Industrial Instrumentation
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Lecture - 3
Dynamic characteristics (Continued)

This is lesson 3 of industrial instrumentation and we will continue with the dynamic characteristics of, of a system especially second order systems. Already we have discussed about the first order system, now we will consider the dynamic characteristics of a second order system.

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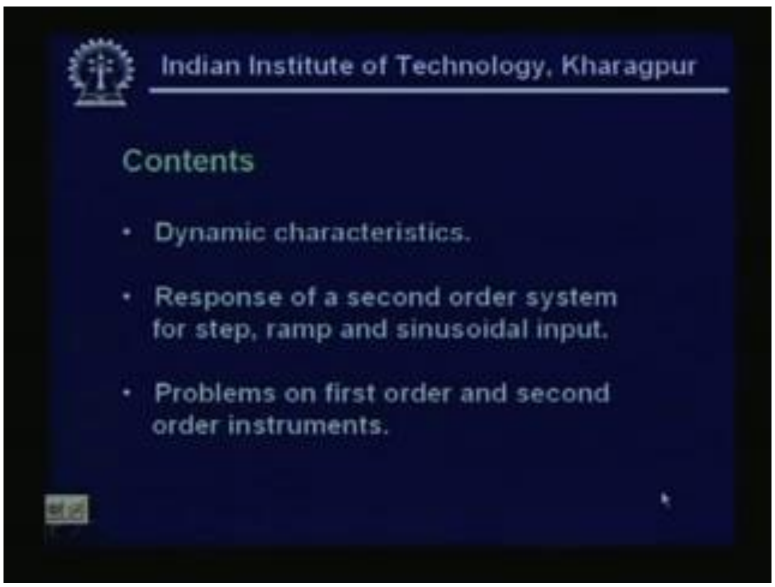

Now, contents of this lessons are - the dynamic characteristics, the response of a second order system for step, ramp and sinusoidal input, where we did it for the first order instruments we will do for the second order instrument. Also we will solve some problems on the first order and second order instruments.

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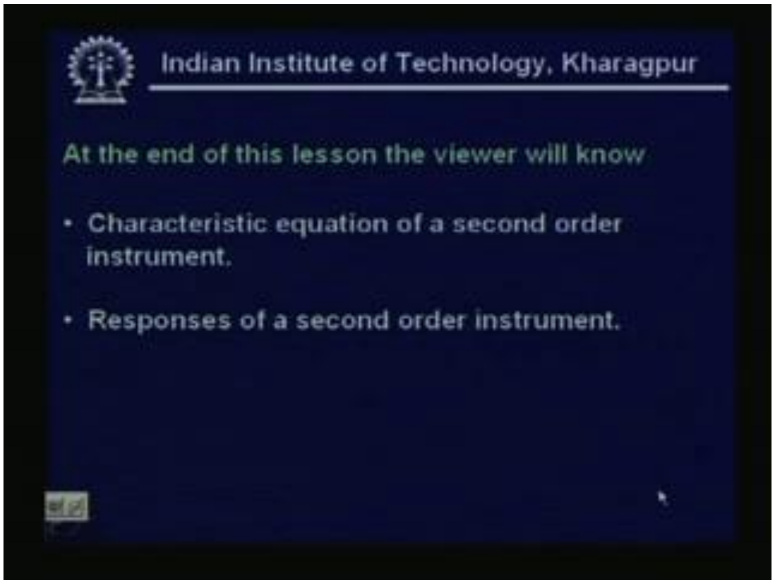

At the end of this lesson the viewer will know the characteristic equation of a second order instrument, responses of a second order instrument.

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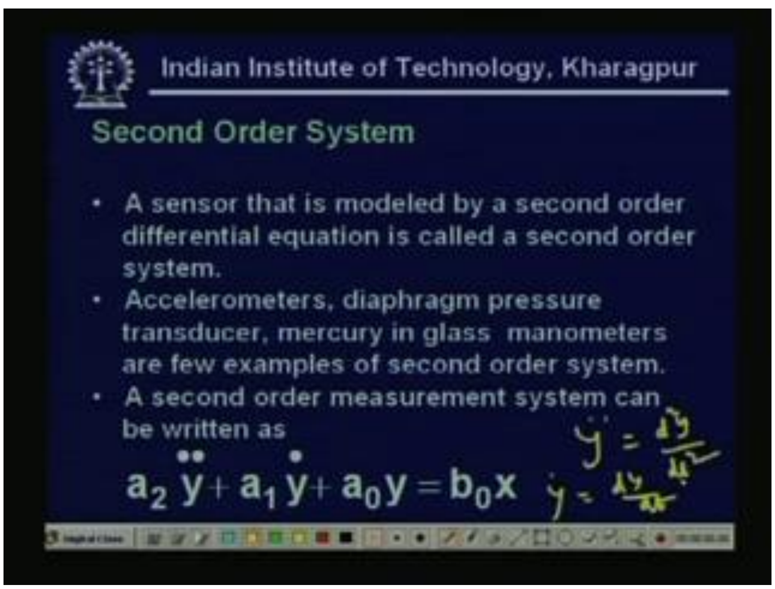

Now, second order system - a sensor that is modeled by a second order differential equation is called a second order system, quite obviously. How will I know whether it is a first order or second order? If I, if I, if I could look at the response and if the, if the characteristics, if the output is coming which is supposed to come, because for some calibrated instruments I can tell whether how the response should look like. I f it, if it cannot be modeled by the first order instruments we have to model by the second order instruments or higher order instruments.

Now, accelerometers, diaphragm pressure transducers, all these things will be discussed later; mercury in glass manometers are few examples of second order system. Now, second order measurement system can be written as a 2 y double dot plus a 1 y dot plus a naught y equal to b naught x.

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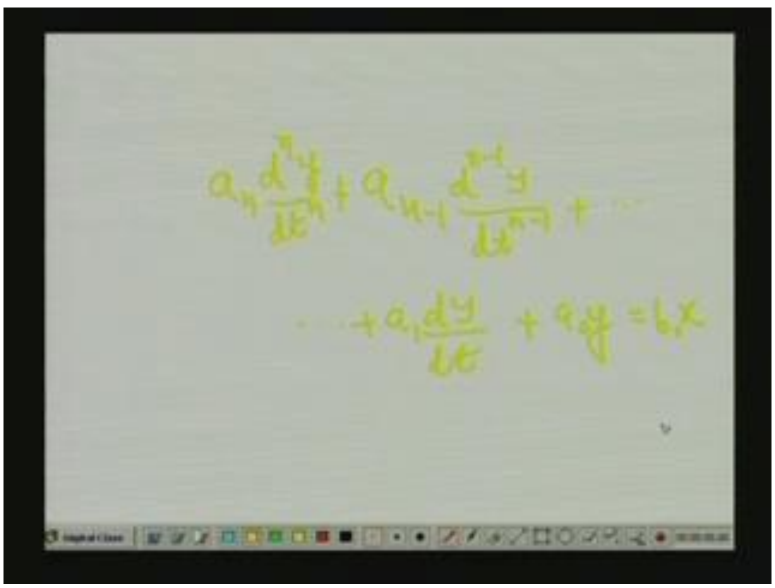

Now, if I go back to our differential equations or the nth order differential equations which looks like this, the …. differential equations looks like a n d n y by dt n plus a n minus 1 d n minus 1 y by dt n minus 1 plus so on … plus a 1 dy by dt plus a naught y equal to b naught x.

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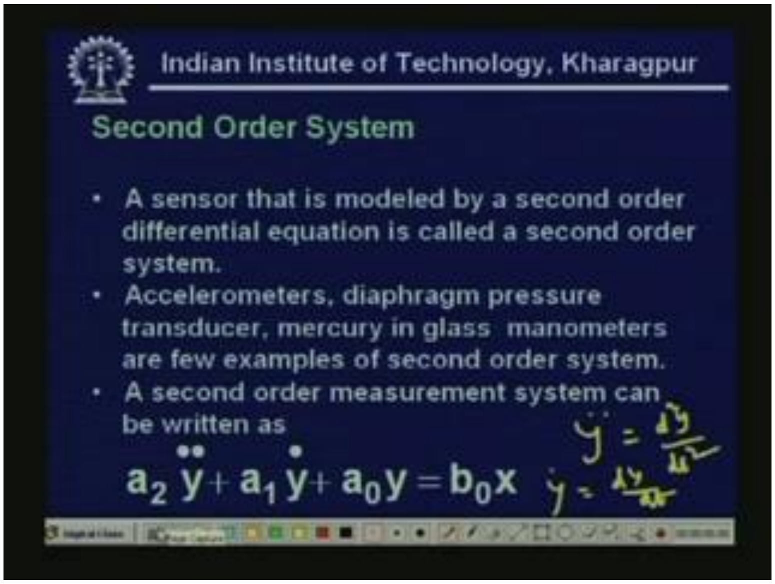

Now, you can see here that if I put n equal to 2, so we will get three terms which looks like a 2 y double dot a 1 y dot plus a naught y equal to b naught x. Quite obviously as we know, all of us know that y double dot is nothing but d square y by dt square. Similarly, y dot is equal to dy by dt.

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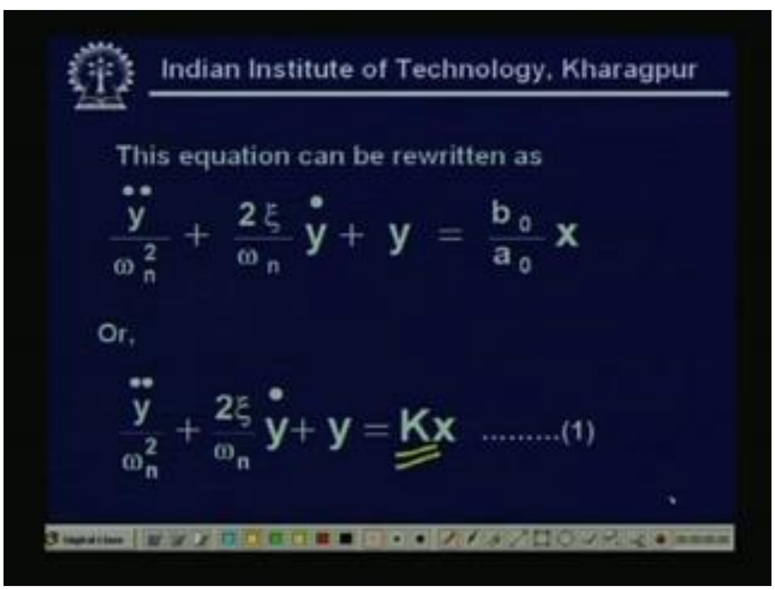

Now, this equation can be written as or rewritten as y double dot upon omega n square plus 2 xi upon omega n y dot plus y equal to b naught upon a naught into x or I can write y double dot upon omega n square 2 xi upon omega n y dot plus y equal Kx. K, we know that in the case of first order systems also K is basically the static sensitivity of the system, which is equal to b naught by a naught.

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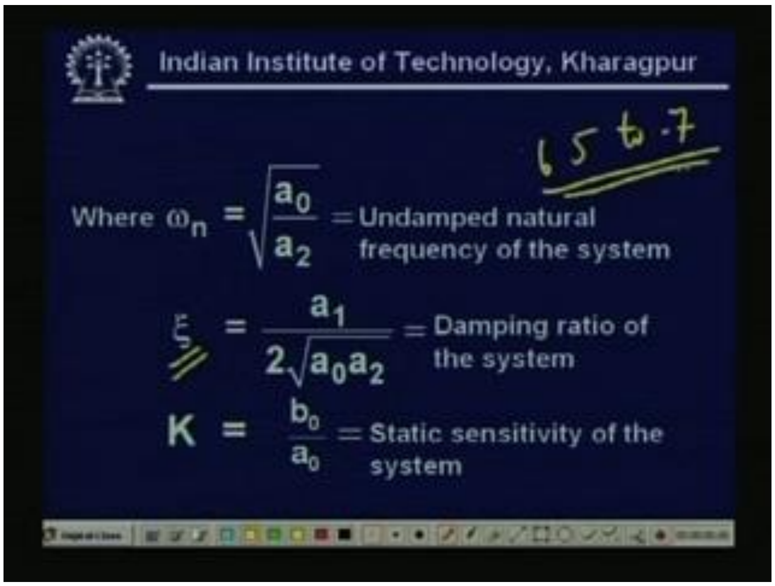

Now, we have introduced some new terms, let us look at all these terms. First of all omega n; omega n is equal to under root a naught upon a 2 which is the undamped natural frequency of the system. We have xi equal to a 1 upon 2 under square root a naught into a 2 which is damping ratio of the system and K is the static sensitivity of the system. Now, we will see that this damping ratio of the system that means xi will play important role for the second order system. Depending on the value of xi, we will find that how much harm will be there in the systems. Typically, you will see in future that in the second order system, in a system, that xi usually we put the value of .65 to .7, it lies between, because we cannot make xi more than 1. Then, in that case, it will be over damped systems and we cannot make xi equal to 1 also, it is critically damped systems; because, if it is critically damped due to aging, because ultimately you know that all omega n, xi, K depends on some physical systems that means some resistance value, some capacitance value, some spring constant.

Now, due to aging problem, this will change, obviously. So, I do not want my system should be over damped, no, I mean we never want until and unless we are forced to. I will give some examples. In future we will find that sometimes even though we are saying that the system is under damped it won't be under damped due to the change of the value of xi. So, always we will try to make xi at least .65 to .7, so that I will get at least 1 over should then we will come back to the rest position.

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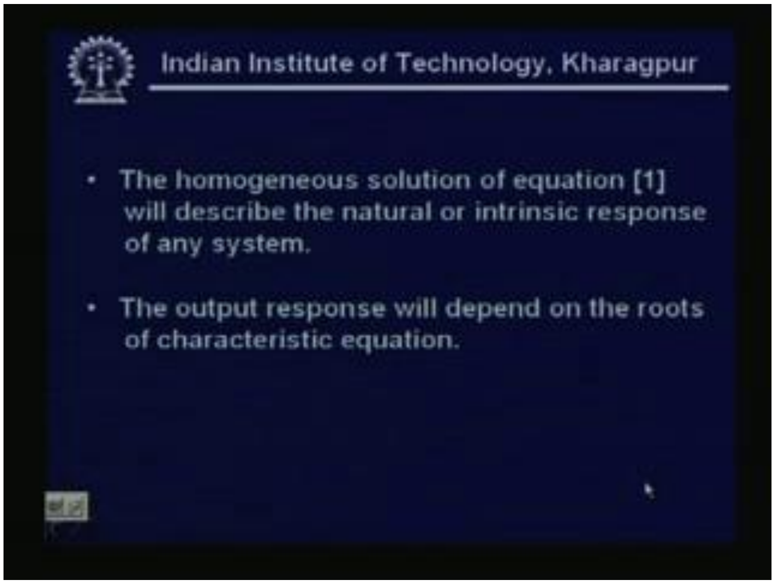

The homogeneous solution of the equation 1 which you have written will describe the natural and intrinsic response of any system. The output response will depend on the roots of the characteristic equation, because the response actually will give, give you the roots. How the, where the root likes, I mean lies for the sigma j omega plane that will tell you the output response.

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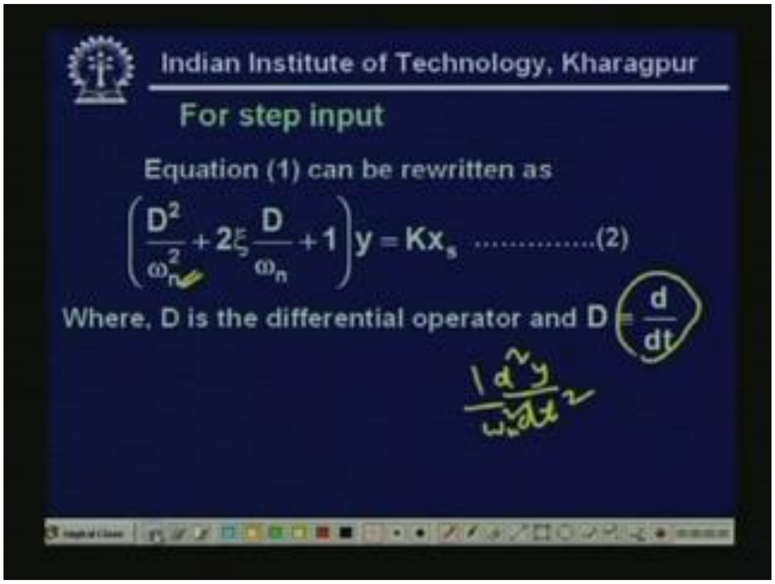

Now for step input, equation 1 can be written as D square upon omega n square plus 2 xi D by omega n plus 1 whole multiplied by y equal to K x s. This is equation number 2. Now, we have introduced a new term which is called differential operator which is equal to d by dt of this one. That means in this case it will be first term for, suppose for the first time it will be d square y by dt square by omega n square. This will be our first term.

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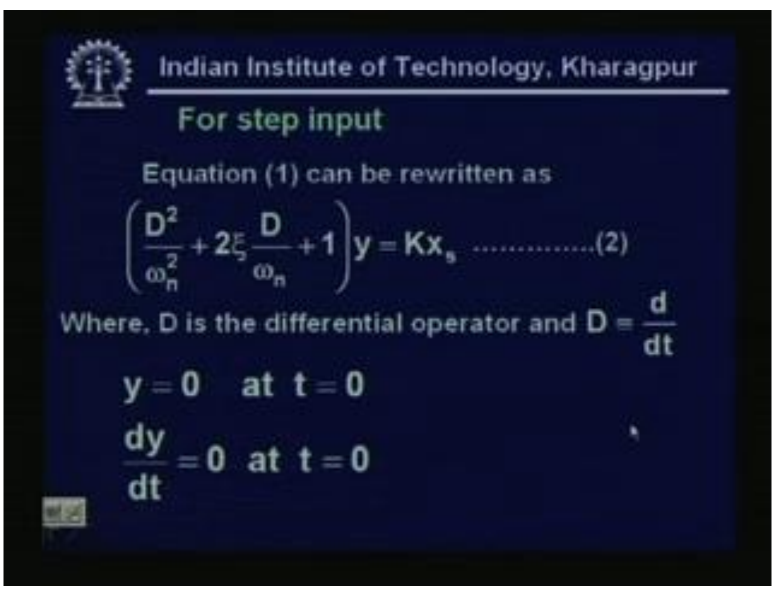

Now, we should have some initial condition, being as you know that for solving any differential equations we need some initial condition to, to solve the equations. The initial conditions are y equal to zero at t equal to zero, dy by dt equal to zero at t equal to zero. So, with this we can solve this differential equation.

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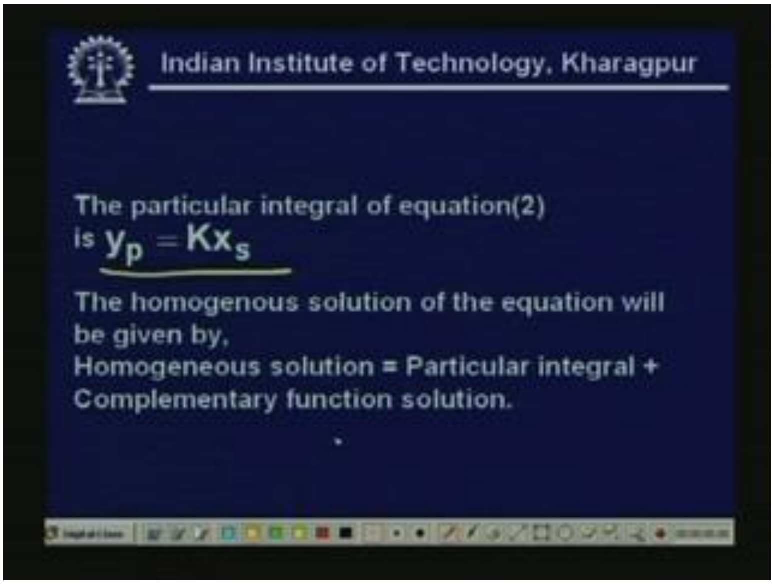

Now, the particular integral of the equation 2 is equal to Y p equal to K x s, but we always, I mean look for the transit response also. So, only a particular integral will not give the complete picture of the response of a system. So, the homogenous solution of this equation will be given by particular integral plus complementary function solution. So, particular integral already we have given and this is the particular integral. So, we have to find the homogenous solution. We need to find a complementary function to give the complete solutions of the homogenous solution for the system.

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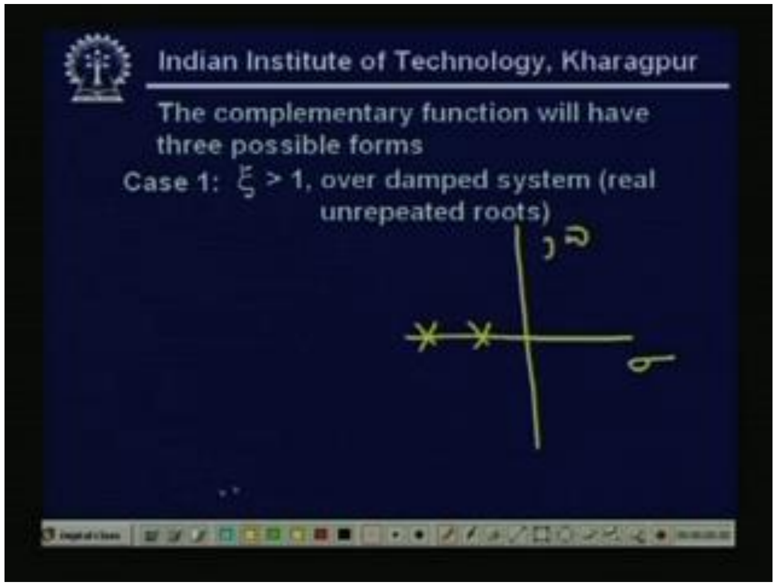

The complementary function, we will have three possible forms. Case 1, when xi greater than 1 our over damped system real unrepeated roots. If you look at the sigma j omega plane, it will look like this. I have sigma j omega plane. So, roots will be real unrepeated roots, fine.

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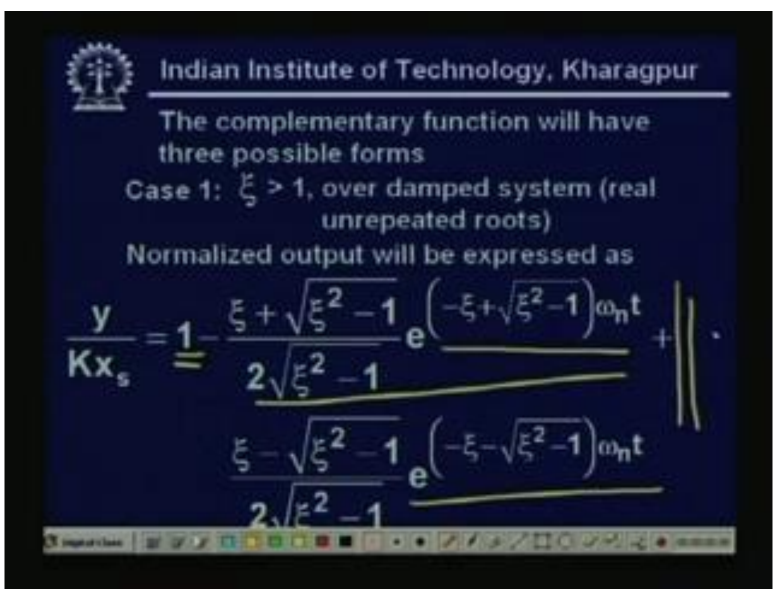

A normalized output will be expressed as y upon K x s equal to 1 minus xi plus square root of xi square minus 1 upon 2 square root of xi square minus 1 multiplied by e to the power minus xi plus root over xi square minus 1 into omega n t plus xi minus under the square root xi square minus 1 upon 2 xi square, square root of xi square minus 1 e to the power minus xi minus square root of xi square minus 1 into omega n t. You can see here this is our, this is our particular integral and this is, that this complete this one, along with this one will give you the complementary function.

Now, if you look at very carefully, you will find this will depend on time. So, at, after sometime when this because of this, because of this term negative so this will die out. Only this Y equal to K x s will remain. But, we should be interested in the transient response of the system also. That means because in all the cases you will find what will be the settling time of the systems, if I have to find the dynamic error of the system. So, this, this, this complementary function is important, because in many of the cases we will find that I cannot allow the system to get the complete, I mean steady state response before it reaches to steady state response. Suppose 5% or 1% of the value, of the final value I have to take some actions. But in some system it is very sluggish. But, I have, if it is allowed that within 5% of the final steady state value that reaches it will take some actions, because the, in instrumentation you will know all these measurements and whether it is you are measuring the temperature, pressure flow, ultimately you have to utilize this to control some final control element which mostly are some valves or some heaters, which will heat the system or some boiler temperature, this, that. So, this is very important.

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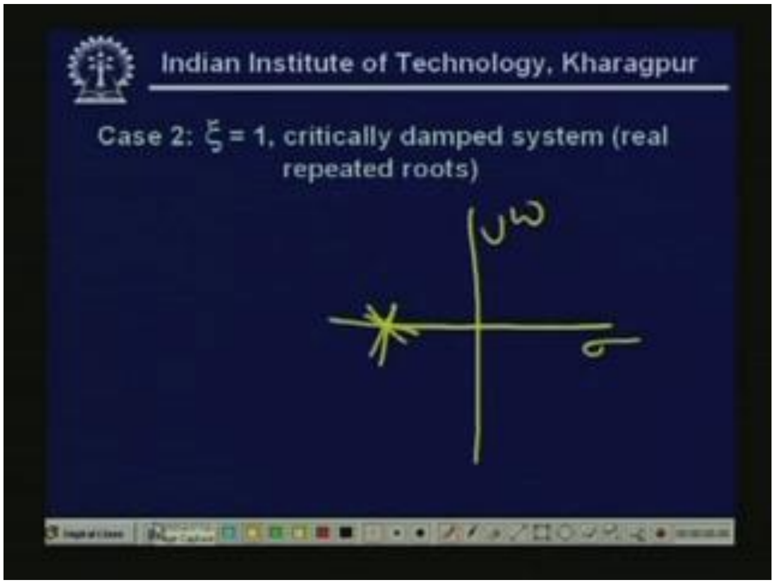

Now, in case 2 when xi is 1, it is a critically damped system, the real repeated roots, how does it look? It looks like that sigma j omega plane, so real repeated roots, one roots over the other. So, it is critically damped system.

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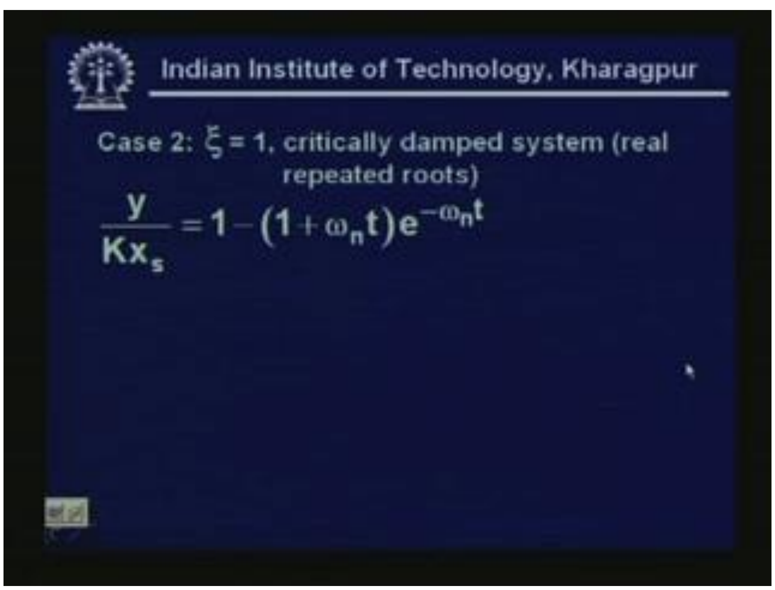

So, in that case, my normalized output will be y upon K x s equal to 1 minus 1 plus omega n t multiplied by e to the power minus omega n t.

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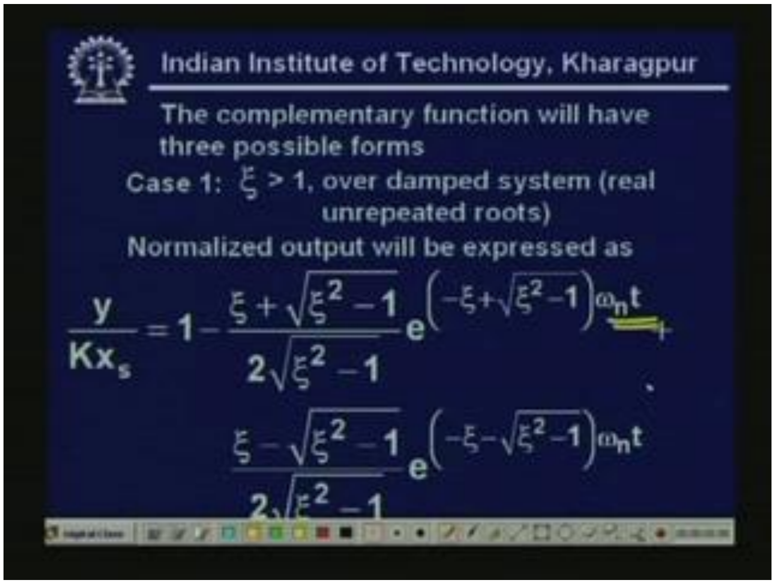

Here also you will find that, in the last cases we have, in the case of, suppose in the case if I go back to our original previous slide you will find some sort of oscillation will come in the case of under damped systems, because of the, you see the omega n t term is there. I am sorry; so, omega n t term is there, so some oscillation will come.

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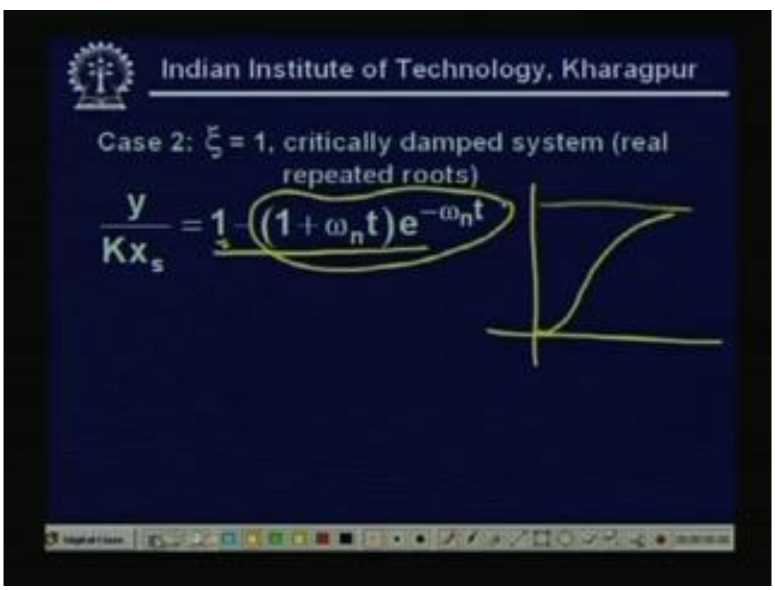

Whereas, in the case of, in the case of critically damped system you will find if I look at the response here, so I have 1 minus this one. So, ultimately the response should look like, so it will go like this and it will go like this one. So, there is no oscillation, but it will take some time before it comes to the steady state conditions or it is 1, because ultimately since it is minus omega n t, you will find this term will die out, only this term will remain.

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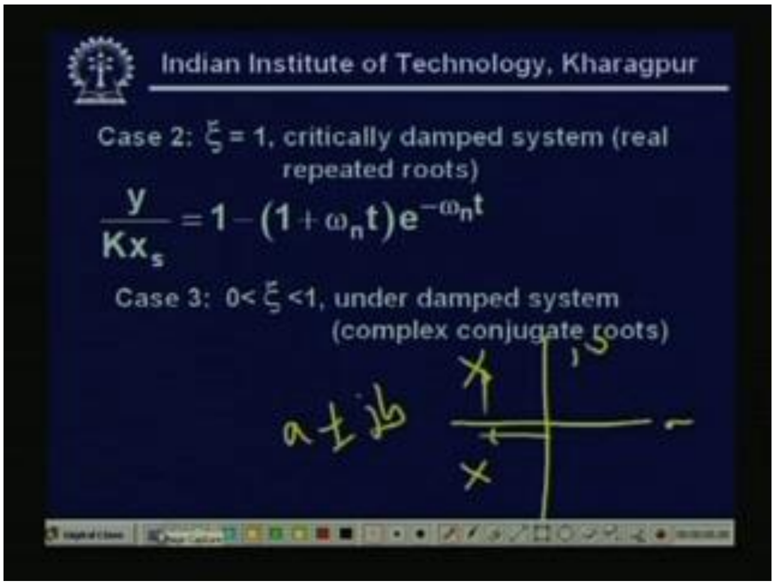

Now case 3, when xi is greater than zero and less than 1, we call it under damped systems when the roots are complex conjugate roots. What does it mean? It means that in the sigma j omega plane, so this will be our complex conjugate roots. Please note that if it is complex, it must be conjugate, otherwise I cannot realize it by any system, any physical systems. So, if the roots are complex, it must be conjugate. That means it will be, suppose it can be a plus minus j b. This is a real part and this is the imaginary part.

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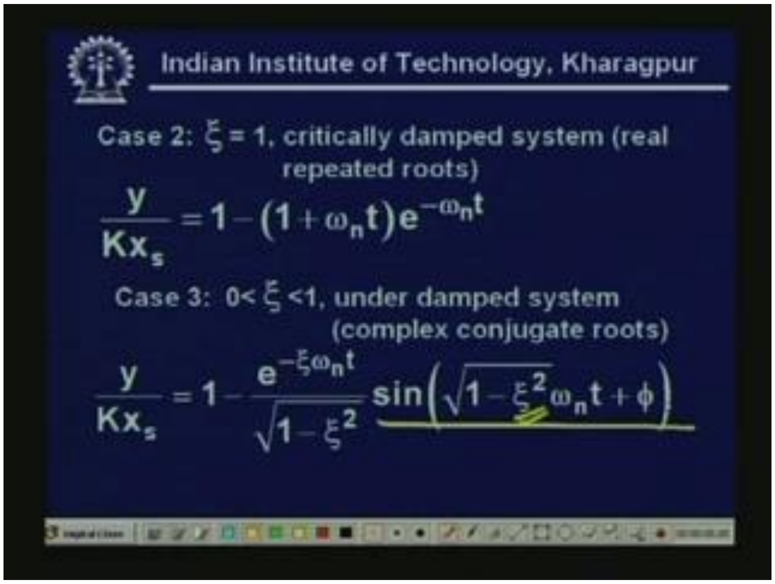

Now, the output in this case y upon K x s equal to 1 minus e to the power xi omega n minus xi omega n t 1 minus xi square. Here you can see, I am sorry, that in the case of second order system, if it is over damped or critically, then there is no question of oscillation. Only in the case of under damped system because of this term you will find the oscillation will come. There will be simple oscillation, after that it will die out.

How much will be the oscillations? How, how many, I mean humps you will get that depends on the value of xi. If the xi is .1 or .01, we will get large hump, otherwise you will get 1. If the xi is .7, .6 to .7, so it will be, it will be always just one hump and …. will come.

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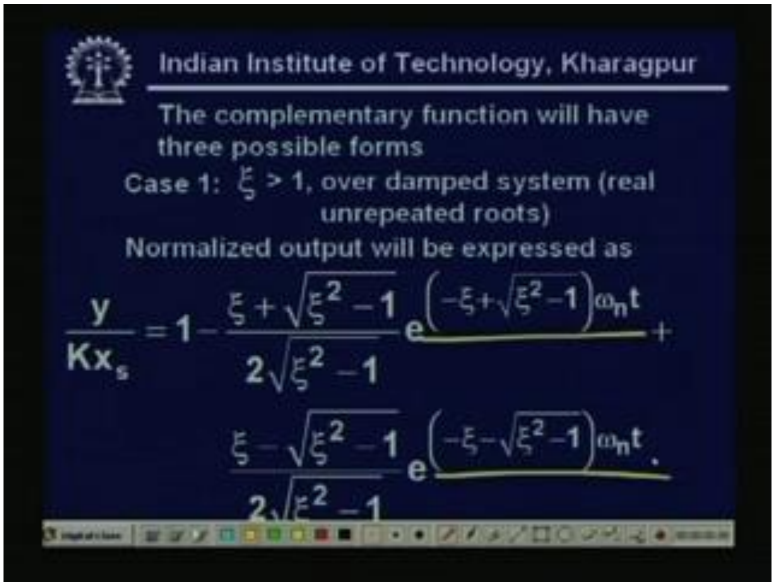

So, if I go back, so it will, I am sorry, so if I go back and have a look in the case of over damped systems, yes, you see, here there is no sine term. So, it is over damped system, no sin term, so there is no hump; only it will take large time to settle down.

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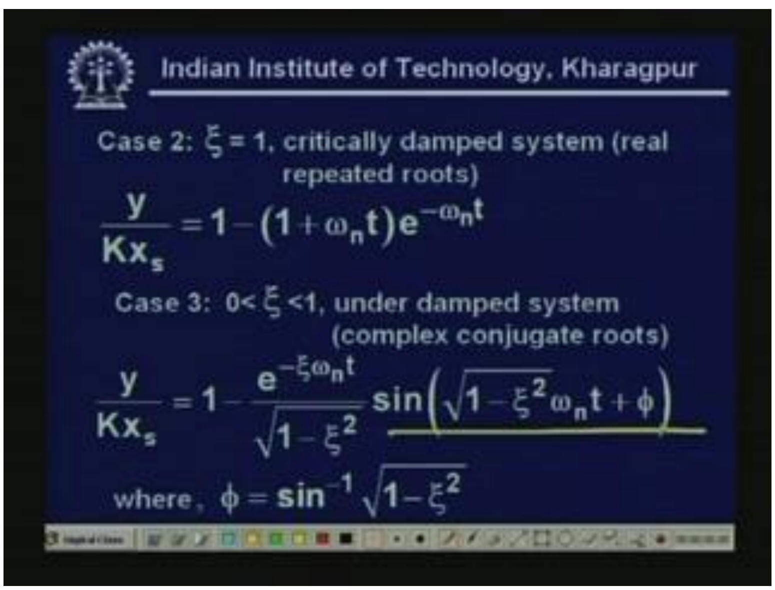

But in the case of, similarly in the case of critically damped system also there is no question of oscillations, there is no sine term, sinusoidal term. But, in the case of, in the case of under damped systems because of this sinusoidal term there will be, I am sorry, there will be oscillation because of this sine term, there will be oscillation. Also, there is a phase shift and the phase shift will be given by sin inverse, phi equal to sin inverse under the root 1 minus xi square. So, phase shift will be there in the case of second order system, when it is in under damped, when working in under damped conditions.

Now, mostly we will try to bias our system as second order system, as an under damped system.

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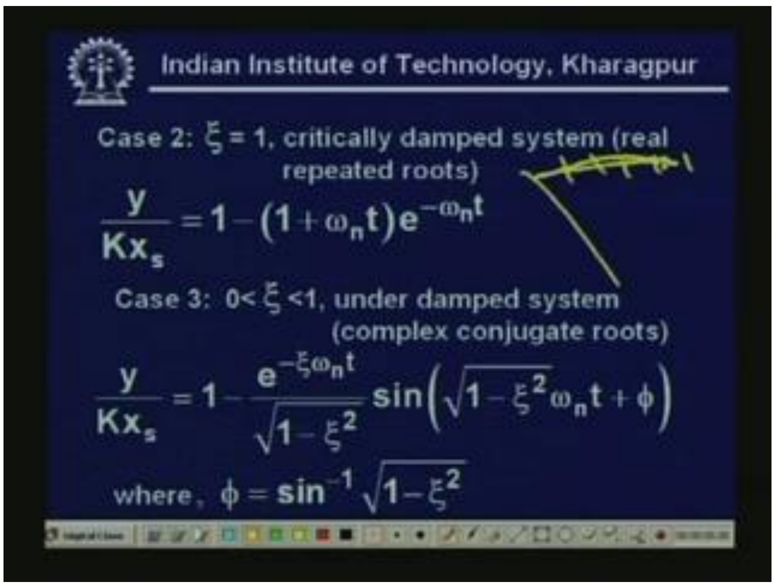

If you look at very carefully in the analog meters, especially the voltmeters and all those things we will find, if you look at very carefully to the needle, we will find that when the needle I have the voltmeters and it is coming. See, the needle will be, if you apply some voltage your needle will go down here. They will make a slight …., then will come back to the original position. That means that is where I will take the measurements?

If you want to make exactly whenever it is crossing that that is you will be wrong, because that is overshoot. So, when it will come back to the original position that is the position where you take the reading. That is the actual reading, because the overshoot we have done purposefully due to spring and all these things that it is under damped systems and the value of the damping coefficient is .6 to .7.

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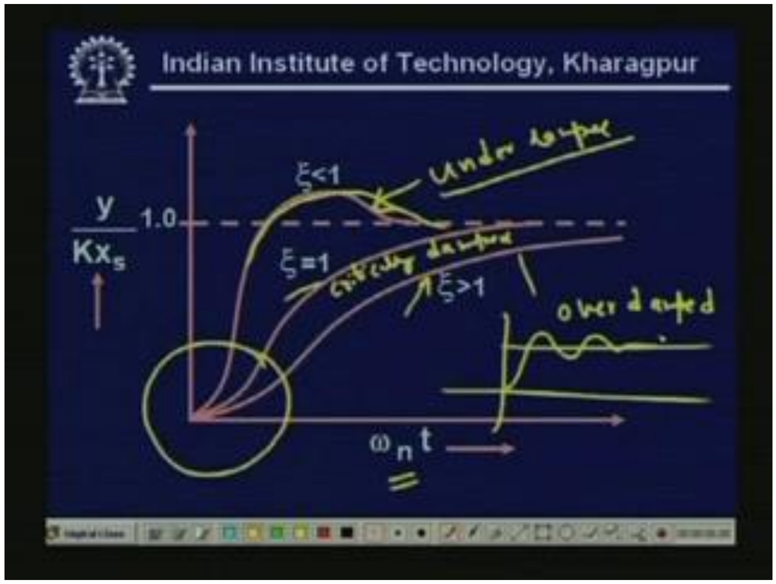

Now, if I look at the response of the system, you see, let it come, there is animation, okay it has come properly. Now, see this is the, I made xi less than 1 and this is xi equal to 1 and this is over damped system. So, this is our over damped system, this is critically damped system and this is our under damped system.

Now, how much will be the overshoot that will depend and number of overshoots also depends on the value of xi and we have plotted the normalized output. Obviously, if it is normalized it will, maximum value will be always 1. So, there is no question about this one. Now, interesting thing if you look at the origin of these systems, here we have normalized output omega n t. It does not matter; we can plot t also, because omega n is a constant. So, I mean, just it is the scale factor we have plotted and here we have plotted also unit less, because we want to plot here unit less that is the reason we have plotted like this and this is also unit less.

Now, look at the origin of the systems. If you look at the origin you see there is a s curve, distinct s curve in the system and this will tell you that even if you do not know whether the system is first order or second order, by looking at the origin I can tell whether it is a first order system or second order system. Well, if the system is under damped there is no question; there will be some overshoot, it will come back. So, because if it is a under damped system I will get overshoot and it will come back to original position. It may happen like this also, so I have a normal …. line, it will go like this.

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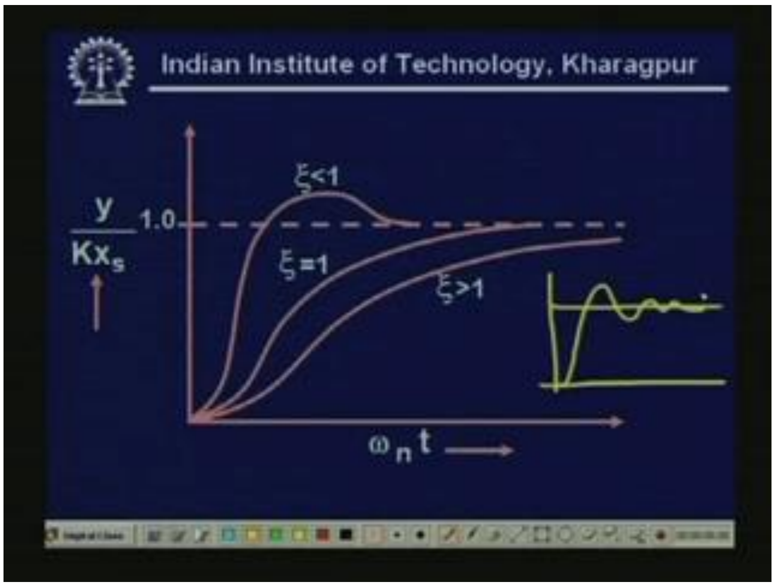

That means suppose I have a second order system, there can be several overshoots, so it will come back, right. So, if it is a under damped system, always I can tell whether it is a first order systems or second orders, because in the case of first order system there will be no resonance, so there will be no overshoot. But in the case of second order system there will be always overshoot.

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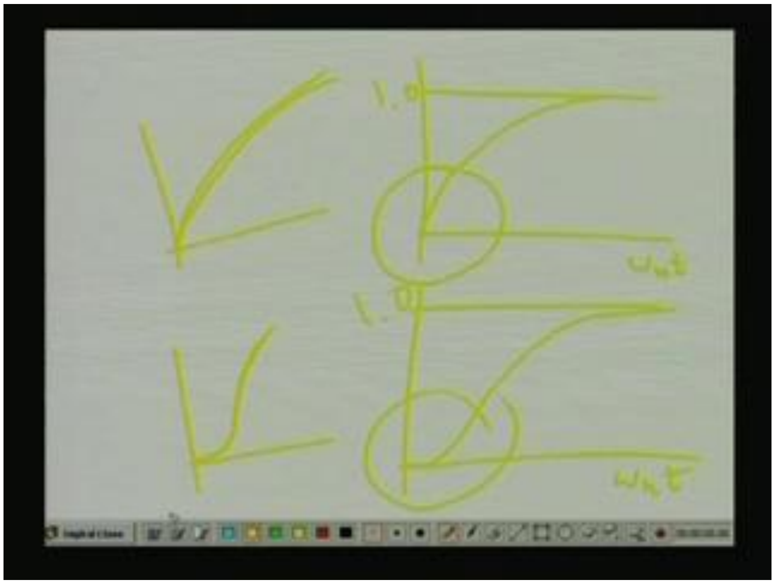

Now, suppose the system is critically damped, right; suppose the system is critically damped, in the case of first order system, I will get a response like this, right normalized output, this is omega n t and in the case of second order system, if it is critically damped I will get a response like this. So, look at the origin in the two cases. So, since it is, it is under damped, I mean it is not under damped it is very difficult to tell, but if somebody I mean does not tell you what is the system whether it is a first order or second order, you can tell by looking at the response. There is a distinct s curve. At the initial you look at, if you zoom it, it will look like this one, whereas in the case of first order it will be always like this, right?

The reason, there is a distinct reason. If you look at the expressions you will find, I will leave it to you; you look at the expressions of the first order and the second order system you will get your answer. Initially it will go like this, then it will go below like this, whereas here it is initially very high change, then it will go like this.

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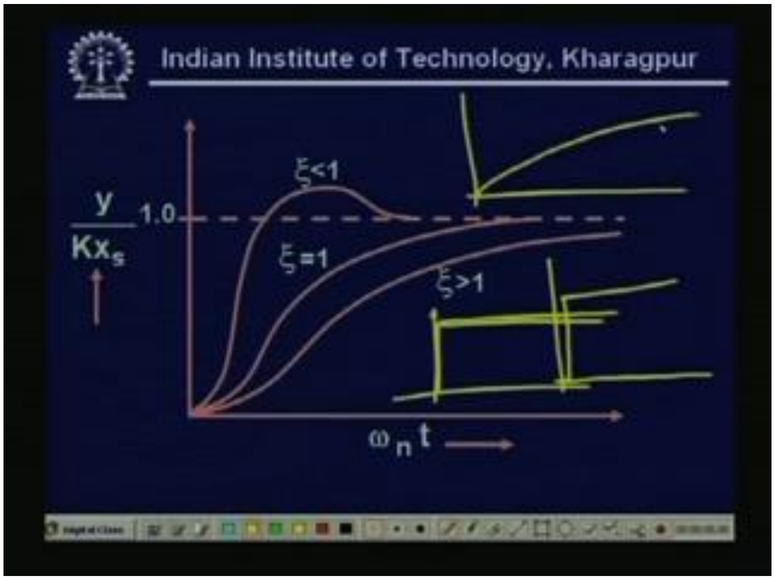

So, this is our response. So, we will try to make, so you can see that the over damped system is taking long time, it is taking long time to get the final steady state value, because any instrument what I want that its response should be 1. So, any instrument if the response is like this, okay, if I am giving the step input, these are all due to step input, is not it? I am giving a step input like this, I want that my output also will look like this. But, in any physical system, no physical system can have an input like this one, output like this one. There is a lag. So, obviously what will happen? So, it will take some time.

Now, you see this will, later on you will see these are plays very important role that this may, how much the error, sorry.

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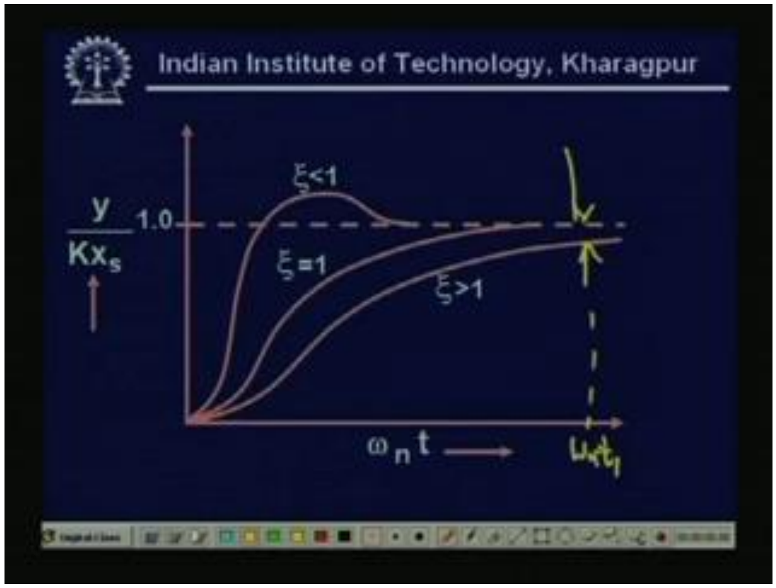

See, this is the errors of measurement. If you look at very carefully, error means that some sort of error has arrived. If suppose at the, at the time t 1 that means omega n t 1, so some error has arrived, right? Now, so this is important, because sometimes we cannot wait for a longtime, that the, when the, it will, it will reach that particular position. So, it will also tell the settling time. We will see the definition of the settling time at the end of the lesson, because always not all necessarily that always you will find that I will allow to have this type of, I mean long time to settle down our system. Why it is important? You see, it is also important in the case for under damped systems.

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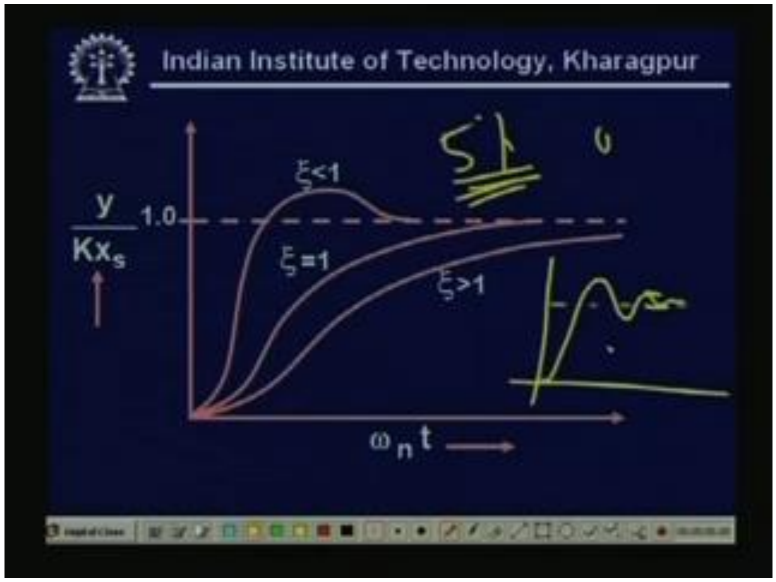

Suppose a under damped systems also what will happen? I will get over, I mean, oscillation. So, in that case, suppose if I say that when it will reach 5% of the final steady state value, you take the reading or take the action, open the control bar or close the control bar. For many industrial process please note that the 5% of the final steady state value is quite good for the control. Ultimately this temperature control or your flow control or pressure control will be reflected in the quality of the product which you are getting. In the case of fertilizer plant, it is the quality of fertilizer which is coming out, in the case of steel plant the quality of steel it is coming out. So, it hardly matters or I mean all the management people will not accept that if you make some sensors till it will reach the final steady state value. I can take the action if my, if my quality of the product does not change even if when it reaches 5% of the final steady, because I know what is my steady state value. So, if it reaches 5% of the steady state value, I can take my action, right, because ultimately it is the quality.

But, in some process we will find, especially in the bioprocess we will find, the precision, the temperature should be very, very tightly controlled. So, 5% of tolerances cannot be allowed there, because cell will die in that cases. So, temperature can be, suppose it is 31 degree centigrade, I have to maintain exactly 31 degree centigrade or it might be only

30.5 to 31.5. If the temperature goes beyond that range, cell will die. In that situations, I cannot allow this 5% of tolerance band. But, in many practical process or industrial processes like plants, these fertilizers, petrochemicals, steel, we can allow this 5%. That means what I am saying that within when it reaches 5%, suppose this is our 5% of the final steady state value, you take action. That means you take the reading, sent to the control valve and whether it is to be opened or closed you take the decision or stop it, whatever it may be, whatever may be the case.

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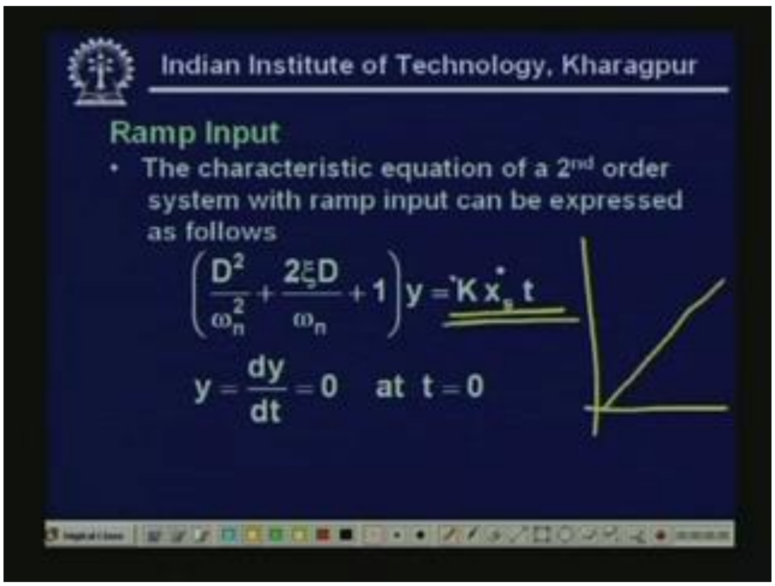

Now, if I look at the ramp input, ramp input we have seen; characteristics equations of a second order system with ramp input can be expressed as follows. Already we have discussed the ramp input in the case of first order system. We are taking same three inputs, so that we can compare putting these two side by side. D square upon omega n square plus 2 xi D by omega n plus 1 y equal to K x s dot into t. y equal to dy by dt equal to zero at t equal zero. This is the initial condition; we have discussed this before also. What does it mean? The x s dot, my input, it looks like this. This is our ramp input. We have discussed thoroughly this when we discussed the first order system, right?

Now, this we can, I mean do it in the case of second order system. So, this is my input, this constant K as it happened before also, x s dot into t, right. It is continuously changing with time, so x s dot, so I made it unit less quantity. So, these are our initial conditions

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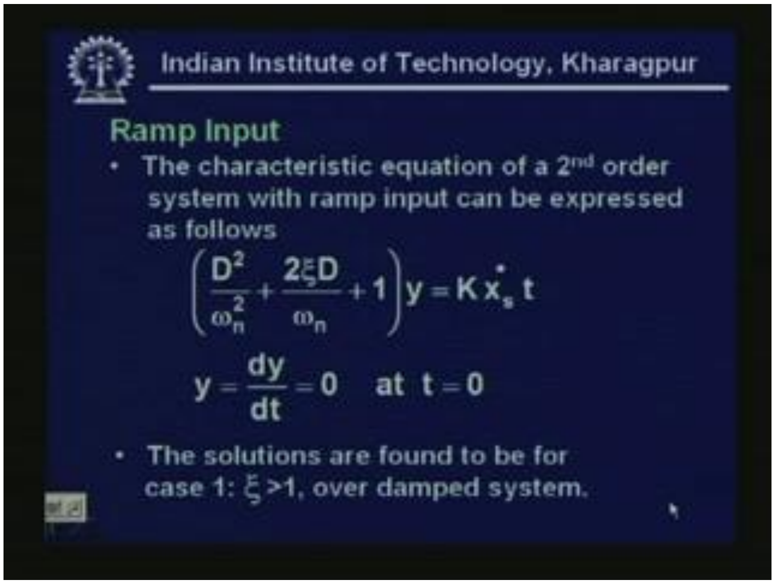

The solutions are found to be for the case 1 when xi is greater than 1, is over damped system.

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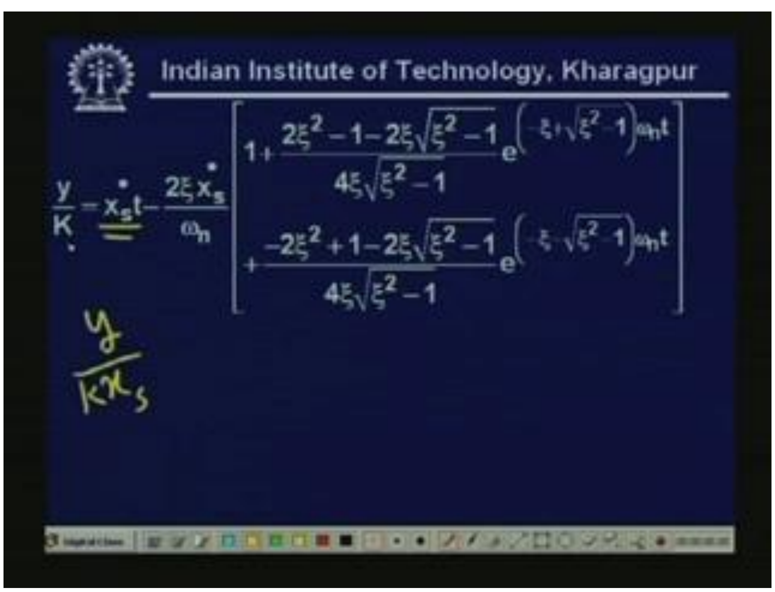

So, this is y by K. I have not put x s on the left hand side as it happened in the case of, I mean, the step input we have seen that we have done like this - y by K x s. These we have not done. The reason is that we have not done because of the fact that we will find there this is unit less quantity. I have to make it unit less quantity. If I put x s dot here, it won't be unit less, because any value when it is normalized is supposed to be unit less. So, we put x s dot t on the right hand side. So, we put just y if it is output divided by the, divided by the static sensitivity on the left hand side and other things on the right hand side. So, this is, as you can see x s dot t 2 xi x s dot upon omega n 1 plus in the bracket 1 plus 2 xi square minus 1 minus 2 xi square root xi square minus 1 upon 4 xi under the square root xi square minus 1 multiplied by e minus xi plus root over xi square minus 1 into omega n t plus minus 2 xi square plus 1 minus 2 xi under the square root xi square minus 1 upon 4 xi square root of xi square minus 1 e to the power minus xi minus square root xi square minus 1 into omega n t, fine.

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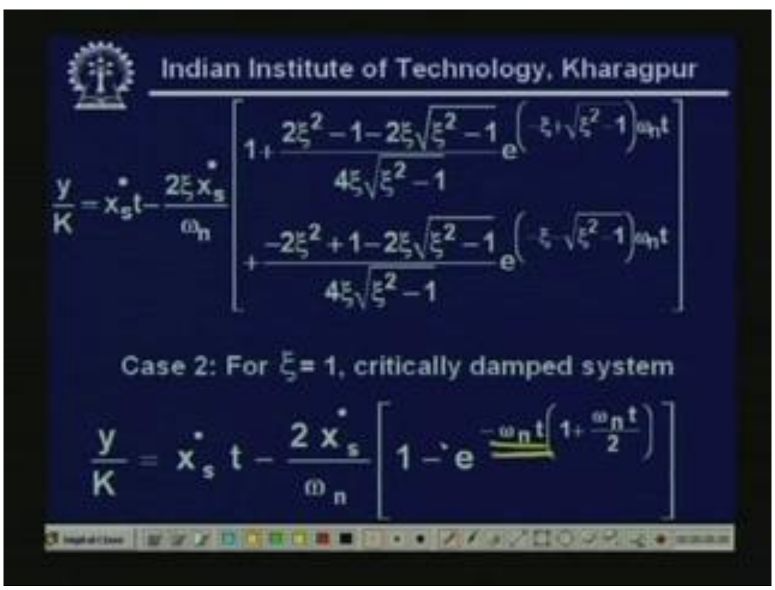

Now case 2, as it happened before also it is critically damped system, when xi equal to 1. In that case the output will look like this. y by K x s dot into t minus 2 x s dot by omega n 1 minus e minus omega n t 1 plus omega n t. You can see here these are all unit less because x s dot by t, omega n is also radiance per, I mean per time, so these all becoming unit less. Nowhere you will find that these, all these terms you can see this is, this steady state, this transient term will die out of the factors, of the minus factors there; here also for this, whereas in the case of, in the case of under damped systems you will find it is something different.

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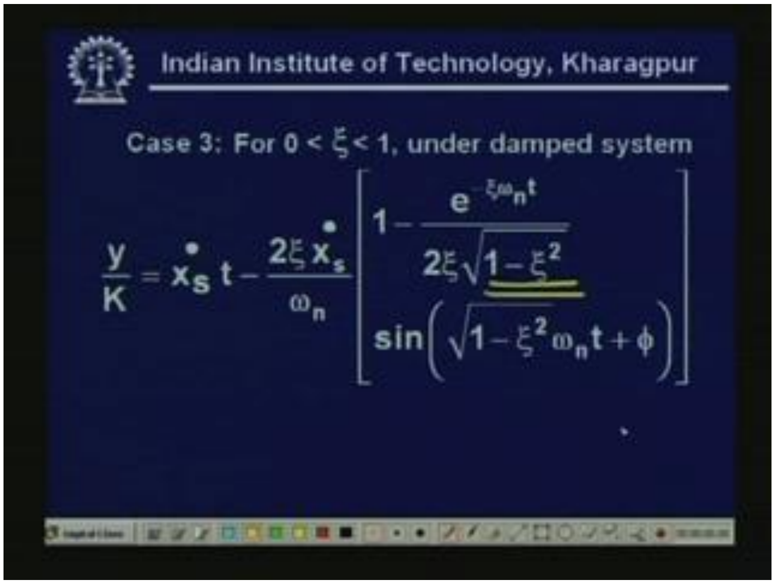

Now, case 3 is a under damped system that means xi is greater than zero but less than 1, as it happened before also, which is more sought after systems, because always we make the system the under damped systems. So, it is y by K x s dot into t minus 2 xi x s dot by omega n in the bracket 1 minus e to the power xi omega n t upon 2 xi under the square root 1 minus xi square sine under the square root 1 minus xi square omega n t plus phi. You can see here that it is, xi is 1 minus xi square because xi is now less than 1, otherwise it will be an imaginary quantity.

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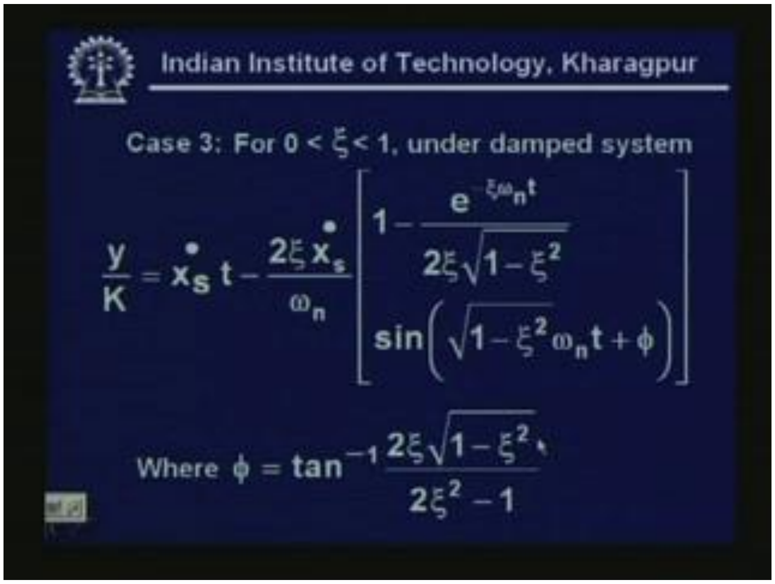

The phase shift, I got a phase shift here and the phase shift is given by tan inverse 2 xi root over 1 minus xi square upon 2 xi square minus 1.

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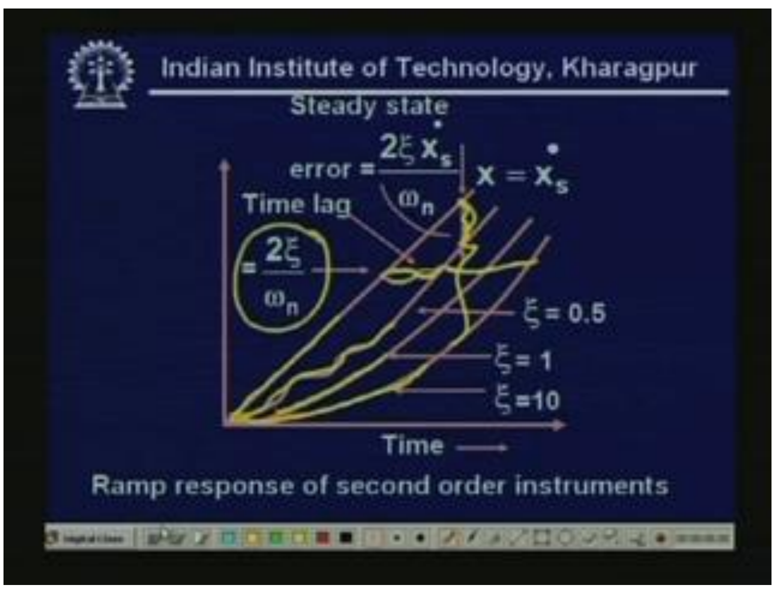

Now, ramp response of the second order instrument, there is an animation, let it come. You can see here that this is my input. This is input; it is going x equal to x s dot and we have plotted for different values of xi. This over damped system is taking large time, so the error is also large and this is for critically damped systems, we can see here and this is for under damped system. We have put xi equal to .5. So, there is a oscillation. Quite obviously there is sine term, sinusoidal term, there is oscillation. After that this will, it will die out.

Now, this is our steady state error. The, we have plotted the, we have shown the steady state error for an under damped system, which is 2 xi x s dot upon omega n. This is the steady state error of the system, right and a time lag of the system that also we have shown for the under damped systems 2 xi by omega n. This is our time lag for the system, this is for this one. So, in the case of over damped system, so this will be our error and our time, our time lag will be this much in the case of over damped systems, right. So, obviously we can see that if it is under damped, after few oscillations it will come, it will start to follow the input. Actually, in fact, this and this should be parallel to each other and asymptotically these, all these two also should be parallel to each other, because at the steady state response all other, I mean, all the transient part will die out, only the particular integral will remain. So, this is our, the response of a second order system for a ramp input. Ramp input, I mean ramp input you will not see that we already discussed also in the case that usually the ramp input will not come.

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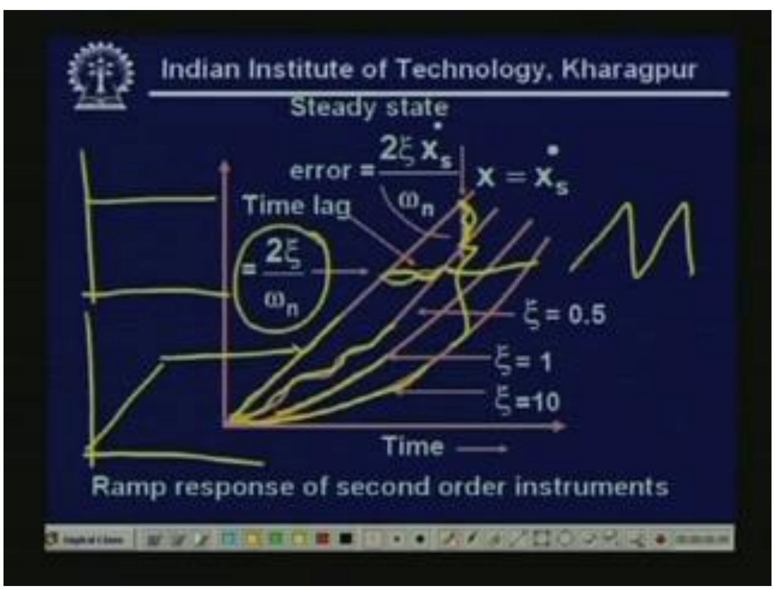

You will not face the ramp; unlike electrical systems, always you will keep ramp input like saw tooth waveforms. Always we know that it is a ramp input which we give in the oscilloscope to synchronize, because in the two vertical plates we give our voltage which we want to measure and to make the signal stationary we give the saw tooth waveforms on the two horizontal plates. So, the signal appears to be stationary on the screen, whereas in this case, the ramp input has not much of significance in the instrumentation, except in some cases, where because in many of the cases you see, because of the large inertias and the spring constant in many of the times you will find that even if you are considering the input as a step input, so it won't be a step input to the systems; it will be a ramp input or terminated ramp input. That means it looks like that I am giving a step input to the system, but the system, to the system it will not appear as a step input, it will appear as a terminated ramp input. This is a very good example.

Now, you see that all of, all of you must be knowing of the load cell. Load cell we will discuss later on in our, of this, of this course. Might be, I don't know what is the exact number of the lesson, but we will cover this in details, these load cells. Those who move in highways also you see the load cell also there, because there is a weigh bridge, because on the states the restrictions of that, the how much the load they will carry in the highway; there is some state restrictions and highway restriction that you cannot. So, usually they want to see the some certificate from the way side road bridge, I mean, weigh bridges where they will and weigh bridge is nothing but the load cell.

Now, when you put on the load cell some weight, so it will never appear as a, even though it is supposed to be a step input it won't appear as a step input, it will work as a ramp input, even though terminated ramp input, rather. Even though we are not much concerned with this transient there, we are more concerned with the steady state response, but the transient is important in the sense that we must know that how long it will take or when it will die out, the system, this transient response will die out. For that reasons we must analyze the transient response of the system also.

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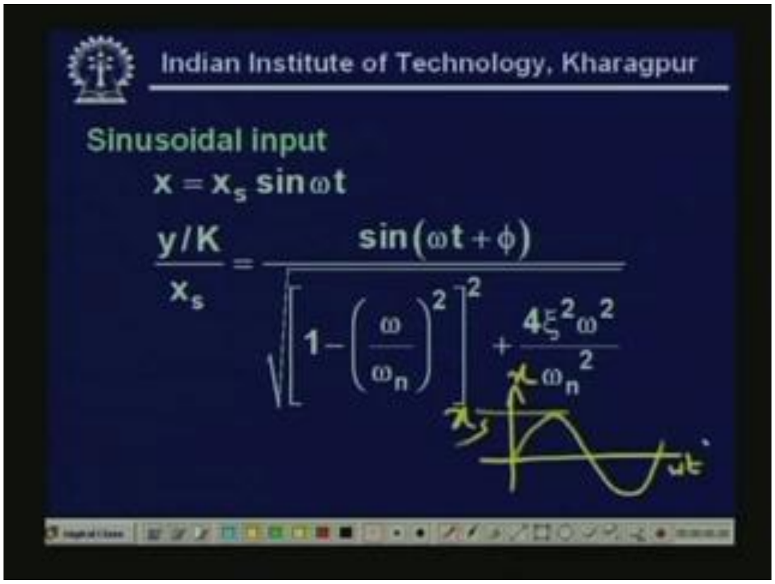

Now, last one is the sinusoidal input. That means the, you are considering the sinusoidal input to a system. The sinusoidal input is equal to x equal to x s sin omega t, say, all of us know from the first year classes how the sinusoidal input. So, this is my sinusoidal input, so this is our x s and this is our x, this is omega t, right and y by K the output, normalized output, rather y by K by x s equal to sin omega t plus phi 1 minus omega by omega n square upon 2 square plus 4 upon square, sorry, whole xi square omega square upon omega n square. Here you will find that is in this case that this omega n plays very important role in the case of sinusoidal input and the frequency is very important, because any system we must know that what is the frequency limit of the system? That means up to what frequency I can operate our, I mean in what frequency it will accept? We cannot put like an anti-analyzing filter that type of things which we usually do in the case of electrical signals, but here always we want that what should be the input? We must determine how much is the input in the system, what should the frequency or the highest frequency of the input to the system which can be tolerated?

So, this is our normalized output of the system. So, in fact what will happen if you take the amplitude? It will be y by K by x s. It will be y by K by x s. Only this part will be the amplitude output.

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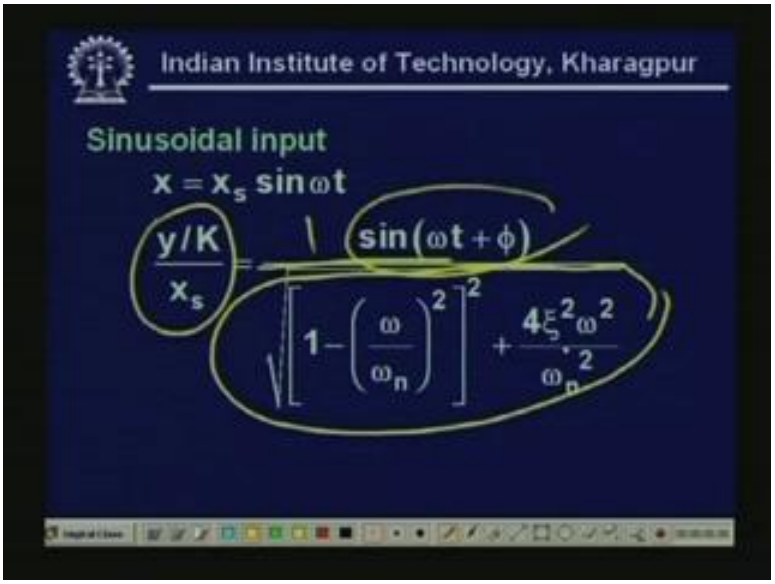

So, if you look at the normalized output y by K x s, so this will be 1 upon, this will be the amplitude output, because this will not go, right. So, may be you will find that in futures and while when you solve the problems, so I need this type of, these values. So, you must remember or you must go through the, this notes while you are solving the problems.

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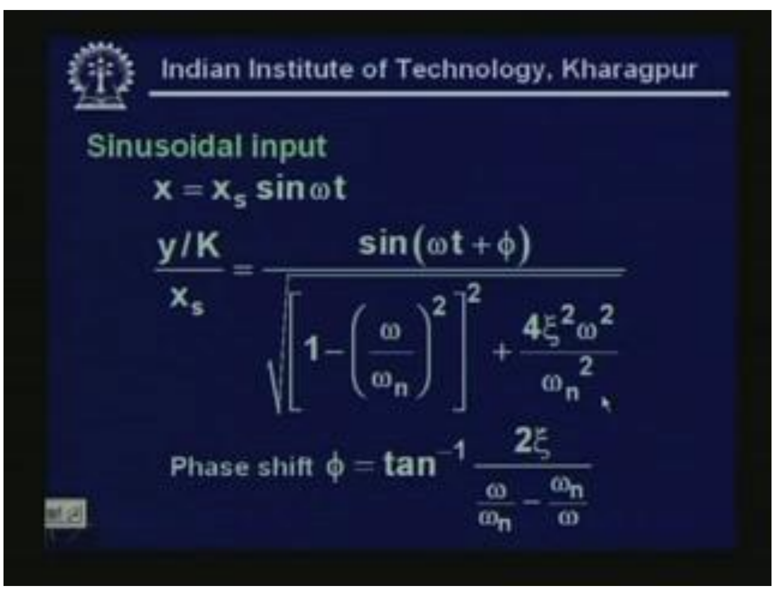

Now, phase shift in this case, in the case of sinusoidal input is tan inverse 2 xi upon omega, upon omega n minus omega n by omega. Phase shift is also important, because any sinusoidal input if you are measuring some sinusoidal signal how much the phase shift of the system you must know. We will see the, how the phase response will look like.

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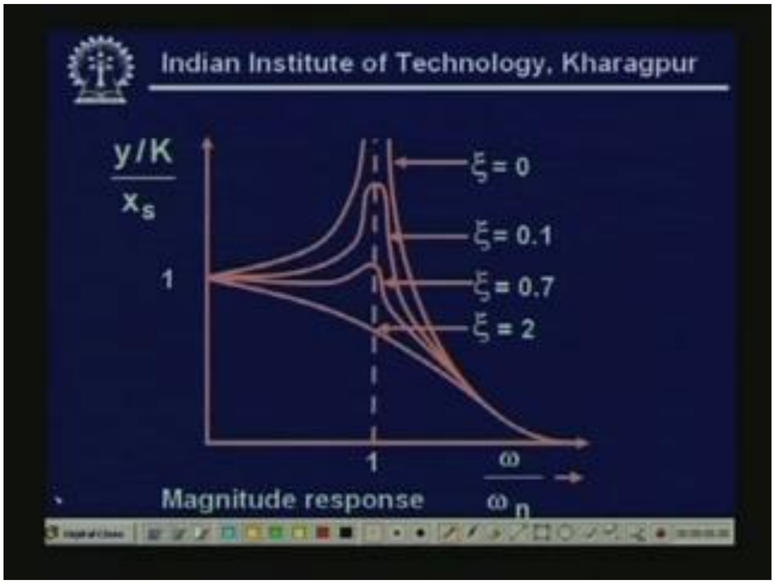

Now, if it is the magnitude response of a first order, I mean second order system for sinusoidal input, it looks like this. Now, in sinusoidal input even though it is very, it is not very common in nature, but you will find the sinusoidal input in many cases. In vibration analysis we will find that we are giving there periodic input, not necessarily always sinusoidal, we are giving some periodic input. In biomedical applications you will find also sinusoidal input. Suppose the variations of the temperature of a patient which is, who is suffering from malaria parasites and it is almost the sinusoidal, even though time period is quite large for 24 hours, you will find that particular temperature will arrive, when temperature will get at a particular time of the day, so that is also sinusoidal input and some other more applications we will find in the sinusoidal; some ambient temperature variation that is also sinusoidal in nature.

So, in the daytime and some temperature night, some other temperature in the morning some other temperature, like this, so it is sinusoidal input. So, it is not very uncommon, but even though the period in most of the cases is very, very large. Unlike electrical systems, any physical systems as mechanical system, has a very large time constants of the sine …. I mean sinusoidal input. Now, if you look at here, this is the magnitude response of our sinusoidal input.

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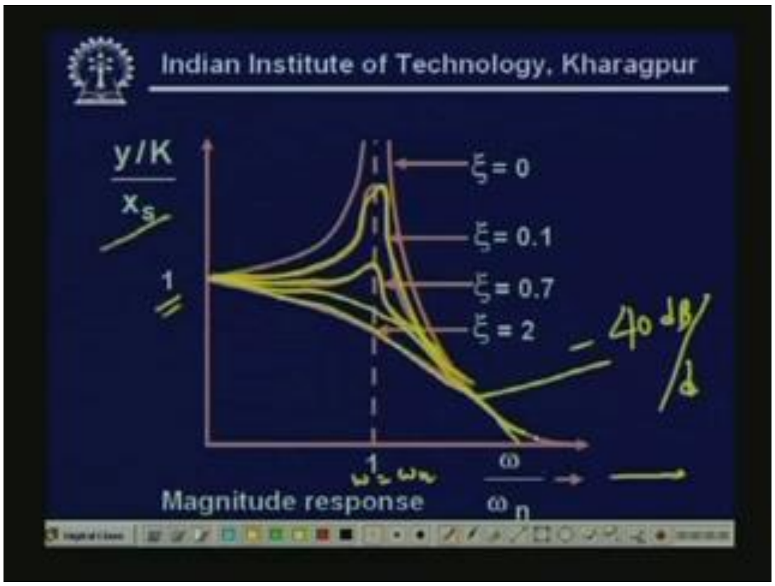

We have plotted here y by K by x s, so maximum is 1. You see here interestingly there is large overshoot here. This happens because of the resonance. So, you have to avoid this thing, because this is important. That is the reason you see we have plotted here in this case omega by omega n on the x axis side and we have normalized the x axis also and you see that here this is the one that means here is the omega equal to omega n. Interestingly when xi equal to zero that means damping coefficient is zero, it is very impractical thing. I mean you cannot make a damping coefficient zero. For academic interest we should consider these things.

You so see that when the damping coefficient is zero, the magnitude becoming infinite asymptotically infinite, infinite cannot be, because it has some supply. Suppose if we say that if I using an op amp, so you cannot get infinite value. See, it will be limited by the nonlinearity that is the power, plus minus power supply of the op amp and if I slowly increase the value of xi from zero to .1, my hump is getting reduced. This is the xi equal to .1. You see, I am getting a hump like this one, then it is coming back. It is xi equal to .7, we are interested in xi equal to .7. We have seen, we told you that in the case of second order systems any instruments we make usually .65 to .7, so I make a xi equal to like this one, so there is one overshoot, then it comes back and when it is over damped systems, xi equal to 2, so it will look like this one.

Now, see here I have not plotted for xi, the critically damped system. It does not matter, because it will be the response for xi equal to 1. It will be, the response should look like this one, something like this and this roll off, you will find always minus 40 dB per decade for the second order system. In the first order system it will be minus 20 dB per decade, it is 40 dB per decade. So, what does it mean? Actually what is the significance of this curve? The significance of this curve it tells that what is the frequency limit of your instrument of a second order instruments?

Suppose in the case of over damped systems you can see that there is a large attenuation if you go far off from the, from the origin. Whereas, in the case of if it is xi equal to .7, you will find there is a very little, there is a single over shoot, then it comes back, so that almost horizontal or flat frequency response I am getting. We are not supposed to use it after the omega n, because at the omega n suppose if it is, xi is .1, there is a large overshoot. In many instruments it is not allowed, it may damage the instrument. So, I will, if I know the omega n, so I can limit my frequency response of my instrument. That is the reason it is necessary we must discuss this point, right.

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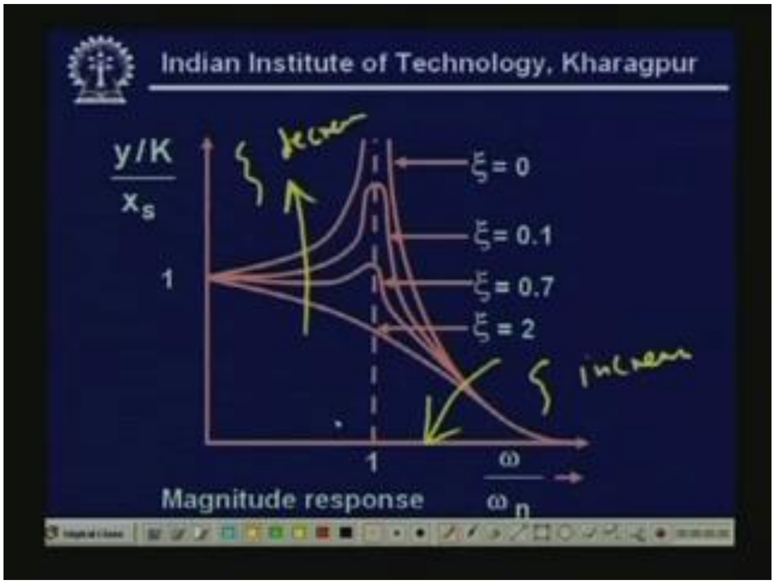

So, as you can see, this way I can see there, let me see, so here I can, I can tell that here it will look like this that the xi decreases means response or here in this case xi increases, so I will get a response like this one. So, I am more interested at omega equal to omega n.

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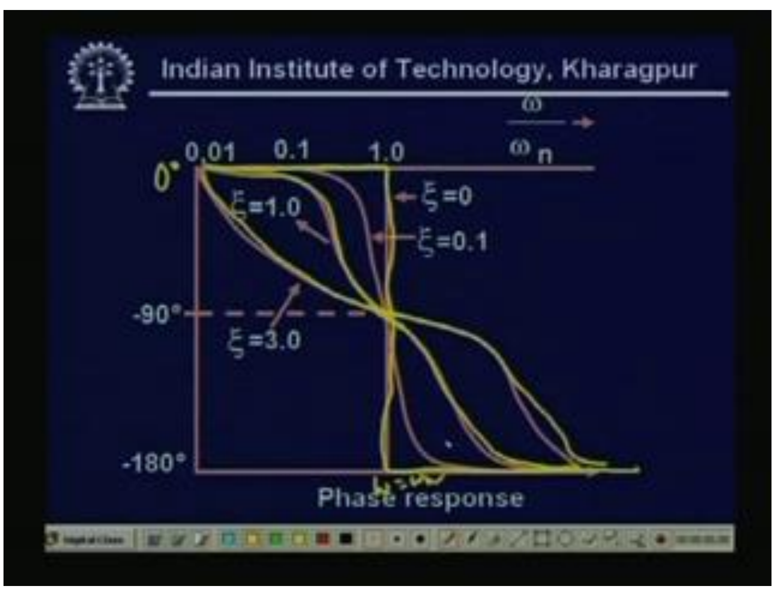

If I look at the phase response of a second order system for sinusoidal input, let it come all the animations, it will take some time, hopefully it is completed, no; you see here that, here that if xi equal to zero that means the damping coefficient is zero. We have already discussed also when the damping coefficient is zero how my magnitude response will look like. My response will go like this, then go, go like this one, total discontinuous curve, it will go like this, come like this; let me go ahead, take it, again it will go like this, xi equal to zero, come like this and response will be this - from zero degree, this is zero degree to minus 180 degree it will follow and if I slowly increase the value of xi, suppose xi I made .1 my response will start to look like this one.

Similarly, if xi equal to 1, it is critically damped systems, so response will look like this, response will look like this and if xi equal to over damped systems, our response will go like this. How? This will go like this. Interestingly you see, for, this is omega equal to omega n, it will, it does not matter what is the value of xi, it will always pass through minus 90 degree at omega equal to omega n.

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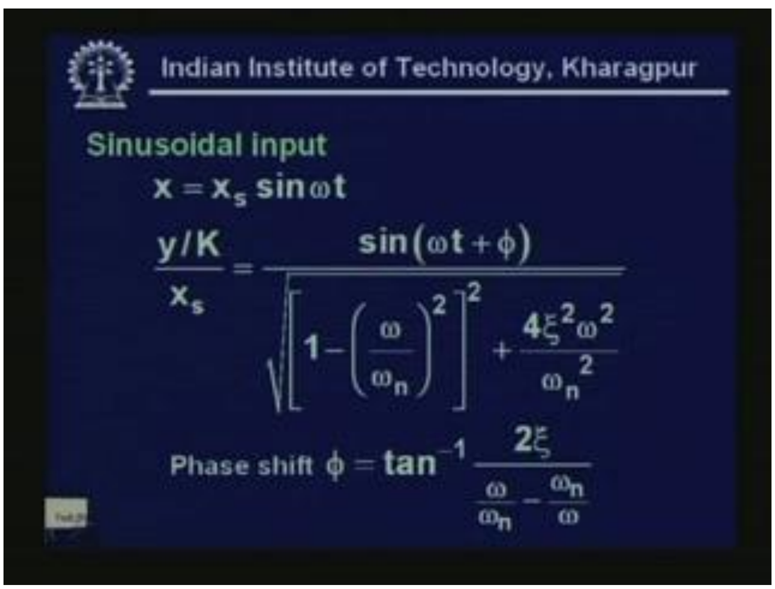

You will be more clear if you look back, if I go back to our, sorry, you see here, you see here, what happen that omega equal to omega, I mean when phi equal to tan inverse 2 xi upon omega by omega n minus omega n by omega. You can see here that when omega equal to omega n, always it is minus 90 degree. That …. depends on the value of xi.

Mathematical fallacy will come, all these things, and what will be zero by zero and all this thing, I am not going to, I mean go into details of that thing. Let us go back; already we have seen this thing.

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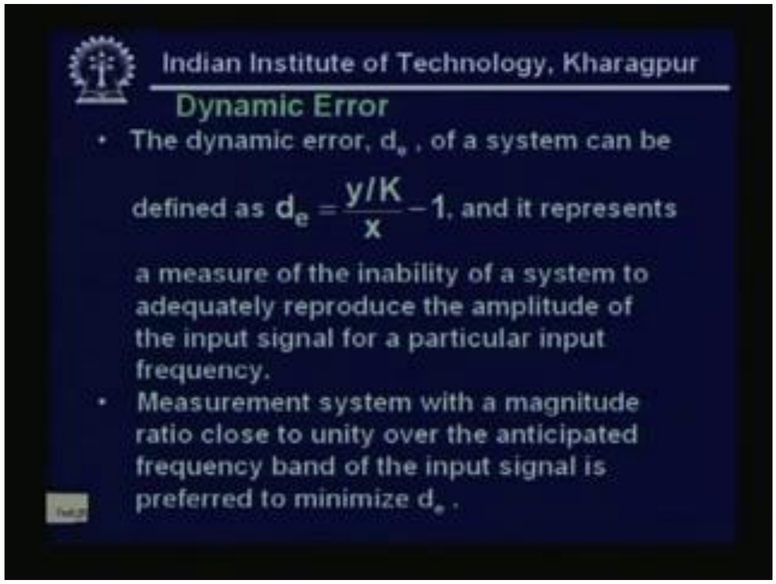

Now, two more terms we will introduce. This is called dynamic error of the system, because we need this while solving the problems. In our future slides we will see that we will solve some problems - first order, second order instruments. Dynamic error, d of a system can be defined as d e equal y by K by x minus 1 and it represents a measure of the inability of a system to adequately reproduce the amplitude of the input signal for a particular input frequency, right. So, its inability that means if you have to look at that what should be the error? Obviously, it depends on time.

If the time goes by, then you will find the error will be getting reduced and reduced, but after certain time if I say that how much is the dynamic error, because this will be utilized to solve some problem, you will find that is very important definitions for any physical systems, a measurement system with a magnitude ratio close to unity over the anticipated frequency band of the input signals is preferred to minimize this. Obviously, we will always make, always try to make d e equal to zero, but that we cannot, so we will take some minimum value of this one.

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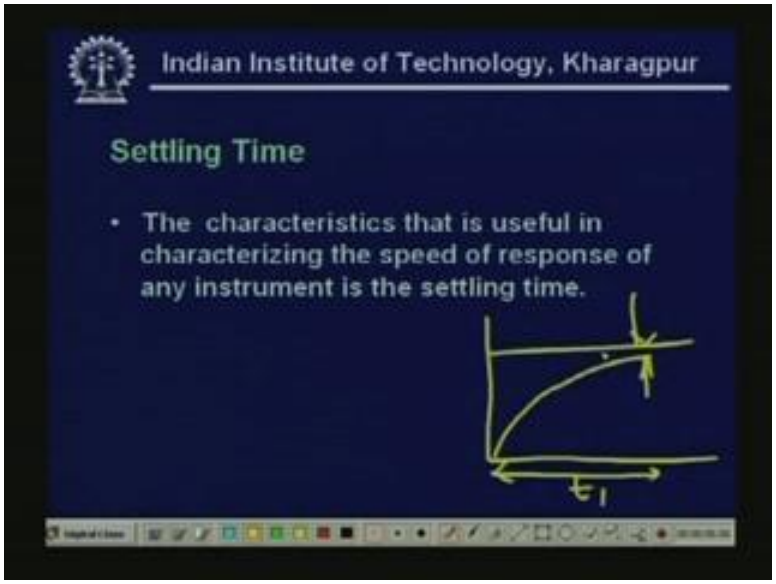

Now, settling time is another important, the characteristics that is used in characterizing the speed of response of an instrument is the settling time. How does it mean? It looks like, you see here, I am sorry, settling time looks like this, I have this one, I am not very, I should draw it very nicely. We have discussed this before also, suppose the 5% of the value, so this is our value t 1, so this is the settling time. If you take infinite settling that does not make any sense that means after the 5%, when it reaches the 5% of the tolerance band whatever the time required that is we call it settling time.

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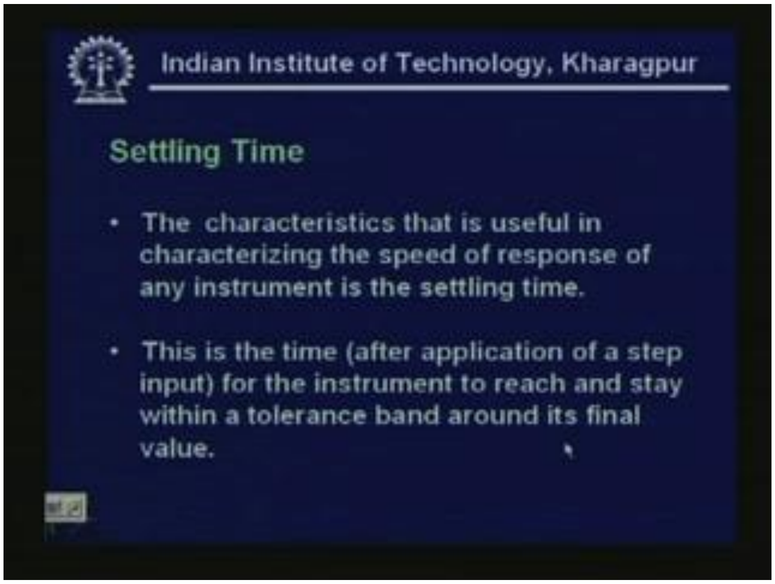

This is the time after application of a step input for the instrument to reach and stay within a tolerance band around its final value.

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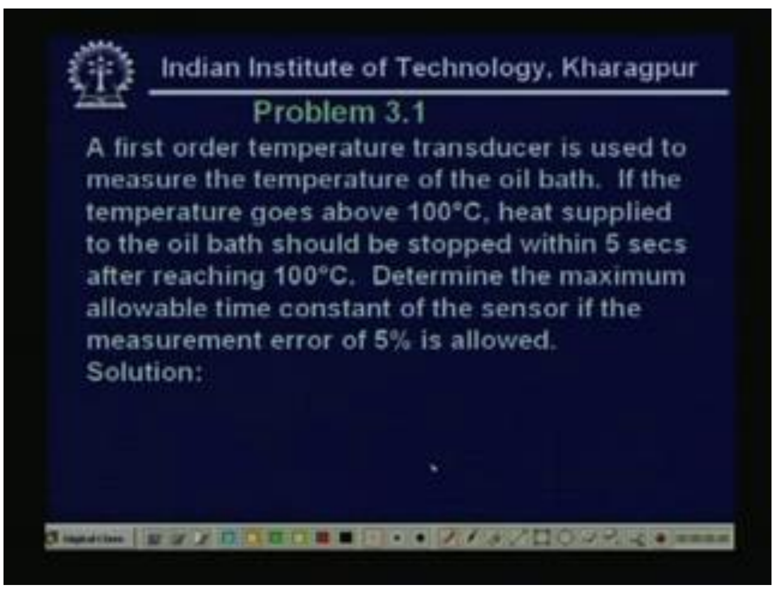

Now, let us start some problem. So, let us look at the problem 3.1. It says, a first order temperature transducer is used to measure the temperature of the oil bath and if the temperature goes above 100 degree centigrade, heat supplied to the oil bath should be stopped within 5 seconds after reaching 100 degree centigrade. Determine the maximum allowable time constant of the sensor, if the measurement error of 5% is allowed, right? So, let us solve this problem.

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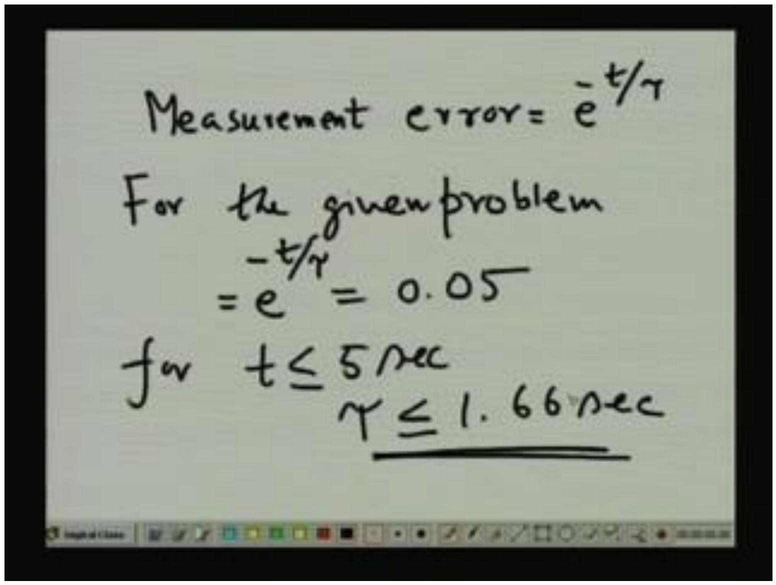

I do not know, yes, it has chosen properly. Problem looks like this, so here, let us go back. So, it looks like this. That measurement error, error will be equal to e to the power minus t by tau. Now, for the given problem, problem e to the power minus t by tau equal to .05, for t less than equal to 5 second, tau will become less than equal to 1.66 second that means the time constants of the system should be less than equal to 1.66 seconds, right? If it, the time constant system is less than equal to 1.66 seconds, so the 5% allowed, I mean after 5 seconds, 5% of tolerance band can be allowed or 5% for the dynamic error can be allowed, right?

Now, let us go to problem number 3.2.

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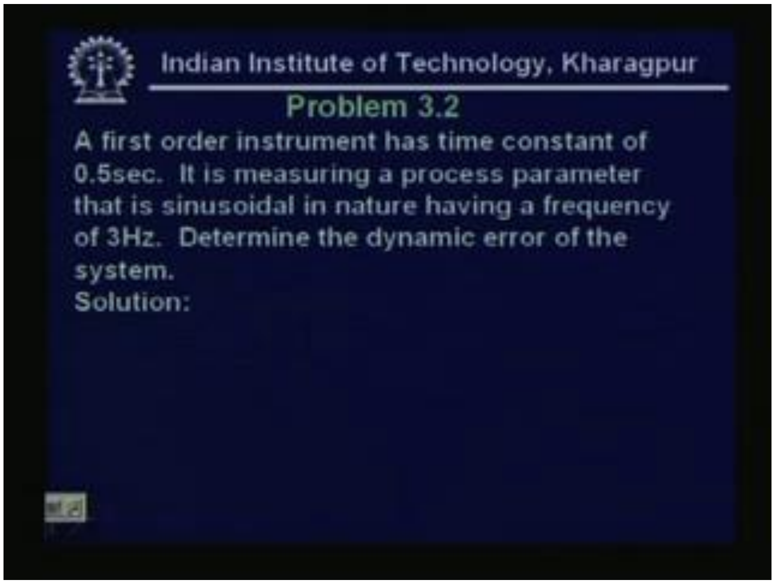

It tells, I will solve here from problem number 3.1to 3.3 and I will give the problem 3.4 to 3.6 and will explain to some extent 3.4 to 3.6, but the solution will be given at the beginning of the lesson 4. So, I will give some hints, so you are supposed to solve it of your own. So, the problem number 2 or 3.2 of the lesson 2, lesson 3 will tell that, like this I mean a first order instrument has a time constant of .5 second, it is measuring a process parameter that is sinusoidal in nature and having a frequency of 3 hertz. Determine the dynamic error of the system, fine.

So, the system is known, it is the first order system, time constant is also known, but the signal is now sinusoidal in nature, instead of step and all those things; frequency is given, so you have to find the dynamic error of the system. So, there will be a constant dynamic error in the first order, because since there is a phase shift, so there is a constant dynamic error in the first order system. Here it looks like this.

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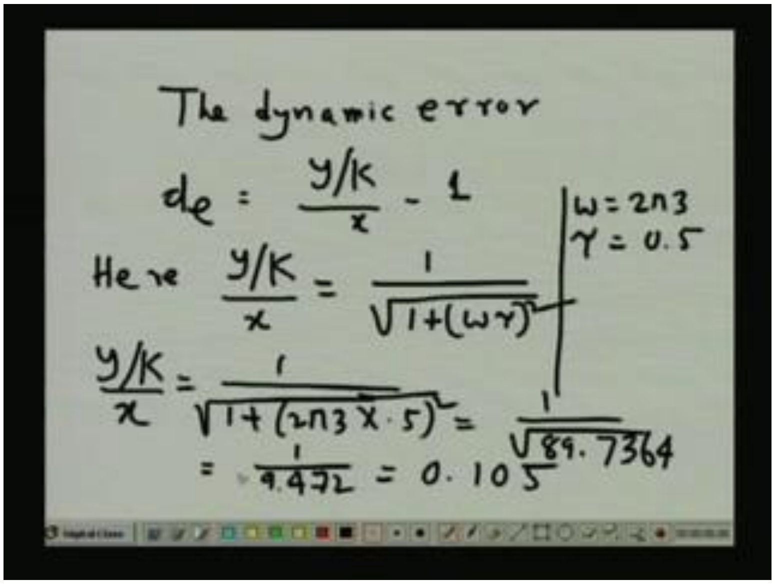

You see here that in the case of first order systems, I can write the dynamic error equal to d e equal to y by K by x minus 1. Here, y by K by x equal to 1 upon root over 1 plus omega tau whole square. Here omega is given. Omega is 2 pi 3 and tau is equal to .5.

So, y by K by x will be equal to 1 upon root over 1 plus 2 pi 3 into .5 whole square, which is coming, I mean showing with the complete calculations, which I have calculated, root over 89.7364, which is equal to 1 upon 9.472, which is 0.105, clear?

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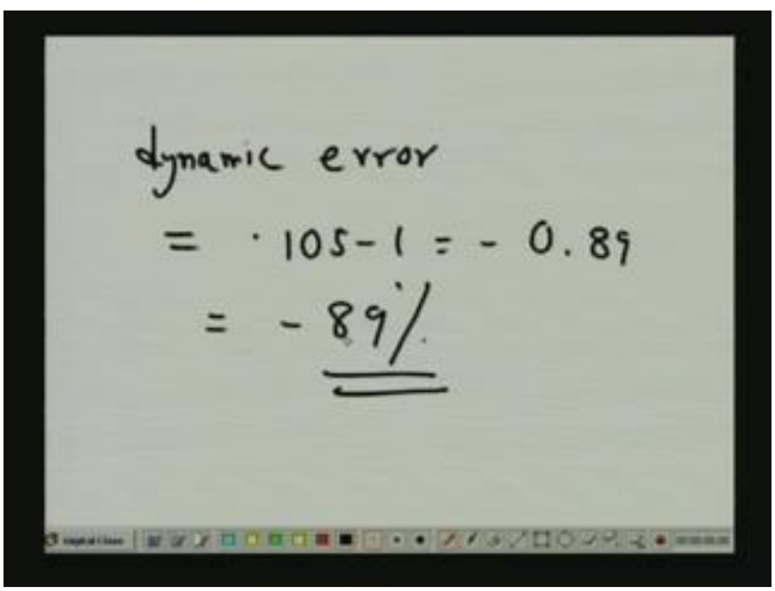

So, the dynamic error, error equal to .105 minus 1 equal to 0.89 or minus 89%. That is the error, right? This is your answer.

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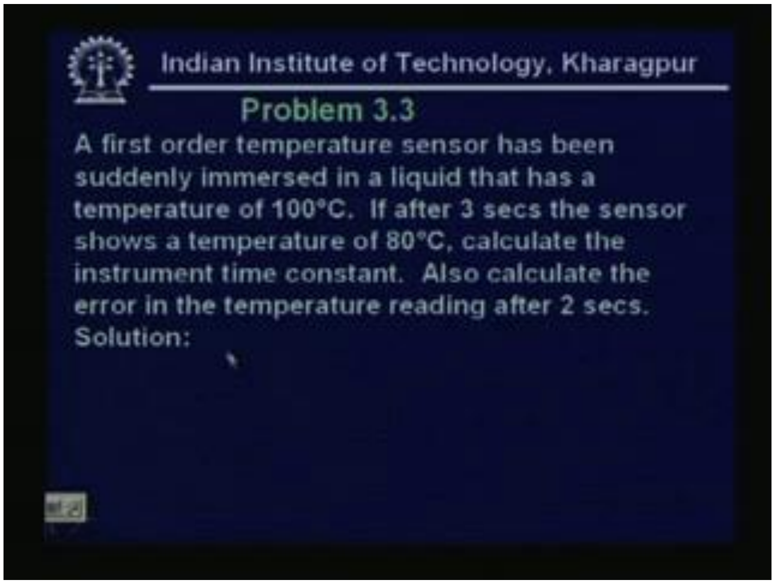

Now, let us go to the problem number 3, which is telling that the first order temperature sensor has been suddenly immersed in a liquid that has a temperature of 100 degree centigrade and if after 3 seconds the sensor shows a temperature of 80 degree centigrade, calculate the instrument time constant. Also calculate the error in the temperature reading after 2 seconds.

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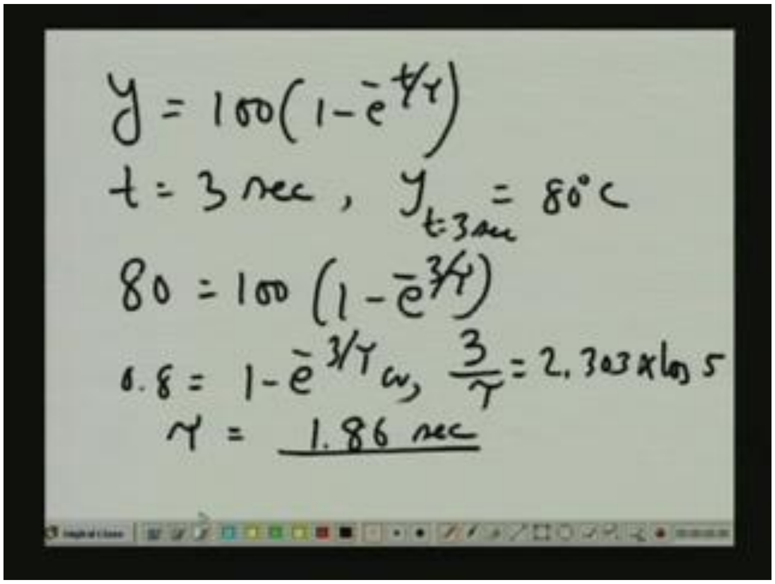

So, here you see that y equal to, let me take the white page, so y equal to 100 1 minus e to the power t by tau. So, t equal to 3 second and y t equal to 3 second equal to 80 degree centigrade. So, 80 equal to 100 1 minus e to the power minus 3 by tau. So, after solving, I can show you the detailed calculations, so it is coming .8 equal to 1 minus e to the power 3 by tau. So, it is coming around 3 by tau or 3 by tau 2.303 into log of 5. So, it is coming around tau equal to 1.86 second.

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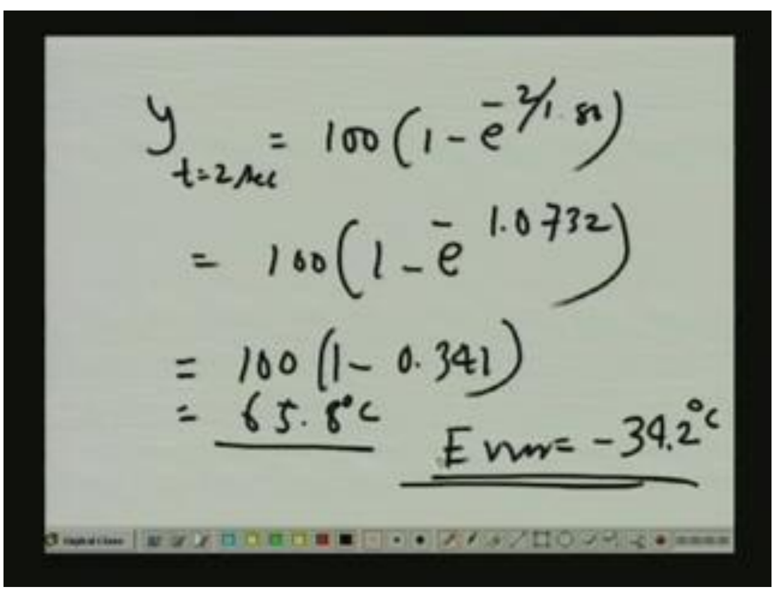

So, now if I put, this is a time constant of the system, now if i put y equal to 2 second to 100 1 minus e to the power minus 2 by 1.86, so equal to 100 1 minus e to the power minus 1.0732, so it is equal to 100 1 minus 0.341 is equal to 65.8 degree centigrade. So, this is the temperatures and at after 2 seconds, so the error will be, you subtract it from 100 or 100 should be subtracted from this. It is minus 34.2 degree centigrade. This is the error, right.

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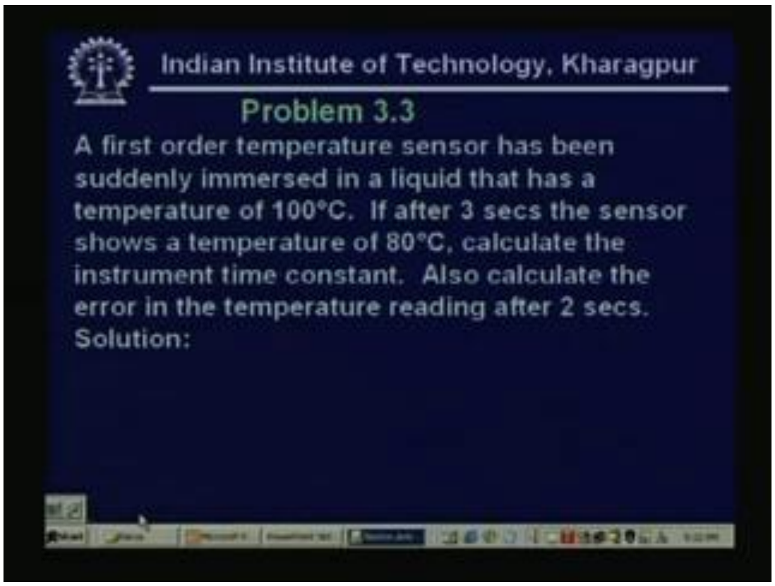

Now, problem 3, I will not make the solution to the problem number 3, which you have to do.

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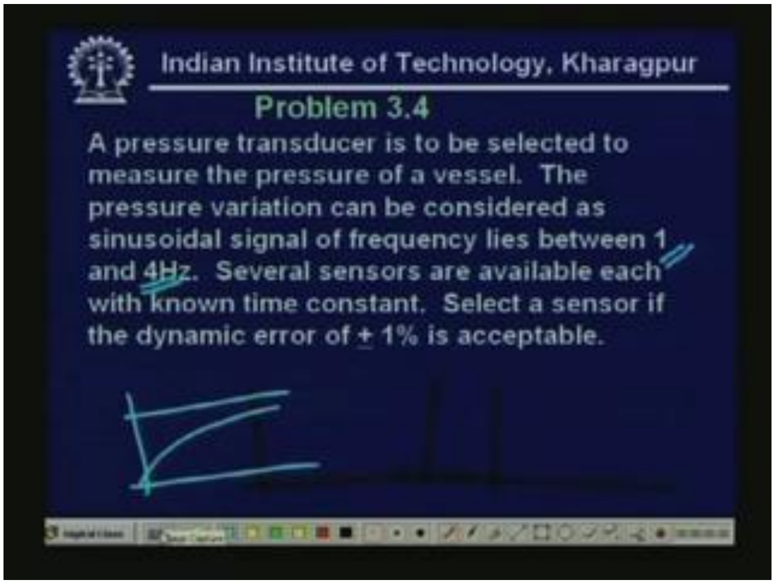

So, a pressure transducer is to be selected to measure the pressure of a vessel and the pressure variations can be considered as a sinusoidal signal or the frequency lies between 1 and 4 hertz. Several sensors are available, each with known time constant. Select a sensor if the dynamic error of plus minus 1% is acceptable. Now, you see, it is first order instrument. So, there is no question of taking the plus 1%, because always it will be, it will go below, if you remember. So, it is the first order systems always, the sorry, so it is a first order system. It can never over shoot like this one. It will always go, so it is, always go up to minus 1% you should take and you have to find the time constant of the system that is your question or what is the range of the time constant, rather? That means that it will, because it is 1 hertz to 4 hertz, so this is 1 hertz to 4 hertz, so within the, if it lies between this, what should be the range of your time constant. This I will solve in the next class.

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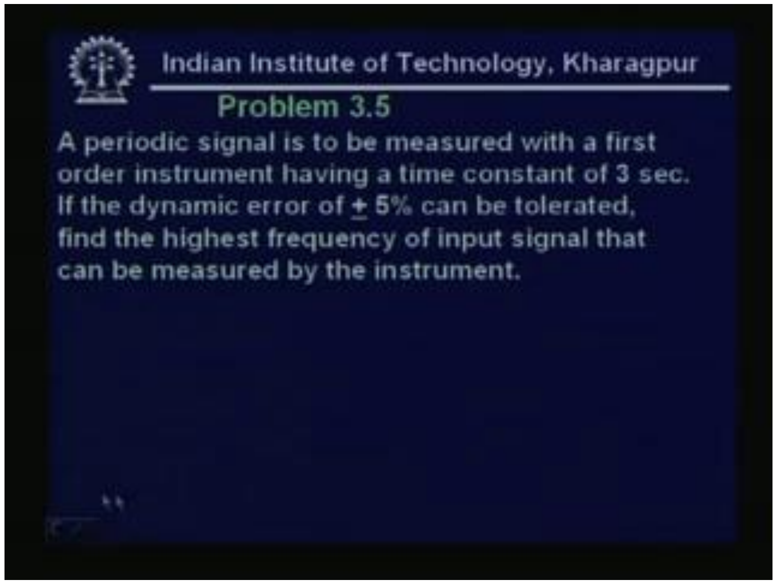

Now, let us look at 3.5. The periodic signal is to be measured with a first order instrument having a time constant of 3 second. If the dynamic error of plus minus 5% can be tolerated, find the highest frequency of input signal that can be measured by the instrument. So, in this case, you must see that this is a, basically we have considered this as a first order instrument and you have to find the value of omega that means the frequency range for which the instrument can be used, right, highest frequency.

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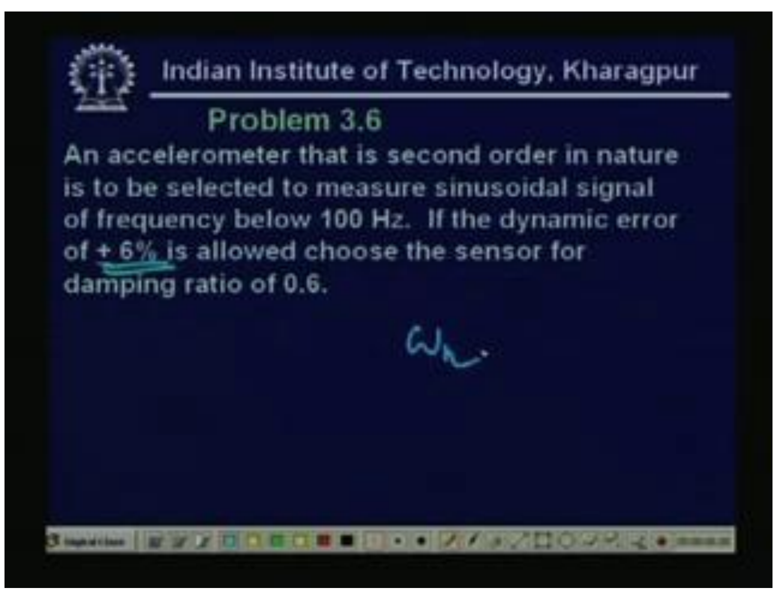

Now, the last problem, you will find a second order system. So, there is no question, so if it is second order system, you have to take both plus and minus of the dynamic error that is the reason we have given. So, you have to take both. That means you have to take both plus and minus of 6%, so accordingly you will keep the value of the omega n. So, the natural frequency of the sensors you have to find here, you have to find the value of omega n here. So, you try to solve these problems and the solutions will be given in the next class and I will remind you that here you see that the problem number 3.4 the instrument is first order, problem number 3.5 also it is the first order system, whereas problem number 3.6 is a second order system. So, with this I come to the end of the lesson 3.