Industrial Instrumentation

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Lecture - 2
Dynamic Characteristics

Lesson 2 of industrial instrumentation, we will discuss basically the dynamic characteristics of instrument or sensor. Now, one thing I told you that a sensor or instruments, we are I mean rather using very loosely here the term, because sometimes we call it sensors, sometimes we call it instruments as such. When I will call the second order instrument or first order instrument, it does not necessarily mean only instrument as a whole, it may mean also the sensor.

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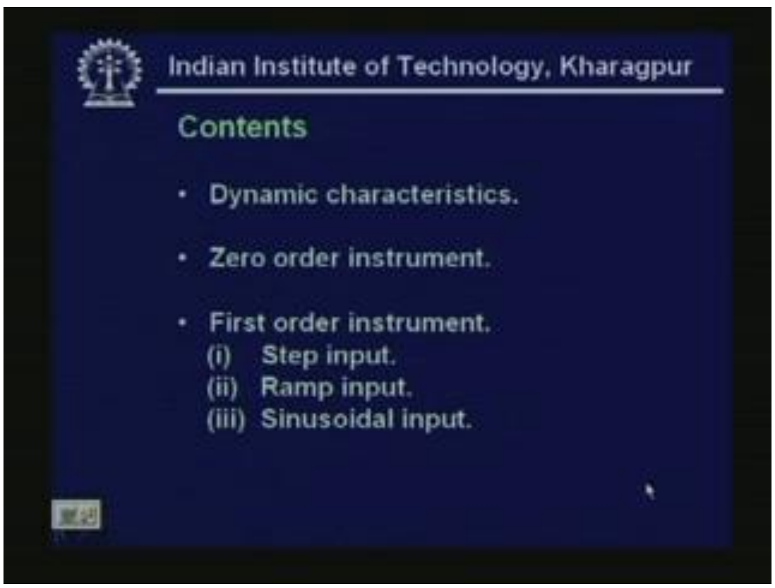

Now, the contents of this lesson are: dynamic characteristics, zero order instrument, first order instrument and its responses to step input to ramp input to sinusoidal input. You see, in both the cases, in the case of first order and second order instruments, we will consider these three different inputs and we will compare putting these side by side.

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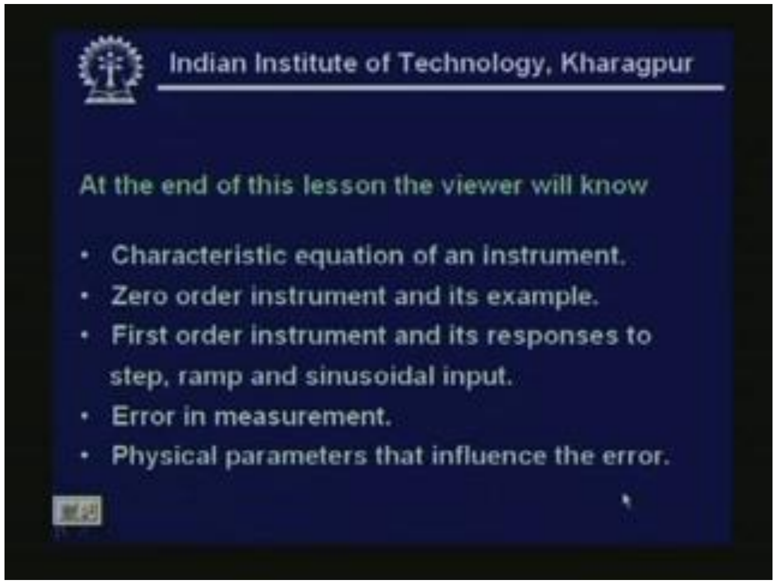

At the end of this lesson, the viewer will know the characteristic equation of an instrument, zero order instrument and example of some zero order instrument, first order instrument and its responses to step, ramp and sinusoidal input. In the instrumentation system, you will find that most of the time you will face these three different inputs. Even though ramp input is not very usual, we will find, instead of ramp the input will be sinusoidal input. However, we will consider the ramp input also and sinusoidal input is very common like ambient temperature variations or in some biomedical applications, we will find these types of inputs will be there.

Step input is very common in the electrical system, because suppose you are using a voltmeter, you are giving the input to that voltmeter, it is considered as a step input and also you consider the error in measurement. How much the error, because until unless you know the error you cannot reduce it. What are the factors on which this error will depend, you must know that thing and how the errors will behave as the time goes that also you should know. For that reasons, we should also consider error in the measurements in all the three different, for the three different inputs and physical parameters that influence the error that I told you just now.

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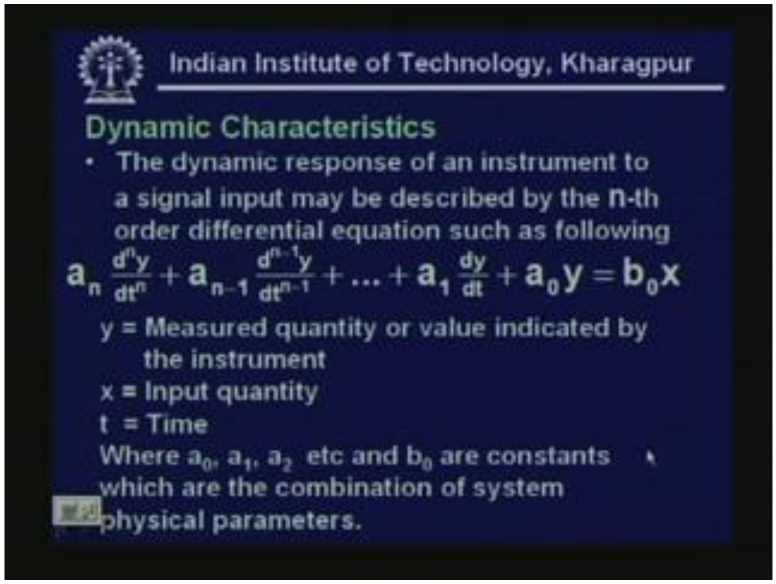

Now, if I look at the dynamic characteristics, the dynamic response of an instrument to a signal input may be described by the nth order differential equation such as following. It looks like a n, nth order derivative of y with the respect to t plus a n minus 1, n minus oneth derivative of y with respect to t, so on until it is a 1 multiplied by dy by dt plus a naught y equal to b naught x, where y is the measured quantity or value indicated by the instrument, right, x is the input quantity.

For any measurement systems or sensor, we will actually, we will want that whatever the x, y should show that without any time delay, without any phase lag. But, this will not happen. You will see that there will be some lag in the system. There will be some, some delay in the system. t is the time and where a naught, a 1, a 2, so on up to a n and b 0 are the constants which are the combination of the system physical parameters. What is the meaning of the system physical parameter? Suppose I have an electrical circuit. That R and C will be the, I mean suppose in the case of, I mean electrical circuit R and C will be the factors which will depend, suppose I have a circuit like this, I have a simple electrical circuit.

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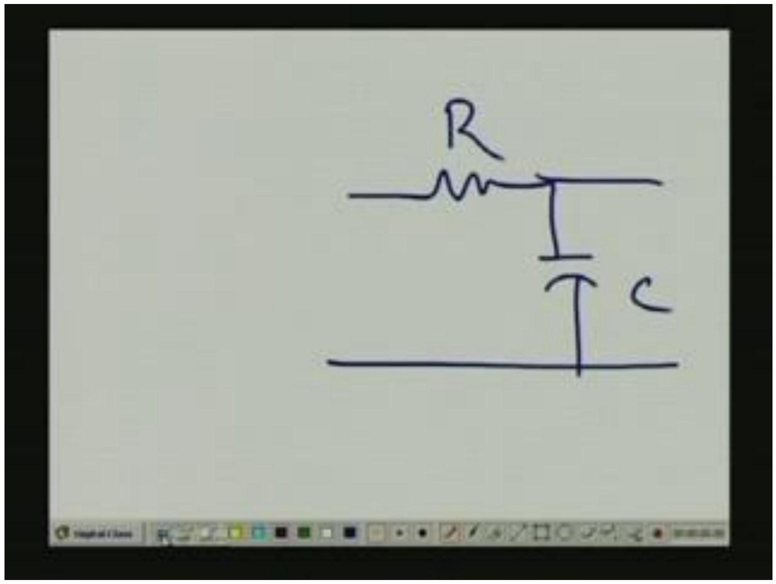

So, I have a resistance capacitance. So, if I draw, so the value, R and C value, so the physical constant will depend on the value of R and C.

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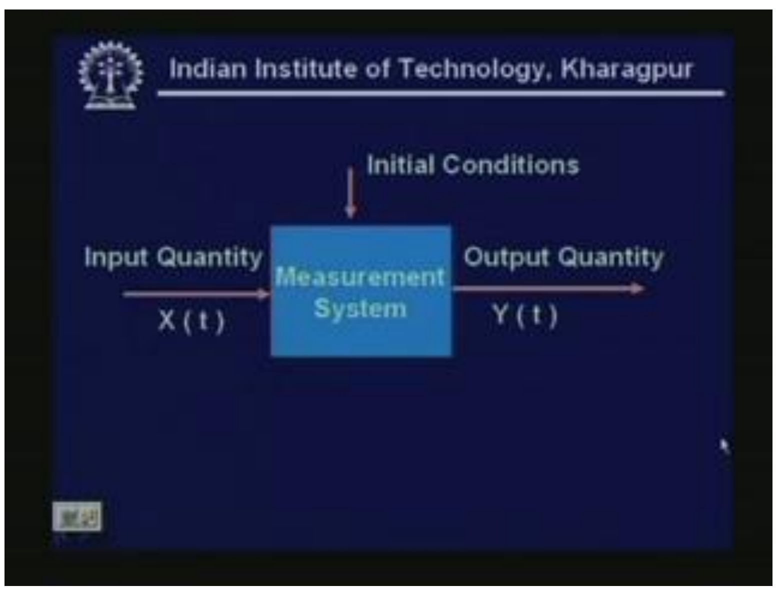

Now, here I will show you a block diagram of a measurement system. I have an input quantity X t, I have an output quantity Y t and I have some initial conditions. These are necessary; these initial conditions are necessary to solve the differential equation. In the case of first order instrument, it will be first order differential equation. Now, zero order instrument is the simplest instrument we have, but there are some instruments also, some sensors in that which are zero, zero order in nature.

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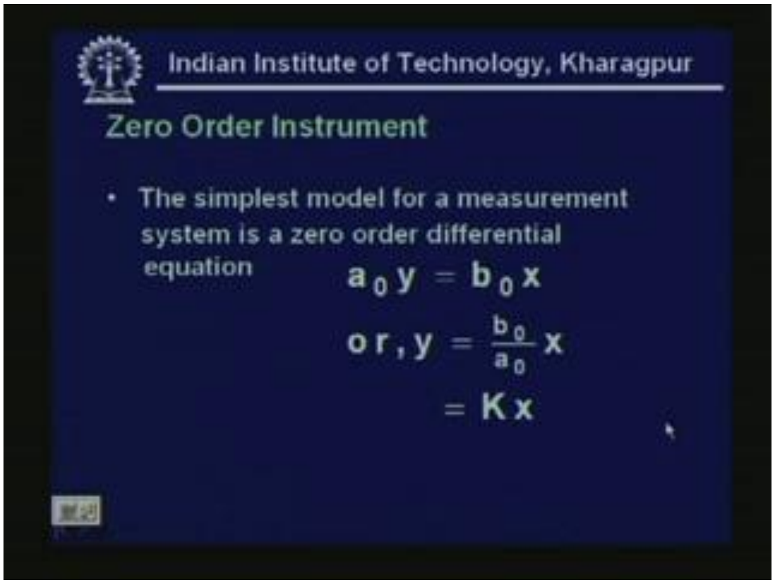

The simplest model of a measurement system is a zero order differential equation. It looks like this: a naught y equal to b naught x or y equal to b naught by a naught x, multiplied by x, equal to K x. You see, if you go back to our initial differential equation, it will look like, you see, this is our differential equation.

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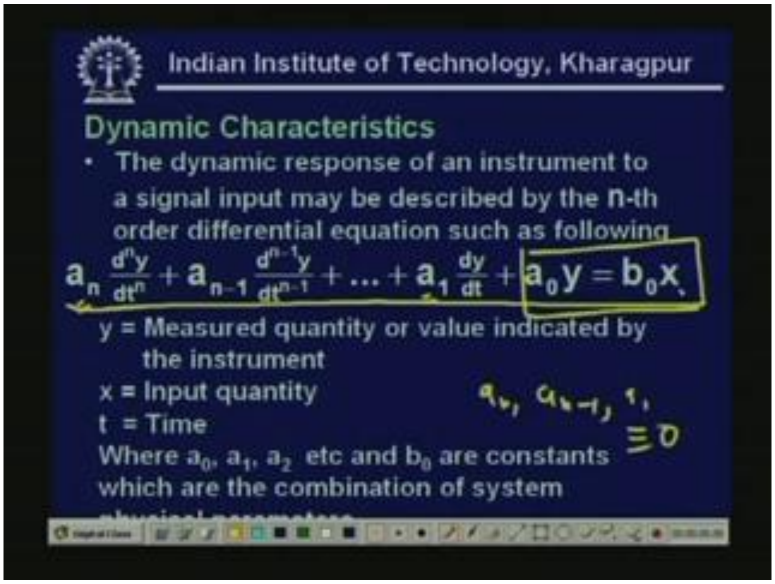

So, this differential equation, nth order differential equation if you put all zero that means from a n up to a 1 that means a n, a n minus 1 up to a 1, if you put all equal to zero, I will get the zero order instrument which looks like this. That actually I have shown just now. So, only this part if you take, it is a zero order instrument, right. So, this is our zero order instrument.

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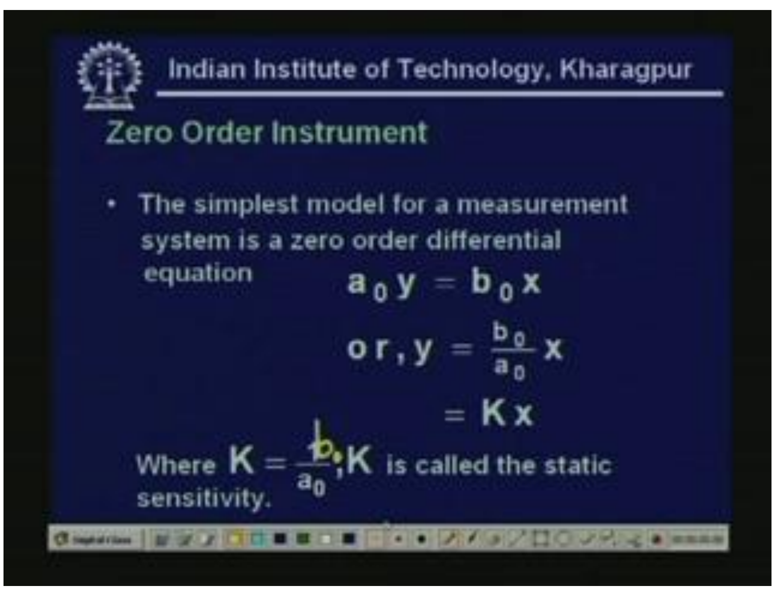

You can see here y output equal to b naught by a naught multiplied by x or equal to K x, where K equal to b naught by a naught. I am sorry, this will be and this will be b naught by a naught, b naught by a naught and K is called the static sensitivity of the system. K is called the static sensitivity of the system. Static sensitivity has a lot of influence. You will find the calibration curve of the entire system will depend on the static sensitivity.

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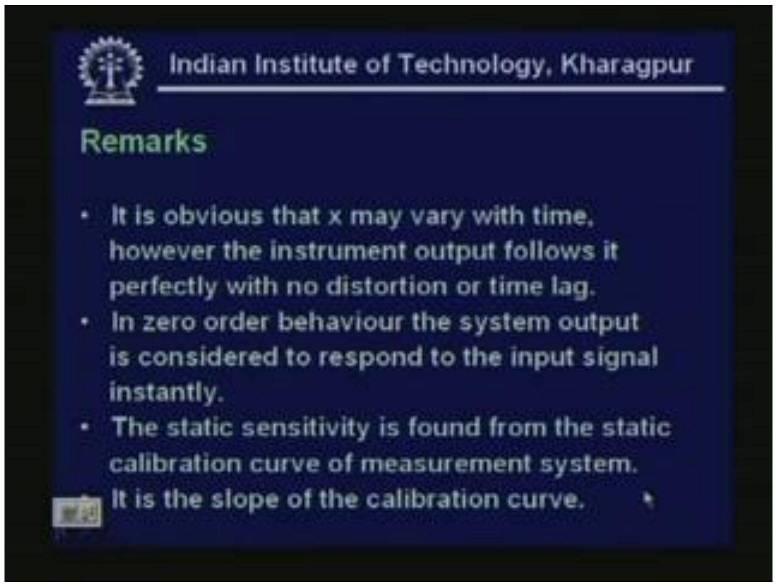

Remarks - you will see that it is obvious that x may vary with time. However, the instrument output follows it perfectly with no distortion or time lag. Whatever the input in the system, it does not matter. It will immediately appear at the output without any phase lag in the system. So, that is a zero order instrument. It does not matter what is the input. It might be step input, it might be ramp input, it might be your sinusoidal input. So, immediately it will appear at the output without any lag, without any phase change. So, that means it is a zero order instrument.

Now, in zero order behaviour, the system output is considered to respond to the input signal instantly. The static sensitivity is found from the static calibration curve of the measurement system. It is the slope of the calibration curve. How does it mean?

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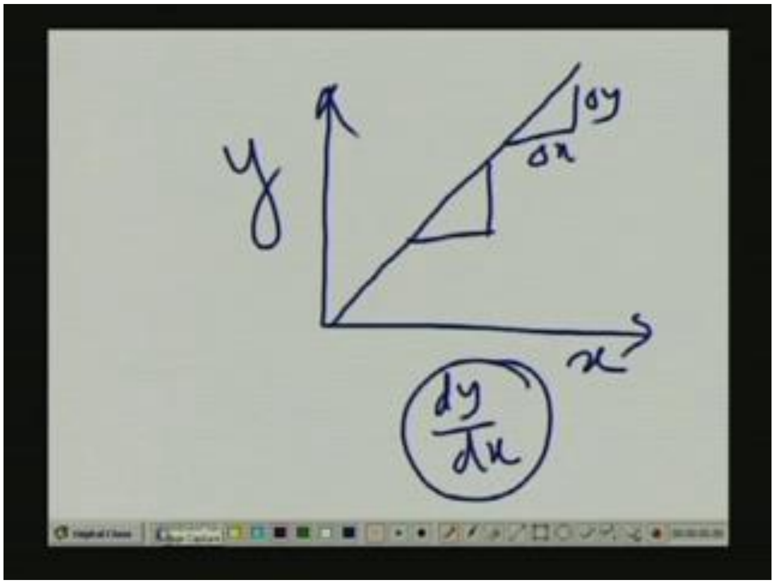

It means that if I look at it will be, suppose I have this characteristics calibration curve, this is x, this is y and I have a linear relation like this one. So, slope of this calibration curve, okay that is my dy by dx will give you the static sensitivity. So, this will give you the static sensitivity and since it is linear, you see anywhere the value of dy by, del y by del x, so if it is del x, it is del y, it is same.

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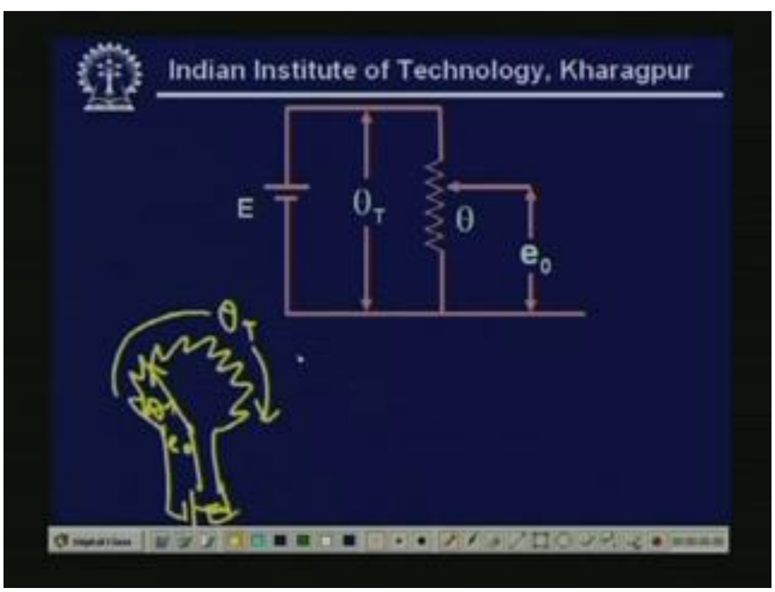

Now, I will take an example of a potentiometer. Potentiometer is a very important sensor you will find in, in all electronic circuits. Also, in instrumentation you will find it is used as position sensors; rotational positions, linear positions for all this different reasons it is used and in the electronic circuits, potentiometers are extensively used. Whether you call it potentiometer or you call preset or you call the rheostat, all are same. These are basically potentiometer.

Potentiometer is a three terminal device. You see here, this is our potentiometer. You can look here. So, I have a, supply excitation voltage I have given and total angular rotation of the potentiometer, because if you look at the physical shape, it looks like this. I have, the potentiometer looks like this, isn't it? Here, so I have a wiper or jokey. In the case of, in the case of potentiometer or preset we call it wiper, in the case of rheostat we call it jockey, but the principle is same. Both are zero order instruments.

Potentiometer is used in electrical, electronic circuits and rheostat is used actually in the case of electrical circuits and presets are used for, it is not for multiple use, but for limited number of ways if you use 1 or 2 or 3 times you can use preset which is basically nothing but a low cost potentiometer. So, I can say, the rheostat, potentiometer and the preset are basically same. So, you see here, the total rotation is theta T. So, I can, may be I have any position theta, so I am giving the voltage, battery voltage here, connected here. So, this is my e, right and I can measure this voltage, voltage between this point and this point. That will be my output voltage e naught. So, this can be e naught, right; this can be my e naught, clear. So, this will give you the, so total is theta T. So, this will give the, I mean symmetric view of a potentiometer.

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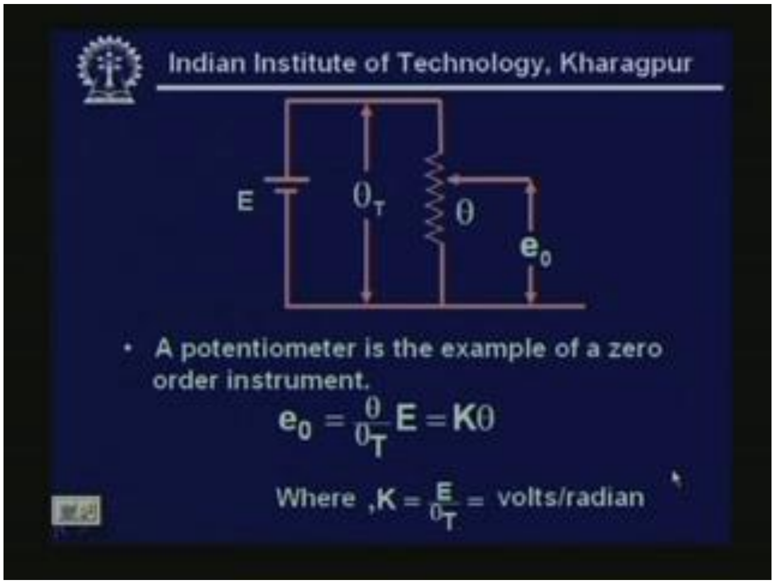

A potentiometer is the example of a zero order instrument and its equation looks like e naught equal to theta by theta T multiplied by a supply voltage or excitation voltage, which is equal to K theta, where K equal to E by theta T which is volts per radian. Because E is a constant, it does not depend on the position of the potentiometer. Theta T is the total angular … These are constant. For a particular potentiometer, you know this is constant. That is K equal to E by theta T which is volts on radian and this is the static sensitivity of a potentiometer.

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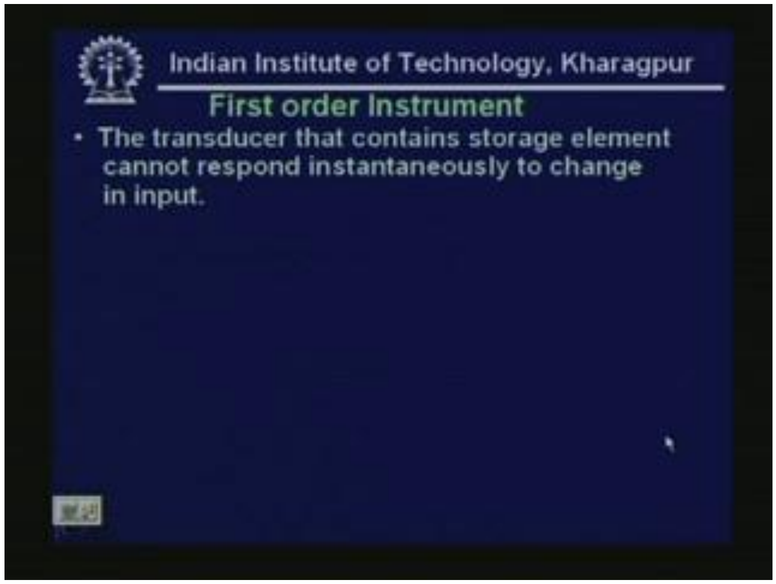

You can see here that the potentiometer, that it always should, should whatever the input it will get, immediately it will be shown at the output without any time lag or without any phase change. That actually happens in the case of potentiometer. That means whatever the inputs you are giving, it immediately appears at the output. I can use it as position sensors also, because as you can see the output voltage is directly proportional to the position. What is that position?

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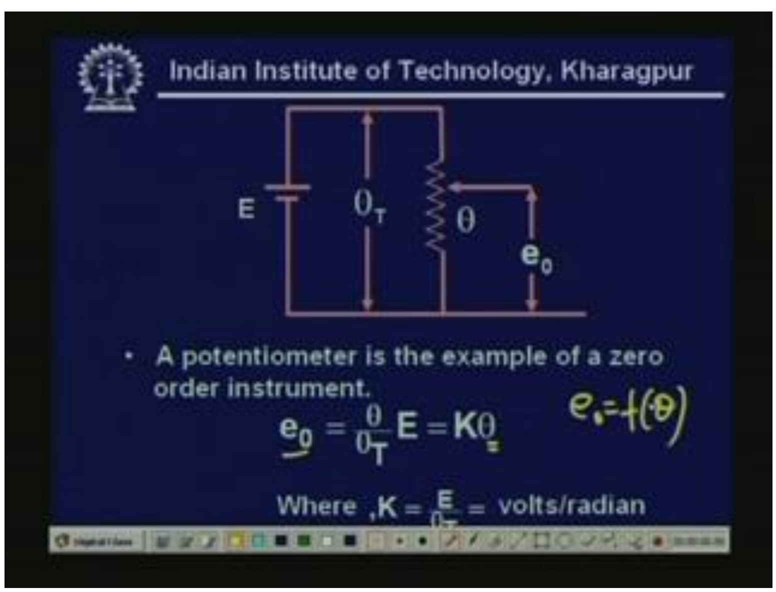

If I go back, I can see here, you see here that output voltage is directly proportional to theta. So, e is a function of theta, so which can be utilized to make a position sensor and it is a zero order instrument. So, obviously there is no phase lag or no time lag, phase change or time lag in the input output, okay.

Now, next we will consider the first order instrument.

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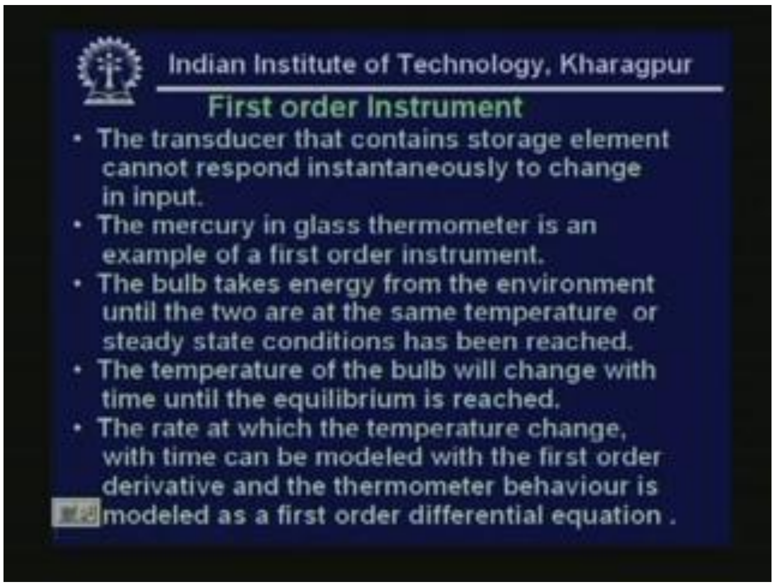

The transducer that contains the storage element cannot respond instantaneously to change in input. The mercury in glass thermometer is an example of a first order instrument. The bulb takes energy from the environment until the two are at the same temperature or steady state condition has been reached, right? The temperature of the bulb will change with the time until the equilibrium is reached. The rate at which the temperature change with time can be modeled with the first order derivative and the thermometer behaviour is modeled as the first order differential equation.

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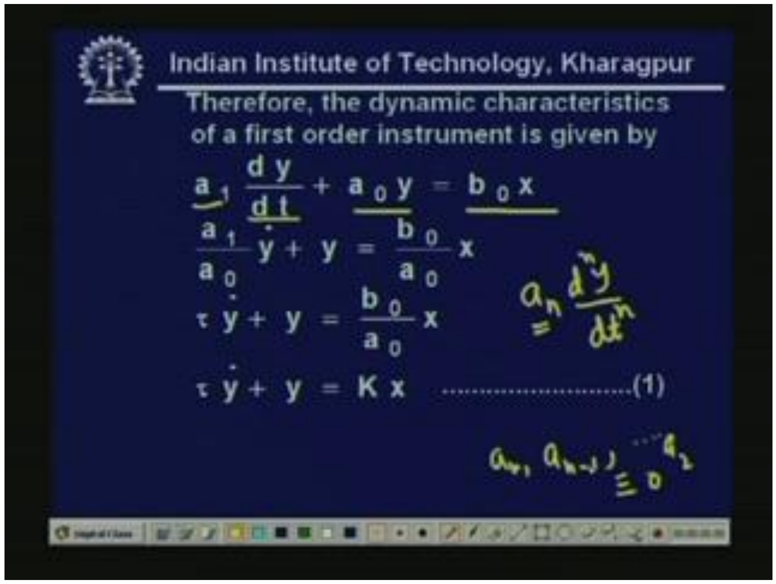

Therefore, the dynamic characteristics of a first order instrument is given by, looks like this: a 1 dy by dt plus a naught y equal to b naught x. This is similar. You see that if you remember, our nth order differential equation that a n, then d n y by dt n, all those things, if you put all the coefficients from, from a n to a 2 from a n, a n minus 1 so on, a 2 equal to all zero, I will get a first order characteristics equation or a first order instrument.

So, exactly we got that thing, you can see here.

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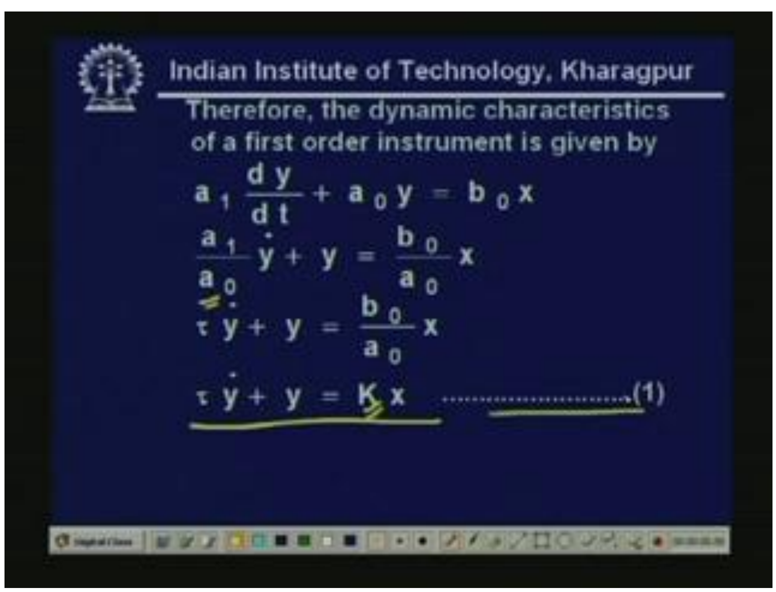

So, a naught, so you can see here, so that is we have multiplied entire equations by a 0. I got the equation a 1 by a 0 y dot plus y equal to b naught by a naught into x. So, dot we just replaced by dy by dt. So, tau, now I replace a 1 by a 0 equal to tau. Tau y dot plus y equal to b naught by a naught, upon a naught x. Finally, we have written tau y dot plus y equal to K x, right. K we remember.

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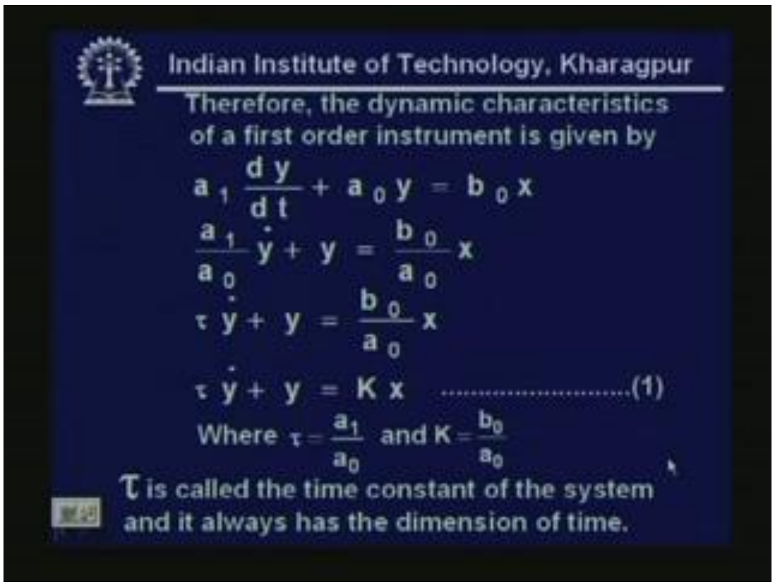

For the zero order instruments also K is a static sensitivity of the system and tau is the time constant of the system and this is the equation number 1, where tau equal to a naught upon a zero and K equal to b 0 by a 0. Obviously all this, I mean time constant of the system like, as you told, because if you take an example of a thermometer this a 1 upon a 0 and static sensitivity b 0 upon a 0, depends on some of the physical parameters of the system. What are those physical parameters? That means what type of, what is the size of the bulb, of the thermometer, materials which you are using as a thermometer liquid, all these factor will actually tell you the value of the tau.

If the, if I use a for an example, just from intuition you can tell if you take a large bulb, so that I, if I have a large, I mean length of the mercury, so in that case I can use, the time constants of the system will also increase. Similarly, the K also will be actually controlled by the physical parameters of the system and if you remember, our first, the nth order differential equations, we have written there clearly this value. That means a 0, a 1, b 0, all are the basically, depends on the physical parameters of the system. Tau is called the time constant of the system and it always has a dimension of time, quite obviously.

Now, now you slowly consider one by one, the inputs. Now, first I consider the step input.

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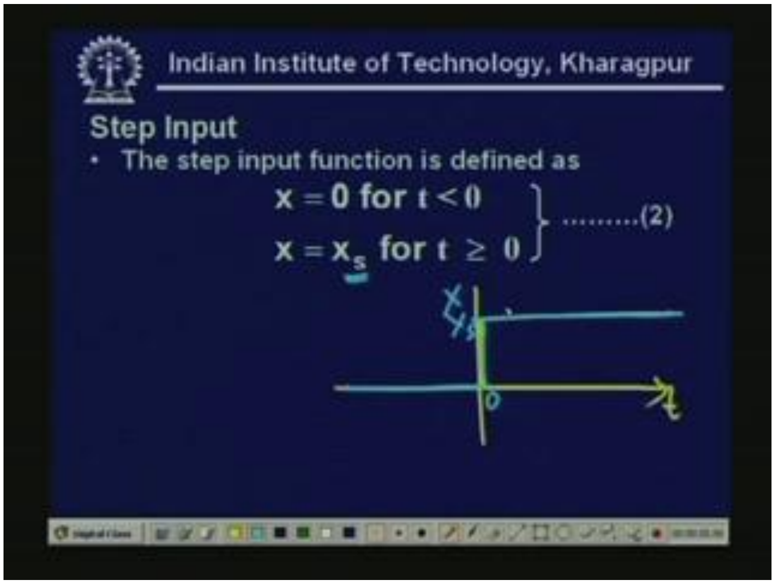

Now, step input is very common to the system. Especially in the electrical systems, step input is very common. Step input function is defined as X equal to 0 for t less than 0 and X equal to x s for t greater than equal to 0. This is equation number 2. These set of equations, we have given the equation number 2. These are necessary to solve the differential equations, right. How does it look? It looks very simple. It looks like that my input, …. time axis if I plot, so it looks like this. Then, suddenly it goes. This is my input. If I take different colours, well little more, so it will, my input looks like this and go like that.

So, this is our zero. That means at t less than 0, it is X equal to 0 and this is our X and for t greater than 0, X is a constant. x s, x s actually I want to signify the steady state value, because this will be our steady state value of the input. I want, my output will be always exactly equal to x s, because if it is x s, …. the steady state value, right.

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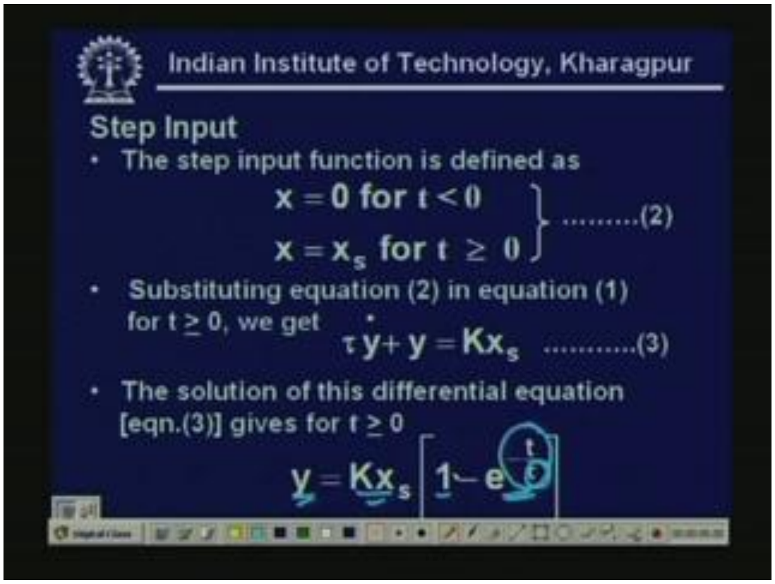

Now, substituting equation 2 in equation 1, for t greater than equal to 0, we get tau y dot plus y, tau y dot plus y equal to Kx s. This, I have given the equation number 3. The solution of this differential equation, equation 3 gives for t greater than equal to zero equal to, y equal to Kx s 1 minus e to the power minus t by tau, right; very carefully, it is e to the power minus t by tau. It has lot of significance, the meaning of this and KX s. As you can see that as the time goes, what will happen to this that as the time goes, this is a transient part, this will die out. So, ultimately we will find that the output will follow the input, because at that time this will be y equal to KX s into 1. This becomes zero, so this equal to 1.

This means that whatever the static sensitivity multiplied by the steady state value of x will give, will be the output, right. But, you will see here that this tau carries a lot of, I mean lot of influence, at what time you will get that y will be equal to KX s. If the tau is large it will be, it will take long time to die out and if the tau is small it will take a very quick time that it will reach the final steady state value. Output will reach the final steady state value, because it is minus term is there. So, let us play it so that, this we have done in flash, so it will be better if you look like this.

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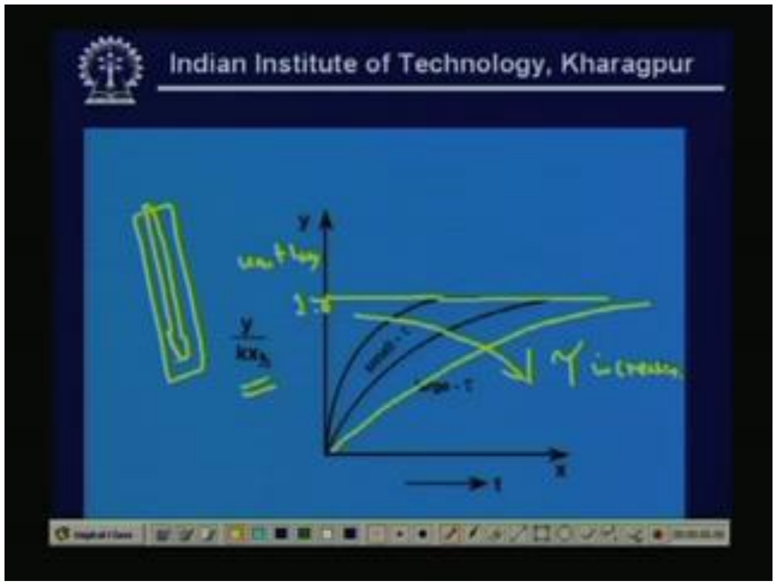

x-axis is my input, y is, my y-axis is my output y by KX s. Now, interestingly, you see one thing. I have normalized it, because this is very much necessary, okay. Let it finish, then we will start again. Hopefully it has finished. Now, see here that I have taken this is my y by K axis. I have taken normalized value. So, y will be the unit less quantity. So, since it is normalized, so it is unit less, right and this value, maximum value will be always 1. Since it is now 1.0, 1.0, so this will be always 1 and you have plotted t in the x-axis.

As you can see that, as I increase the value of the tau, it is taking longer and longer time to reach the final steady state. This is our final steady state value. So, as I increase the tau, so I will get more and more time. So, I can have, if I still further increase the tau, so it will go like this. Suppose I have the, one of my example of this that suppose I have a thermometer; thermometer, a simple mercury in glass thermometer, fine. It has some, depending on the type of I mean liquid which you are using, the type of the thickness of the glass and all those things it will have some value of the tau, because physical parameters will control the value of tau.

Now, what will happen, you see here that if the, if I for protection suppose that mercury and glass I put on a steel sleevings that means I have mercury in glass manometer, sorry mercury in glass thermometer, so I put on a, for protection I put on sleevings, so that it will not break. For that type of situations we will find that the time constant of the system will increase. Later on you see that we have various types of, I mean thermometers. We have thermocouple, we have thermistor, we have RTD. You find that in the case of RTD, because of its large, huge size you will find it is the, time constant of the system is very large compared to the thermocouple or thermistor. So, time constant will control that how quickly your output will reach the input. So, in this case you will find that the tau increases that means your time constant increases, right.

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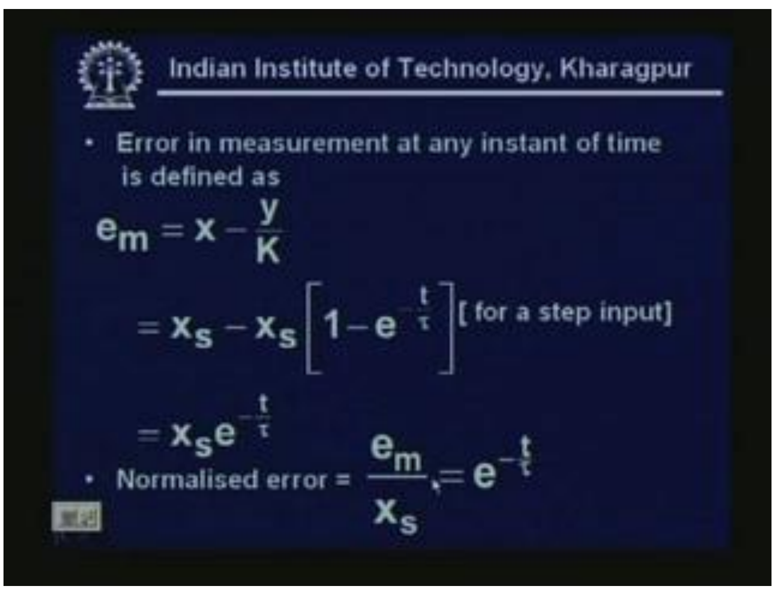

Error in measurement at any instant of time is defined as e m. We have used the subscript m that is to tell that it is an error in the measurement x minus y by K. So, it will be ultimately X s minus X s multiplied by 1 minus e to the power minus t by tau and it is for a step input equal to X s into e to the power minus t by tau. We have seen that, you see that as the time goes, so this error will be smaller and smaller, because it is exponential term and there is negative, so obviously as the t will be large and large, so this will die out. At the initial stage when t is zero, so it has large value. Its value is almost equal to, it is exactly equal to X s, the steady state value. As the t goes, so it will die out.

So, it will be better visualized if you run in a flash. So, the normalized error if have I normalized, if we make it unit less, so it will be e m by X s equal to e to the power minus t by tau. It will be better visualized, I mean in a flash it will, you can see here.

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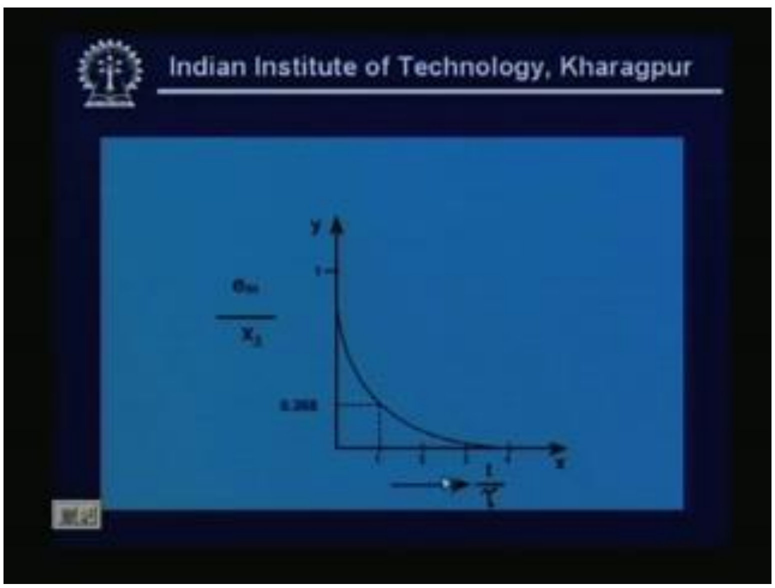

So, x-axis we have plotted t by tau instead of t. It does not matter, because it is the normalized ….. Because t is a dimension of time, so we have now plotted normalized value. It does not matter, because it is t just multiplied by some constant factor of 1 by tau. You can see here that at a very low value of t or at very low value of t by tau or when it is zero, initial stage we have just given step input, my output is maximum. This is our error, error is maximum. So, as the time goes, so obviously the error will fall down. It is exponential decay and you will find that as the time goes, so my error will get decreased or the normalized error is getting small and small and ultimately it will die out. So, this is very important in the sense you will find that in a system that you must know that when the error will die out.

In some cases I cannot; my entire reading will be erroneous, because from a sensor, the reading from the sensor, because if I take reading, suppose if I take the reading of a sensor after one of t by tau, so it is totally error. So, I cannot accept. So, I should take the reading when the t by tau is 4 or t is 4, when 4 seconds, suppose if it is, I mean if it is, I mean if I plot it in, in terms of t instead of t by tau or if it is suppose tau is 1. So, after 4 seconds I should take the reading, okay.

Now, we should consider the ramp input. Ramp input is not very usual. I mean in the instrumentation system, it is very difficult to give a ramp input, but electrical system it is very necessary. You find in some of the cases you have to give the ramp input. One of the good examples of the ramp input is saw tooth waveform.

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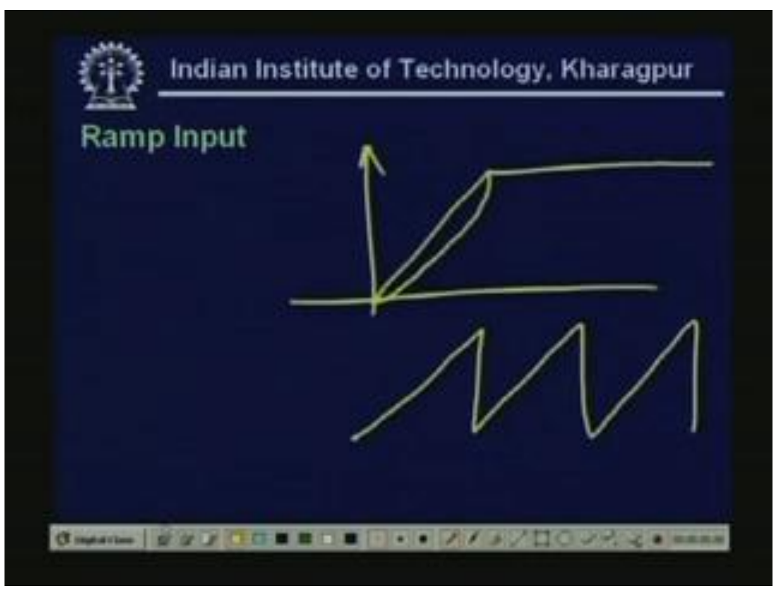

See, you are aware of that in the case of, always in the, this is the saw tooth waveform you give in many electrical circuits, this type of waveform. So, one of the common examples of the saw tooth waveforms when we give in the, in the oscilloscope, in the cathode ray oscilloscope to measure some voltages. Voltages, amplitude we are giving between the plates y, particle plates and the saw tooth waveforms we are giving in the x plates, so that to synchronize or to make the wave stationary.

In instrumentation system, saw tooth, I mean raw, I mean ramp input, you will find it is not very usual, even though we will consider the ramp input. In many cases we will find that instead of ramp input, the input which I will give is basically a terminated ramp input. In the case of step input also, sometimes very difficult to give in instrumentation system. So, it will appear actually the terminated ramp input like this one instead of step input. So, in this case, you must consider this portion which looks like a ramp, okay. So, for that reasons we consider here the ramp input.

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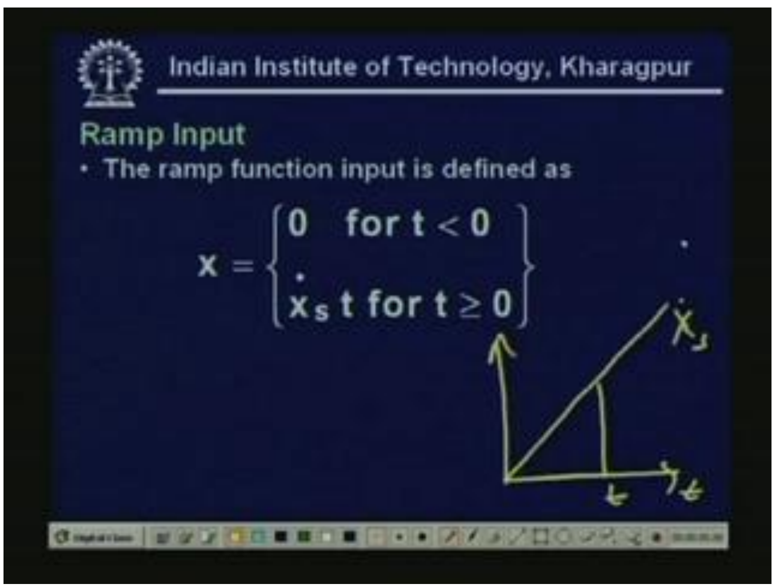

Now, the ramp function input is defined as X equal to 0 for t less than 0 and X equal to X s dot into t for t greater than equal to 0, right. It looks like this that if the time is varying, so it is going on increasing like this, right. This is t. So, this is at any step time, what will be the value of X that can be multiplied by t. So, this is our X s, X s dot. Slowly it is increasing, right. So, this is our ramp input.

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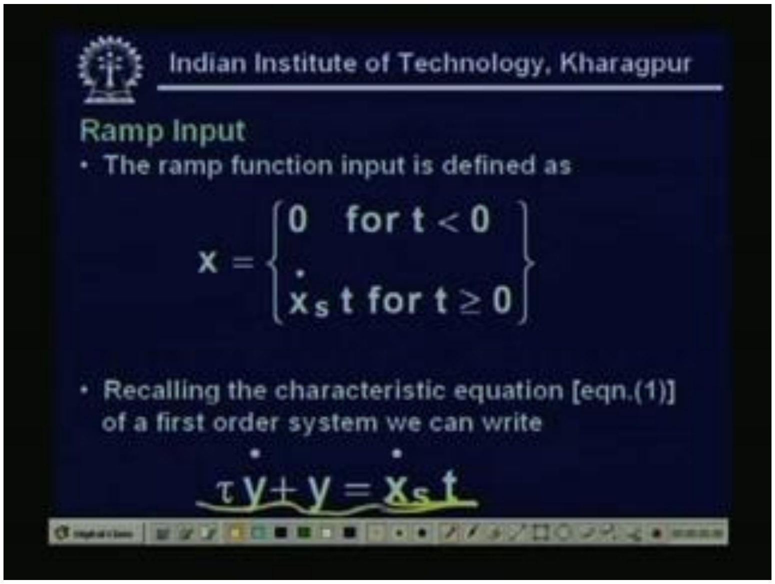

Recalling the characteristics equation, equation 1 of the first order systems, we call that first order systems here, we just replace X by X s dot t. X s dot means dX s by dt into t. So, tau y dot plus y equal to X s dot into t. This is our first order differential equation for ramp input. We will solve this equation and see how the output will come, how the response will come and how my, how my error in the measurement system will come?

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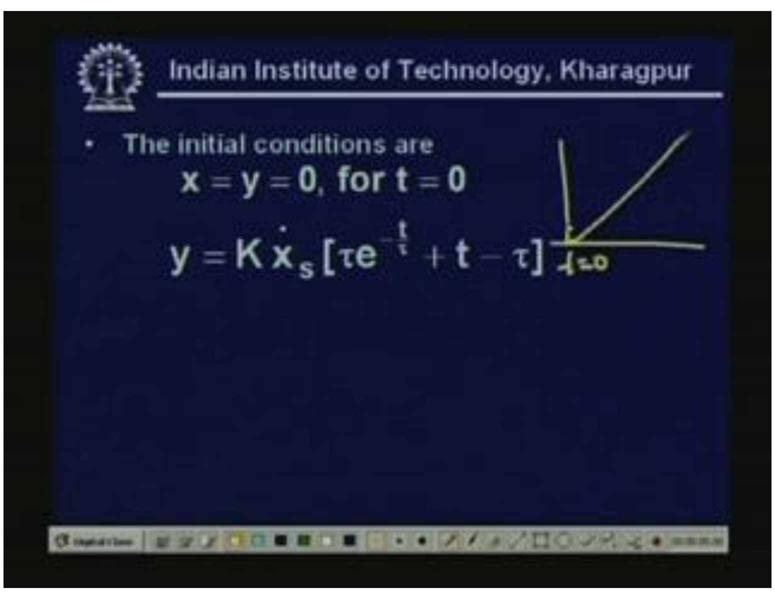

The initial conditions are x equal to y equal to 0 for t equal to 0. That is quite obvious. That means we may assume that the ramp input, it does not matter that it will give an input like this one. So, at t equal to 0, so quite obviously I can say that the x is equal to 0 and y is equal to 0.

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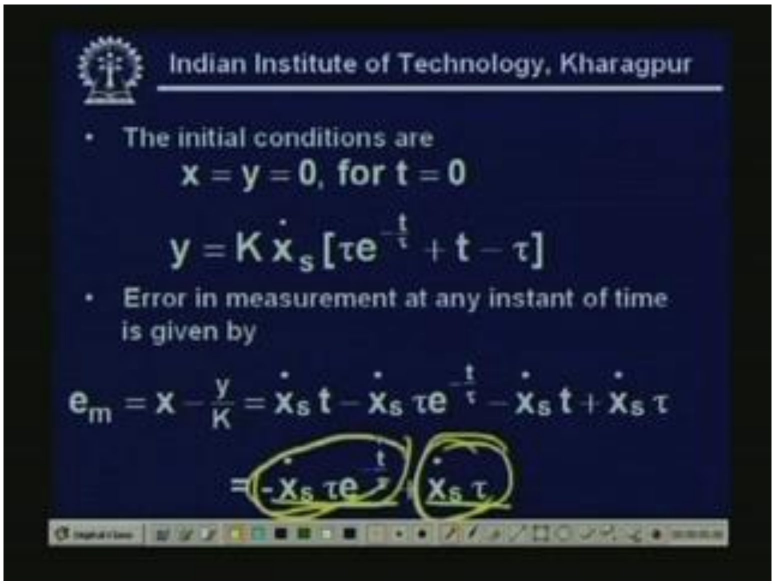

And my output will look like y equal to KX s dot tau e to the power minus t by tau plus t minus tau, right. Now, error in the measurement at any instant of time will be given by, see, e m equal to X minus y by K equal to X s dot into t minus X s dot tau e to the power minus t by tau minus X s dot into t plus X s dot into tau equal to minus X s dot tau e to the power minus t by tau plus X s dot into tau, right. So, let us go back. So, this is our, you see, there are two parts. One is the transient part and there is a steady state part. Why it is transient? You see that, this as the time goes, this function will be, this function is going to reduce. As the time goes on, so this will become, almost approaches to zero, but this will remain as it is. So, this is our steady state error and this is our transient error, right?

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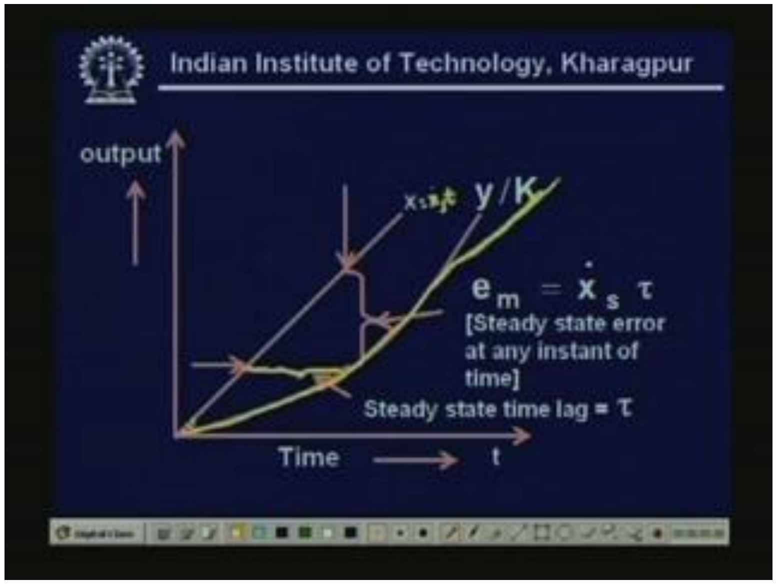

You see, if I plot it, it will look like this that steady state error at any instant of time, okay, hopefully it is finalized, it looks like this. So, x-axis we have plotted time and this is our output. These I have plotted. This is my input, so this is equal to actually X s dot into t, right and this is y by K. I have plotted the output. Actually this should be parallel, I mean it is, I cannot draw it nicely. I think it should be like this then. It should be parallel to the input.

Here the steady state time lag you can see; it is the steady state time lag of the system. Initially time lag is very small, but as the time goes, so we have a steady state time lag which is tau and steady state error also you can set. The initial stage it is, steady state error is very small.

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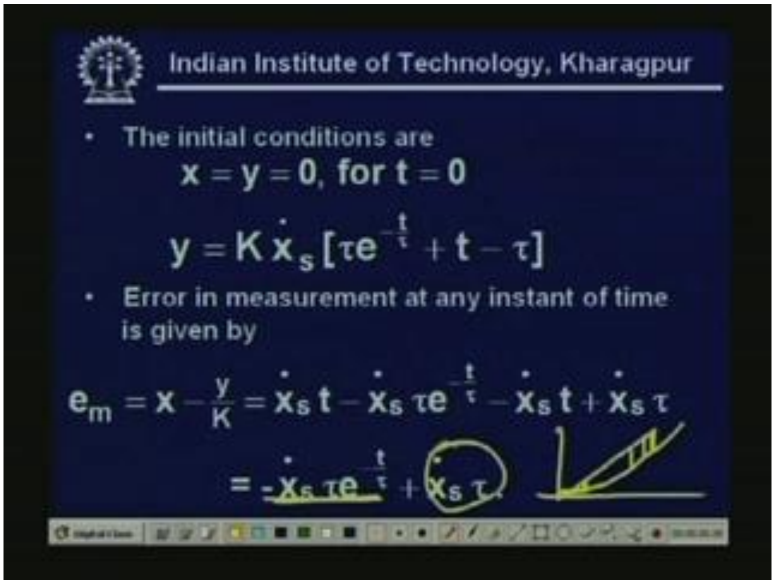

Why? If you look at, if you go back, if I go back to the previous slide, you see here, you have a negative term here. So, that will contribute, so this will become almost zero for some value and as it goes, as the time goes, so this will become insignificant, so this will be dominant. So, that is the reason we got the characteristics like this. Here, characteristics, this is my output and this is my input and this is, my output will look like this. So, error initially is very small and as the time goes we have a steady state error which is X s dot into tau.

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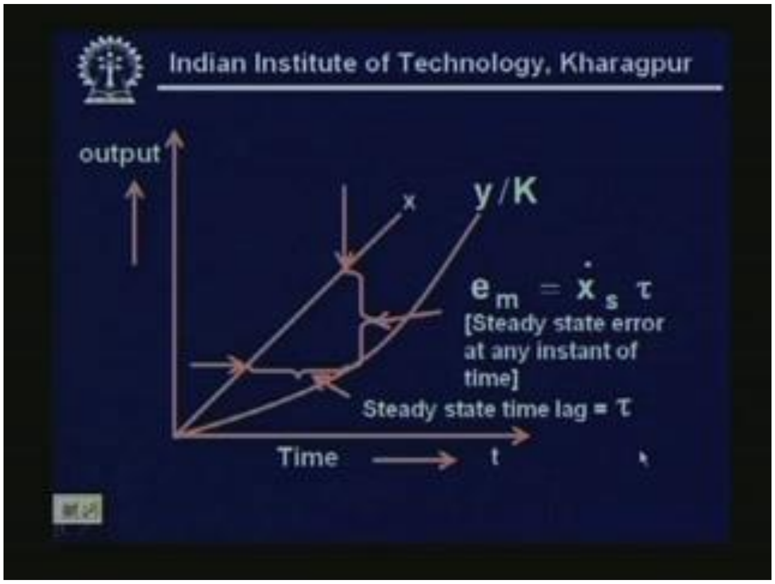

And interestingly, you see this steady state time lag is always tau. So, it means that steady state error will be at any instant of time, any instant of time X s dot into tau and so it is, time lag is tau.

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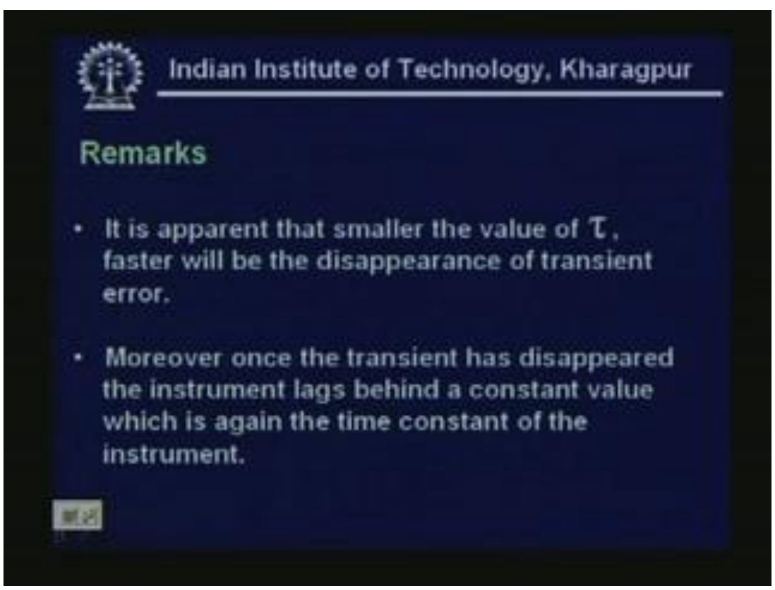

What is the remarks of a, I mean for a system with a step input? It is apparent that the smaller the value of tau, the faster will be the disappearance of the transient error. This is everywhere I found that the, if the, in the case of step input also you find that if the tau is small, so I will get, immediately I will reach the final steady state value. If the tau is large, it takes long time, right? Long time means what? The steady state error will take long time, to make the steady state error zero. It is our goal for any instrumentation system or any sensor that the steady state error should be zero as quickly as possible.

Now, it is apparent that the smaller the value of tau, the faster will be the disappearance of the transient error. Moreover, once the transient has disappeared, the instrument lags behind a constant value which is again the time constant of the instrument, tau. That is already we have seen, right?

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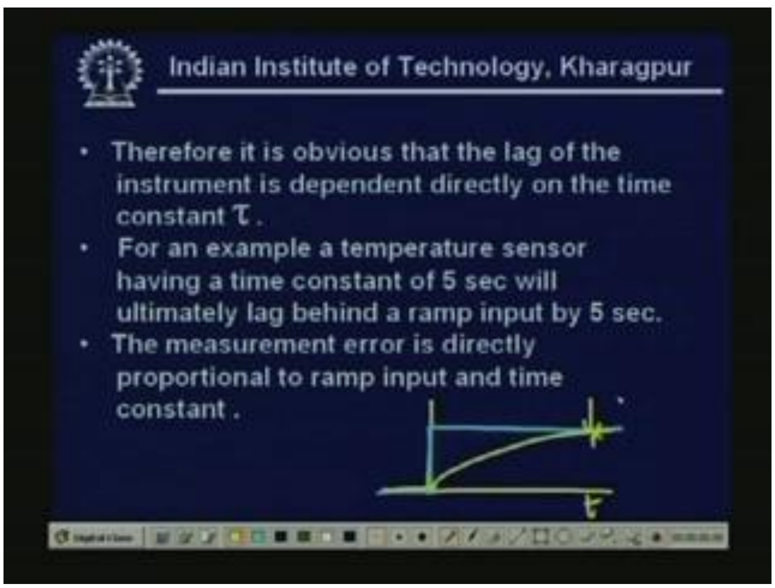

Therefore, it is obvious that the lag of the instrument is dependent directly on the time constant tau. For an example, a temperature sensor having a time constant of 5 second will ultimately lag behind a ramp input by 5 second. Initially lag will be small, but after sometime we will find the lag will have steady state value, say it will be 5 second. So, whatever the time constant of the system, our lag also will be the same. So, the measurement error is directly proportional to the ramp input and time, okay. This is also very important.

We have seen that the measurement error depends on the ramp input as well as the time constant of the system. So, if the time constant is small, the measurement error will be small. If the time constant is large, the measurement error will be large and you see, unlike the step input this measurement error will remain forever. If you look at the, in the case of step input, I have an input, I am giving an input like this, right, I am getting output like … So, after sometime, there is no steady state error. This will be zero.

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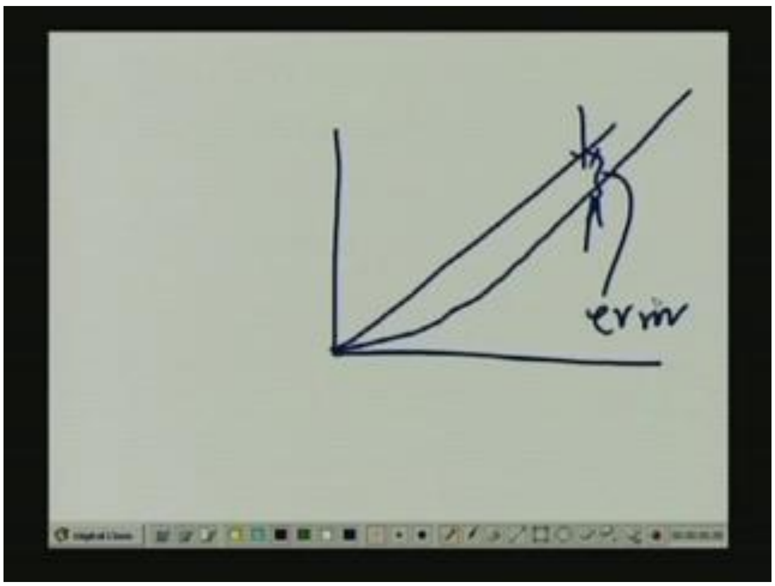

But, whereas in the case of ramp input, you see here, if I take a white page, in the case of ramp input, so there is always a steady state error, initially small. So, it is like that, so there is always a steady state error. This is our error. So, it will remain forever in the case of ramp input, since it is a time varying input function, right.

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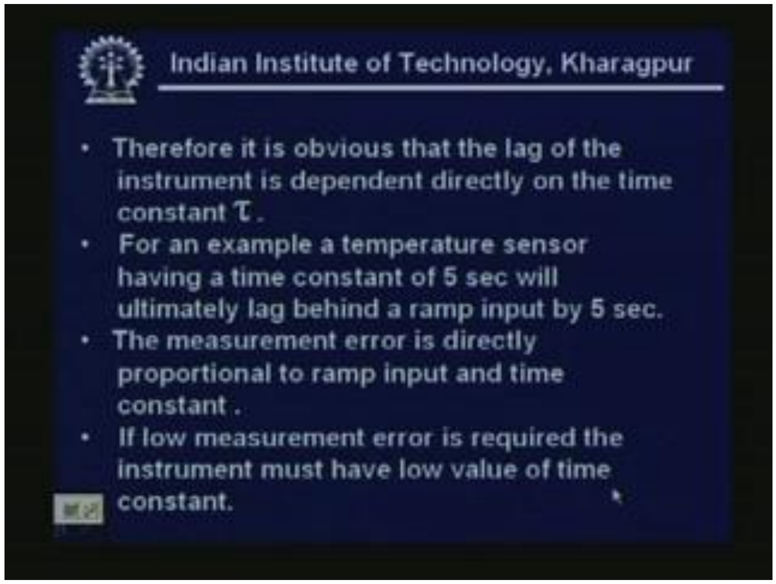

If low measurement error is required, the instrument must have a low value of the time constant; it is quite obvious, if the measurement error low is required. It is not necessary in the case of the step input and all these things, because ultimately it will reach …. Only time constant will control the value that means after what time the steady state error will be zero. Whereas, in the case of, I mean ramp input, I mean if you want to reduce the steady state error, it will be, always you have to choose the value of the time constant smaller and smaller.

Now, I will consider the sinusoidal input. Sinusoidal input is common that the ambient temperature variations or some, in some particular applications like the biomedical applications, you will find, when the patient is infected with malaria parasites, we will find the temperature usually for the first two three days that that particular time only the particular temperature appears, so very periodic. So, ambient temperatures also you will find the daytime there is some temperature, nighttime some other temperature and interestingly, the time constant of the system is obviously very large, right? So, sinusoidal input, we will face a lot like vibration analysis and all those things.

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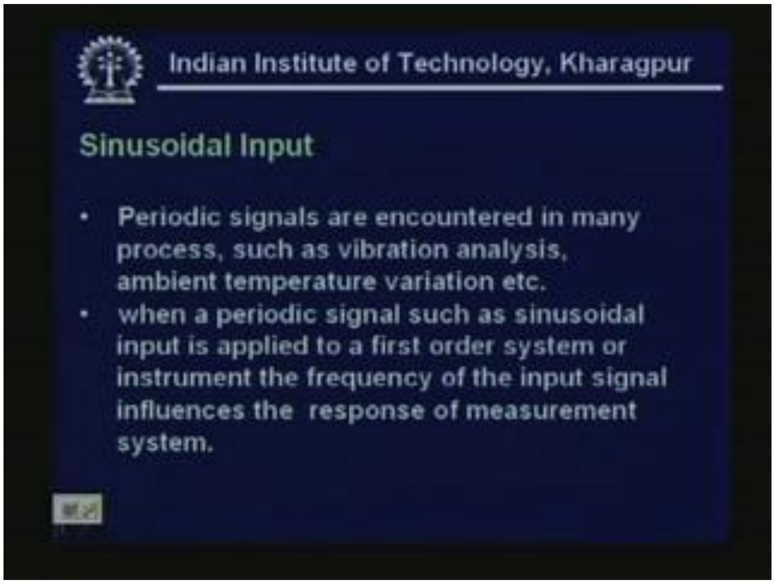

You see, the periodic signals are encountered in many process such as vibration analysis, ambient temperature variation, then, etc. When a periodic signal such as sinusoidal input is applied to a first order system or instrument, the frequency of the input signal influence the response of the measurement system. So, the frequency will control a lot of parameters; we will find that. So, frequency is of prime importance in the case of sinusoidal system.

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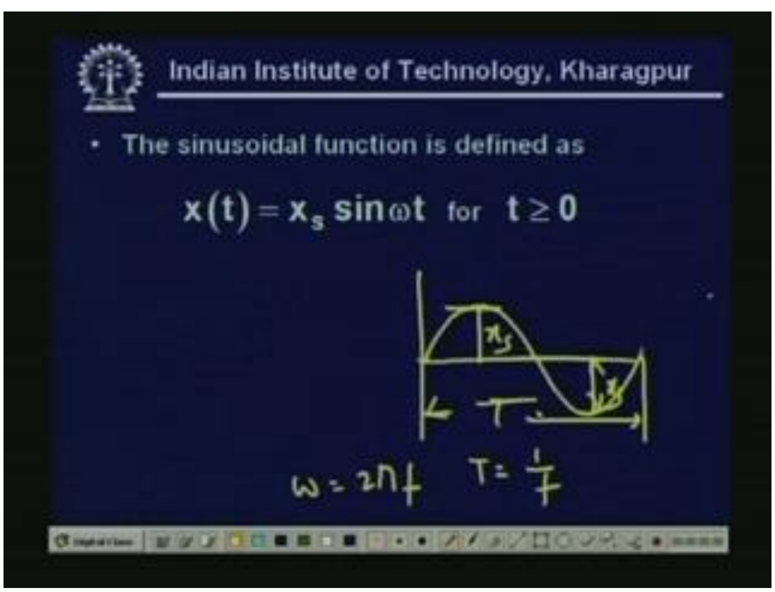

The sinusoidal function is defined as, we can write the sinusoidal function like the X equal to X s sin omega t; very simple, we all know. That means if I have a sinusoidal function, sorry, so this is my X s, okay. So, this is total time period, T where T equal to 1 by f and omega equal to 2 pi f, circular frequency equal to f and T is the total time period and time period 1 by inverse. So, this is X s value on both side, this is also X s, so for T greater than equal to zero and for T less than zero, it is zero. So, this our sinusoidal input.

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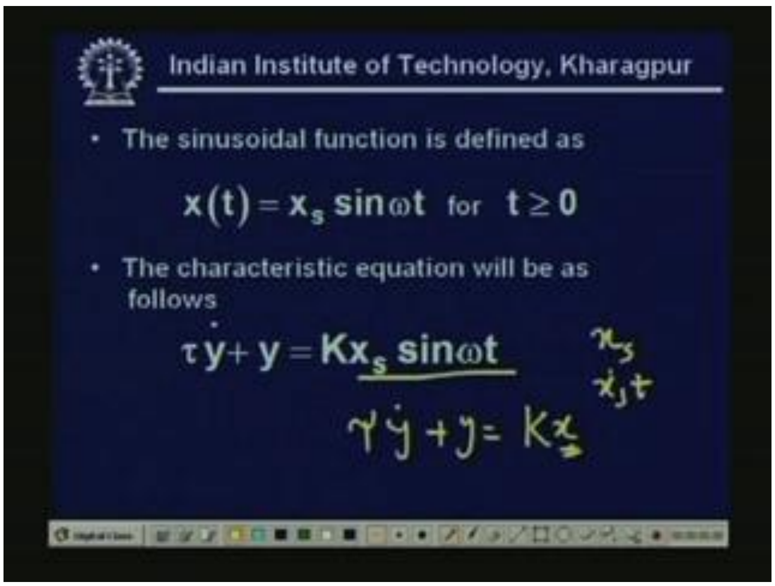

So, the characteristics equations will be as follows. Obviously, I have just replaced on the right hand side x by Kx s. Actually it was there, so I have replaced tau y dot plus y equal to Kx s into sin omega t. Initially our basic equations, do you remember in the very beginning of the lecture, it is tau y dot plus y equal to Kx s. In the case step up, in the case of step input we have replaced with K equal to x s. In the case of ramp input, we have replaced x equal to x s into t. In the case of sinusoidal input, we have replaced x by x s sin omega t, right?

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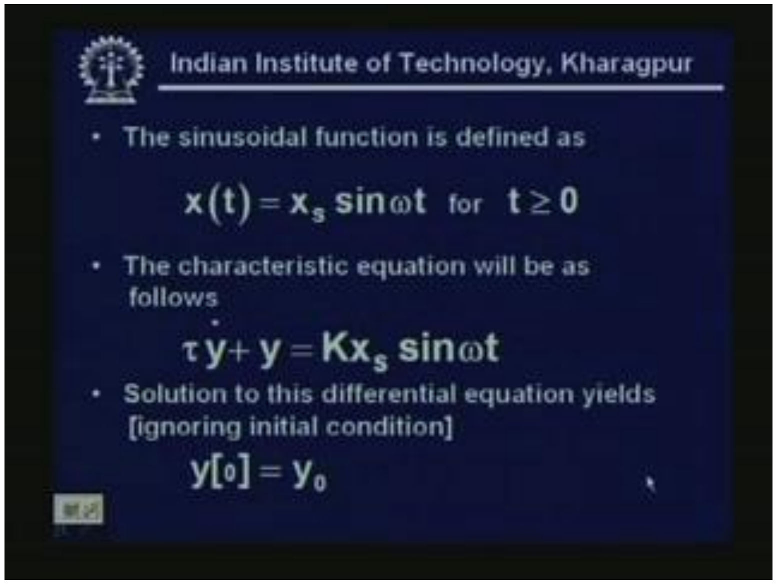

So, the solution to this differential equation yields, ignoring initial condition that y zero equal to y zero, get Kx s upon under square root 1 plus omega square into tau square sin omega t minus phi, where phi equal to tan inverse omega tau.

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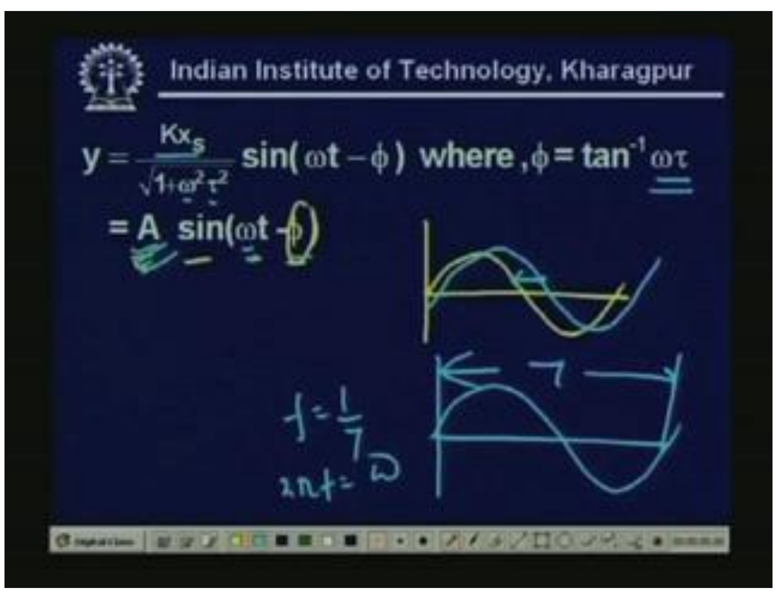

This we can write as equal to A sin omega t minus phi; phi is the phase shift of the input signal. There will be, there will be some phase shift that means that my, if my input is like this, so my output will look like some other, so it will look like a, phase shift like this one, right? So, there is a shift in phase. Sometimes, in many instrumentation systems you will find that this phase shift is a nuisance. That means we have to kill this phase shift. In the case of leading phase angle we have to use a lag network and in the case of lagging phase angle we have to use a lead network.

Even though the circuit is very simple with one resistance and one capacitance, I can make a lead lag network, right? So, this is my output, you can see here. So, the phase change is coming and as I told you earlier, you see, this omega is very important in the case of sinusoidal signal. That means omega equal to 2 pi f. What is omega? Omega is, if I draw it here again, so if my input signal is like that, so this is my T, total time period, so f equal to 1 by T and 2 pi f equal to circular frequency omega. So, this omega will influence a lot. It depends on, the phase shift also influenced by both omega and tau.

You can see that the phase shift is equal to tan inverse omega tau and the amplitude of the signal which is A, which is I mean K into x s under the square root 1 plus omega square tau square, we will find that if that omega is large, obviously your value of y also will be small, value of A that means the amplitude also will be small, right?

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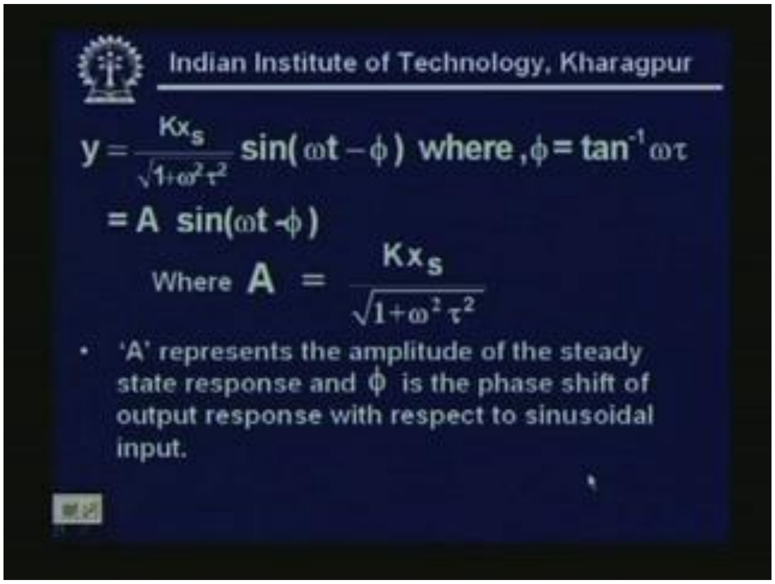

Where A equal to Kx s upon under the square root 1 plus omega square into tau square that A represent the amplitude of the steady state response and phi is the phase shift of the output response with respect to sinusoidal input, right?

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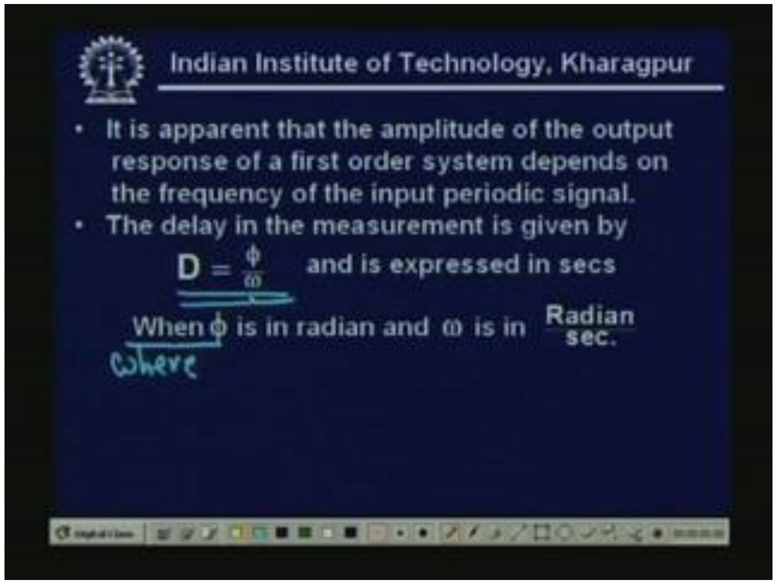

It is apparent that the amplitude of the output response of a first order system depends on the frequency of the input periodic signal, right? The delay in the measurement is given by D equal phi by omega and is expressed in seconds, where phi is the radian. I am sorry, this will be, this will be, where phi is the radian and omega is in radian per second, right? Now, delay in a system is important. In many cases we will find we cannot allow delay. In some systems, we will find that we have to, I mean we have to accept this delay, because this delay is a natural phenomenon; like a phase shift, it is a natural phenomenon. Only thing we have to make the delay possibly to, so that the delay will be same for all the frequencies. We cannot make delay zero, but if the frequency, for all the frequency range the delay is same, so that will satisfy our goal.

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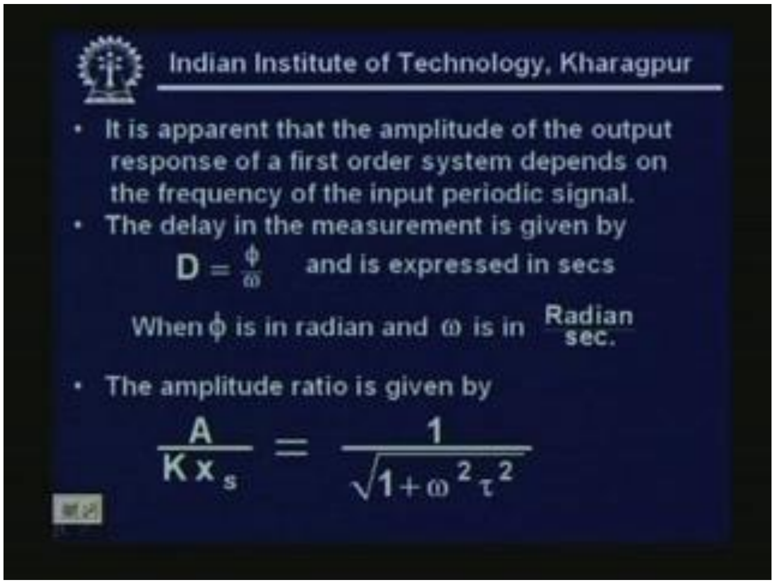

The amplitude ratio is given by A by Kx s equal to 1 upon under the square root 1 plus omega square tau square, we can see here.

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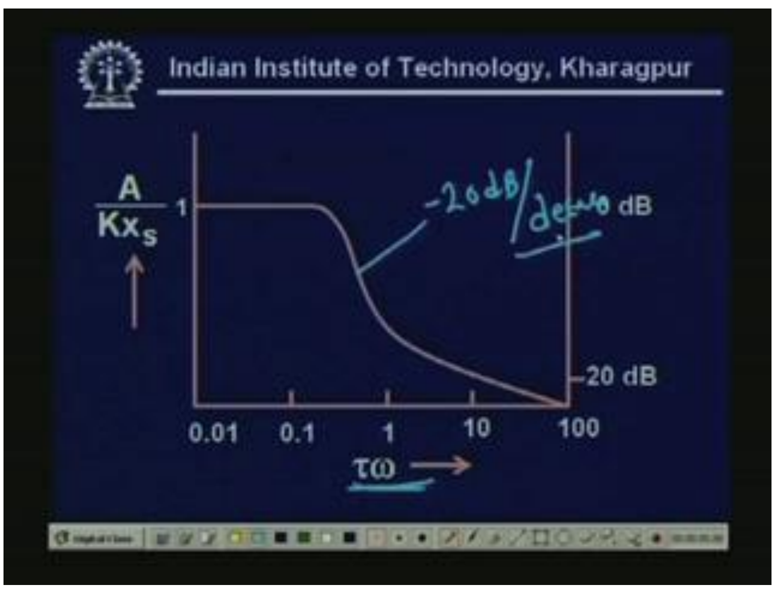

Now, we have plotted here, you see that, plotted normalized value of the output, okay versus omega tau. Instead of omega, we have plotted omega tau. We want to make it also unit less that is the reason we have plotted. It is dimensionless. Omega is in radian per second, this is seconds, so it is dimensionless we have plotted. So, this is our response of a first order system or the frequency response of a first order system, okay with sinusoidal input, right? So obviously, since it is normalized, this output will be 1 and this will be 20 log 10 of A by Kx s will be zero degree, right?

You can see here that if we increase the frequency, so our, there is a roll off and for the first order system, this will be minus 20 db per decay, right? So, this is a roll off and that is the phase. So, it is the flat response, after that it will start to fall.

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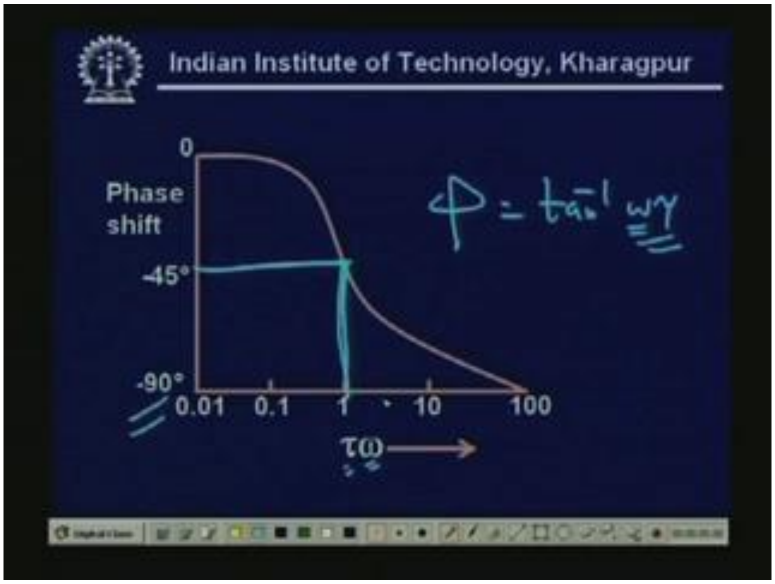

Now if I, if I plot the phase, you will find of a first order system with, I mean, sinusoidal input, it looks like a phase shift of 45 degree. You see the phase shift of 45 degree at omega t equal to …, because you see, you remember that phi, phase shift is tan inverse omega tau. So, if omega tau equal to 1, so phase shift will be obviously 45 degree. So, it is 45 degree at omega equal to 1, right and at very low frequency it is zero phase shift as the frequency is becoming high. We have a, this omega is high …… I mean, tau is constant. Omega is getting higher and higher, so if it is infinite, obviously I will get a 90 degree phase shift, right. So, this is about the amplitude and the phase response of a, of a first order system with sinusoidal input.

Now you see, this is important, because that I told you that earlier that omega plays a key role in the case of response, in finding the response of the first order and second order instruments with the sinusoidal input. So, the frequency will determine or the tau will determine at what frequency you can use your instrument, because in other way, because if the, if you fix the omega, the tau will be determined and if you know that my signal has some particular frequency, I can tell that what should be the value of tau, for that reason that I will get a reasonable response of the input to the system.

Now, remarks will appear like this.

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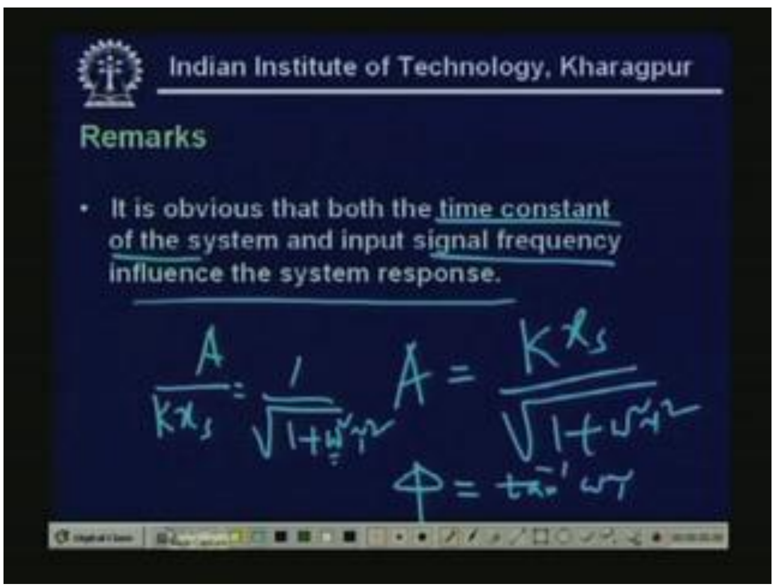

It is obvious that both the time constant of the system, both the time constant of the system and the input signal frequency influence the system response. So, the system response will depend on both omega and tau. We have seen, you see, why because you see, the amplitude also, you remember A equal to Kx s root over 1 plus omega square tau square. Isn't it, we have seen; so, this is my response or if I take a normalized, so it will be A by Kx s equal to 1 upon root over 1 plus omega square tau square. So, as we say that both the time constant of the system and the input signal frequency which is omega, influence the system response, because it will control the value of the amplitude or the normalized output as well as the phase shift, which basically depends on tan inverse omega tau, right?

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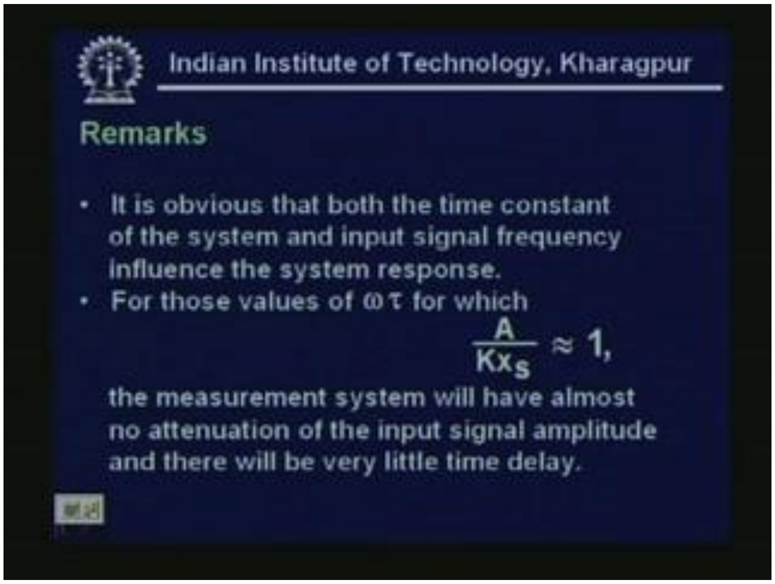

For those values of omega tau for which A equal to A by Kx s is almost equal to 1, the measurement system will have almost no attenuations of the input signal amplitude and there will be very little time delay. That is always we want that we have shown the response. Isn't it?

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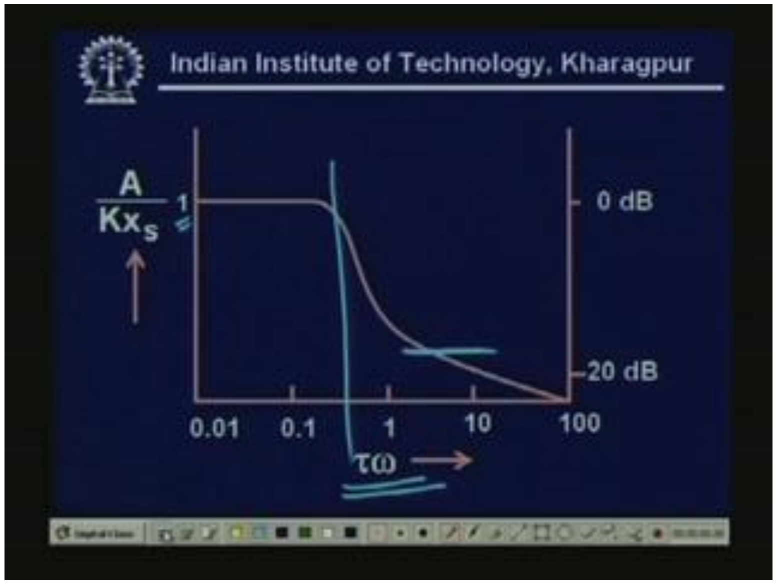

If I go back and see, our response we look at, for the frequency when I am getting the response here 1, so that is this desirable property. I cannot make the measurement at this region. I want to make the measurement in this region only, right, isn't it? So, this will be my frequency response of the operation, so that the omega tau has a lot of influence.

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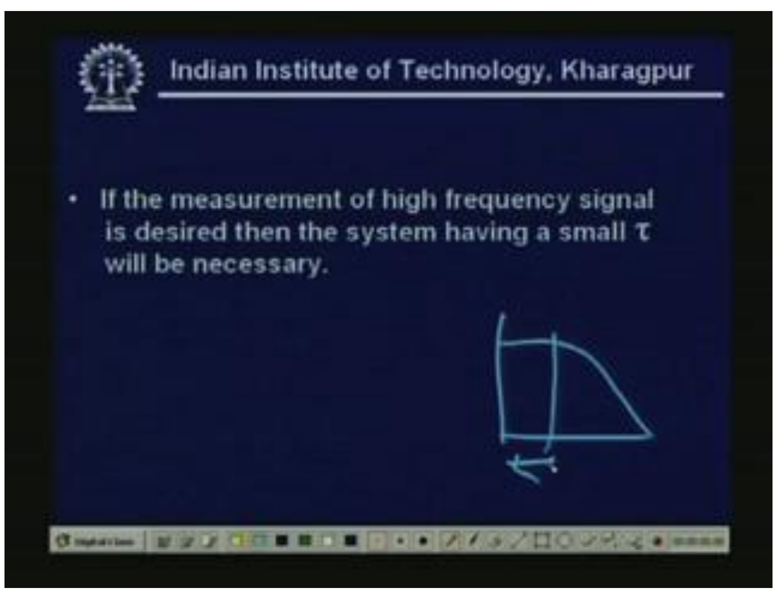

Now, if the measurement of high frequency signal is desired, then the system having a small tau will be necessary. Quite obviously, because our response we have seen again that the response falls down. So, if I want to take the response at the higher frequency, I must use a small value of tau, so that the omega tau will be, I can accommodate omega tau, will not be large, so that I can accommodate my frequency in this region, right?

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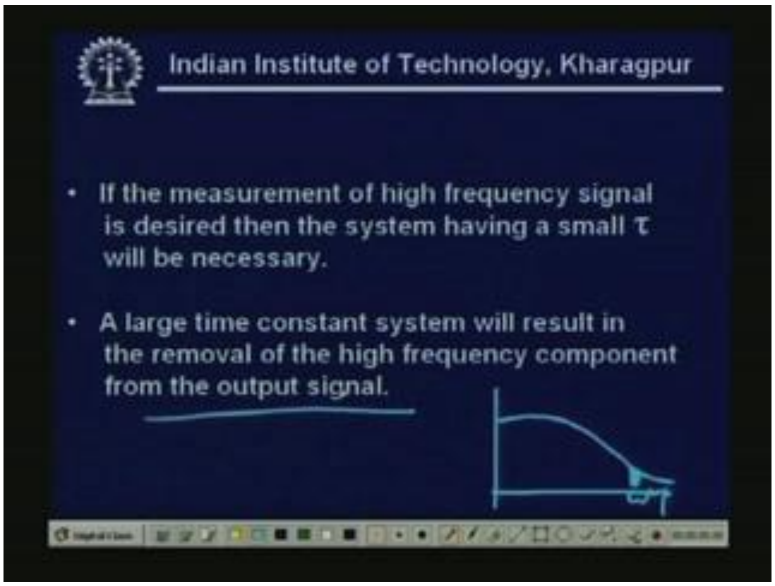

A large time constant system will result in the removal of the high frequency component from the output signal. Quite obviously, I mean if the frequency is high, we have seen that the, if we look at the plot, right, so that what will happen? That means I have a plot like this. So, you see, here it is falling down. So, if the omega tau is large, so if the tau is constant, what will happen? That the frequency component of the output signal, so it falls. If I want to take the reading here, so obviously my output attenuations will be quite large. So, I won't get any output, right. So, that is we are telling; a large time constant system result in the removal of high frequency component from the output signal, right?

Now, you see that in measurement systems, so the first order instrument is, even though is not very common, but there are some instruments which you will find first order. But suppose, if I take a mercury in glass manometer, I mean thermometer, it is a first order instrument and the other one if we consider that the mercury in glass manometer, then we can consider as a second order instrument. Now, first order instruments and second order instruments the difference is we will find that as I told you that if you look at the output, if you know the outputs of the systems and if I can or if I have a calibrated output, in that type of situations I can tell that what is my output and if my differential equations I have approximated as the first order systems, I am getting the desired output or the output which is supposed to I get, because if it is calibrated, I must know what is the output. So, I can tell the system is first order.

See, if i am not getting, say I have to go for higher order instruments, for a second order instruments or third order instruments, but in the entire course we will find that we will concentrate only on the first order and second order instruments. If the system is non-interactive, so I can go for a higher order instrument. Suppose I have a thermometer which is cascaded with a second order recorder, so in that case it will be a, I will, suppose I have an example I am giving.

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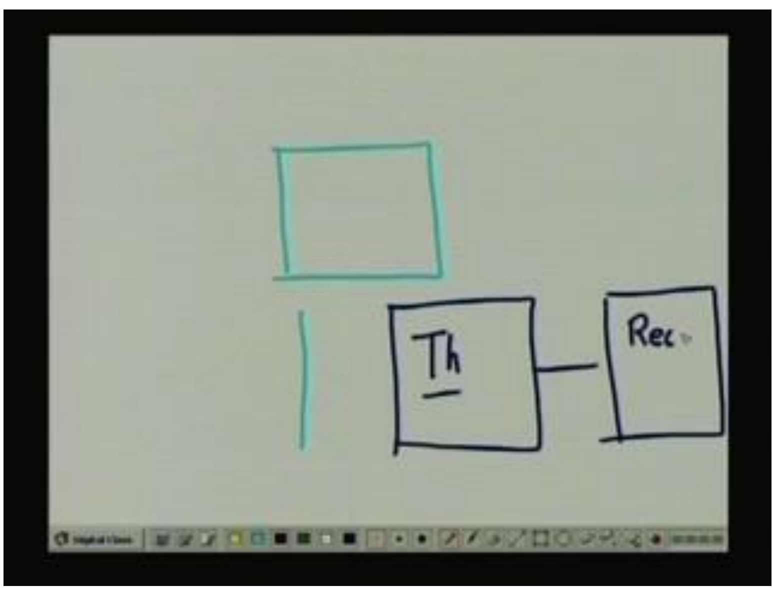

Suppose I have a, suppose I have a first order instrument, what happened? I have a first order instrument cascaded with a second order instrument. Suppose it is a thermometer and I have a recorder. So, in that case it will be, if it is non-interacting, I can consider it as a second order third order instrument. But mostly we will consider these two separately, first order and second order instrument as a, separately.

So, this we have considered the, here in this we have considered the dynamic characteristics and the first order instrument and these dynamic characteristics of the first order, I mean of the instrument, not for the first order, for the second order instruments will be considered in the lesson, lesson 3 of this course, industrial instrumentation. This ends the lesson 2 of industrial instrumentation.

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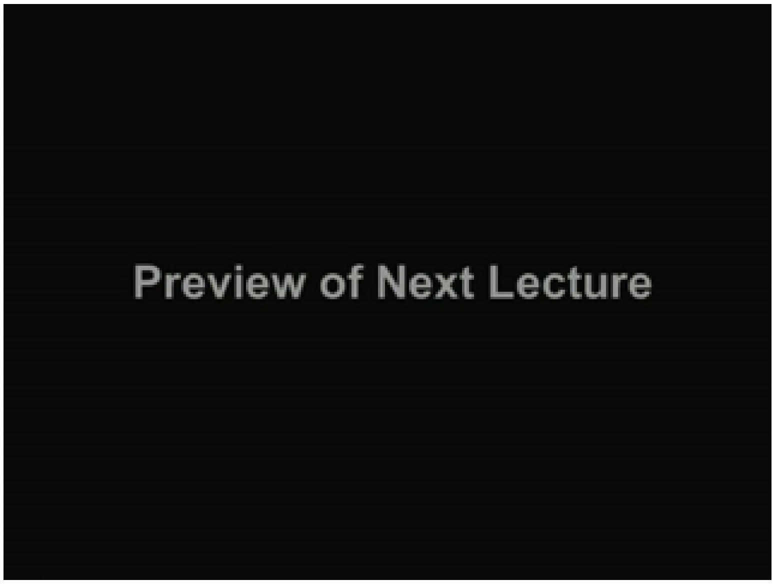

This is lesson 3 of industrial instrumentation and we will continue with the dynamic characteristics of, of a system especially second order systems. Already we have discussed about the first order system. Now, we will consider the dynamic characteristics of a second order system.

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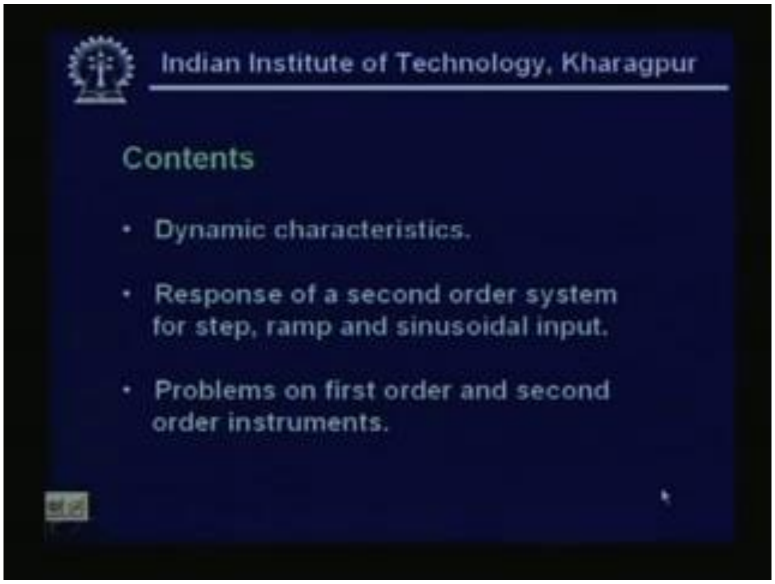

Now, contents of this lesson are the dynamic characteristics, the response of a second order system for step, ramp and sinusoidal input, where we did it for the first order instruments we will do for the second order instrument. Also, we will solve some problems on the first order and second order instruments.

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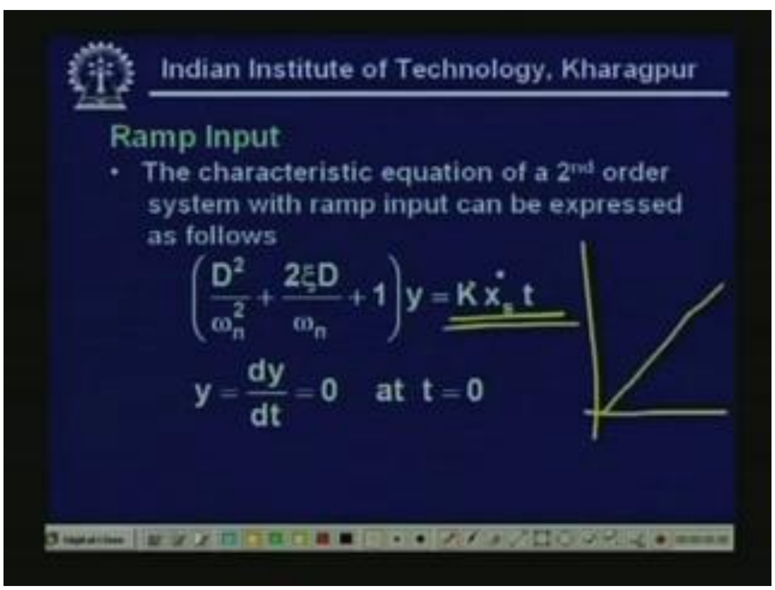

If I look at the ramp input, ramp input we have seen, characteristic equations of a second order system with ramp input can be expressed as follows. Already we have discussed the ramp input in the case of first order system. So, we are taking same three inputs, so that we can compare putting these two side by side. D square upon omega n square plus 2 psi D by omega n plus 1 y equal to Kx s dot into t. y equal to dy by dt equal to 0 at t equal to 0. This is initial condition, so we have discussed this before also. What does it mean? …. x s dot, but my input looks like this.

This is our ramp input. We have discussed thoroughly these when we discussed the first order system, right? Now, this we can, I mean do it in the case of second order systems. So this is my input. This constant K, as it happened before also, the x s dot into t, right? It is continuously changing with time, so x s dot, so I made it unit less quantity. So, these are our initial conditions.

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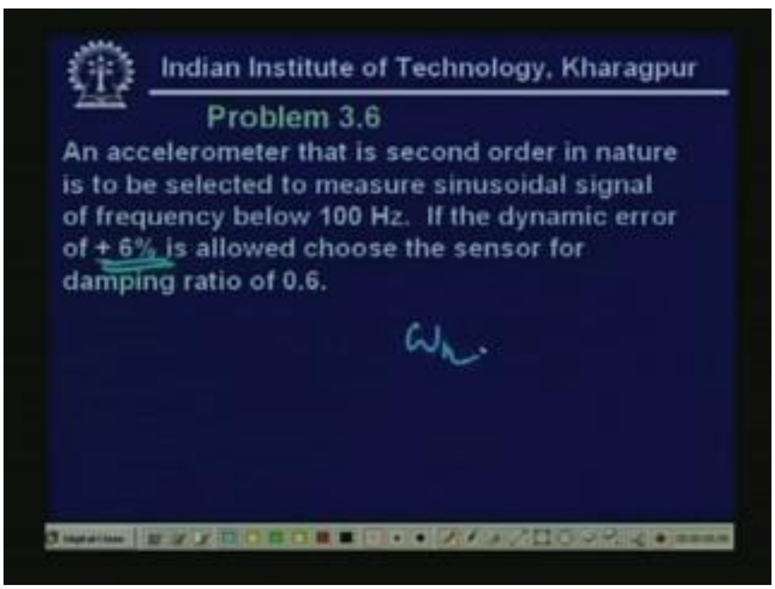

Now, the last problem; we will find in the second order system, so there is no question. So, if it is second order system we have to take both plus and minus of the dynamic error. That is the reason we have given. So, you have to take both. That means you have to take both plus or minus of 6%, so accordingly you will keep the value of the omega n. So, the natural frequency of the sensors you have to find here. You have to find the value of omega n here. So, you try to solve these problems and the solutions will be given in the next class and I will remind you that, here you see that the problem number 3.4, the instrument is first order, problem number 3.5 also it is a first order system and whereas, problem number 3.6 is a second order system. So, with this I come to the end of the lesson 3.