Estimation of Signals and Systems
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Lecture - 17
State Estimation

So, welcome back after Pooja holidays and in Bengal we say 'Shubh Pooja' which means happy Pooja, greetings.

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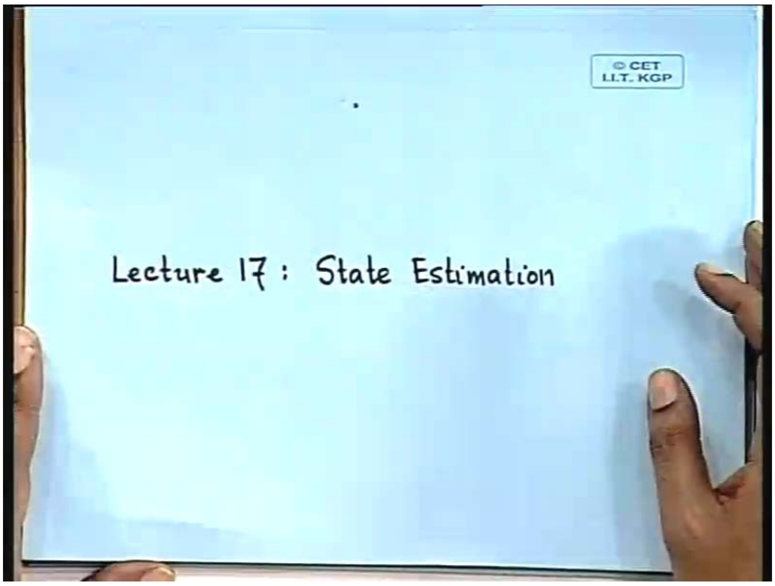

So, we are going to start our module of the course, you can say first module was on probability, second module was on filtering; based on basically based on input, output models. And so now we are coming to the third model which is on which is called which is state estimation. And basically we will study for the large part of this module the Kalman filter.

So, today we will recapitulate and try to you know; I mean motivate the Kalman filter as to why it is needed, and in and in what circumstances it is needed? By starting right from the basics that is we will we will take a look at the problem, and see there are there are various kinds of solutions proposed for various kinds of situations. So, we will start from a very simple situation where we know most of the things are known and something is unknown, and I mean we will try to estimate the state in that situation. I mean estimation is always applicable where there is some uncertainty about something.

So, we will gradually see that as we remove you know assumptions of certainty about many things, situation becomes more and more complex and then more and more complex algorithms are needed, right. So, so we will go that way and we will we will we will first just for the sake of completeness, this is probably is very familiar to you all as to what is a state? That is I want to distinguish between, basically I want to distinguish between the output and the state, right.

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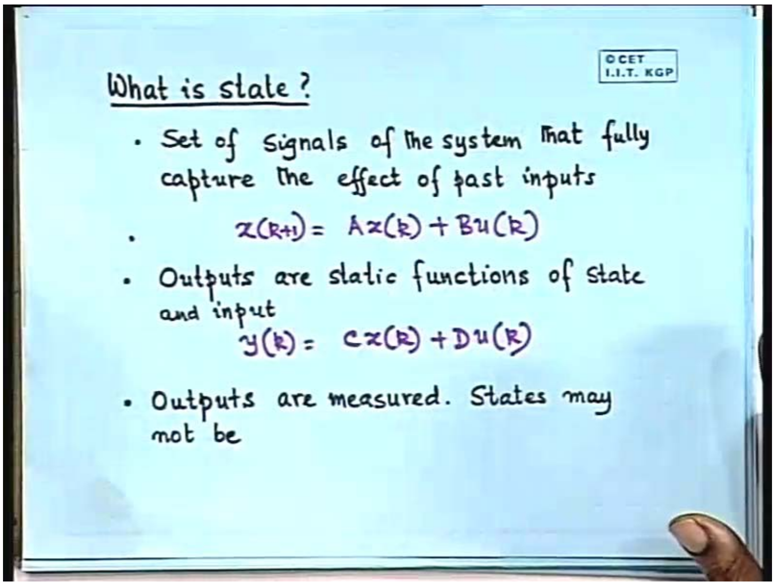

So as we all know that, the state is the set of signals of the system, that fully captures the the effect of past inputs; you know why this is so important because of the fact, that we always when when we when we try to characterise systems we always look for finite descriptions. You know something which can be described, using a finite number of quantities, if you have to describe something using a very large number of quantities then then it is very inconvenient, right.

Now, there is a there is a problem of dynamic systems, I mean the the the the basic characteristic of dynamic systems by which dynamic systems are different from static systems is that they have memory, right. So, their they actually remember in in some fashion; they they will remember the effect of all past inputs. So the so the so the behaviour, that is the future behaviour of the system, cannot be described just by giving the future inputs; ideally speaking, theoretically speaking, you need along with that all the past inputs, right from the state when the, right from the state when when the system was first constructed, a time is equal to probably minus infinity or something.

Now, this is a very very inconvenient situation, I mean if you want to I mean how do you deal with it? You you need you need an infinite number of samples of inputs to be able to to to able to compute any, I mean let us say the output and it a at any instant. So, therefore this kind of this property has to be circumvented, so people proposed models by which this property can be circumvented and the concept of state came in.

So now we say that, if you know the states then they hold all the information about the past. So, you only need to know the state at a given point of time and the inputs from there on. So the past is encoded into a finite number of quantities, which is the biggest advantage, right. So now you have x one, x two, x three, x four, x n; five quantities which will hold the effect of all the past inputs, right.

So, so so you can say that; the that the future, now the future state can the can the now expressed very, you know succinctly in terms of the past state and this input. So it is a very finite finite description, though the system has infinite memory but that memory has been encoded into a finite number of quantities; that is the that is the beauty of it. And well we are we are we are we are obviously interested in outputs, in fact we are not interested outputs; we call those signals as outputs, which we are specifically interested in and which we generally which we might measure.

So, it is it is by definition, that the that the outputs are measured. I mean, those signals are called outputs which are measured generally and which are of interest. For example, if you want to control some specific quantity in a process, we would like to call them call that as an output; while there may be many others which also will affect that, but we will not call them as outputs because because they are not of primary interest to us, right. So and it so happens that the the output in which we are interested and which we measure are actually related by the state and the input. So this is the this is the basic advantage of the state space model, as we know which with which which we are we are we are all familiar that number one, that the past can be encoded into a finite number of quantities which is very easy to handle.

If we have a finite dimensional system; we could also have infinite dimensional systems where even a even if we want to construct a state space model, we will we will require an infinite number of states, we are not talking about such such systems. We are talking of systems in which the state vector is a finite dimensional vector. And so outputs are measured but states but but all states may not be measured, right. So, there may be some other quantities which also affect the dynamics of the system, which will also affect the output, but they may not be measured.

Now, this is what actually makes the problem of estimation nontrivial. If we had measured all states, then we have no problem of estimation, I mean in most cases. But we have we have a nontrivial problem here, because we are interested in calculating some of those states for some purpose, even if they are not measured that is what brings in estimation, right. So, so so then we have this we have this question, as to why we need state estimation. Basically, because because of two facts; firstly that states may not be measured and we we may still like to have them.

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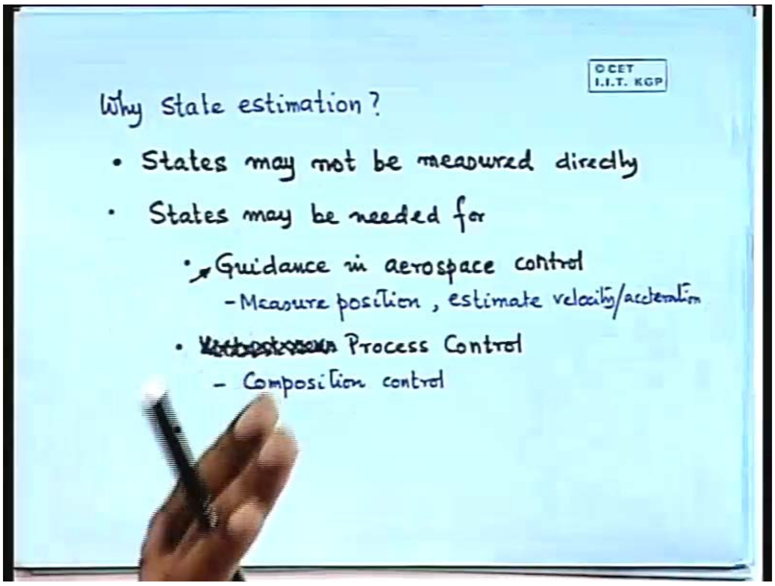

So, why do we like to have them? We like to have we may like to have them for for various purposes; for example, typically this is an example with which I am I mean in which I am interested, that is if you want to let us say, if you want to if you want to construct a control mechanism, for let us say a rocket or let us say a missile, which is supposed to hit, let us say another missile or may be an aircraft.

So, now now the question is that, this this missile will have a controller of course and naturally it has to be given not only not only the controller; you see in in our course, somehow we we we we focussed too much attention on the on the controls part, that is we are we are we are we are always concerned about the control. And we never bother as to this this this R reference signal, that that usually comes in a control, where does it come from? We do not spend much time on that. But it so happens that; in in aerospace control, that is that is extremely important in other words. You must tell the you must tell the missile as to where to go, which direction to go.

Then the if you if you can tell the missile that, then the then the missiles controllers' job is to is to obey that faithfully. See the job of the any controller is to obey its command faithfully in the face of disturbance. So, it essentially need to have mixture of two properties; one is that it must follow reference faithfully, secondly is that it must reject all disturbances, this is the this is the desired property of all controllers. But but there is this there is this nontrivial problem of try to compute the reference itself. That is which direction should it go, after all one missile is coming from here.

So which which direction should it goes, so that at some point they should meet, right. So that comes from what is known as guidance. Now now obviously you can understand, that if you want to if you want to hit something which is moving, then you need to know all its motion parameters; in other words you you will need to know its present position, you will know you will need to know its velocity. If it is an if it is an accelerating target, then you need to know it's accelerate; otherwise how do you, how do you hit it? You you cannot hit it, so but it so happens that, the kinds of sensors that are that are that are put on these vehicles, basically sense position.

So so there is this non trivial problem of trying to estimate velocity and acceleration given the position which is not measured, right. So, that invariably calls for having a Kalman filter, because you need to know the position and position and position velocity and acceleration of the target, right. Similarly, there is a good amount of application in process control, right in in in a process control; let us say; if you want to if you want to control a distillation column, now distillation column has thirty, forty, fifty, sixty variables. All of which are actually coupled, they actually affect both the bottom product quality as well as the top product quality. And there are only a few let us say, typical typical distillation column control will have five six sensors.

So but for but for effective control, you need to be able to estimate other variables; because otherwise actually what happens is typically what happens, I mean the problem in process control is that because of the because of the large nature of the process and because of this slow and slow and accumulative nature of the process. What happens is that; by the time you you find that a that a problem has actually reflected itself in the output, the system has a lot of storage capacity, lot of storage capacities so the so the input, that you are giving now does not affect the output immediately.

So, if you if you base your if you if you base your control only on the input, only by sensing the output then then what happens is that by the time you have seen the output, you have done lot of wrong things. So all those are actually stored and and and you cannot; I mean if you even if you change the input now, you cannot correct the output now. Even if you change the input now, that effect will come later. So so in such situation there is a there there there is a lot of requirement for sensing signals which are not measured. I mean these are these are very I mean; some some typical examples but there are there are many other examples.

There are there are examples in motor control. For example, nowadays you have you have brushless D C motors are I mean, very very common you know; they have they they are become very important drive machines. Now one of the one of the major difficulties of of having a brushless D C motor, you you need to sense its position; I mean if you could do away with that position sensor, then it will be good. So, I mean lot of people actually put state estimators by which you can you can actually estimate the position based on the current measurements, I mean typically current is the easiest thing to measure.

So, there you can have an you have an estimator, I mean similarly you can have lots of other examples; where I mean measurement of state, I mean rather estimation of state based on measurements is required for effective control. So I mean, that is why a state estimators are very important; for example typically I mean this distillation column control problem is is is essentially a composition control problem. That is you want to produce from the crude, you want to produce petrol of a certain quality. So, if you want to produce petrol of a certain quality, it has a certain composition only then it will have certain parameters like octane numbers, right. So, essentially it is a composition control problem. So, so for such cases you need to have state estimators.

There are as I said that, there are various kinds of state estimation problems and even, I mean they they they differ on on on a on a number of cases, I just have mentioned three classes. For example, initially I mean, this simplest state estimation problems are known as this state observation problems, okay. What what is the situation there?

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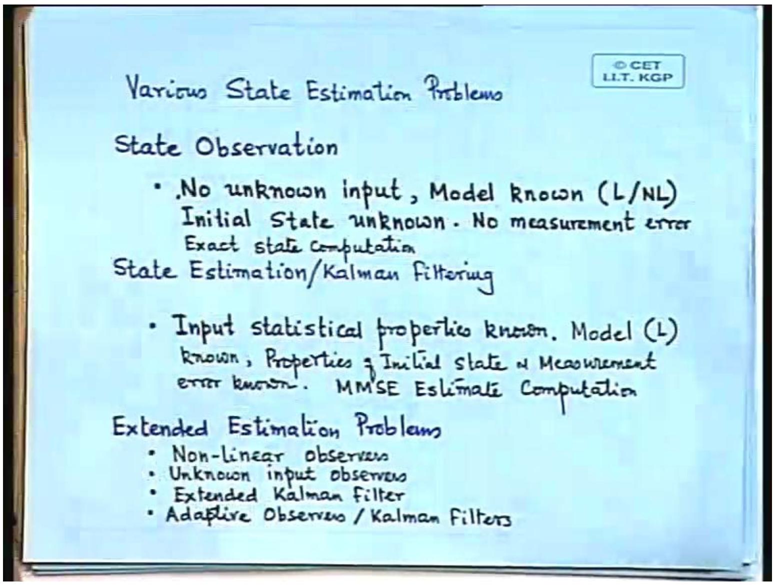

First of all you have a known model, generally completely it assumes to be known, okay. It could be linear it could be non-linear. We are going to concern ourselves mainly with linear and and, I mean that is what we have or we are going to study in a standard linear system theory courses, okay. So, you have a model you have the model known; now the model, what is the model? The model is the simplest model, that is x k plus one equal to x k plus B u k kind of model or x dot equal to x plus Bu. Now, in that second assumption is that the input is known, so we are assuming that the total input vector u is known.

In general, a system has two kinds of inputs; one kind of input is called the control input, these things are known which is actually applied. The other kinds of inputs are called disturbance inputs, which is which which are not applied which are which are actually apply applied by the environment. So, so you have you in general have no control over them and and often they are immeasurable. So in this case for this state observation problem, we are not considering disturbance input in it is in its in simplest form, okay.

So, all inputs are assumed known, what is unknown then? What is unknown is the initial state. So, the essentially the problem of I mean, estimation in in the in the in the in the state observer problem is basically a problem of I mean; whether you can estimate given a set of input, output data, whether you can estimate, whatever the state before that, that is essentially the problem okay, which actually leads to observability as we will see.

I mean, similarly know I mean many many practical things are not considered, for example no measurement error. I mean we we will we will actually have measurement error, but typical treatment will not consider measurement error. Similarly, now since we have so many things, the problem that is consider is that, whether we can asymptotically; that is which time whether we can whether we can estimate the actual state, the exact value of the state.

So, the so the problem that we I mean, attempt to solve, tries to get the actual value of the state with the error going to zero. This is what is attempted because the situation is so nice, I mean we have we know all the inputs; we have no noise in in in the measurement, we know the model fully, only the initial state is not known, right.

So, now the clearly this situation is not realistic, right. So then we we we put the next set of problem that is okay. We now assume that, let there be some noise in the in the measurement and we also assume, that let there be some uncertainty in the inputs; that is some input may be known but then there there may be some other inputs which are which are unknown. So at even if we consider them as unknown, let us at least assume that now one one common way of characterising unknown and uncertain things is to have a probability distribution. I mean when having a probabilistic description.

So, here in this problem, now we now we increase the complexity little bit to make it more realistic. So we say that, input is not exactly known but the statistical properties of the input will be known; that is it may have a mean, it may have some variance, we will assume those things. So, we are now reducing our our assumption of noise, our assumption of knowledge model is still known. Properties of initial state and measurement error, I mean we now assume that that there may be measurement error, but we assume their properties, what could be theirs variance?

How much measurement error? And we also assume some properties of the initial state; that is what could be our initial state? I mean by how much can be our initial state guess known? So, once we introduced these uncertainties; obviously we can never hope for computing the state exactly, because there is noise. So, our estimates are always going to be a erroneous. So therefore; it is not meaningful to try to solve a problem, where the where the estimation error will go to zero, it cannot go to zero. So, therefore now we try for a minimum mean square error estimate, that okay error is going to stay there, but can I make an estimator which is going to make my error power minimum.

So, now that is the problem which is attempted; now once you have this now you can have various other variables; for example, you can have you can have a non-linear version of the observer which is which is much more complicated, because non-linear dynamics is much more complicated. So, same case except for the fact that, the known model is is nonlinear. Similarly, you could have you you could construct unknown input observers; that is whether I can whether it is possible, that is the effects of some of the inputs at least can be totally decoupled from the state.

That is even if I even if I do not know them at all, can I in by some way can I produce an estimate which will which will not depend on these, not depend on these inputs, right. So, we sometimes we make unknown input observer, sometimes we make we try to estimate the input separately because they are unknown, but we.. we this is this is another class of estimator, where a large large I mean I mean a significant input which is not probabilistically described is is assumed to exist, okay.

Then there are there are extended versions of the Kalman filter, mainly to mainly to take care of non-linear, non-linear cases and they are also adaptive versions of it, when the when the system tends to change from one type to another. So, whether we can have whether we can have state estimators of systems which are varying, so these are you know somewhat advanced problems.

So in our in the coming few lectures, we will basically be be be concerned with this class of this class of estimators; but today we will take a take a look at these, just to you know kind of warm up. So, we will try to see, what is the what are the results on the simplest problem of state estimation, okay? And this results I think you will also come across in your control theory class, these these I mean, observers and I mean observers based controllers et cetera.

So, we will not derive results but we will see the basic principles, against these results today.

Student: State estimates and the states are known?

No, that properties of initial state known known means, that the that some initial state error variance may be known, right. My exactly initial state is not known, if if the if exactly the initial state is known and if all inputs are known, then then you can simulate it, right. If exactly initial state is not not assumed to be known, yes.

Student; state observer what is the..

State observer no properties is known, because because in the state observer generally we generally; we we we try to design the observer in such a manner as we will see that, that as long as the error is finite, eventually it will come to zero. That is how we want to design it. So, the question is so the first question that is, when the first concept which is generally introduced is that, when is this theoretically possible, that is now we are considering this particular case.

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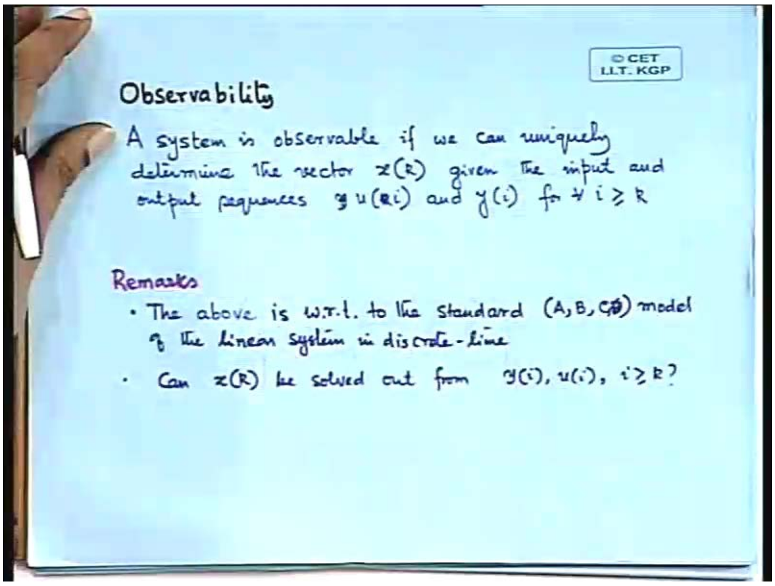

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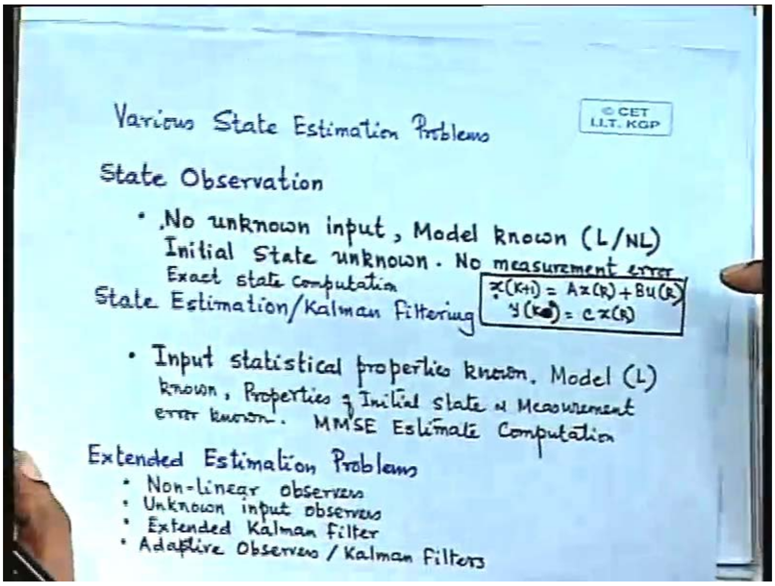

That is we have a standard model, x k plus one equal to A x k plus B u k. You can have you know you can have; so the model is x k plus one, we are considering discrete because that is simpler continuous creates many problems, basically because of it because of the derivative operator. Especially, if you have a if you have a statistical signal, defining is derivative is not simple, so we are avoiding all that.

So, this is our model and you can have either y k plus one or rather y k is equal to C x k or you can have y k, y k equal to C x plus B u k; it makes no difference because u k is known, so you can always construct some other y dash k equal to y k minus d u k. So, even then the left hand side will be known, so we will we will generally assume that this is our model. In this A B C is known, u is known, we want to estimate x but initial x zero is not known, okay.

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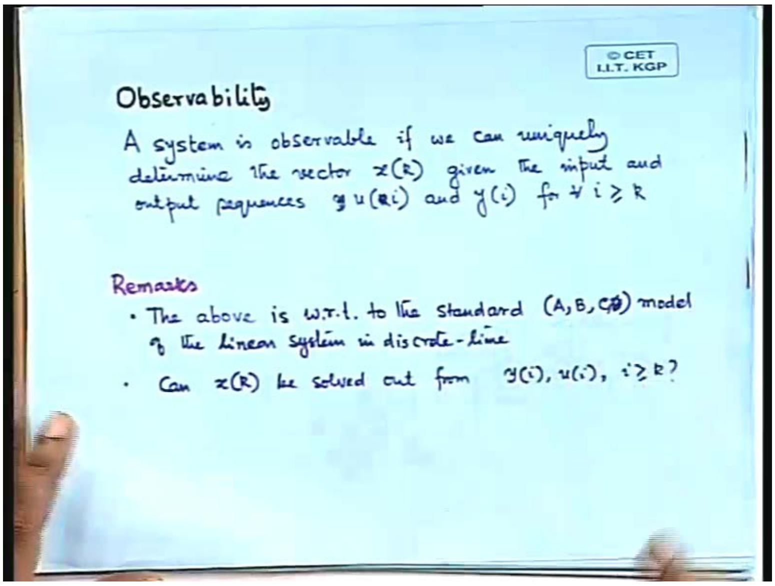

So, the question is when do we when can we even when hope to solve this problem? So that is given by what is known as the condition of observability. So, a system is is said to be observable; if we can uniquely determine the vector x k at time k, given the input and the output sequences u i and y I, for all i greater than k. That is suppose the suppose; the system started from k, now if you are given all the future inputs and outputs, can you, for using them can you compute what was x k? It is a kind of a initial state problem, right.

So, if if if that is possible then the system is said to be observable, right. Whether you can solve out the state from which it started, I mean actually there are if you see books; you will find there are many definitions of observability. They are actually they can be shown to be mathematical equivalent, this is probably the one which is the most popular but there are others many others. So, note that I mean obviously this is we will assume that, that this is with respect to a standard A, B, C or even A, B, C, D I have cut this D but you could put this D as well, does not matter. So, we will assume that to the standard A, B, C, D model of the linear system in discrete time.

So, we will treat this problem there and the and so the basic question is basic basic question of observability is that, if you are given u k, y k, u k plus one y k plus one et cetera, whether you can solve out x k? It is a question of solving out from a set of linear equations. So, basically it boils down to the solvability of a set of equations. And what are those equations, these are the equations, why these are the equations?

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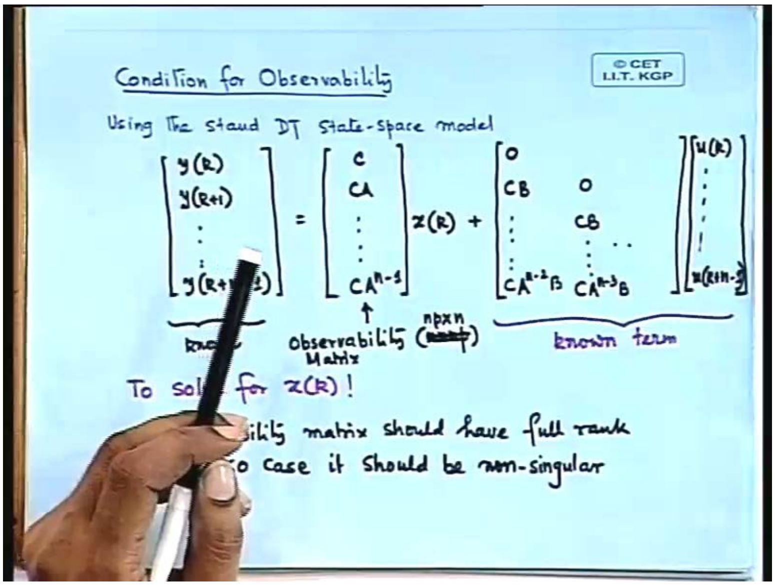

That is very simple, because you cannot…see you want to find out x k, x k has how many components? X k has n components. Let us say it is a nth order system. So, if you want to solve out n components, you need how many equations? You need n equations, you need not only n equations; you need n linearly independent equations. So, so so the first thing that we have to do is that, we have to set up those equations and then solve it, right. So, so basically and these equations must be set up with u and y, because they are they are what we know.

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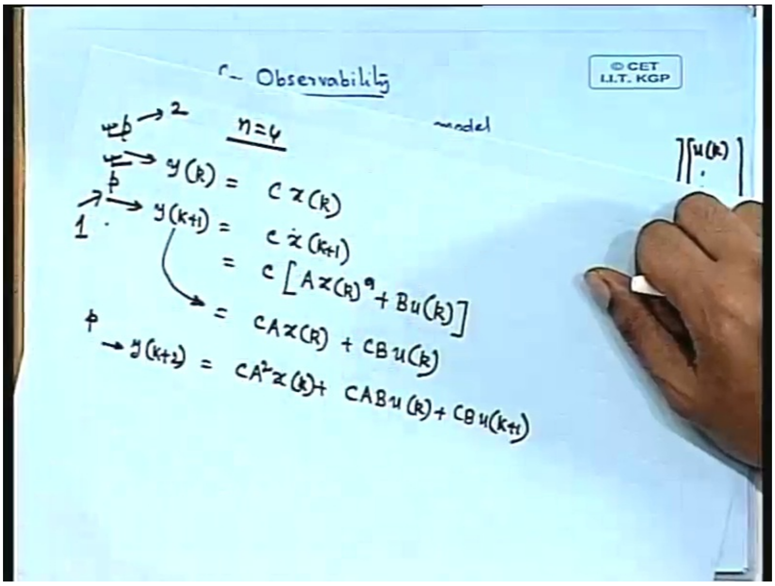

So then, so you can always write that, y k is equal to C x k this is the first equation. Second equation is now, what is y k plus 1? It is equal to C x k plus 1; what is x k plus 1? That is equal to C into A x k plus B u k; that is equal to CA x k plus CB u k. So, this gives you the second equation, see all equation must involve x. Similarly; you can have y k plus two, go on doing it then, if you do it you will get C A square x k plus C A B u k plus C B u k plus 1.

This is this gives you a third equation, right. Now, there is a there is something, this is not the third equation; I mean ideally speaking if you have multi input, multi-output system; then this is p equations, this is p equations this is p equations, each of them are actually because why could be a p dimensional vector, right.

So it could be, if it if it is single input, single output then one equation you need n, but if all you need are n independent equations. So, how many of these equations you will consider depends on, if you go on doing it how many of them you need? First get a set n, for example suppose p is 2 n is 4 could very well happen, then may be from the from the first two you will get 2 linearly independent, one from the second one you will get1.

So, the that means that, from the second set you will get two equations but one of them will be linearly dependent on either these and the third one. So, that is not an independent equation. So, you have to find out how many independent equations, by doing this you have to find out n independent equations, they should not be linearly dependent, right. So, now the now the question is that; how many of them you actually need to consider? Whether you should take p is equal to, I mean a whether you should, whether this will this will clearly not do, because generally p is less than n. So, you have to have another one, but whether by considering y k, plus one only you will be able to find or or whether you need to go for y k plus two, how do you determine that?

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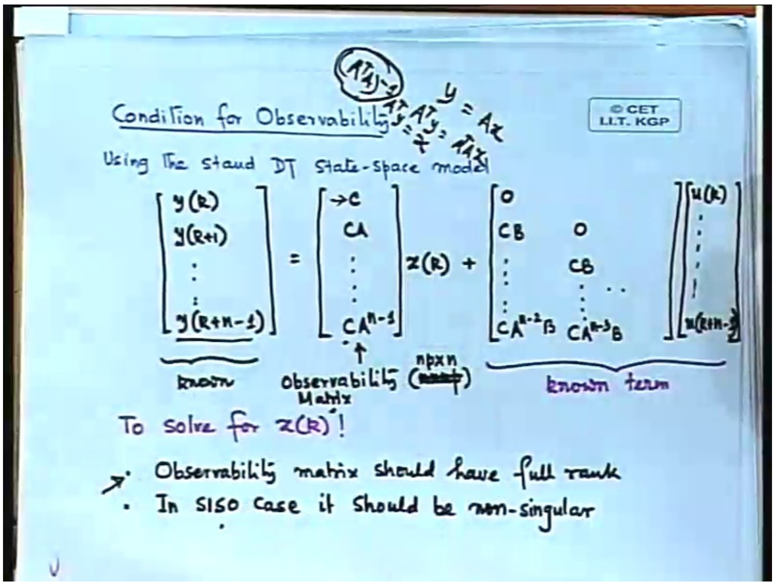

So so then there is this, see it can be shown by by a theorem which is called the Cayley-Hamilton theorem, which says that you you you need to consider maximum up to C A to the power n minus 1; that is you need to consider only these because, if you take more then by by taking more of them, you cannot generate another linearly independent equations or whatever linearly independent equations you can generate, you can hope to generate you can generate only with among these these.

If you take C A to the power n C A to the power n plus one, no new now new linearly independent equation will be generated; that follows by a theorem which is known as the Cayley-Hamilton theorem, which is I do not know you must have heard about it. So, if there are n independent equations, they will definitely occur among up to these; if there are, if there are not then you then then bad luck, you cannot solve for x, okay. So, now it it turns out that, if you so this matrix is called the observability matrix. So when we say that, that celebrated condition of observability that that that this matrix should have rank n, what we basically mean is that there should be, you should be able to set up n linearly independent equations, to be able to solve for the n linearly independent unknowns. That is the that is the meaning, okay.

So, so it should have full rank, why full rank? Because what is this matrix? This matrix is n p into n, there are n p number of columns; each one of them is p, so there are n of them. So, there are n p number of rows and there are n number of columns, C has n number of columns, correct. So, obviously an an n p into n matrix, cannot have rank more than n. Rank is minimum number, minimum of maximum possible rank is minimum of row and column number.

So, so that is why since since this matrix should have rank n, so it is called full rank, it is actually a non-square matrix. If it is if if this is a single input, single output system then this matrix happen to be n by n, because p is one in which case full rank means invertible or non-singular. So in so it is in the S I S O case; this matrix is required to be non-singular, so obviously you can you can know that, here this this this is known. We have assumed that, we have exact values of output without any measurement error. This is known we have assumed that, we have exact values of input without any error. So then that means, you can bring it to the left hand side and then the left hand side will be known.

So, if you have, if you bring it to the left hand side, left hand side is known and here you have a matrix into x. So so to get x, what you have to do? Invert it. So but you cannot invert, it because it is a square because a non-square matrix. So then you have to do pseudo inverse, that is you have to do theta transpose, theta inverse. You understand, what i am saying? If you have y is equal to A x and then, x is non-square then how do you solve for x? You say A transpose y is equal to A transpose A x and then you say A transpose A inverse, A transpose y is equal to x, so you can do that.

Now, now who says that, A transpose A A A inverse will exist; that is actually establish by this, by this if it has n rows it will happen that, this will be inverted. So, so this is the way you essentially can solve for x. So, if it is observable; its simply means that, this equations can be uniquely solved for x, all right. But, but but but that is not the way we we want to do it, we want to do it in a different way, we will see that.

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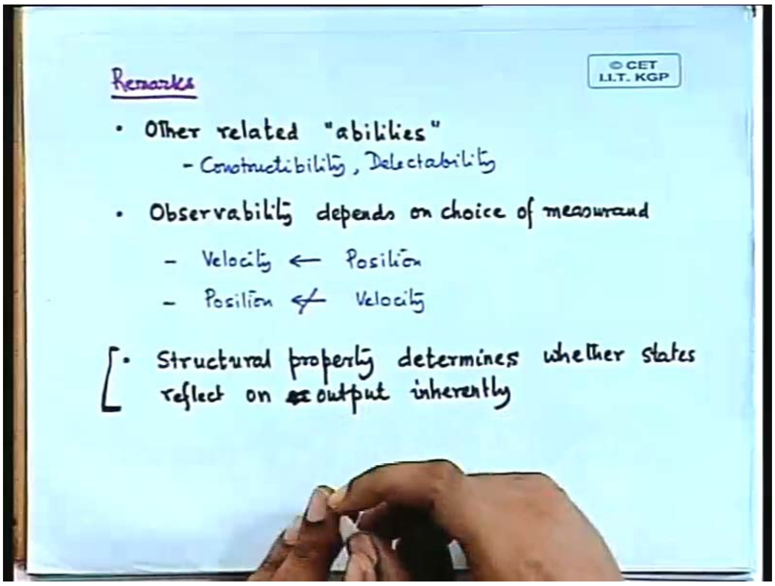

So now, so at this point let us let us also mention that; though I have talked about observability, if you you know if you see a slightly advanced text, you will find many other abilities. You know, you will find, for example there is control ability, there is rich ability, there stabilize ability, there is just like there are these three for control, there is there is the on the estimation side.

So, you have observability, you have constructability and you have detect ability, okay. They are actually one, one and actually counter parts of others. So, any way let's not, we are not too much concerned about this particular problem; I am just trying to impress that, this state estimation problem just basically trying to get its character, our main aim, I mean procedure of interest is under stochastic conditions, that is Kalman filtering.

So, we will not going to this but just want to mention, I always like to mention; what what what what is not mentioned in the class. So so these also exists basic variance of the same thing. Interestingly, remember that, observability depends on choice of measurement; you see many times you know, this is these things are generally not treated in the sense that, if you are let us say, when you are when you are when you are planning a system, probably system will be given I mean a discretion column is a discretion column you you cannot do much about it, but but but you probably can have a choice as to where you are going to put the sensor, right.

So choice of measurand is important. For example, you can you can easily show that choice of measurand can make a system observable or unobservable, okay. A very simple example is that if you measure position, you can extract velocity from it, but if you extract velocity you can never extract position from it because because you will not know the initial position, right. So sensing sensing velocity you will you can never hope to get position.

So, it is so that state can never be observed; for example, suppose you suppose you set up a model in which position and velocity are the are the two state variables. Then if you if you measure velocity, then the system is observable, non-observable because you can never get position from it. But if you measure position, if you measure that state variable, if your y is one zero rather than zero one, C matrix; then you will find that your system is observable, because from position you can get velocity.

So, we we must remember the that, the choice of the which variable you are going to measure can affect whether you can estimate the state or not, right. So, so basically observability is is more of a theoretical interest in the sense that, it it it tells you, it is its its important because it tells you that it that structurally the whether the system is such that; you know when when a when a system is not observable, what happens? What happens is that, because of the speculative of the A and C matrix; some of the state variables are not reflected on the outputs, structurally, they are not reflected. If you if you multiply C into x; it will become it will tend to become cancelled, right.

So, so so it basically gives you a kind of a structural in formation, more than a you know I mean in a in a in a very idealistic case, that is that is what usually it is. So so let us see that, what are we going to do with this observability thing? We are going to construct an observer, okay. So, what is an observer? It is an algorithm, that produces an estimate of state called x hat k. I am differentiating it from x k, x k is the true quantity, x hat is the estimate and and it does it you know recursively. As you get inputs and outputs, it it continuously gives you a new new estimate, okay, that that that is an observer. So again assume is that, model is known x, zero is is unknown; so even if x zero is unknown, we want to set up a systems such that; eventually even with unknown x zero, my estimate of x k will come close to the true one, that is what I want to do. So, what do we do? Obviously, we have to do this part of the things.

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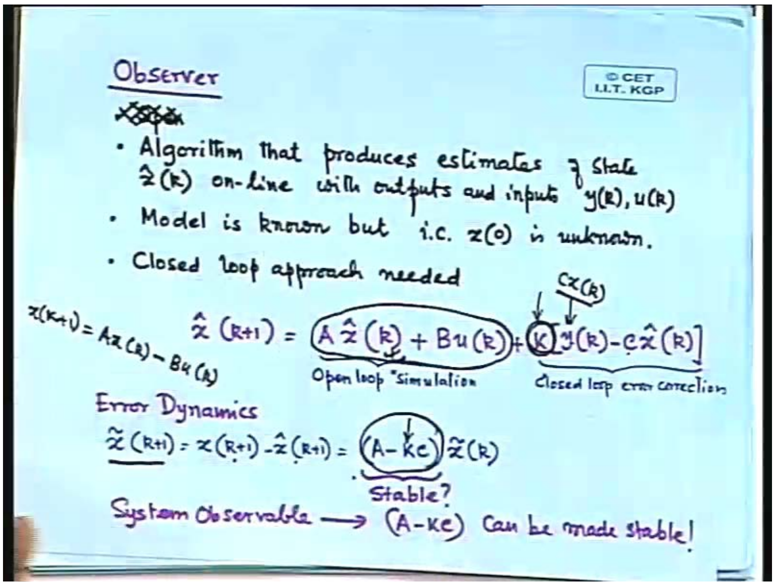

So, so this is the basic equation, which says how am I going to update my estimate. So, I had an old estimate, from that old estimate I want to compute a new estimate at a new time instant. So, obviously since since the true state goes through A x k plus B u k, obviously I have to do that with my new estimate, also with my old estimate also only then, so so this is a kind of you know, open loop simulation. So, I had an old estimate, I am assuming as if it is true and normal system dynamics; I am doing a one step ahead simulation of that, but then just that will not do, why?

It will not do because, if I keep on doing this then my initial condition error will not die, I want to make my initial condition error die. So to do that, I must always correct, see every time I am producing an estimate; I want to check how good it is. So, so how do I check that? I do not have a measurement. So I cannot check that directly, but I can but I have y. So, I can at at least check whether C x came close to y. I cannot check I I do not have a measurement for x, so I cannot check whether my x hat was good; but I can at least check whether my C x hat was good with respect to my y. So, this is the feedback that I am giving, that the every time I am getting an error; I am trying to correct my estimate, not only with an open loop simulation but with a but with a correction turn, right.

So, so that is how I am I am updating my estimate. So so this is my observer and the crack of the problem is; how do you choose these? In in in linear state estimation, this is the cracks of the problem. Most estimators will have the same structure; even the even the Kalman filter has this structure, but the all the trick is how do you choose this k, right? Right now in the observer case, we will choose k as a constant k. Now, how do we choose this k, so let us see that what is our what is my objective of choosing k? My objective of choosing k is that, my estimation error dies down, that is my objective. So, so now let us define define my estimation error; this is this is x tilde k plus 1 is my estimation error which is x k plus 1 minus x hat k plus 1.

So, how does that evolves? So, if you put if you just put x k plus one equal to A A x k plus B u k and then, rearrange you will get this term. So, you see that this this becomes a homogeneous equation, Bu term will get cancelled. Why? Okay, so x k plus 1, first of all note that that that that this is C x k. So then this term is actually k C into x k minus x hat k. So, it is k C into x tilde k, this term and if you do x k plus 1 minus this equal to A x k minus B u k. So, if you subtract then what will happen? It will be A into x minus x hat, B will get cut. So it will be A minus k C, that is how this is very standard; I do not think I need to elaborate this.

So, now the question is you, this is this was your error of estimation in the kth step and this is the error in the k plus oneth step; so it shows the one step evolution of the error. Now, why should this error die? This error will die if this matrix is stable, in the sense that it has Eigen values on the inside the unit circle then it will contract, correct. And so the whole problem of choosing k is such that; you choose k such that this has Eigen values inside the unit circle and you can theoretically proof that, if the system is observable then you can choose k always to make this to be have its Eigen values, that is arbitrary pole placement is possible, that is the good result.

So, if the system is observable then you can always place the poles, always make its stable. Now, the question is so, so make a stable means what? Makes a stable; means that error will eventually die. Eventually; means how long? We want the error to quickly die, so so if you want to make this error quickly go to zero; what we should do? We should this should this should have poles in the deep in the left of plane. If it does poles close to the imaginary axis, even in the even if in the left of plane, then this not going to die fast; this will die slowly, right, right or not?

Obviously, because all its poles will will will I mean, if you find a time domain solution of this; you will get e to the power minus lambda one t, e to the power minus lambda two t et cetera. Where those lambda one, lambda two are the Eigen values. So, you so if lambda one, lambda two are have high values, e to the power minus lambda one t will quickly fall to zero, correct. Now, the question is so then why do not you make it to infinity? Make it terribly high, after all this is all in your computer. This is not like control, that you have to apply actual physical signals; in the case of control you cannot make this very high, why because if you make this very very high, then your control input become high and and you cannot apply such control inputs to the plant; because you will have limitations, you will have actuated saturation.

You cannot apply suddenly, you cannot apply twenty thousand Newton metre torque. So, even if you compute that control, the plant is not going to get that, right. There is a there u is a physical limitation. Now, here there is there is no such thing. You are doing it all in the computers. So, so so have a thirty-two metre ward and make a high, what stops you? What will stop you actually is that, what we have not considered? We have not considered any measurement noise, but if you have measurement noise and if you make this k high then all that measurement noise will actually come in your estimates. That is in in a in a theoretically; you will get a very good estimate, if you make k high.

I I mean, if you make if you make k such that, the poles are deep in the left of plane. This is all theoretical, but actually what will happen is that is there will be some sensor noise and those sensor noises, those sensor noises are here, right. So, if you have if you have some noise here, you can imagine what will happen, if you make k high. Poles will go to the deep left of plane, if you have high values of k right.

So, that is why practically you cannot make k too high. So, it is always an optimisation of how close you want to, I mean achieve your estimation error and how much measurement noise you want to pass, right. So, that is the that is the practicality of observer design. Now, the the last concept; that we want to consider is that, why should we here we were estimating the whole state? So, we have an we have an observer which is an end dimensional system. So, we have to update n equations, right. This equation has to be computed in real time, so why should we update n equation; we already have p of them in my hand, we have p measurements.

So, why do not I update only n minus p of them? p of them I always, I mean I already have as the output. So, why I should again update them? So, so there may be a situation in which you may not like to update, may not like to estimate the full state; rather you would like to estimate a an n minus p dimensional quantity which is the smaller dimensional quantity, and then take the output and these quantities to estimate the full state. So, so only estimate n minus p of them p of them, you you already have as measurement. So, make them and then give some linear transformation or something to make the vector.

Then then at every time instant, you have to update only an n minus p dimensional equation. So, it is much simpler, it will take less computation time, right. So, so when we do that, we have what is known as the reduced order observer. So, so what do, we actually this statement is exactly correct in the sense that, observer only those states for which they are no outputs.

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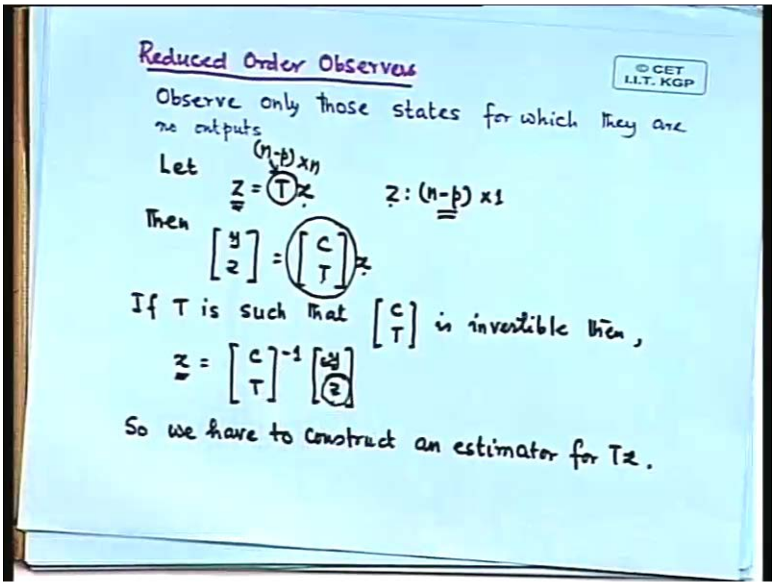

I mean, in this question in this this sentence, I feel, I I wrote it then I had misgivings about it. That I mean there it is not that, you have you have I mean output, corresponding to each state; this output is for state number two nothing like that, but the basic idea is that if you have p outputs then estimate n minus p of them and then try to make up the state from that without having an observer. So, so how do you do that?

So, now define a new variable called Z, which is T into x. Where T is this, T is an where this Z is an n minus p dimensional vector; so smaller dimensional vector; which means that this is an n minus p into n dimensional matrix. So, basically you are considering linear combinations of x, of the elements of x and you are considering n minus p such linear combination that is the whole idea. So, now this is the quantity, now try to estimate Z. Now now the question is that, what is the beauty of this this Z? Beauty of this Z is that, you choose now y and z if you make this vector, it will be C T x because y is equal to C x and you have chosen Z such that; Z is equal to T x.

Now, if you choose T in such a manner, that this matrix is invertible. Then you could write x is equal to C T inverse y z, you could get x. So, you see if you can estimate this lower dimensional thing this you already have you do not have to do anything for that. So, you estimate this lower dimensional thing and then from there you can get x. So, the estimator only needs to have n minus p dimensions. So, now the problem is how to construct an estimator for z, all right.

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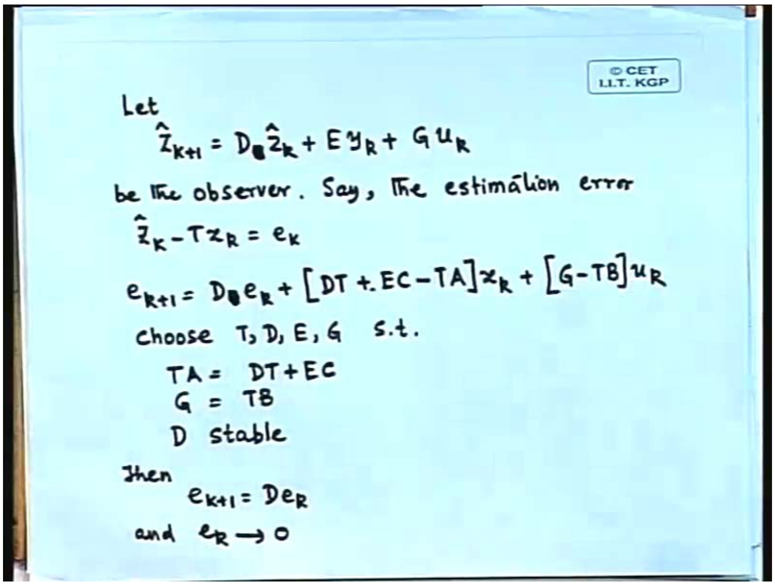

So, how do you do that? That is that is diltively simple; we will just finish it now. So, let us to construct a, after all if you want to use an estimator; what all can use? You can use u values, you can use y values and you can use the past estimate values, other than that you do not have anything. So, I am just assuming that and and we are we are assuming a linear structure of the estimator. So, I am just assuming that, let Z, Z hat k plus one, be some be a some D E into Z hat, k plus some E, some G. I do not not know, I have just made the most general possible estimator structure.

Then I will find that, what are the properties of D E and G required, but this is a very general form after all you could not use anything else, correct. So, now now let now let us see, what happens to error? So, if what so, now my objective is to estimate this. So, this is the error; so now, how does this error propagate? Just like previous time, I am doing. So, now if you same thing you write Z minus Z hat k plus 1; like that you write, so you will get e e k plus 1 is equal to D into e k plus these terms. This is just this is just algebra, you just put Z k is equal to, this is simple just just simple algebra.

Now, the question is that this is your, you see that this is the this the homogeneous dynamics and these two are there, these are like inputs. So, if y u can make these zero, then it will again have in the that form. So, first of all you choose so now is basically a design problem. So, you choose T D, what are the your, what what are the things that you choose; when you design this observer? You first choose this T.

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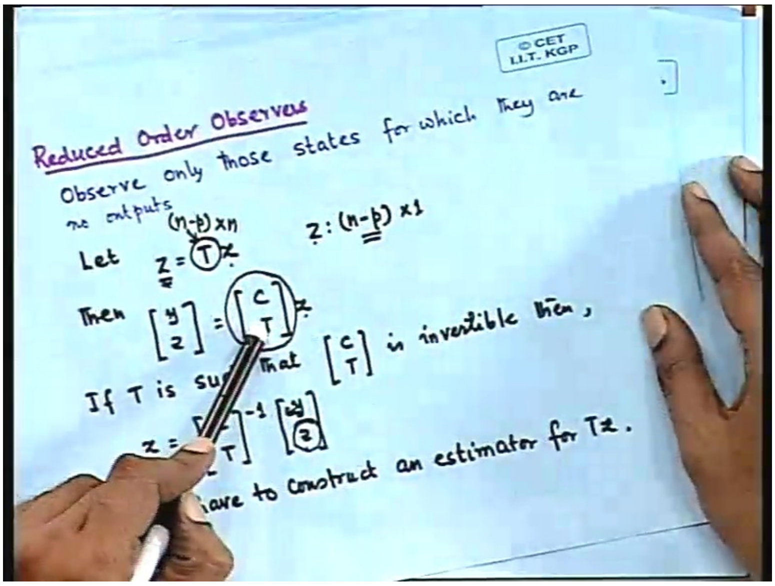

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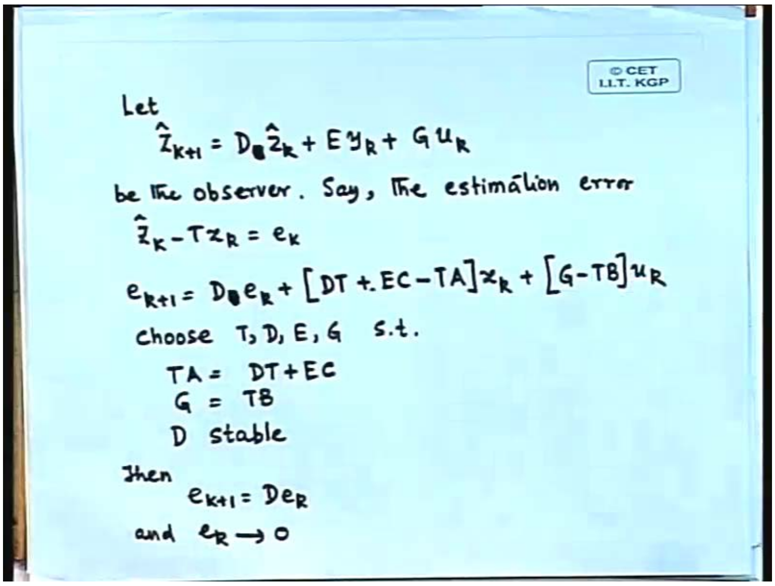

First choose this T then corresponding to that T, you choose D E and G. This is at your matrix is that, you have to choose then you get your observer. So, you choose these, such that T A is equal to D T plus E C. That will make this term zero and put G is equal to T B, that will make this term zero and and also choose in such a manner that, that D is stable then you will get this term will become zero. This term will become zero and you will only only we left with D and then you choose D is stable, that is all.

So, then you get this e k extend to zero. So, so this is the this is the problem, now now how do you do it? This is the I mean, you have you have various kinds of algorithms by which you can actually make a choice; that those algorithms I am not discussing, but this is the essential principle of a reduced order observer.

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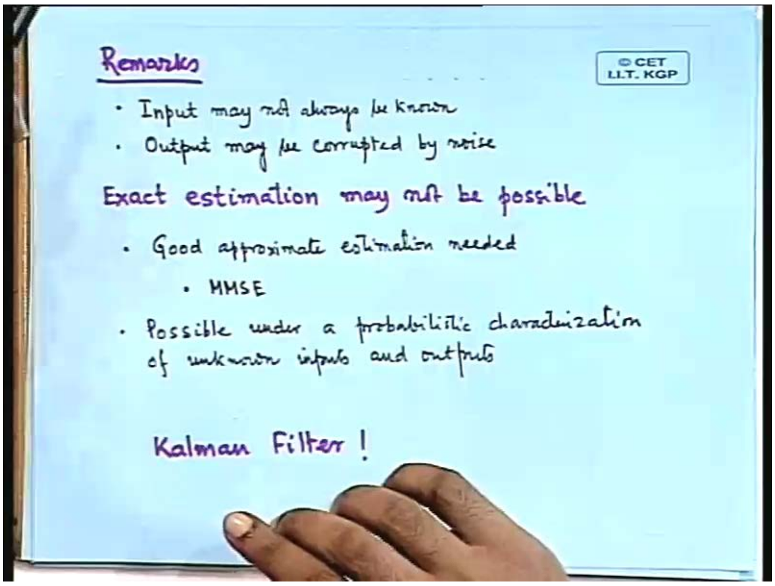

So, we are going to end this, this is the concluding slide. So see that, so now so so this is our observer theory. There is there is a significantly very interesting theory; which says that if you use these observer estimates in control, then what happens? That is another story, as very very very interesting results on that, especially in the context of robustness but anyway we are not going to that. So, now our final remarks will be that, that that theory is nice but not really applicable in practice because of the fact that input may not always been known.

Firstly, there may be non-idealness in the actuators, secondly there may be disturbance inputs; for example, on a on on a aircraft there may be wind ousts which are not measurable. Similarly, output will always be corrupted with noise, I mean if you are I mean; for example, if you are using a radar for target tracking, it it there are n number of sources of noise in a radar okay various kinds. So, output may be corrupted by noise, so therefore these assumptions that we made are rather unrealistic and cannot be actually used.

And and because these are not this is not the case, so it is also unrealistic to actually try to match the state exactly. So, we can never hope to make the error go to zero. So, so so that is not the problem that should be formulated. So rather we should should formulate an an MMAC problem, which will give me minimum mean square error; error will persist but its amplitude will be low. And and now since since we may we may not know it, I mean if we if we assume, nothing is known then then we will never get any solution.

So, we will always have to say that something is known, something is unknown which is the practical case. So, the so the known part we will generally use we generally I mean express in the term B u k, that is that will like like previous times, we still assume that that that that is the known part; but we will assume that there is something more on that, and and for that we will assume a probabilistic description. Because if you have no description, then we cannot produce any result again. So therefore; we will use a probabilistic characterization of the unknown inputs and outputs, and as we shall see in the next class, that will lead us to the Kalman filter, okay. So, that is all today, thank you very much.