared plus S plus 4 whole squared what will be the sketch like 20 log of G, so the first 1 we will plot here 1 by S squared what will be the plot like.

Suppose this is 1, this is 10 and so on at 1 S square, 1 by S_1 by S squared etcetera will be passing through one and at what slope 40 db per decade40 db per decade it is S squared 40 db per decade will be the slope here all right. Let me draw it clearly and then at omega equal to 4 how much is it again another 40 db so it will be 80 db per decade this will be the slope, this is 40 db per decade and 80 db per decade.

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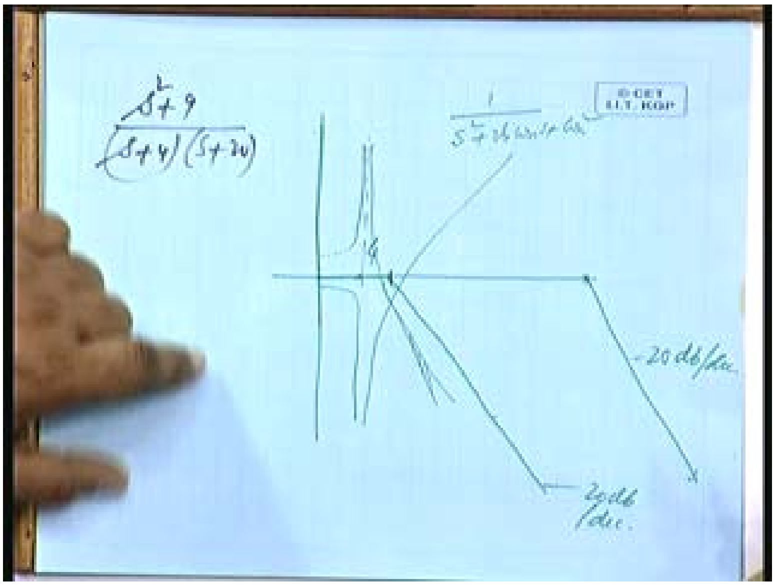

It is all right, if it is written as 1 by S squared into 4 squared taken out side 1 plus S by 4 whole squared into 1by 16 okay. So log of 1 by 16 will have to be added with this so it will go down further a similar curve log of 1 by 16 will be negative minus log of 16, how much is log of 16, 2 to the power 4, so 4 into .3, 1.2 into 20 minus 24 db. So the whole thing has to be brought down by minus 24 db, is that all right? It will be somewhat like this say at this point it is minus 24 db, this is minus 24. Let us see this function S squared plus 9 by S plus 4 into S plus 20 how does it look like S squared plus 9 by S plus 4 into S plus 20. So S plus 4, so if I take 4 here this will be one component these are the asymptotes S plus 20 if take 20 here again this will be another component both which slopes 20 db per decade, 20 db per decade and S squared plus 9, S squared plus 9 will be at 9, 9 3, 3 at 3 square root of 9 is 3.

So before this what will be like some constant it will be shooting to infinity and then form there it will be falling at 40 db per decade it will be falling at much faster. So it will be, will it be this way, will it be this way. Last time when you studied it was 1 by S squared plus zeta omega N_s a side plus omega N squared, now it is in the numerator so it will the other way and then 40 db per decade okay. See if you are having if you are having a numerator function then the peak will go down and then go up if it is in the denominator then it will go up first and then it will go down by 40 db per decade is that all right.

So now it will be some total of these 3 components this peak occurs at omega equal to 3 then this one at omega into 420 db per decade and omega into 20, 40 db per 20 db per decade. So 20 plus 20, 40 db fall and this is 40 db raise so at the end the resultant if I can sketch the resultant.

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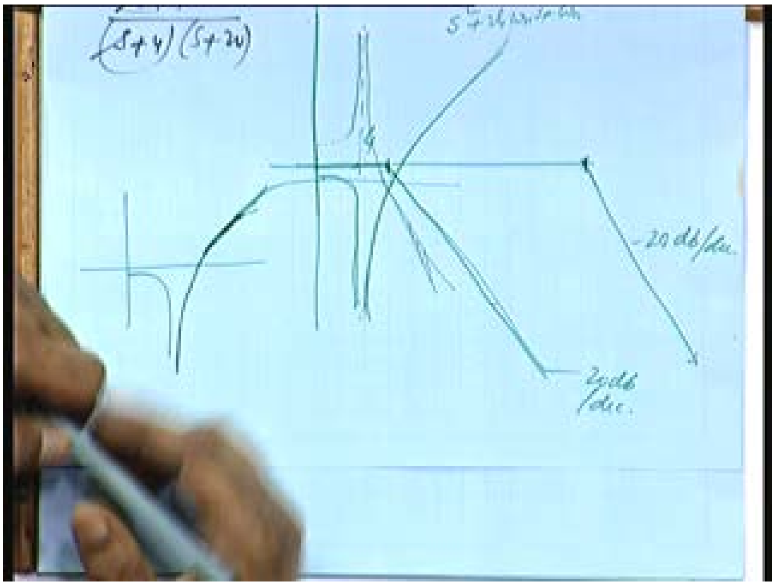

It will look like this it will be a function which will be going like this and then 20 db fall and 40 db increase so from 4 onward it will be 20 db per decade net result all right and then from 20 omega equal to 20 onward, it will become horizontal. So the net result will be somewhat like this 3 and 4 are very close so you will not able to see 40 db slope anywhere it will be gradually going up to 20 db per decade slope and then finally catching up with the horizontal line.

Another interesting function suppose you have S squared plus twice zeta 1 omega n_1 plus omega n_1 squared divide by S squared plus twice zeta 2 omega n_2 plus omega n_2 sorry 2 squared that means you are having 2 sets quadratics with complex roots suppose omega n is greater than much greater than omega n_2, omega n_1 is such greater then omega n_2. So the numerator starts appearing first is it not. So that will give me a function like this depending on the value of zeta this will be very peaky or may be flat all right. We are not interested in actual value of zeta at this moment similarly, the other one it will have this is somewhere close to omega n_1 say approximately, omega n_1 and somewhere here omega n_2, omega n_2 is far off, it will be falling like this. So I am just putting 2 different dotted lines so what will be the resultant like it will be this one going up like this and then if I add up these 2, it will be going up and then gradually this is 40 db per decade and this is 40 db per decade. So there will be cancelling.

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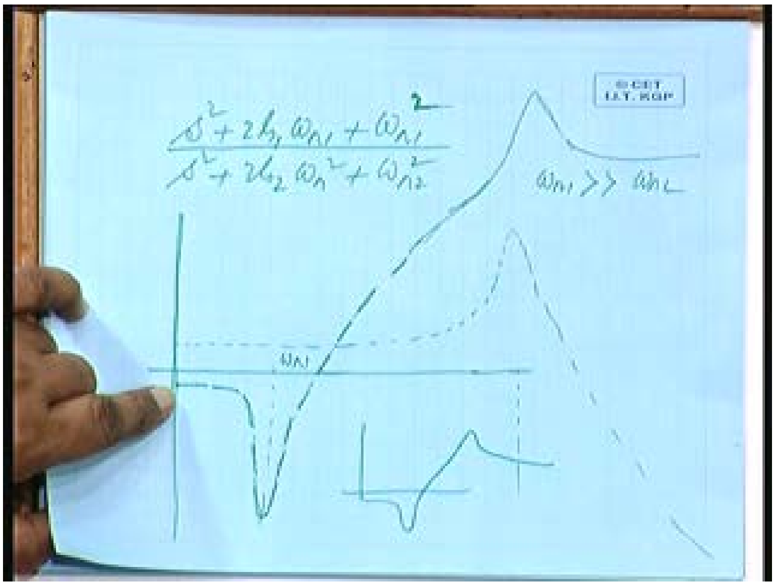

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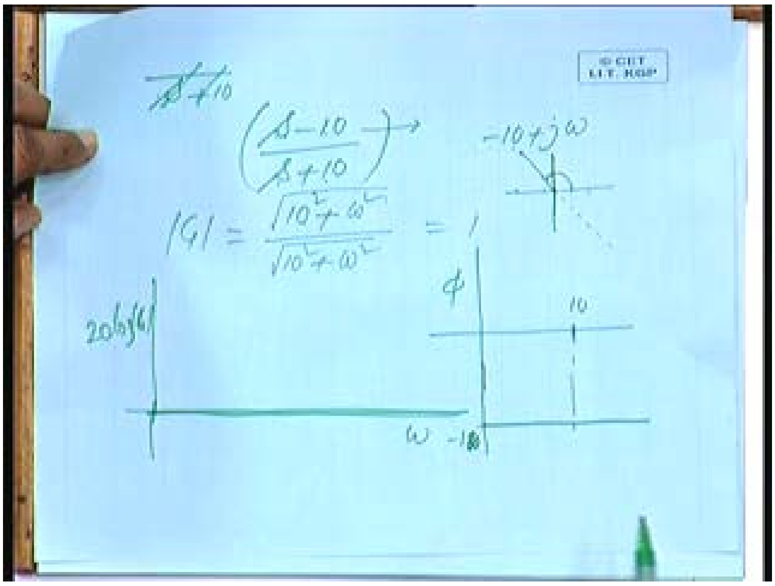

Okay if they are reasonably separated then the resultant will look like this okay. Now tell me if I have S plus sorry, S minus 10 by S plus 10 what will be the gain plot what will be the gain plot for this S plus 10 by S minus 10. So let us compute G magnitude, G magnitude is 10 plus how much is it 10 squared plus omega squared under root for the numerator divided by 10 squared plus omega square under root that is always equal to 1, what is it? all the time gain is 1, what is this called? What is this called ? Non- minimum, non-minimum phase it is an all pass filter, all the frequencies are allowed with a same gain all right.

So for an all pass filter what is the bode plot 0 that means if I draw horizontal line it can be S minus 10 by S plus 10, it can be S plus 12, S minus 12 by S plus 12 anything can you identify where can I identify phase, phase that this function is phase, phase. So what will be the phase plot like, so 20 log of G and omega it will be 0 db all the time and for phase, yes S minus 10 what is the angle? Let us see what will be the angle it is minus 10 plus j omega minus 10 plus j omega is what, what would be the value which quadrant is it, if you take a calculator if you take the tan inverse b by a you will be always getting the prime value as say omega equal to 10 minus 45 degrees but actual value is minus 135 degrees because it is minus 10 plus j 10.

So it will be this angle but if you take plain ratio, you will be misled you will be taking this angle, is to not? 10 inverse of minus 1 is minus 45 degrees but it is also 135 degrees. So it is this 135 degrees that has to be taken because it is minus plus 10 j omega and 10 plus j 10 that is 45 degrees, so 45 degrees in the denominator and numerator minus 135 degrees will give me minus 180, so at omega equal to 10 it will be minus 180. So like that you can make a plot okay. So if you take omega equal to 10 say it will be minus 180 omega tending to 0 say omega equal to 1, so it will be 1 minus 10 plus j_1 how much is it? minus 10 plus j_1 almost minus 180 minus 6 degrees, 174 degrees.

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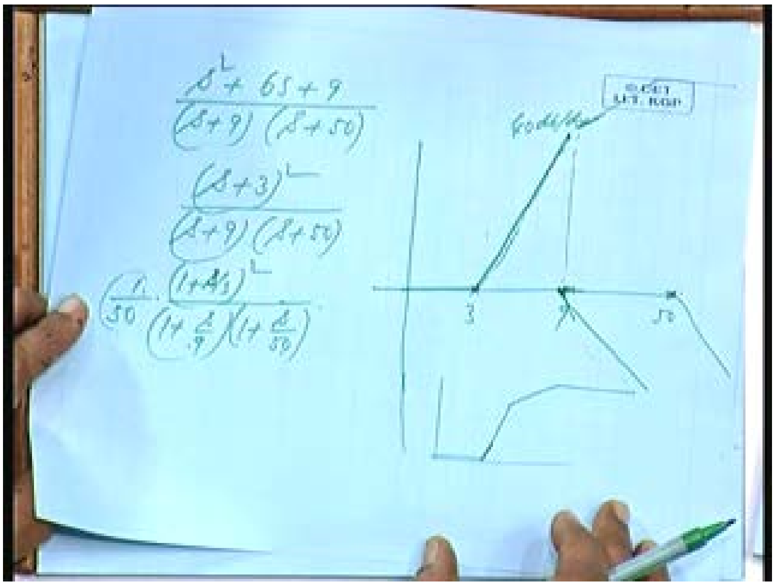

So that will be minus 174 degrees and this one will be, this one will be plus 6, 10 plus j_1 that be plus 60. So that will become minus 180 degree so it will be always minus 180 always minus 180, I will request all of I mean all of you to think over it and we will have a discussion on this phase plot okay and also what will be nature of variation of the phase for a quadratic like this how will the phase change. So when you search examples can be taken S squared plus 6, S plus 9 divided by S plus 9 into S plus 50 what will be the bode plot for this what will be the gain plot like numerator is I can see b square is equal to 4 Sc it is S plus 3 whole squared divided by S plus 9 into S plus 50. So what be the gain plot like I can always take the constants out, so 9 by 9 and 1 by 50 so it will be 1 by 50 into 1 plus S by 3 whole squared divided by 1 plus S by 9 into 1 plus S by 50 okay.

So 20 log of 1 by 50 can be taken separately and let us make sketches for these 3 components so 1 by 1 plus S by 3 whole squared. So let us take this as 3, this as 9, this as 50 okay. So 1 plus S by 3 whole squared will be plus 40 db per decade very good this slope and at 9 it will be 20, 20 db per decade minus minus 20 db per decade and then again at fort 50 minus 20 db per decade all right.

So what is the resultant of these 3 resultant will be if I am permitted to draw by a dotted line like this it will be actually this line and then at 9, okay at 9 this will be minus 20, minus 20 so it will be plus 20 plus 20 db per decade it will go like this and at 50 it will become horizontal. So the net function will look like 40 db, 20 db and then horizontal one and then depending on the constant 20 log of 1 by 50 it will be shifted up or down depending on its magnitude here, it is 1by 50. So it will be negative so it be brought down by this factor 20 log of 50 is that all right, so we will stop here for today. We will take some examples in the next class. Thank you very much.

Preview of next Lecture

Lecture - 12

Bode Plot (contd)-Poles & Zeros

Okay good morning friends, we will be continuing our discussion on bode plot before you start I would like to just bring to your notice some small slip that I have made yesterday, I was mentioning about 2 frequencies omega n_1 and omega n_2 remember somebody pointed out later on, is that any way it was like this.

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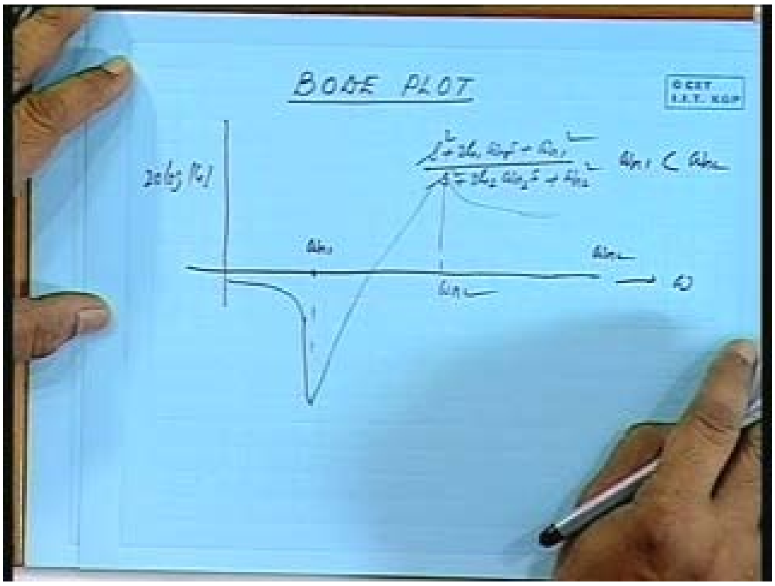

Omega n_1 there was a frequency omega n_2, so what I meant was omega n_1 is less then omega n_2 and it was in the numerator S squared plus twice zeta 1 omega n_1 S plus omega n_1 squared divided by S squared plus twice zeta 2, omega n_2 S plus omega n_2 squared. So the curve was somewhat like this okay this was omega n_2, this was omega n_1 okay. In the phase plot this was 20 log of G, in the phase plot we discussed for a function like 1 by S plus omega c approximately 110th of omega C, approximately 110th of omega c the angle was about 6 degrees minus 6 degrees and at 10 times this it is 84 degrees all right. So it is close to 0 degree and this is close to 90 degrees. So almost in 2 decades you are having a fall of 90 degree approximately. So if one draws a line 45 degrees per decade in many books they write 45 degrees per decade straight line and this will be slightly over this and slightly below this okay, it is one decade around omega c okay.

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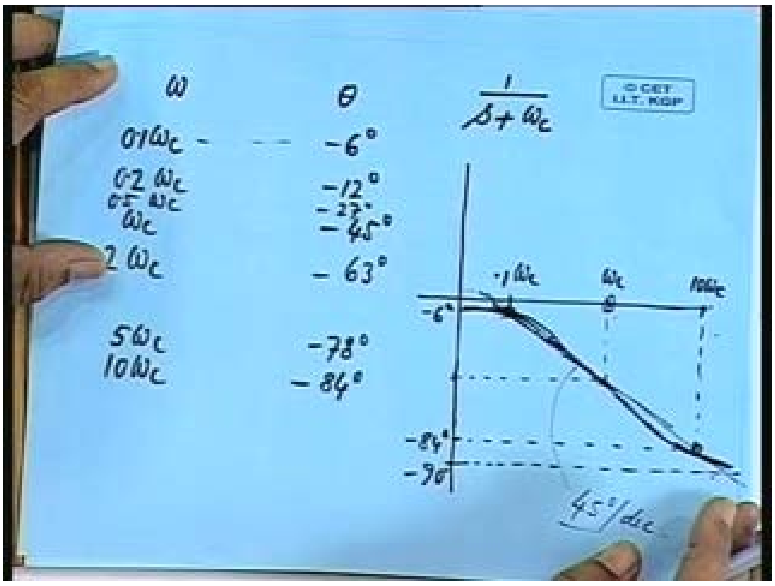

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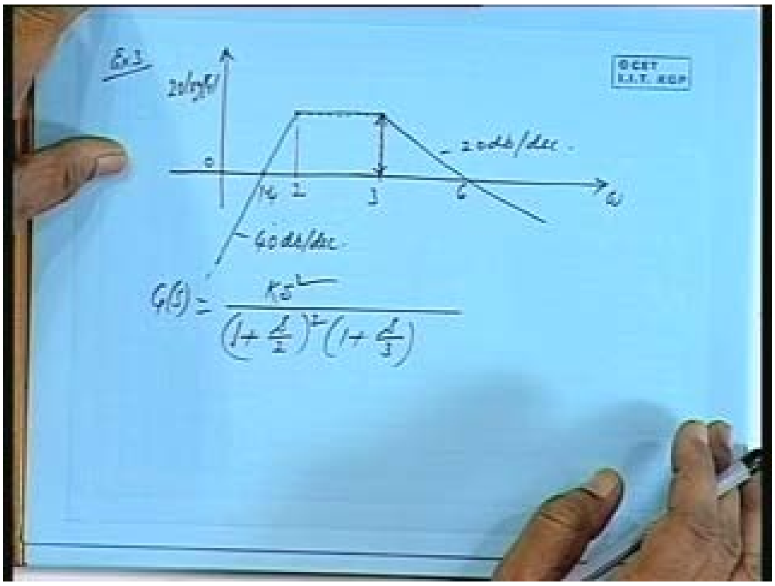

So this is again another approximate sketch in the phase G (S) what do the frequencies they will be some gain K, there is a line that is starting with a slope of 40 db per decade. So what will be the nature of the function will it be K by S or KS or K by S squared, as you said K by

S squared, KS squared, KS squared it is going up so KS squared thank you very much and then it was going up like this and then it takes a bend at 2 is equal to j2 then 1 plus S by 2 whole squared there was a raise of 40 db per decade and you are bringing it to 0 that that means it will be whole squared, is it not? Then 1 plus S by 3 here you are having a further fall of 20 db per decade then anything else, nothing else it continues.

Okay then how do identify K see you have to identify this height, how do you do that actually there are two information is which are redundant, I could have asked for this frequency, I could have asked for this frequency or I could have asked for this frequency. The question that was given actually given one excess information the same information so you calculate either from this side or from this side.

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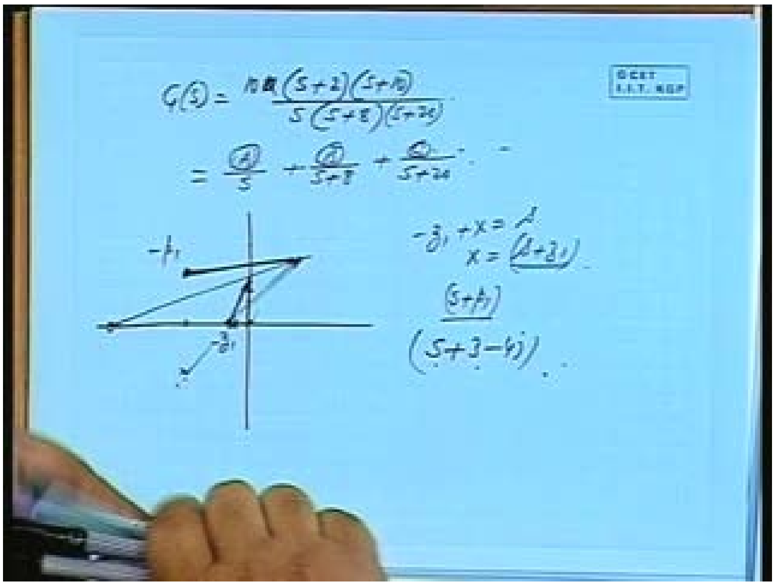

No no there is no pole I am taking any value of S I am varying along this imagery axis all right then what does it show me? If I vary S along this what does it show. Now for that method anywhere if I take what will be this affect alike this is S, so minus z plus this unknown vector X minus z_1 plus unknown vector x is equal to S. So how much is that unknown vector S plus z, S plus z all right. So if I take any S it need not be on this line any S then a distance this vector connecting minus z_1 and this will give me S plus z_1 vector all right.

Similarly, for the pole P_1 if I draw this what will be this representing this P_1 is basically minus P_1 is it not, suppose it is at minus P_1, P_1 is a complex quantity all right then it will be representing S plus P_1 vector if S plus P_1 is a factor then S equal to minus P_1 is a root all right. So if this is minus 3 plus 4 j then the factor corresponding this vector will be giving me S plus 3 minus 4 j, if minus 3 plus 4 j is this point then this vector is S plus 3 minus 4 j okay. Similarly, for this 1 this 1 and so on okay.

So in the next class we shall discuss about how to compute these A, B, C these constants with the help of these phases or these vectors, these vectors okay. So we know the location of poles and 0s from the factors and then we will find out how to compute these residues geometrically okay. Thank you very much.