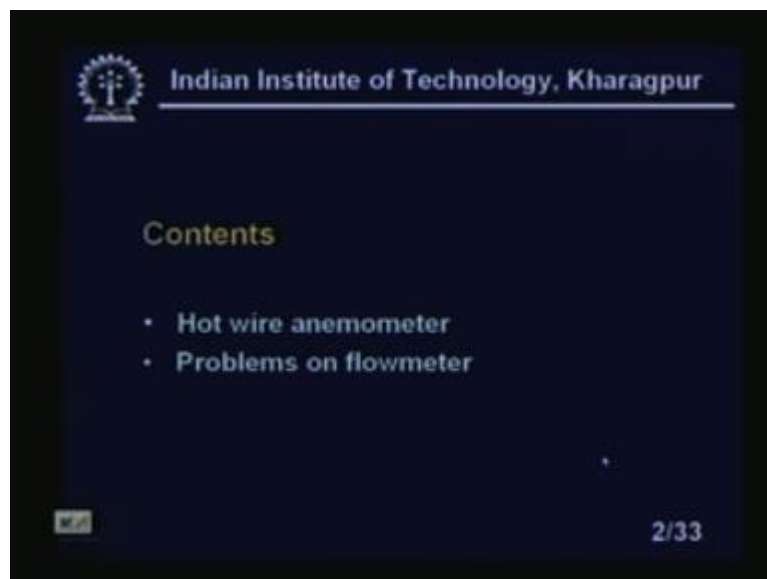


**Industrial Instrumentation**  
**Prof. Dr. Alok Barua**  
**Department of Electrical Engineering**  
**Indian Institute of Technology – Kharagpur**

**Lecture - 16**  
**Flowmeter – V**

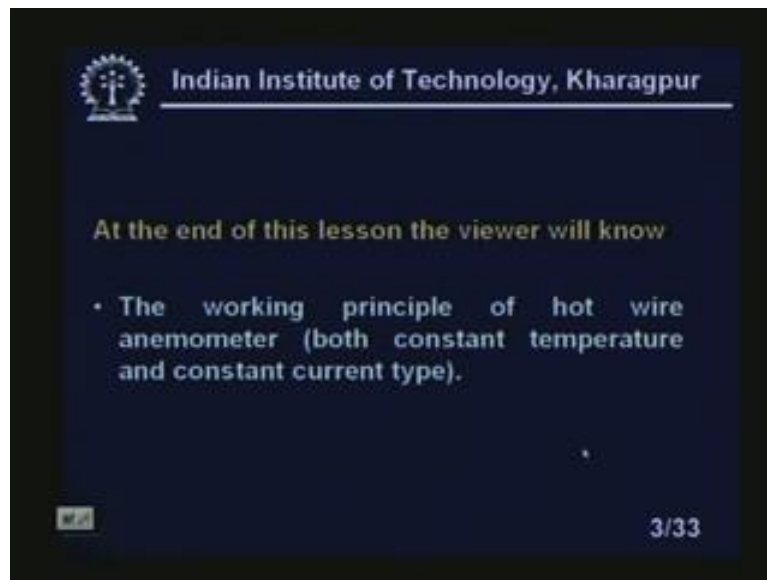
Welcome to the class of Industrial Instrumentation. So, this is lesson 16. In this lesson, we will consider flowmeter. Because we are continuing for last several lesson flowmeters, this lesson also will cover basically the flowmeter and the contents of this lesson:

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Hot wire anemometer, one of the most widely used industrial flowmeter both for liquids and gases that we will discuss in this particular lesson. Also, we will solve several problems on the flowmeter, various types of flowmeter we will solve problems. We will give the problems and also we will provide the solution to these problems.

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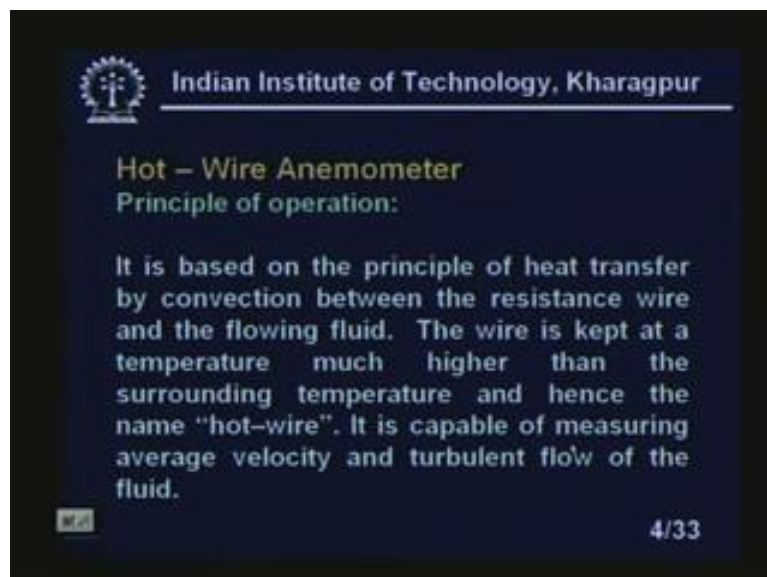
At the end of this lesson the viewer will know

- The working principle of hot wire anemometer (both constant temperature and constant current type).

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Obviously at the end of the lesson, the viewer will know the working principle of hot wire anemometer, both constant temperature and constant current type. This is the basic two types of hot wire anemometer we will find, so we will cover both. How it works, what is the signal conditioning circuitry and advantage, disadvantage, everything will be covered in this particular lesson, along with the problems and solution to the different flowmeters.

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Hot - Wire Anemometer

Principle of operation:

It is based on the principle of heat transfer by convection between the resistance wire and the flowing fluid. The wire is kept at a temperature much higher than the surrounding temperature and hence the name "hot-wire". It is capable of measuring average velocity and turbulent flow of the fluid.

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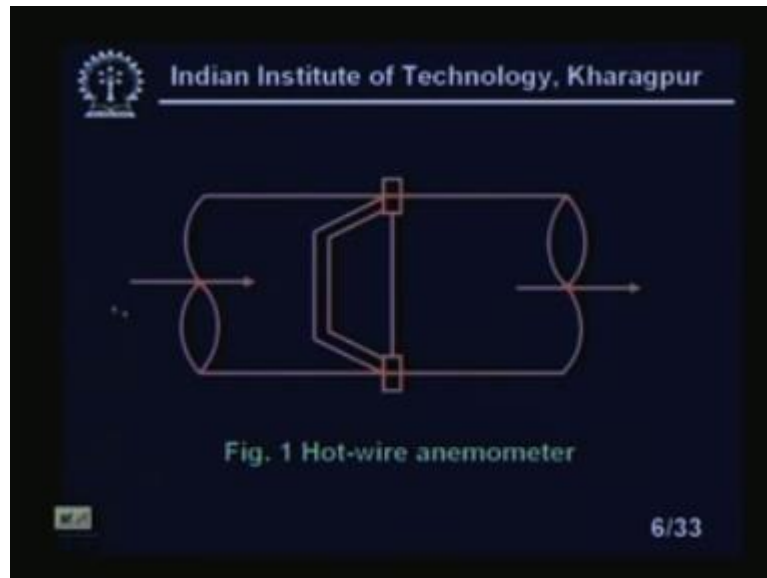
Now, hot wire anemometer if I look at, the principle of operation, it is based on the principle of heat transfer by convection between the resistance wire and the flowing fluid. The wire is kept at a temperature which is much higher than the surrounding temperature and hence the name hot-wire is given. It is capable of measuring average velocity and turbulent flow of the fluid. It can measure both, I mean average velocity as well as turbulent flow of the measurement. This is a great advantage of this particular flowmeter and moreover, time constant of this flowmeter is also quite small compared to the other flowmeters and it is direct electrical output flowmeters. So, if you compare this flowmeter with the DP transmitter, DP transmitter based flowmeters, so it is a great advantage of this particular flowmeter.

(Refer Slide Time: 2:32)



Now element used is, basically hot wire filament is usually a fine wire of platinum and tungsten. This is a, basically two wires used. I should say the tungsten is most widely used as a hot wire probe, right? Typical dimensions are: diameter 5 millimeter, 5 micrometer to 300 micrometer. So, it is quite thin wire. It is very difficult to even see in a bear eyes, it is a very thin wire and the length is usually 1 millimeter to 10 millimeters. It is to be installed inside the pipe. Please note that one thing is very important that it is to be installed, installed inside the pipe. It is not a invasive type of flowmeter, so you have to cut the flowmeters of the pipe and install with a **fork** sort of arrangement, so that it will, the length of the flowmeter will be opposite at 90 degree to the direction of the flow of the fluid in the pipe. You see, this is our pipe.

(Refer Slide Time: 3:28)



We can see here this is the pipe and this is a holder and this is our hot wire anemometer. This is our hot wire. You can see here this is a hot wire and liquids are coming in this and liquids will go down, right? So, what is the, this basically we will see th, this basically the heat balance equations, I mean is involved in this particular ... So, that is basic principle of this one. That we will discuss later on.

(Refer Slide Time: 3:55)

Care needs to be taken so that loss of heat energy due to conduction or radiation is minimized. The element is thus jacketed and placed inside the pipe with the help of a fork type support as shown in Fig 1.

At equilibrium, the heat balance equation can be written as

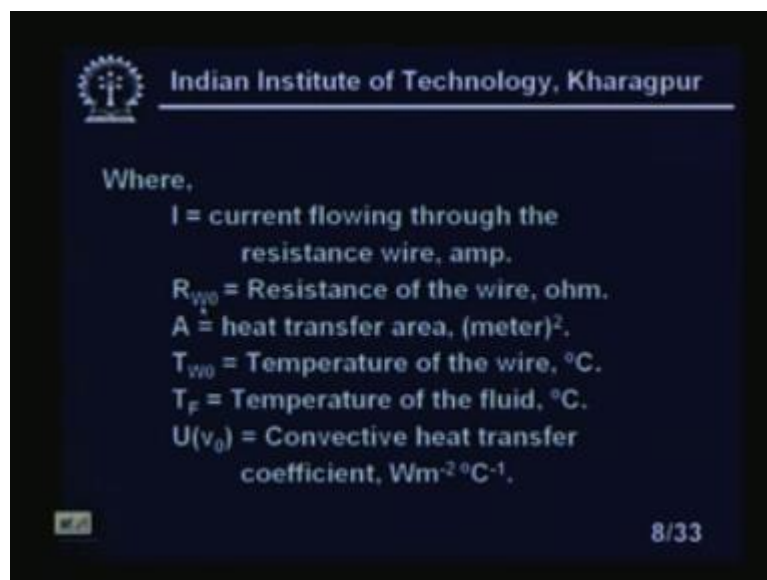
$$I_0^2 R_{W0} = U(v_0)A(T_{W0} - T_F) \dots\dots\dots(1)$$

i.e. heat generated in wire = convective loss of heat from its surface.

Care needs to be taken, so that the loss of heat energy due to conduction or radiation is minimized, because basically we are depending on the convection type of heat

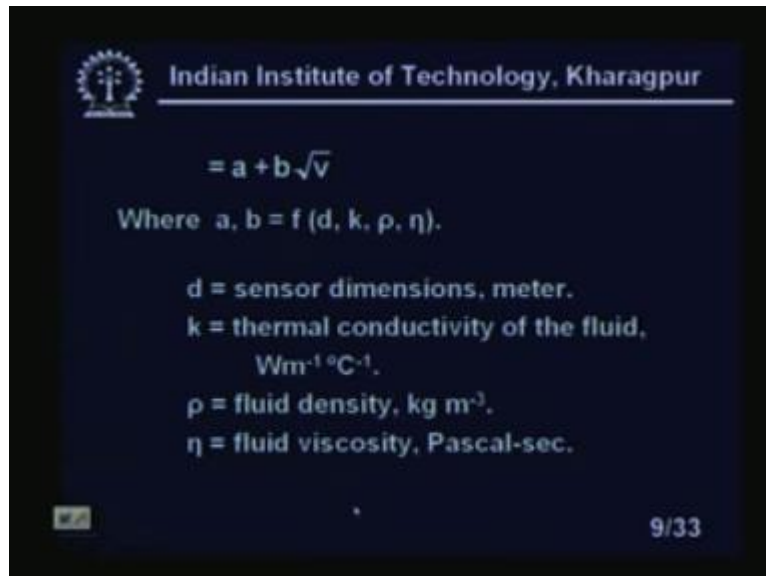
transfer, so the conduction loss and the radiation loss is to be minimized. The element is thus jacketed and placed inside the pipe with the help of a fork type support as shown in the figure, last figure we have shown. So, this figure we can see, so it is a forked type of arrangement, so it is placed inside the pipe, right? At equilibrium the heat balance equation can be written as  $I^2 R_{WO} = U_c A (T_{WO} - T_F)$ . This is equation number 1. That is heat generated in the wire equal to convective heat loss of heat from its surface, right? What is this?

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The legends are  $I$  is the current flowing through the resistance wire in ampere,  $R_{WO}$  is the resistance of the wire in ohm,  $A$  is the heat transfer area that is in meter square,  $T_{WO}$  is the temperature of the wire in degree centigrade,  $T_F$  is the temperature of the fluid in degree, in degree centigrade,  $U_c$  is the convective heat transfer coefficient which is in Watt per meter square per degree centigrade. You will find this is for the, when there is no, I mean flow in the liquid what is the resistance of the wire we are talking about, so that will come later on, right? In addition to that, some resistance will come whenever there is a change of flow or the flow starts to flow, I mean flow starts to act.

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$$= a + b\sqrt{v}$$

Where  $a, b = f(d, k, \rho, \eta)$ .

$d$  = sensor dimensions, meter.  
 $k$  = thermal conductivity of the fluid,  
 $\text{Wm}^{-1}\text{ }^\circ\text{C}^{-1}$ .  
 $\rho$  = fluid density,  $\text{kg m}^{-3}$ .  
 $\eta$  = fluid viscosity, Pascal-sec.

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Now, this also can be written as that means this equation I am talking about, this equation this  $U v$  naught can be written as that means I am saying that it looks like this,  $U v$  naught so if I take a yellow pen,  $U v$  naught equal to  $a + b$  under the square root  $V$ , where  $a + b$  depends on  $d, k, \rho$  and  $\eta$ , viscosity. This we will discuss. See here, next is let us come here, where  $d$  is the sensor dimensions that is in meter,  $k$  is the thermal conductivity of the fluid which is in Watt per meter per degree centigrade,  $\rho$  is the fluid density that is in kg per meter cube and  $\eta$  is the viscosity, fluid viscosity that is in the Pascal second, right?

For the sake of the, I mean SI units we are using Pascal second. Even though in the industry we will find still people are using the terms centipoise, poise and all those things. Since we are, here we are actually following SI unit, because the SI unit of the viscosity is the Pascal second.

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**Dynamic characteristics**

Taking into account the dynamics of the system, we can write

Rate of heat input - Rate of heat dissipation  
= Rate of rise of heat energy.

We can write,  $I^2 R_w - U(v)A | T_w - T_f | = MC \frac{dT_w}{dt}$

Where, M = Mass of heating element.  
C = Specific heat.

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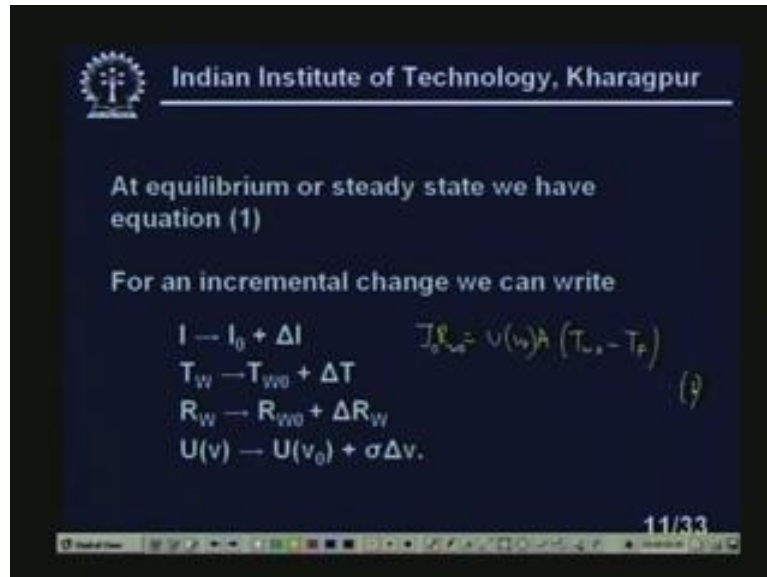
Now, dynamic characteristics, now we are talking about when it is in steady state. When it is not in steady state condition what will be the dynamic characteristics? Steady state, there is a convection heat loss equal to the heat generated in the, thermally heat generated in the wire. But, what will happen when there is a certain change? Liquid is flowing in a pipe in a steady state condition, now sudden there is a change in the fluid velocity. So, what will happen? So, there is a rate of change of the heat generation.

So, this is no more in steady state condition. So, during that time what will be our, I mean equation, heat balance equations, let us look at that. Taking into account the dynamics of the system we can write, you see, rate of heat input minus the rate of heat dissipation equal to the rate of rise of heat energy **or** rate of rise of temperature. So, rate of heat input which we are giving thermally and rate of heat dissipation due to the convecting heat loss from the wire to the surrounding that is in the fluid medium equal to the rate of rise of heat energy in the wire itself, right?

So, we can write here,  $I^2 R_w - U(v)A | T_w - T_f | = MC \frac{dT_w}{dt}$ . You see, all the units are slightly different previous equations, we can see here. You see these are all different there. If you look at this  $R_w$ ,  $T_w$ ,  $T_f$ ,  $T_f$  will remain same because, it is the fluid temperature it does not depend on the temperature of the wire. But the, all these units, all the legends you will find  $R_w$ ,  $U(v)$ ,  $T_w$ , all things have

been changed there if I look at the previous expression, so where M is the mass of heating element and c is the specific heat of the element.

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At equilibrium or steady state, we can have the equation 1 as, for incremental change we can write first there is a change in the wire temperature, so change of velocity. What will happen? Let us, for incremental change I has changed to I naught plus del I. T W, wire temperature previously it was constant, steady state condition U (v zero) in the first equation we have written like this. Then plus delta T change of temperature of the wire, R W is the resistance of the wire. In the first equation, in the steady state equation we have written in terms of R WO, here we are writing del R W plus del R W, then U(v) equal to U(v naught) delta v naught.

You see, what was our first equation let us look at. If I, I think this is already chosen or very first equation if I look at, it was like this I naught R W equal to U (v naught) A 2 minus T F. This is equation 1, is not it? Now we have a change, because we say that it is no more in steady state condition. That means there is heat loss, heat generated minus heat loss equal to rate of heat energy I mean generated (inside or in site). So, it has previously now we have, so now it has new values. We are, so it is time varying value. So, this T W, R W are all time varying. I, R are also time varying. This, all these elements are time varying. These are steady state conditions.



If it is time varying I can write incremental increase  $I$  naught plus  $\Delta I$  T W plus  $\Delta T$ , R W equal to, R W equal to R W O plus R W. So, all these expressions, you see these expressions  $I$  naught, T W, RW, V naught were used in the steady state conditions, right? When it is not the steady state condition, condition it will be something different. That is in that case you have to use these time varying parameters, right?

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Where,

$$\sigma = \left. \frac{\partial U}{\partial v} \right|_{v=v_0, T=T_0}$$

Simplifying, we get

$$[2I_0 \Delta R_{w0} + I_0^2 \Delta R_w] - U(v_0) A \Delta T - \sigma \Delta v A (T_{w0} - T_f) = MC \frac{d}{dt} (\Delta T)$$

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Sigma we write  $\Delta U$  del of U upon del of v, where v equal to v naught, T W equal to T naught. Simplifying, we are getting this expression, but we must derive how we are getting this expression. After simplifications we will get these expressions, right? So, let us look at how you are getting these expressions. Expression looks like this. You see initially that you take a white page that will be more clear. You see, we had previously if I take the, change the colour of the pen, yes our equation is, looks like this.

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$$I^2 R_w - U(v) A (T_w - T_f) = MC \frac{dT_w}{dt}$$

$$I = I_0 + \Delta I$$

$$T_w = T_{w0} + \Delta T$$

$$R_w = R_{w0} + \Delta R_w$$

$$U(v) = U(v_0) + \sigma \Delta v$$

Equation is  $I^2 R_w - U(v) A (T_w - T_f) = MC \frac{dT_w}{dt}$ . This, this was our new equations or dynamic equations of the hot wire anemometer. Now, I replace  $I$  by  $I_0 + \Delta I$  I replaced  $T_w$  by  $T_{w0} + \Delta T$ , yes,  $R_w$  I replaced by  $R_{w0} + \Delta R_w$  and  $U(v)$  I replace  $U(v_0) + \sigma \Delta v$ , right? See, if I put all these  $I$  here, I will get a expressions we have to take a new page.

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$$\begin{aligned} & (I_0 + \Delta I)^2 (R_{w0} + \Delta R_w) - \{U(v_0) + \sigma \Delta v\} A (T_{w0} + \Delta T - T_f) = \frac{MC d(T_{w0} + \Delta T)}{dt} \end{aligned}$$

$$\approx (I_0^2 + 2I_0 \Delta I) R_{w0} + I_0^2 \Delta R_w - U(v_0) A (T_{w0} - T_f) - U(v_0) A \Delta T - \sigma A (T_{w0} - T_f) \Delta v = \frac{MC \Delta T}{dt}$$

$$I_0^2 R_{w0} = U(v_0) A (T_{w0} - T_f) \dots (1)$$

I naught plus delta I whole square R WO plus del R W minus U v naught plus sigma delta v A multiplied by T WO plus delta T minus T F equal to MC derivative of T WO plus delta T by dt, right? So, I can simplify this equation neglecting all as multiplication of the terms like delta I, delta R W, so these are all quite small. So, we can neglect this that means we are neglecting delta I into delta R W, small quantity; we are assuming these are all equal to zero. So, I will get a expressions I naught or I will get sorry or I will get I naught square 2 I naught delta I R WO plus I naught square delta W minus U v naught T WO minus T F into A minus U v naught A delta T minus sigma A T W minus T F delta v equal to MC delta t, sorry, this will be delta T, see if I take the eraser, right, so I take the pen again, pen again, C delta T by dt.

Now from this expression, I subtract, excuse me, I subtract the equation number 1. What is that equations? Let me write down the equation if I recall the equation number 1 that equation number 1 was, so from this equation if I give it equation number 2 and our equation number 1 was I naught square R W equal to U v naught A T WO minus T F. This was equation 1, right? So, what we are doing?

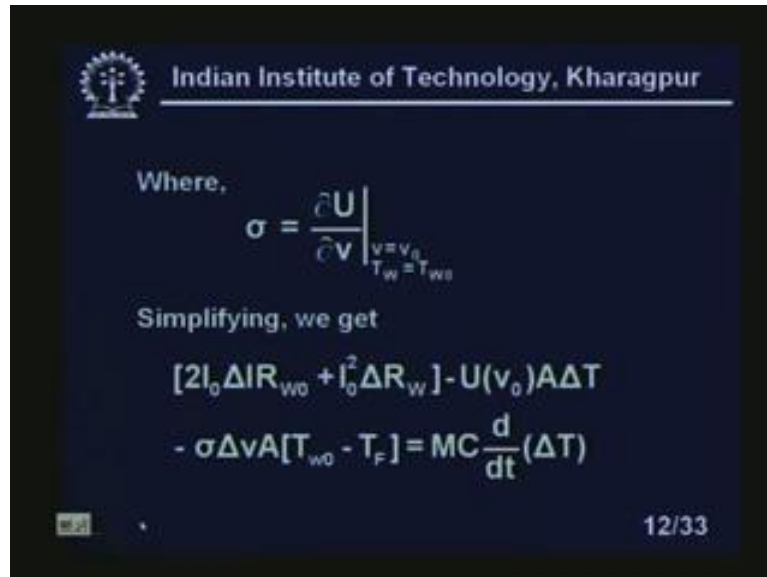
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$$\begin{aligned}
 & (2) - (1) \\
 & 2I_0 \Delta I R_{WO} + I_0^2 \Delta R_W - U(v_0) A \Delta T \\
 & - \sigma A (T_{WO} - T_F) \Delta v = MC \frac{\Delta T}{dt}
 \end{aligned}$$

Taking a new page, so I making 2 minus 1, I will get the expression which looks like this, which looks like 2 I naught del I R WO plus I naught square delta R W minus U v naught A delta T minus sigma A T WO minus T F del v equal to MC delta T by dt,

right, clear? If this is our expressions, so let me go back to the power point presentation again.

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Where,

$$\sigma = \left. \frac{\partial U}{\partial v} \right|_{v=v_0, T_w=T_{w0}}$$

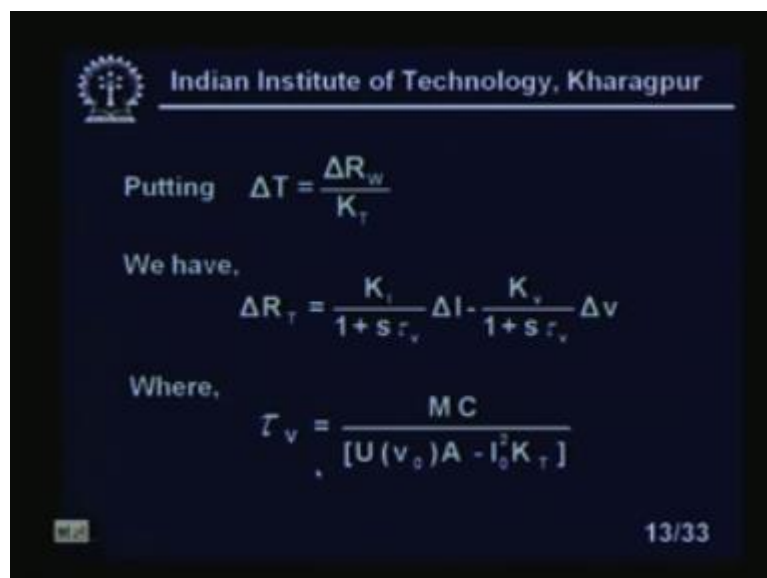
Simplifying, we get

$$[2I_0 \Delta R_{w0} + I_0^2 \Delta R_w] - U(v_0)A \Delta T - \sigma \Delta v A [T_{w0} - T_f] = MC \frac{d}{dt} (\Delta T)$$

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So, this is, so this is our expression, right? Yes, this is the sigma equal to partial derivative of U with respect to v when v equal to v naught and T W equal to T naught.

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Putting  $\Delta T = \frac{\Delta R_w}{K_t}$

We have,

$$\Delta R_T = \frac{K_i}{1+s\tau_v} \Delta I - \frac{K_v}{1+s\tau_v} \Delta v$$

Where,

$$\tau_v = \frac{MC}{[U(v_0)A - I_0^2 K_T]}$$

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If we have this expression, putting, we put delta T equal to delta R W by K t, we have delta R T K i 1 plus S tow v into delta I minus K v by 1 plus tow v into delta v, so

where  $\tau_v$  is given by  $\frac{K_T \sigma A (T_{W0} - T_F)}{U(v_0)A - I_0^2 K_T}$ , excuse me.

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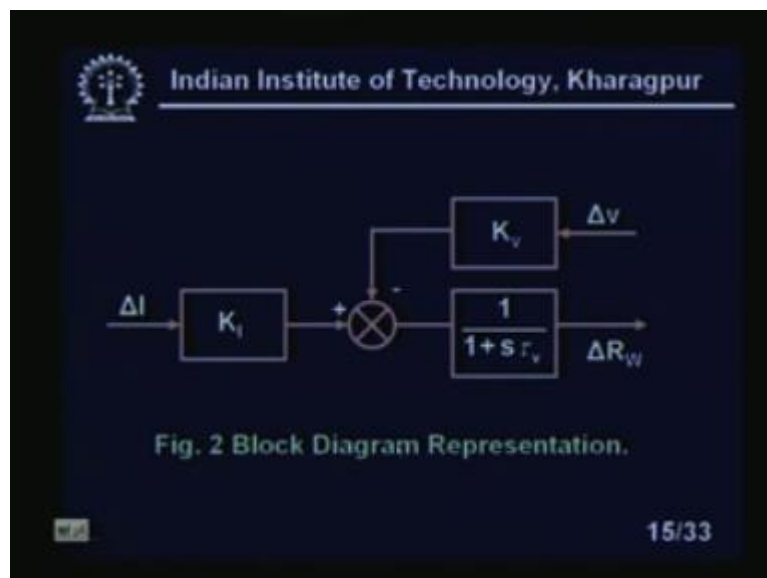
$$K_v = \frac{K_T \sigma A (T_{W0} - T_F)}{U(v_0)A - I_0^2 K_T}$$

$$K_i = \frac{2K_T I_0 R_{W0}}{U(v_0)A - I_0^2 K_T}$$

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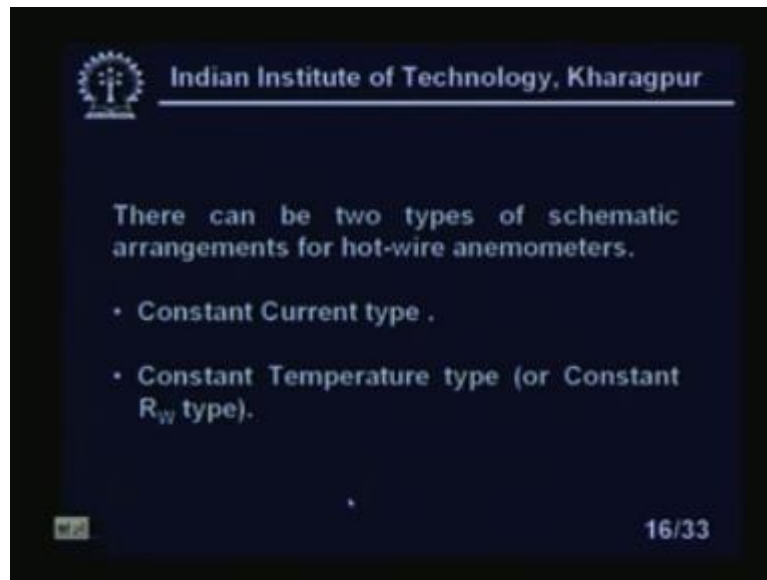
Where,  $K_v$  we can write is equal to  $\frac{K_T \sigma A (T_{W0} - T_F)}{U(v_0)A - I_0^2 K_T}$  when  $U(v_0)A - I_0^2 K_T$  and  $K_i$  equal to  $\frac{2K_T I_0 R_{W0}}{U(v_0)A - I_0^2 K_T}$ .

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Now, you see the block diagram representation of the hot wire anemometer. This  $\Delta I$  I am giving, this is change of velocity  $\Delta v$  and this is my change of resistance of the hot wire, right? This is the first order system as you can see.

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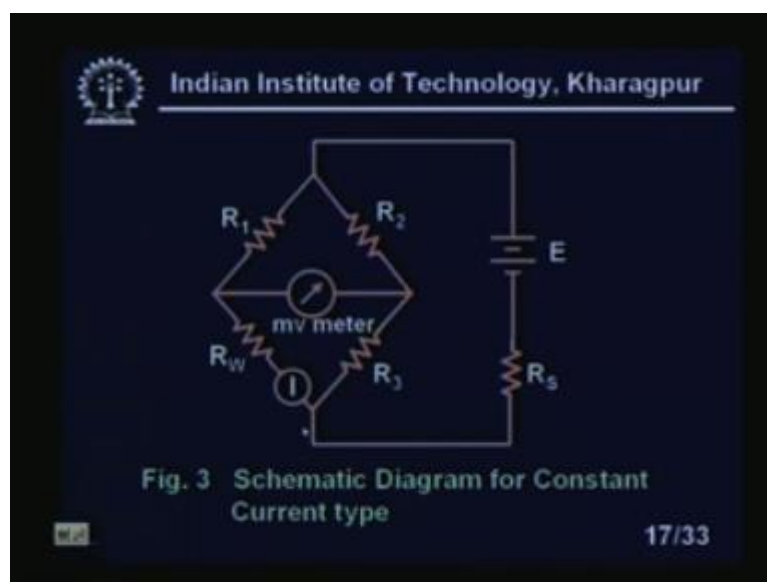
There can be two types of schematic arrangements for hot-wire anemometers.

- Constant Current type .
- Constant Temperature type (or Constant  $R_W$  type).

16/33

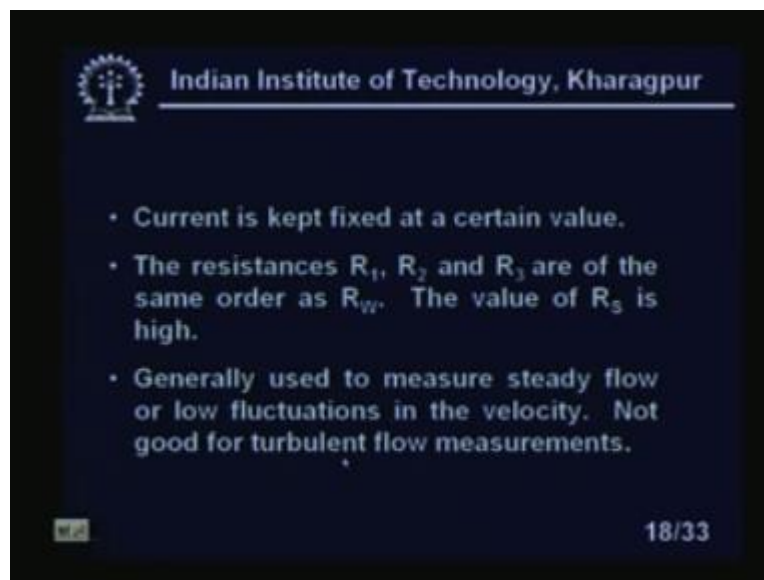
So, there can be two types of schematic arrangements of hot wire anemometers. That is I am telling either it will be a constant current or constant temperature, right? So, constant current type and constant temperature type or constant  $R_W$  type.

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A schematic diagram of the constant current type, you see here actually what we are doing that initially we are taking all the resistance equal  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_W$  and this  $R_s$  is quite high, right and we can measure, we can see the current which is passing through the, through this hot wire. Now, this resistance is so high compared to this resistance you will find, due to the change of flow the resistance even though the resistance will change, the change of current will be insignificant, right?

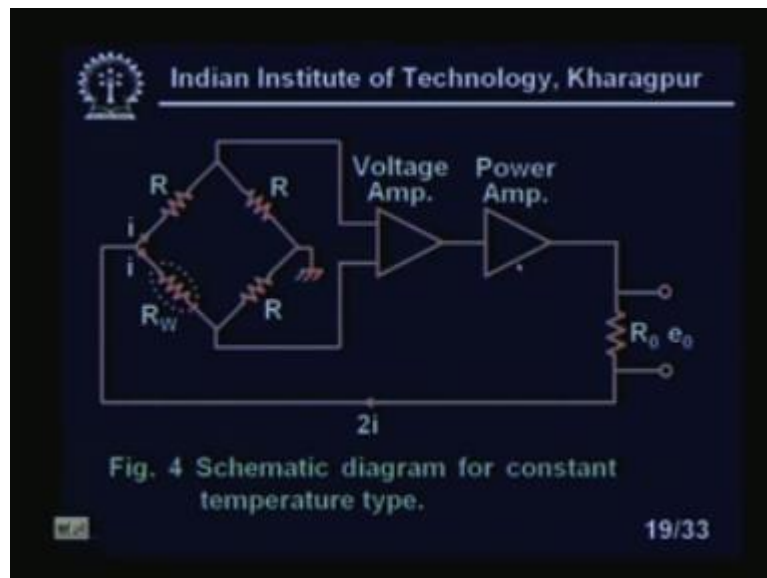
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So, current is kept fixed at a certain value that can be measured by that resistance, that by the, by the ammeter. What is that ammeter? This ammeter, with the ammeter we can see whether the currents are getting, remaining constant or not. The resistance  $R_1$ ,  $R_2$ ,  $R_3$  are of the same order as  $R_W$ . Typically it should be quite small compared to the  $R_s$  and the value of  $R_s$  is high, right? Generally used to, this is, this constant current type is generally used to measure the steady flow or low fluctuations in the velocity, not good for very high frequency turbulent flow.

We need some compensation. If you do the compensation, then it is possible to, with the lead lag network using I can make the circuit in such a way, so that I can measure the much higher frequency. Otherwise normally it is not suitable for the **low**, high frequency fluctuations. So, high frequency turbulent, turbulent of the flow

(Refer Slide Time: 20:20)



Now, this is a schematic diagram of a constant temperature type of a hot wire anemometer. You see here, it is auto balancing systems. This  $R_w$  is put on arm of a bridge which is a, our hot wire and all other resistances are put, all are same Now, you see what will happen that if there is a, if there is a unbalance that means suppose due to resistance change, then what will happen? You see that if there is, there is a unbalance, so you will get a, initially it is balanced, there is a no output voltage, no error voltage. But, some steady state current will flow through this resistance.

Now, what will happen? You see that if there is a change, so that change will be a, if there is, there is unbalanced voltage here and that will amplify this voltage amplifier, then power amplifier. So, it will give a current. So, it will change the current through this, through this bridge and it will continue till we achieve the balance, right? So, it is auto balancing systems we can see here.



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- Here it keeps the Resistance  $R_w$  constant by incorporating feedback.

Feedback

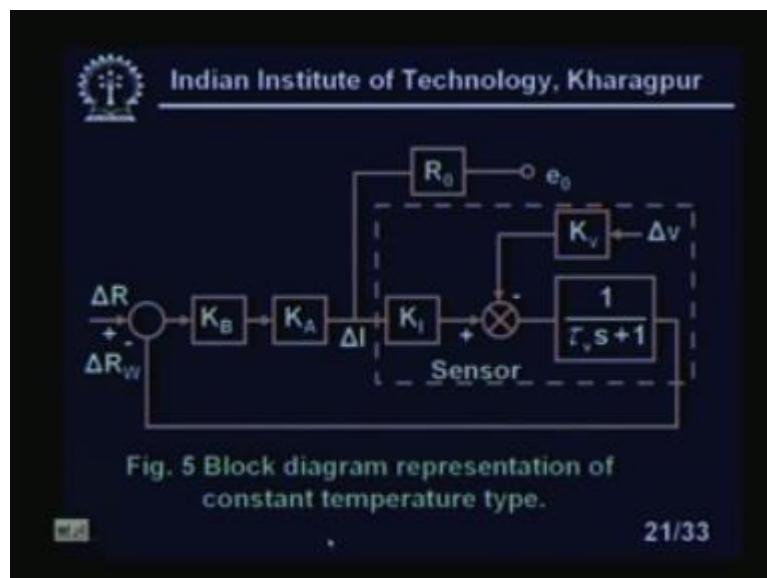
As the velocity increases,  $R_w$  decreases thereby creating an unbalance voltage. The current increases which brings back the resistance to the initial value.

- It increases bandwidth and thereby suitable for turbulent flow measurements also.

20/33

Here, it keeps the resistance  $R_w$  constant by incorporating the feedback. Now, as the velocity increases,  $R_w$  decreases thereby creating an unbalance voltage and the current increases, which brings back the resistance to the initial value, right? It increases the bandwidth and thereby suitable for turbulent flow measurements also. So, it is also for increasing, since we are using feedback so it is, it is suitable for measurements of the high frequency .... and turbulent flow.

(Refer Slide Time: 21:57)



Now, this is a block diagram of a constant temperature type of a sensor we can see. We have  $K_B$ ,  $K_A$ ,  $e$  is going out, so this is our typical system.

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Thus we get

$$\frac{\Delta e_o}{\Delta v} = \frac{K_v K_B K_A R_o}{\tau_v s + (1 + K_B K_A K_i)} = \frac{K'}{\tau_v' s + 1}$$

Where,  $K' = \frac{K_v K_B K_A R_o}{1 + K_B K_A K_i}$

$$\tau_v' = \frac{\tau_v}{1 + K_B K_A K_i}$$

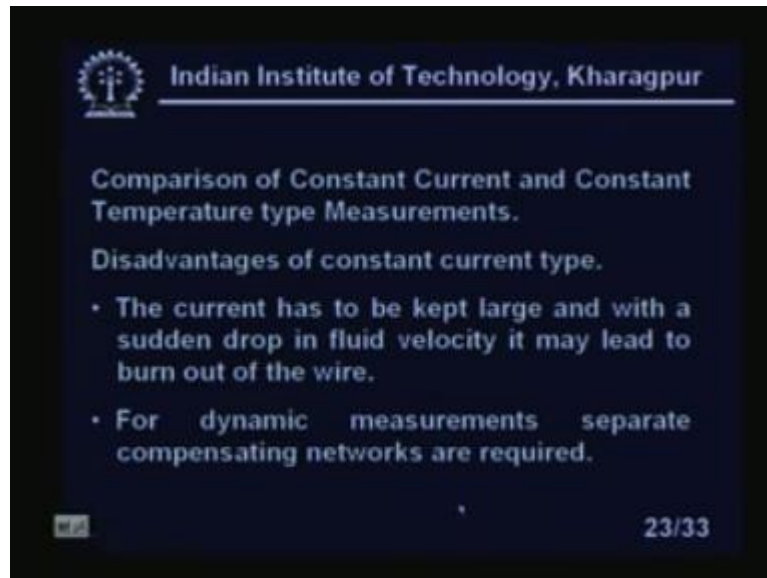
Thus, we see that the system becomes faster.

22/33

Thus we get you see,  $\Delta e_o / \Delta v$ , this output voltage divided by change of output voltage due to the change of velocity  $K_v K_B K_A R_o / \tau_v s + 1 + K_B K_A K_i$ , which is equal to  $K' / \tau_v' s + 1$ . Previously it is only  $\tau_v s + 1$ . So, what is  $\tau_v$ ,  $\tau_v$  let us look at. So,  $K'$  equal to  $K_v K_B K_A R_o / (1 + K_B K_A K_i)$  and  $\tau_v'$  equal to, now previously we have  $\tau_v s$  and now actually what will happen if I decrease the time constants of the system or my frequency response will increase, is not it?

**Lower** the time constants I can measure the high frequency. This is the basic fundamental, I mean fundamental equations of any, any instruments or any systems. Even if the **thermodynamic** time constant is large I can measure the response very fast for a step input. If it is cyclic, obviously that advantage is there. Thus, you see that the system becomes faster. Since we are dividing this  $\tau_v$  by these factors, obviously what will happen, by this factor what will happen is frequency is  $\tau_v$ ,  $\tau_v$  will decrease and I can measure the higher frequency.

(Refer Slide Time: 23:27)



The slide features the IIT Kharagpur logo and name at the top. The main text is centered and discusses the disadvantages of constant current type measurements. It lists two points: the need for a large current that can lead to wire burnout if fluid velocity drops, and the requirement for compensating networks for dynamic measurements. A small icon and the slide number '23/33' are visible in the bottom left and right corners, respectively.

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Comparison of Constant Current and Constant Temperature type Measurements.

Disadvantages of constant current type.

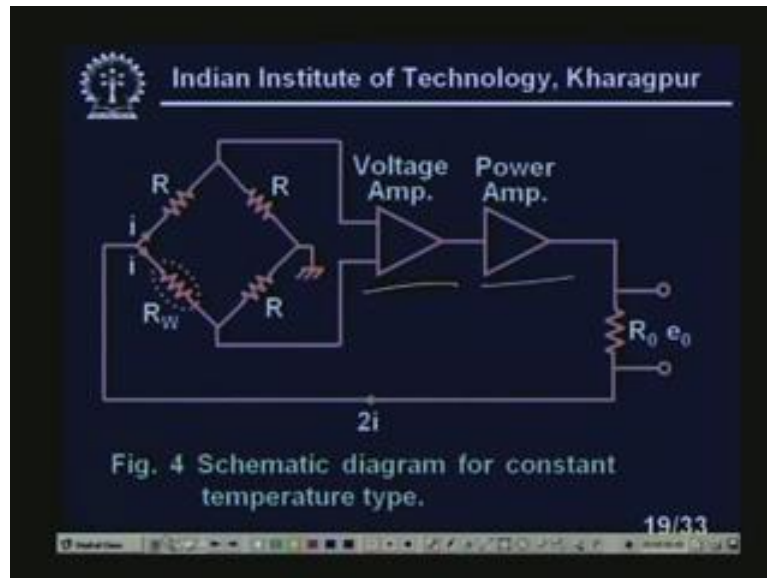
- The current has to be kept large and with a sudden drop in fluid velocity it may lead to burn out of the wire.
- For dynamic measurements separate compensating networks are required.

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Now comparison of the constant current and constant temperature type measurements, disadvantage of the constant current type - the current has to be kept large and with a sudden drop in fluid velocity it may lead to the burn out of the wire. So, this is a typical problem of the constant current type. That means the current has to be kept large. Otherwise what will happen? So, usually it is large and if there is a sudden fall in the fluid velocity, so temperature may rise. Because it is constant temperature, some constant current must flow through these ones. It will burn out the wire.

For dynamic measurements separate compensating networks are required. That means I need a lead lag network, as I told you earlier, to make the phase compensation, so that I can go for the higher frequency of measurement.

(Refer Slide Time: 24:17)



You see the drift of the amplifier will come in picture. If I have a drift here, suppose if I have a drift here, drift here, so that will come in the amplifier output and we will take output from these terminals anyway.

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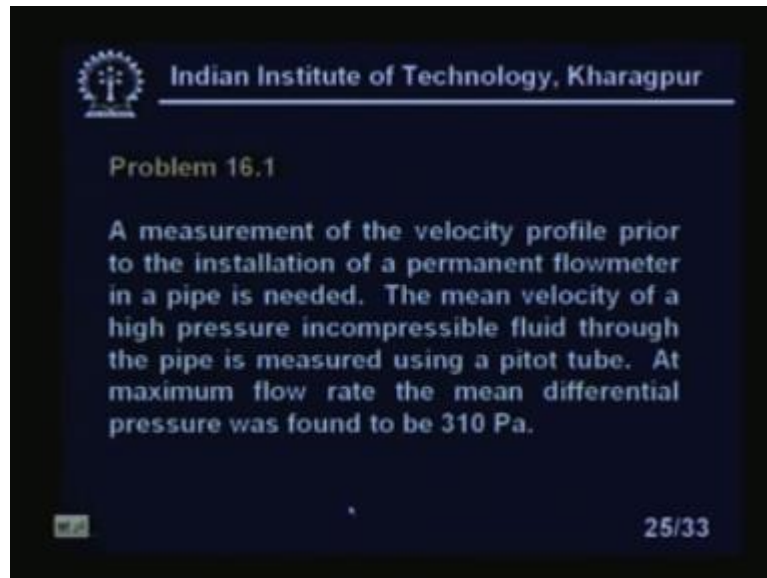
Constant temperature type removes the above problem with the introduction of a feedback loop. However there are other problems

- Instability and drift problems.
- For small velocity change noise has to be taken care of separately or the measurements are erroneous.

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Instability and drift problems and for small velocity change noise has to be taken care of separately or the measurements are erroneous, right? That also we have to think of.

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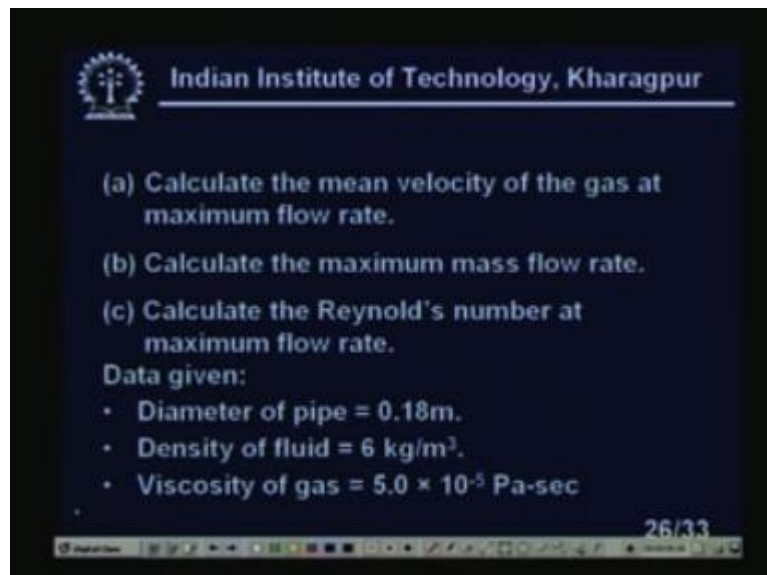
**Problem 16.1**

A measurement of the velocity profile prior to the installation of a permanent flowmeter in a pipe is needed. The mean velocity of a high pressure incompressible fluid through the pipe is measured using a pitot tube. At maximum flow rate the mean differential pressure was found to be 310 Pa.

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Now, let us solve few problems on the flowmeter. Let us start with the problem number 16.1. We will solve, we will give the questions, then we will solve the problems. So, problem number 16.1 you see here, a measurement of the velocity profile prior to the installation of a permanent flowmeter in a pipe is needed, right? So we, we first want to get estimate of the flow velocity, right? The mean velocity of a high pressure incompressible fluid through the pipe is measured using the pitot tube. At max, at maximum flow rate the mean differential pressure was found to be 310 Pascals, right?

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(a) Calculate the mean velocity of the gas at maximum flow rate.

(b) Calculate the maximum mass flow rate.

(c) Calculate the Reynold's number at maximum flow rate.

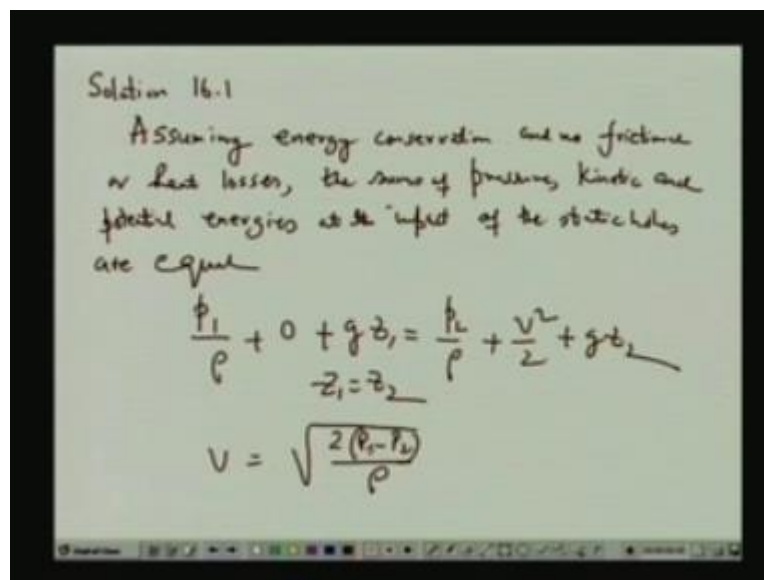
Data given:

- Diameter of pipe = 0.18m.
- Density of fluid = 6 kg/m<sup>3</sup>.
- Viscosity of gas =  $5.0 \times 10^{-5}$  Pa-sec

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We have to measure, number a, calculate the mean velocity of the gas at maximum flow rate, calculate the maximum mass flow rate, calculate the Reynolds's number at the maximum flow rate and the data given are: the diameter of pipe which is 0.18 meter, then density of the fluid 6 kg per meter cube, viscosity of the gas 5.0 into 10 to the power minus 5 **Pascal** per Pascal second; this is given, right? So, let us go one by one, let us take a white page here, so we will take a pen.

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Assuming, solution 16.1, we have assuming energy conservation and no frictional or heat loss, the sums of pressure, kinetic and potential energies, potential energies, at the impact of the static holes are equal. This already we discussed in the case of, in the case of when we discussed actually the pitot tube. Anyway, let me write away, straight away write the equations:  $p_1$  by row plus zero  $g z_1$   $p_2$  by row plus  $v^2$  square by plus  $g z_2$ , right?

So,  $p_2$  is a static, static pressure and this is a stagnation pressure. Obviously,  $p_1$  will be greater than  $p_2$ , right, where  $z_1$ ,  $z_2$  are the elevations of the holes above the datum levels that means static hole and the and the stagnation hole. So, we assume initially that  $z_1$ , now we assume  $z_1$  equal to  $z_2$ . Hence, we can write that  $v$  equal to, velocity will be equal to, velocity will be equal to  $2 p_1$  minus  $p_2$  by row, right?

(Refer Slide Time: 29:16)

Handwritten mathematical derivation on a whiteboard:

$$\text{Mean velocity} = v = 10.16 \text{ m/sec}$$

The maximum mass flow rate is given by

$$\frac{dm}{dt} = (vA)\rho$$
$$\dot{m} = v \frac{\pi D^2}{4} \times \rho \quad , \quad D = 0.18 \text{ m}$$
$$\rho = 6 \text{ kg/m}^3$$
$$\text{Mass flow rate} = \underline{\underline{1.55 \text{ kg/sec}}}$$

Therefore the mean velocity will be given by, the mean velocity will be given by equal to 10.16 meter per second. The maximum mass flow rate is given by  $\frac{dm}{dt}$  equal to  $vA$  into  $\rho$ , where  $vA$  is the volumetric flow rate. So it will become, sorry, so it will become  $\dot{m}$  equal to  $v \pi D^2$  by 4, where  $D$  is the diameter of the pipe into row. Now,  $D$  here is 0.18 meter. If I put everything there, because  $\rho$  is,  $\rho$  is given by 6 kg per meter cube, density of the fluid, so I will get that mass flow rate equal to 1.55 kg per second. This is one answer, this is another answer, right?

Now, we have to find the Reynolds number. Let us take a new page.




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$$Re = \frac{\rho v D}{\eta}$$
$$= \frac{6 \times 10.16 \times 0.18}{5 \times 10^{-5}}$$
$$= \underline{21.94 \times 10^4}$$

Now, Reynolds number is given by  $Re = \frac{\rho v D}{\eta}$ . Viscosity is  $\eta$ , we are writing, where  $\eta$  is the viscosity. So, this will become  $6 \times 10.16 \times 0.18$  by, already given  $5 \times 10^{-5}$ ,  $5 \times 10^{-5}$  Pascal seconds. So, this will give you  $21.94 \times 10^4$ . So, obviously you can see this is a turbulent flow, right? So, this ends the problem number 1. Let us go to the problem number 2.

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**Problem 16.2**

A turbine flowmeter consisting of a rotor with six blades is suspended in the fluid stream, the axis of the rotation of the rotor being parallel to the direction of flow. The blades are rotating at an angular velocity of  $\omega$  rad/sec. Where,  $\omega = 4.2 \times 10^4 Q$

Where  $Q$   $m^3/sec$  is the volume flow rate of the fluid. The total flux  $\phi_T$  linked by the coil of the magnetic transducer is given by

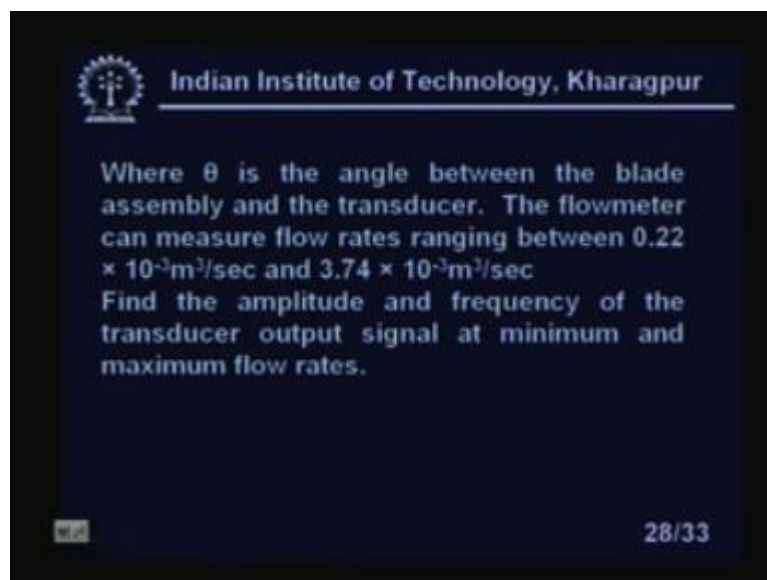
$$\phi_T = 4.637 + 0.92 \cos(6\theta) \text{ milliwēbers.}$$

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Problem number 2 is on turbine flowmeter, very straight forward problem; just plug in the equations, you will get the answer. A turbine flowmeter consisting of a rotor with 6 blades is suspended in the fluid stream, the axis of the rotation of the rotor being parallel to the direction of the flow of the fluid, right? The blades are rotating at an angular velocity of  $\omega$  radian per second, where  $\omega$  equal to  $4.2 \times 10^4$  rad/s, right, where  $Q$  meter cube per second is the volume flow rate of the fluid and the total flux  $\phi$  T linked by the coil of the magnetic transducer is given by  $4.637 \times 10^{-3} \cos 6\theta$ . This is in milliwebers, right?

(Refer Slide Time: 33:01)



Theta is the angle between the blade assembly and the transducer. These are all very standard notation which we have given in the, our particular lessons. So, exactly we use the same notations here, right and the flowmeter can measure the flow rates ranging from  $0.22 \times 10^{-3}$  meter cube per second and  $3.74 \times 10^{-3}$  meter cube per second, right? Find the amplitude. Find the amplitude and the angle, sorry; find the amplitude and the frequency of the transducer output signal at minimum and maximum flow rate, right? So, this is our question number 2 on the turbine flowmeter. Let us look at the solution, right? Let us take a white page.

(Refer Slide Time: 33:54)

Prob 16.2

$$\Phi_T(\theta) = \alpha + \beta \cos(n\theta)$$

$\alpha$  = Mean flux  
 $\beta$  = Amplitude of flux variation  
 $n$  = Number of blades

$$E = -\frac{d\Phi_T}{dt} = -\frac{d\Phi_T}{d\theta} \cdot \frac{d\theta}{dt}$$

$$\frac{d\Phi_T}{d\theta} = -\beta n \sin(n\theta) \quad , \quad \frac{d\theta}{dt} = \omega$$

$\theta = \omega t$   
 $\omega = 0 \text{ to } 2\pi$

So, this is problem number 16.2, right? Here we know that phi T theta equal to alpha plus beta cos n theta, right, where alpha is the mean flux, beta is the amplitude of the flux variation and n is the number of blades. So, E equal to, we know equal to minus d phi T by dt minus d phi T by d theta into d theta by dt. So, it will give you now d theta by dt by dt is equal to minus beta n sin n theta, right and d theta by dt is equal to omega, which is angular velocity. So, theta equal to omega t assuming that at theta equal to zero, at t equal to zero right? Now we can obviously write, let us take a new page.

(Refer Slide Time: 35:42)

$$e = \beta n \omega \sin \omega t$$

Amplitude =  $\beta n \omega$       Min. Flux  
 frequency =  $\frac{n\omega}{2\pi}$

$\omega = 9.24 \text{ rad/sec}$   
 Amplitude =  $0.92 \times 6 \times 9.24 = 51 \text{ mV}$   
 Frequency =  $\frac{6 \times 9.24}{2\pi} = 8.82 \text{ Hz}$

} Mean flux rate

That  $e$  equal to  $\beta n \omega \sin n \omega t$ , right? So, the amplitude of this signal is  $\beta n \omega$  and the frequency is equal to  $n \omega$  by  $2\pi$ , linear frequency not circular frequency, by  $2\pi$ , right? So,  $\omega$  will be, frequency for the minimum flow rate will be, if I plug in all the expressions will become 9.24 radians per, per second, right and amplitude will be given by 0.92, right, 0.92, multiplied by number of blades, 6 into 9.24 which is, will be 51 milli volt, right and the frequency will be, this should be milliwabers, so this should be in millivolts. Frequency will be 6.6 into 9.24 by  $2\pi$ . This will be equal to 8.82 Hertz. So, this is the amplitude, this is the frequency, right and this is for the minimum flow, right, minimum flow rate, minimum flow.

Now, let us do for the maximum flow. Take a white page.

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For maximum flow rate

$$\omega = 157.1 \text{ rad/sec}$$

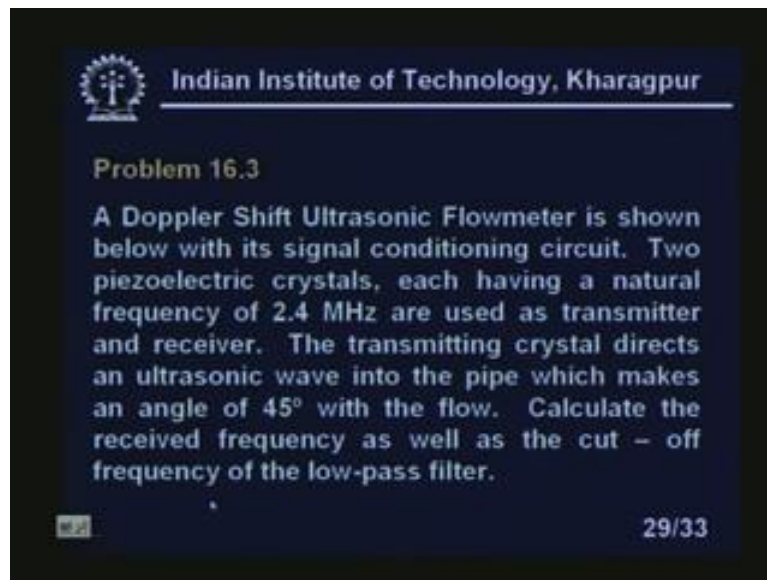
$$\text{Amplitude} = \frac{867.2 \text{ mV}}{1000} = 867.2 \text{ mV}$$

$$\text{Frequency} = \frac{150 \text{ Hz}}{1000}$$

For maximum flow, for maximum flow rate I will get  $\omega$  equal to, if I plug in all these values, 157.1 radian per second and the amplitude will come up, whereas if I calculate equal to 867.2 milli volt and frequency equal to 150 Hertz, right? So, this is one answer, this is another answer, right? So, these are another problem, right? So, this is 800, I do not know whether is legible, 867.2 milli volt, right? So, the frequency is 150 Hertz.

Let us do the other problem. Let us go to the problem number 6.3.

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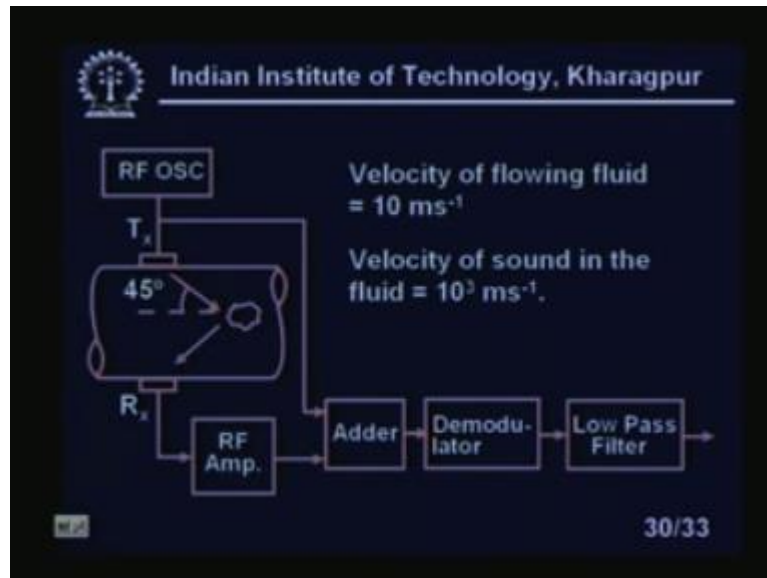
**Problem 16.3**

A Doppler Shift Ultrasonic Flowmeter is shown below with its signal conditioning circuit. Two piezoelectric crystals, each having a natural frequency of 2.4 MHz are used as transmitter and receiver. The transmitting crystal directs an ultrasonic wave into the pipe which makes an angle of  $45^\circ$  with the flow. Calculate the received frequency as well as the cut – off frequency of the low-pass filter.

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Problem 6.3 is on the Doppler flowmeter. Doppler shift, as you know there is two types of Doppler flowmeters. Time shift measurements and the ultrasonic and the Doppler shift measurements. So, let us first do the Doppler shift ultrasonic flowmeter, problem on the, on Doppler shift ultrasonic flowmeter. A Doppler shift ultrasonic flowmeter is shown in figure below which its, with its signal conditioning circuitry. Two piezoelectric crystals each having a natural frequency of 2.4 mega Hertz are used as transmitter and receiver. The transmitting crystal directs an ultrasonic wave into the pipe which makes an angle of 45 degree with the flow. Calculate the received frequency as well as the cut-off frequency of the low-pass filter. Why should I .... low-pass filter, let us look at.

(Refer Slide Time: 40:16)



This is our diagram of the system that means you see that RF oscillator. So the, we are launching the ultrasonic wave. It is getting refracted from liquid, some suspended particles. As you know this Doppler shift ultrasonic flowmeter is not for the clean liquids that there should be some particles inside the liquids for which the ultrasonic wave should be reflected by the particular particle. These ultrasonic waves are launched from this transmitter. It is coming here, it is getting reflected and coming in this directions.

(Refer Slide Time: 40:52)

$$f_r = f_i \frac{1 - \frac{v \cos \theta}{c}}{1 + \frac{v \cos \theta}{c}}$$
$$\approx f_i \left(1 - \frac{v \cos \theta}{c}\right)^2$$
$$f_r = f_i \left(1 - \frac{2v \cos \theta}{c}\right) \quad v \cos \theta \ll c$$
$$f_r = 2.344 \text{ MHz}$$

You see we can write here from our previous expression,  $f_r$ , frequency of the reflected wave equal to  $f_i \frac{1 - v \cos \theta}{C}$  upon  $1 + v \cos \theta$  by  $C$ . right? This we can write, simplify, equal to  $f_i \frac{1 - v \cos \theta}{C}$  whole square, where  $f_i$  is the frequency of the incident beam and  $f_r$  is the frequency of the reflected beam, right, of the ultrasonic wave. So, we can obviously write  $f_r$  equal to  $f_i$  equal to multiplied by  $1 - 2v \cos \theta$  by  $C$ , right? Now, one thing you please note that  $v \cos \theta$  is much, much less than  $C$ . So,  $f_r$  will come up as 2.366 mega Hertz, right? So this is our, even though it is small, so it is,  $2.4 f_i$ . So, received signal is like this one.

Now, the input signal we can write like this one. Let us take a blank page.

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Handwritten mathematical derivation on a whiteboard:

$$\begin{aligned} \text{Input signal} &= V \sin 2\pi f_i t \\ \text{Reflected signal} &= V \sin 2\pi f_r t \\ \text{Output of the adder} &= V (\sin 2\pi f_i t + \sin 2\pi f_r t) \\ &= 2V \cos \left[ \frac{2\pi (f_r - f_i)t}{2} \right] \sin \left[ \frac{2\pi (f_r + f_i)t}{2} \right] \\ &= 2V \cos \left( \frac{2\pi \Delta f t}{2} \right) \sin \left[ \frac{2\pi (f_r + f_i)t}{2} \right] \end{aligned}$$

Input signal is equal to  $v \sin 2\pi f_i$  into  $t$ , right and the reflected signal equal to  $v \sin 2\pi f_r$  into  $t$ . So, output of the adder, adder is equal to  $v \sin 2\pi f_i t$  plus  $\sin 2\pi f_r t$ , right? So, this I can simplify as  $2v \cos$  of  $2\pi f_r$  minus  $f_i$  by 2 into  $t$  multiplied by  $\sin 2\pi f_r$  plus  $f_i$  by 2 into  $t$ . So, this will give you  $2v \cos$  of  $2\pi \Delta f$  by 2 into  $t$  multiplied by  $\sin 2\pi f_r$  plus  $f_i$  by 2 if I put in the first bracket, this will be in the second bracket into  $t$ , right? So, you see these equations I have modulated with, with the carrier frequency of this one. This is the carrier frequency, right and the modulating frequency is  $\Delta f$  by 2, right?

Now let us suppose we write that, take a blank page again.

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$$\frac{\Delta f}{2} = \frac{\omega_m}{2\pi} \quad \frac{f_r + f_i}{2} = \frac{\omega_c}{2\pi}$$

input to the demodulator

$$= v [\sin(\omega_c + \omega_m)t + \sin(\omega_c - \omega_m)t]$$

Demodulate with  $k \sin \omega_c t$

$$\text{Output} = k v [\sin \omega_c t \sin(\omega_c + \omega_m)t + \sin \omega_c t \sin(\omega_c - \omega_m)t]$$

$$= \frac{kv}{2} [\cos(\omega_c + \omega_m)t - \cos \omega_c t + \cos(\omega_c - \omega_m)t - \cos \omega_c t]$$

Let me suppose that we are telling  $\Delta f$  by  $2\omega_m$  and  $f_r$  plus  $f_i$  by  $2$  is equal to  $\omega_c$  by  $2\pi$ ,  $\omega_c$  by  $2\pi$ . If it is there, so quite obviously I can write here the input to the demodulator is equal to  $v \sin(\omega_c + \omega_m)t + \sin(\omega_c - \omega_m)t$ . If I demodulate with, demodulate with  $k \sin \omega_c t$ , so the output of the modulator will be  $K v \sin \omega_c t \sin(\omega_c + \omega_m)t + \sin \omega_c t \sin(\omega_c - \omega_m)t$ , right?

So, this will give you  $K v$  by  $2 \cos$  of  $2\omega_c + \omega_m$  into  $t$  minus  $\cos$  of  $\omega_m$  into  $t$  plus  $\cos$  of  $2\omega_c - \omega_m$  into  $t$  minus  $\cos$  of  $\omega_m$  into  $t$ .

(Refer Slide Time: 47:53)

The image shows a whiteboard with handwritten mathematical work. At the top, the expression  $\frac{Kv}{2} \cos \omega_m t$  is written. To its right,  $\frac{\Delta f}{2}$  is written. Below this, the calculation  $\Delta f = f_r - f_i = 33.93 \text{ kHz}$  is shown. This is followed by  $\therefore \frac{\Delta f}{2} = 16.96 \text{ kHz}$ . The final line, which is underlined, states Cut off frequency = 20 kHz (Ans). At the bottom of the whiteboard, a standard software interface with various icons is visible.

So, thus when passed through a low-pass filter, see this signal is passed through a low-pass filter, output I will get,  $K v$  by 2  $\cos$  of  $\omega_m t$ . So, that means we have only signals which consists of only  $\Delta f$  by 2. Now, the cut-off frequency of this low-pass filter must be greater than that of  $\Delta f$  by 2. So, what is  $\Delta f$ ?  $\Delta f$  we have seen  $f_r$  minus  $f_i$  is actually coming as 33.93 kilo Hertz. So,  $\Delta f$  by 2, I can write 16.96 kilo Hertz. So, the cut-off frequency we have chosen, frequency we have chosen is equal to 20 kilo Hertz, right say, so this is answer of the third problem.

Now, you see the, I mean fourth problem is very straight forward.



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**Problem 16.4**

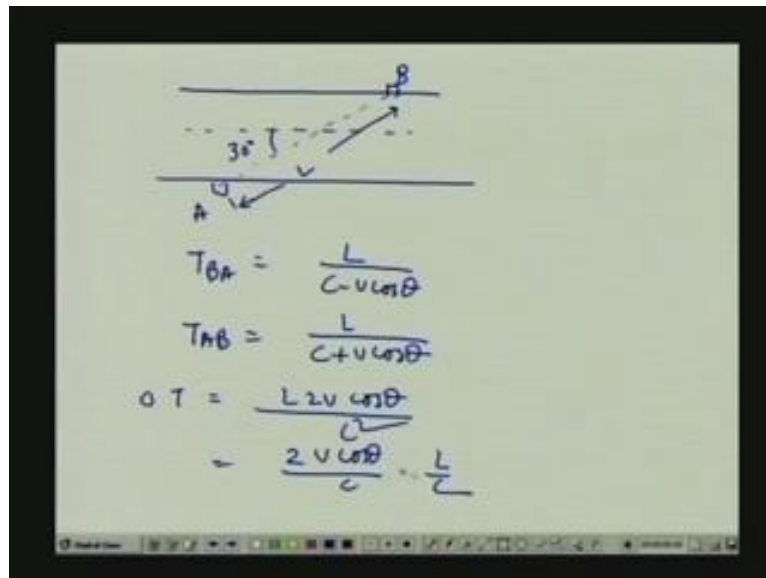
A transit time ultrasonic flowmeter is used to measure velocity of a gas flowing in a pipe. In such a case it was found that the zero flow transit time as 1.2 ms. Whereas when there was flow the differential transit time was 115  $\mu$ s. The angle between the line connecting the transmitter/ the receiver and the direction flow of fluid is  $30^\circ$ . Find the velocity of the gas. By what will the transit time change for  $\pm 2\%$  change in the velocity of sound. Velocity of sound in the gas is  $500 \text{ ms}^{-1}$ .

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The problem is 16.4. A transit time ultrasonic flowmeter is used to measure velocity of a gas flowing in a pipe. In such a case it was found that the zero flow transit time as 1.2 milli second. That means when there is a, there is no flow in the liquid, so transit time, the transit of the signal between the transmitter and receiver is 1.2 milli second, whereas when there is a flow the differential transit time was 115 micro seconds. The angle between the line connecting the transmitter and the receiver and the direction of flow of the fluid is 30 degree.

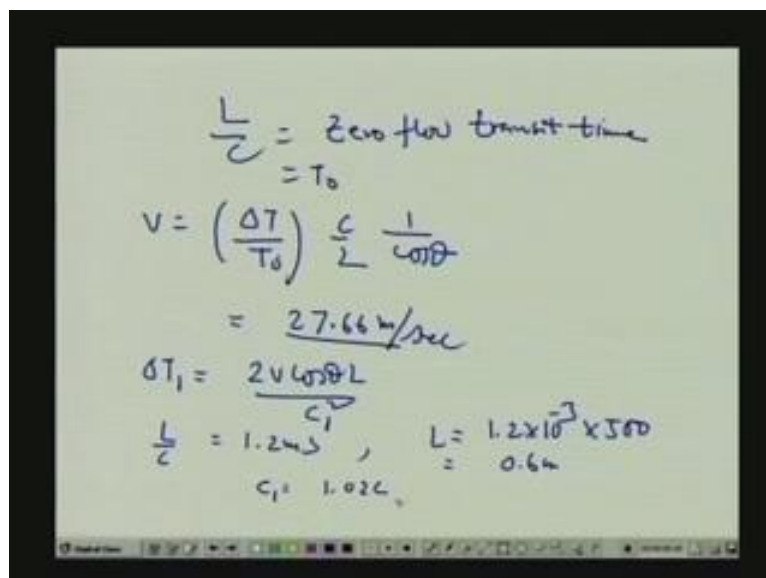
Find the velocity of the gas and by what will the transit time change for percentage of 2% change, plus minus 2% change in the velocity of the sound. Velocity of the sound in the gas is 500 meter per second. So, this is our problem. So, this obviously is very familiar with, so let us take a blank page.

(Refer Slide Time: 50:06)



So, it looks like this. I have a pipe, so I have a transmitter here, I have a transmitter here. So, this angle making with this is 45 degree. What is the angle? No, this is 30 degree, right? So, this is our L, length. So, we see that, see if you say A, transmitter B, so T BA is coming L by minus v cos theta. T AB equal to L by c plus v cos theta. So, delta T, I can write L 2v cos theta by c square, right? So, this we can write 2 V cos theta by c into L by c. So, L by c is the zero flow transit time.

(Refer Slide Time: 51:17)



L by c is the zero flow transit time. That means when there is no flow, transit time is equal to T naught. So v equal to, I can write delta T by T naught into c by 2 into 1 by cos theta. So, if I put, plug in all the value, I will get v equal to 27.66 meter per second, right because T naught also we know, T naught already given. Now, we see that there is a persistent change of the, I mean flow of the fluid, right? A persistent change in the measure, no sorry, there is a persistent change in the velocity of the sound, how my measurement will be affected by that. So, let us look at that.

So, initially when it is, there is no change that times I ..... with T 1. So, delta T 1 equal to 2v cos theta into L by c 1 square, right? So, L by c c 1 equal to 1.2 milli seconds, is not it, right and L, I can write taking that the, there is no change on the velocity of sound that means if it is 500 meter per seconds, so I will get L equal to 1.2 into 10 to the power minus 3 into 500, so it is coming as 0.6 meter, right? So, c 1 will come up as 1.02, 1.02 c 1. Actually this should be c 1 I think. So, because this is a no change so let me take the eraser, so c 1 equal to 1.02 c. Let me take the pen again, right? C 1 equal to 1.02 C. Now, there is a change, right? So, there is a change of the fluid velocity. So, let us take again a new page.

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$$\Delta T_1 = \frac{2v \cos \theta L}{c}$$

$$= 110.5 \mu s$$

$$v/c_2 = 0.98c,$$

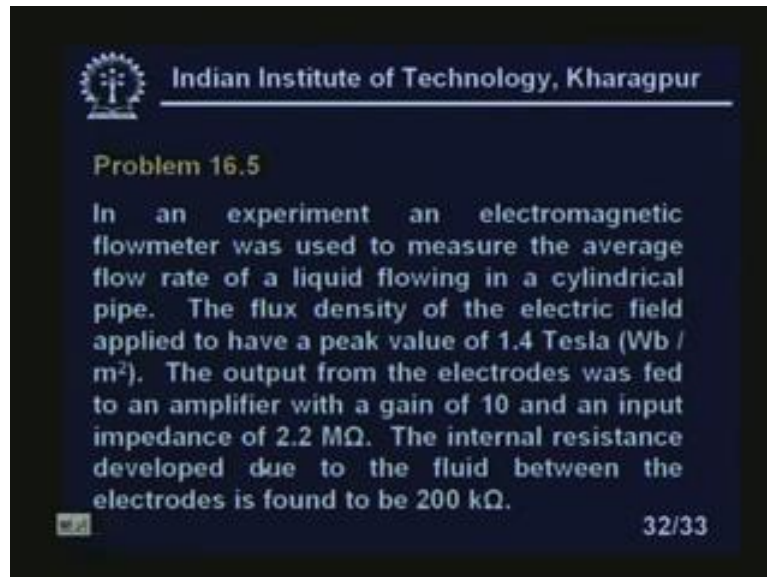
$$\Delta T_2 = \frac{1}{0.98} \Delta T = 119.2 \mu s$$

$$\text{Transit time error} = \pm \underline{\underline{3.9\% \Delta t}}$$

So, second case delta T 1 equal to 2v cos theta L by c square. So, this will become or c 1 square rather, this should be c 1 square equal to 2v. So, this will be, if I take, sorry if I take, I am sorry, so it will be c 1, if I write delta T 1 equal to 2v cos theta L by c

square. So, this will give you, if I plug in all the values, 110.5 micro second. So, for  $c^2$  equal to 0.98 c, so we have  $\Delta T$ ,  $T^2$  equal to one upon 0.98 whole square into  $\Delta T$  equal to 119.7 micro second. So, this will lead to the transit time variation. So, the transit time variation, transit time variation, transit time variation is equal to plus minus 3.9%. This is our answer, right?

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**Problem 16.5**

In an experiment an electromagnetic flowmeter was used to measure the average flow rate of a liquid flowing in a cylindrical pipe. The flux density of the electric field applied to have a peak value of 1.4 Tesla ( $\text{Wb} / \text{m}^2$ ). The output from the electrodes was fed to an amplifier with a gain of 10 and an input impedance of 2.2  $\text{M}\Omega$ . The internal resistance developed due to the fluid between the electrodes is found to be 200  $\text{k}\Omega$ .

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Now, let us look at the problem number 5. Problem number 5 is on the electromagnetic flowmeter. In an experiment, an electromagnetic flowmeter was used to measure the average flow rate of a liquid flowing in a cylindrical pipe. The flux density of the electric field applied to have a peak value of 1.4 Tesla Weber per meter square. The output from the electrodes was fed to an amplifier with a gain of 10, an input impedance of 2.2 mega ohm. The internal resistance developed due to the fluid between the electrodes is found to be 200 kilo ohm, right?

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(a) Determine the velocity of the liquid when the peak to peak output voltage of the amplifier was found to be 4V.

(b) Find the percentage change in the reading of the amplifier for a 10% increase in the conductivity of the flowing fluid.

Given, the diameter of pipe = 0.1 m.

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Now, determine the velocity of the liquid when peak to peak output voltage of the amplifier was found to be 4 volt and find the percentage change in the reading of the amplifier for a 10% increase in the conductivity of the flowing fluid, given the diameter of pipe as 0.1 meter.

(Refer Slide Time: 56:30)

$$E = 2V$$
$$E = e - iR_{int} = i \times R_{amp}$$
$$e = E \left(1 + \frac{R_{int}}{R_{amp}}\right)$$
$$e = 2 \left(1 + \frac{200 \times 10^7}{2.2 \times 10^6}\right) = 2.18V$$
$$e = BLV \times g_{int}$$
$$V = \frac{e}{BL \times g_{int}} = 1.517 \text{ m/sec} = 1.557 \text{ m/sec}$$

So, it will look like that E is equal to here 2 volt. Now, the output voltage will be given by e minus i R internal equal to i into R amplifier, right? So, e equal to E 1 plus R internal by R amplifier, where e is the voltage induced to the magnetic field and

also the open circuit voltage. This basically is the open circuit voltage. So, this will become, if I plug in all the values it will become  $2 \times 10^3 + 200$  into  $10^6$  equal to 2.18 volt.

Now, we know that the  $e$  equal to  $BLv$  into the gain of the amplifier. If I have the amplifier, amplifier gain or the usual notation, flux density, length or diameter of the pipe and  $v$  is the velocity of the fluid, so this will give you obviously, if I manipulate that  $v$  equal to  $e / BL$  multiplied by gain of the amplifier. So, it will give you 1.55 seconds, sorry meter per second. I will write again, 1.557 meter per second, right? So, if I have that thing, we will take a new page.

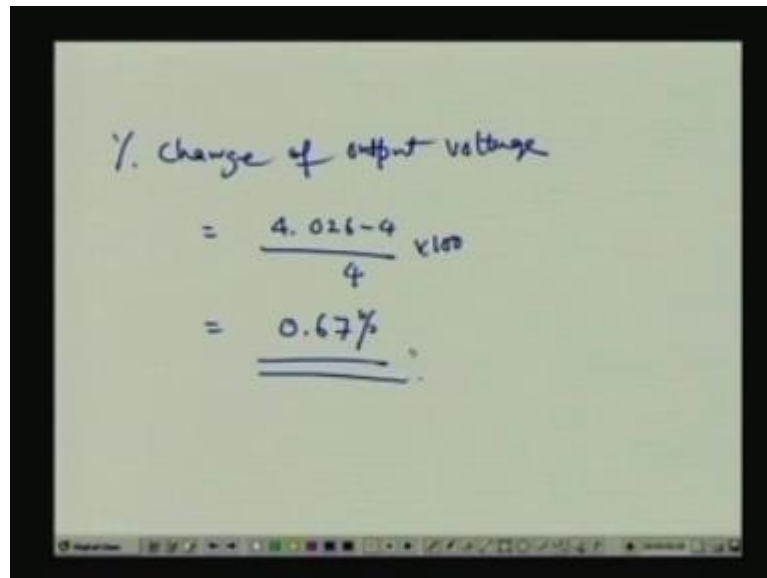
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Handwritten calculations on a greenboard:

Conductivity increases by 10%.  
 Resistance decreases by 9.1%.  
 $R' = 0.91 \times 200 = 182 \text{ k}\Omega$   
 Output voltage =  $\frac{2 \times 2.18}{\sqrt{1 + \frac{182 \times 10^3}{2.2 \times 10^3}}}$   
 $= 4.026$

Conductivity increases by 9% that means conductivity increases, increases by 10%. So, the resistance change, resistance decreases by 9.1%, right? So,  $R'$  will be 0.91 into 200 equal to 182 kilo ohm. So, the output voltage will be equal to 2 multiplied by 2.18 upon  $1 + 182$  into  $10^3$  divided by  $2.2$  into  $10^3$ , like this one. So, this will give you 4.026, right?

(Refer Slide Time: 59:24)



The image shows a handwritten calculation on a whiteboard. The text reads: '% change of output voltage', followed by the equation  $= \frac{4.026 - 4}{4} \times 100$ , and the final result  $= \underline{\underline{0.67\%}}$ .

So, the change of output voltage, if I take a white page again, so percentage change of output voltage is equal to 4.026 minus 4 by 4 into 100. So, this will give you 0.67%. This is the answer.

So, with this I come to the end of the lesson 16 of Industrial Instrumentation.