Estimation of Signals and Systems

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Lecture - 10
Linear Signal Models

Good morning, so today we will discuss linear signal models.

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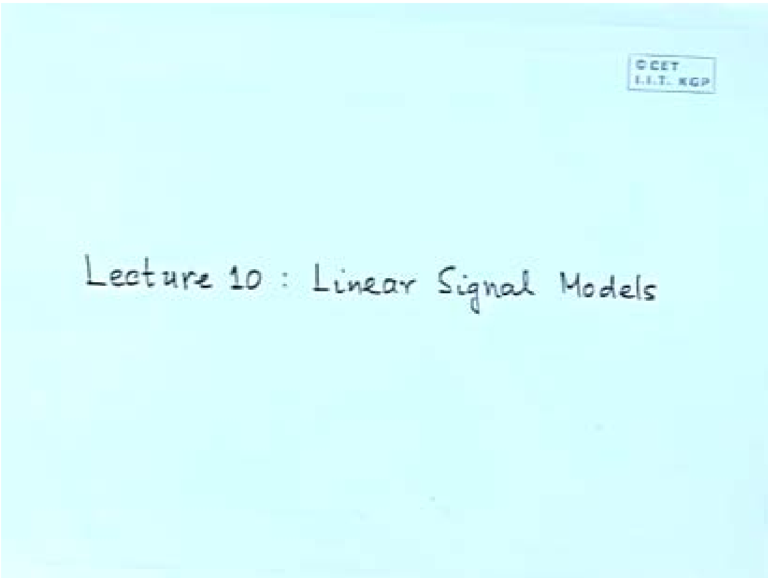

We are trying to get closer and closer to the actual problems; but still there is something to understand, before we can actually handle the the problem of estimation. We have not yet arrived at any estimation problem so for. So we will begin with, the question of what is a signal model? A signal model simple stated, is a mathematical way of describing a signal. We may be describing in some cases, we may be interested in describing the signal values themselves. For example, I mean a very simple, these pens creating a trouble. For example, we write that say let us say, when we describe a signal as a Fourier Series, right.

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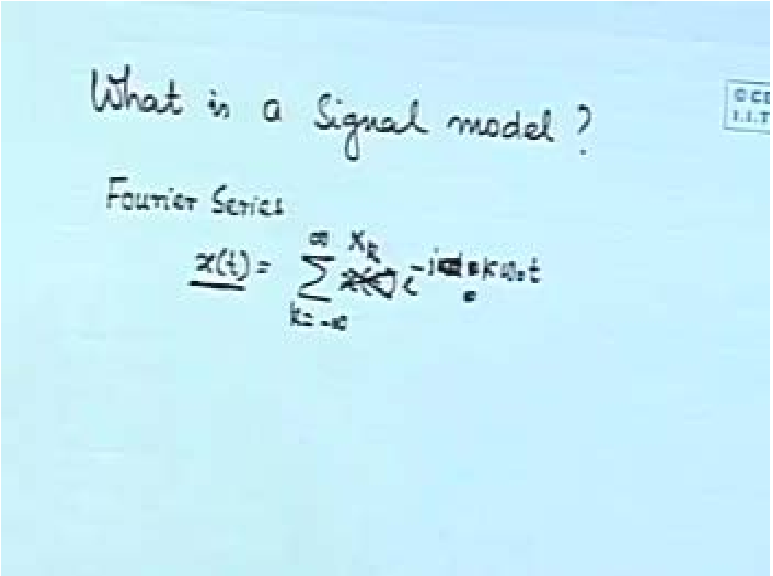

So we say that the signal x t; any periodic signal x t can be broken up, as x t into the power minus j omega t, actually it should be written as, if it is periodic it can be written as k omega zero t, k is equal to minus infinity to plus infinity. So, we what are we trying to do? We are trying to express, rather no this is not x t this is the this is x k. So we are saying that, a signal which is periodic can be expressed as an infinite sum of some complex exponentials; which are like like, you know no no you know in our audio classes, we write in terms of sins and coses. If we add them together, we can write in terms of a complex exponential j k omega, zero k is a index which varies, which denotes the various frequencies and into the power g omega zero can be written as sin and cos. So, what are we trying to say, we are trying to express a signal in some mathematical form. So Fourier series is a is a kind of signal model, so is a Fourier transform.

So this is this is a deterministic signal model. If we use try to model random signals, then we say then we cannot express values, because there are no values signal can be something depending on the outcome of the experiment. Then we what we try to do is we try to..in the case of random signals, we try to either characterise its auto co-relation function; since it is an ensemble average, so it becomes a deterministic function. So so there we are trying to characterise; some property of the system of the of the of the signal, because the values of the signals cannot be directly described. Similarly we can we can describe corresponding to this, we can write its power spectral density, which is another mathematical description only written in terms of omega now.

So in each of these, we are trying to describe either the signal values themselves or some of its properties in a mathematical way, that is a signal model. So, why do you have to understand signal models, because signal models are going to be very much used in our course. We are we are we are always we will always, be dealing with certain kinds of signal models for a for the purpose of estimation. Why do we need signal models? For the purpose of estimation, for various reasons, for example, suppose for example suppose we have we have discussed that, what problem of Active Noise Cancelation.

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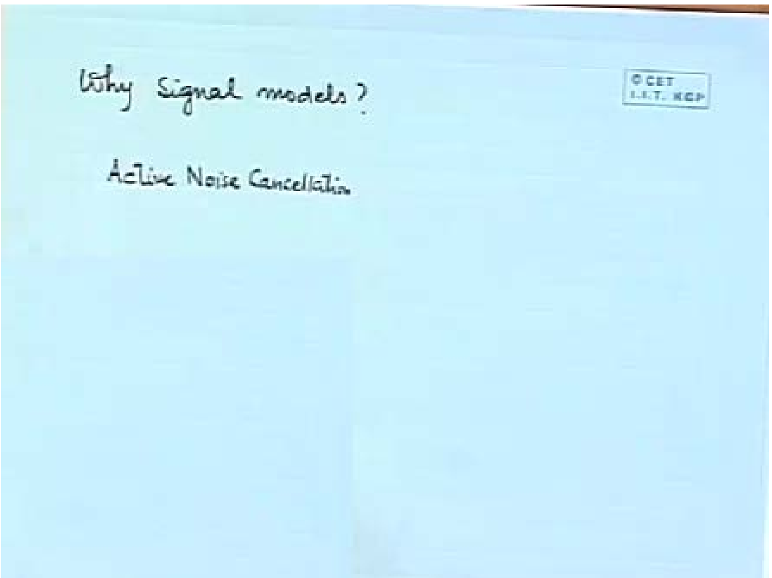

So if you want to do that then, what we what do we essentially need? We essentially need to predict, what is the next signal of.. that, is the noise which is being created, what is going to be its amplitude and phase in the next instant, so that we can create another signal, which will destructively interfere with that and then reduce the noise. So so so essentially, what is involved is a prediction. So some prediction of the next noise signal value is needed, so that we can cancel it. Now how are we going predict it? We will predict it, using some mathematical form, based on its past value.

So the movement we try to do that, we are going to need a model because; we we must have a computational rule by which we are going to say that, since this this this this noise samples occurred in the past, that or or or if you know that, the that the noise that is being created has this kind of characteristics, if you had given that, then we must have a rule of calculating, what signals we should generate, such that the noise is cancelled. So we need to describe properties of signals, that is why we need signal model, right. Similarly suppose we regularly, do when we when we let us say, send a speech signal. Let’s say, we described the you know the concept of voice recognition systems. So voice recognition system; once you speak something to the system from that signal sample, it tries to recognise whether who is the speaker, right? So so how does it do that? It does that because, it tries to extract, it has already some properties of the speech signal of a speaker, has been stored in the system. And when when a given speech signal is I mean supplied to the system, then it creates a signal model for that speech signal.

For example, suppose it will try to model the power spectral density of the speech that, you have given to it. Now it knows that the, what is the nature what is the nature of the power spectral density of of Mr x. So you see, it has got a model of a signal that it has got, it already has a model stored of various speeches of Mr x and then it tries to compare the models. Because this because the signal values will the signal values of a speech signal, if you if you actually record a signal, it can be done very easily in the PC, and you if you plot it if you if you say that, same sentence n number of times, and none of the times the signal values are going to match. Similarly when you want to transmit speech signals to a long distance. Let us say over a network, you do not transmit each and every signal value, because because that is going to take a lot of bandwidth. So what you do is you actually create a signal model out of that speech, and you transmit only the properties of that model to the other end and then using the other end using those model coefficients you recreate the signal. You do not send the signal every sample by sample because, that will take too much of communication bandwidth. So for all these kinds of problems; you need to create a model of a signal, without a model of a signal you cannot do anything virtually.

So that is why, we will we all our estimation problems, we will pose in terms of a signal model. So we are so we need to understand, what are the typical commonly used signal models and some of their properties. It itself is a somewhat vast area, but some of it you are already know, learning in some of the other courses. Like for example, you are doing d s p, so some of what I am going to talk about today, you will do in much detail in d s p and some of it you are you may be doing in a linear systems course and some things like that. So we will you now, sort of scan over I am just to be just to be able to start our estimation problems. So typically speaking, signals are of various types. They can be this is a just one classification and you can have various types of classification for signals. For example, you can have a continuous, these are very very important classifications;

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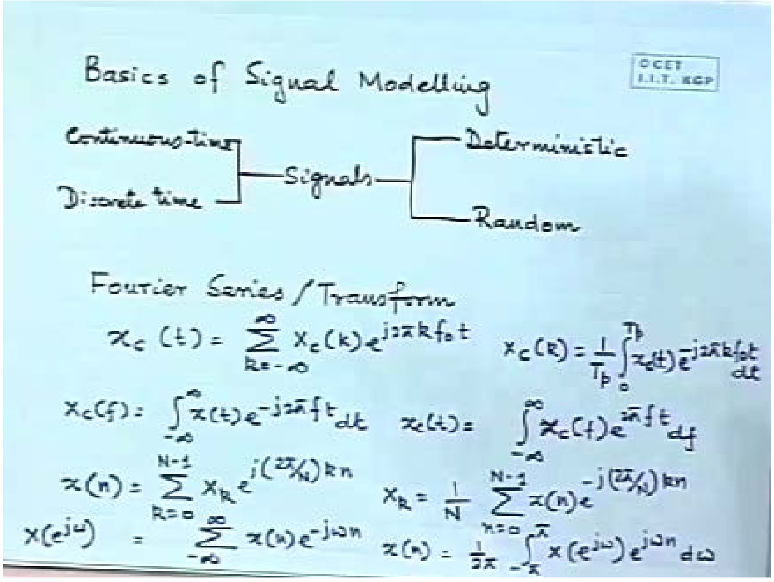

One is continuous time, and other is discreet time. When you are talking of a discreet time signal, you are only given the values at certain points and you are and you do not at all know the values at the other points, right. You can make certain assumptions about them, but as such when you are dealing with a discreet time signal, you are whatever you are doing you are doing filtering, whatever I mean all that you do in d s p. You are you are actually talking about only those points of time, at the other instance of time between sampling instance. What is happening we are not so much concerned, of course they come because I mean real signals have to be continuous time, but they come through other means. In our in the coral cases, you will always be talking about the signal at certain points of time and that signals are called discreet time.

Similarly as we have seen that, we have we we may have deterministic signal or random signals. Typically in this course we will we will be talking about this particular combination, that is random discreet time signal, right. Now random… these are this is just a recap, typically as I was saying that you can have for example, if you have a periodic continuous time signal. Signals can also be classified as periodic and aperiodic let us say, so if you have this is one of the earliest signal models that, we learnt so far in our engineering. So if you have a periodic continuous time signal, then then it can be expressed as a Fourier Series and the Fourier components, can be obtained from this integral this is well known. Similarly but, what what what is the signal is aperiodic? If a signal is aperiodic, then you cannot express it as a Fourier Series or rather you have to express it as a Fourier transform. So for a aperiodic.. signal this is a Fourier transform, which we are so familiar with and you can again get back the time signal, using what is known as a inverse Fourier transform. Similarly for discreet, exactly similarly for discreet periodic signals, you can have a periodic, you can have a discreet Fourier series. Well now now I am only talking about n here, I was talking about t, t is a continuous variable, here n is a discreet variable. I mean, numbers like minus two, minus one, zero, seven only integers, okay. And similarly it is so so these are the discreet Fourier components and this is the I mean, you can you can you can get the this is this is how a signals times time values, can be expressed in terms of its Fourier components. And if you want to express the Fourier components, if you want to compute the Fourier components in terms of the sequence, actual time sequence of the signal then this is the inverse Fourier series.

Similarly again so just like here, you can have an aperiodic signal, here also you can have an aperiodic signal. You can have an aperiodic discreet time signal, which which is not periodic, which we does not repeat itself, in which case you have to define, this is known as the this is these celebrated discreet Fourier transform or d f t. So so so the discreet Fourier transform is actually defined like this. And you can again get back the sample values from the discreet Fourier transform, using an inwards inverse d f t, okay. So these are these are some of the very early signal descriptions, which are constructed and I mean, Fourier transform is so important because, it is because it is analytically simple as well as enlightening to study signals, not in the time domain but in the frequency domain, okay. So I mean studying signals in the frequency domain is is is so popular because, we can very simply understand some of its properties which is very difficult to visualise in the time domain, okay.

So now since; we are basically concerned about discreet time signals in this course, so we have to, it is it is good to understand the the connection between the discreet time signals and continuous time signals because, after all what we see in real life everything is continuous time, right. They are discreet time signals as well I mean they are discreet time quantities, say like for example, if you if you take a if you take the stock market, let us say the the the the b s c index at ten o clock every day, for n number of days it will form a discreet time sequence, right. That since that essentially, even even physically it does not have a continuous time counterpart, it is an it is an inherently discreet signal. But normally the physical signals current, voltage, temperature, that we see are essentially continuous time, but when we talk when we create a discreet time description of them and we when we deal with them, which will do so often because especially in the context of this course because, we you whenever we are talking about signal processing, we are we are we are generally talking about physical signal. See I mean, single type voltage then current typically, okay? So so such signals are a I mean inherently continuous time but, still we will deal with their discreet time description.

So we need to understand, what does the discreet time description actually mean, right. So so we need to understand that, I mean the descript time, description is actually obtained from the continuous time description by a process called sampling, right.

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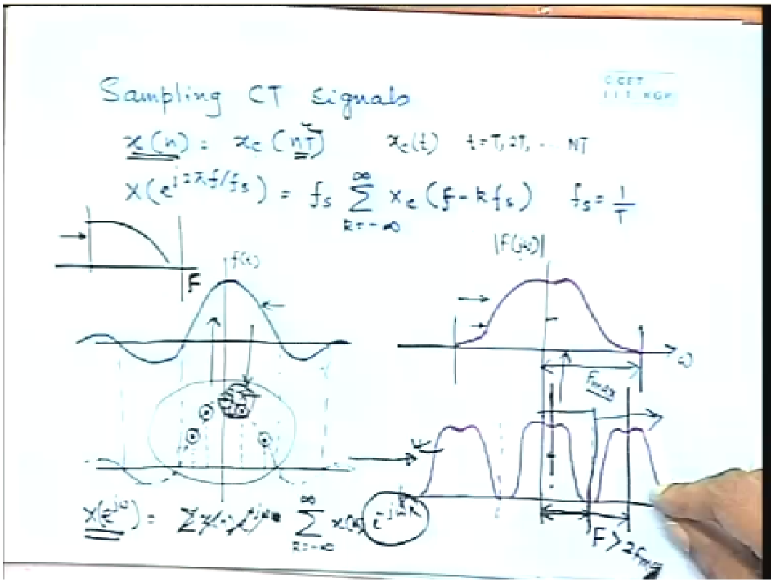

So when we say x n, what we actually mean is x I mean c for continuous, that is there is a continuous.. time signal x c of t of small. This pens give you, so actually there is a continuous..time signal x c of t and you are merely sampling it, at t is equal to T, two T in general NT. So we are talking about a periodic sampling here and then, when we say x c n, what we actually mean is that x c t act t is equal to n T, that is what we mean. Now the question is that, so so fine so this is how the discreet time signals sequence and the continuous time signal are are related, so very very I mean a very valid question is ask that, how are their transforms related because, we can we can easily construct a d f t of this discreet time sequence. We can also construct a Fourier transform of this continuous time sequence. So how are those two related? That is a very important question to to ask, and it has I mean there are very some of the most beautiful and a.. profound results of signal theory are I mean actually let to that. So it says that this is a very fundamental result. This what does it say? It says that if you sample a signal at using at a sampling frequency f s, these things will be I think will be covered in much more detail in your d f t course. We cannot this has a great deal of mathematical on it, we we cannot do that, what we will do is we will learn the result and we will understand the I mean understand, the meaning of the result without actually going through proofs and other things, right.

So this so this says that, now the question is a very important question is that, if you are if you have a continuous time signal and if you are going to take samples of it at certain instance of time and then work with the samples, and then arrive at certain results, are those results going to be valid for a valid for the actual continuous time signal? We must ensure that the that the.. that the result, that we arrive at using the discreet time sequence are actually meaningful, in the final real world. If if if that does not happen, what is the use of using the discreet time model? So so we say that now that, has so what what do we mean by it has meaning? What we mean by is it has meaning? It is that first of all that suppose we take a signal, okay. So we take a signal and this pen is giving me trouble, so so for what we mean is that suppose; you have a continuous time signal and we have taken samples of it, so we have got a sequence. Now we are going to manipulate that sequence, we will say that suppose a signal is suppose a actual continuous time signal, is pass through a filter, so you will go so I will get some other signal at the output. Now the point is that, I I need to know that, how can I use this sequence and pass it through some other box, which we call a digital filter and get another sequence out; and that sequence should actually match the sequence of the of my of my analogue filter, which is fed by the analogue signal?

We ask such questions, because unless it is unless they they match, what is point of digital filtering? So it it turns out, first of all the first to ask is that, once I have sampled a signal, if I have given a sequence from the sequence, can I go back here? Going back from here to here is very simple just take back; just pick up some points but, given in the points, can you get back the original signal? So that is called the problem of reconstruction. Now it turns out that; as we know this is the the the famous Nyquist theorem which says that, it is theoretically possible, remember the word theoretically, because even the even if you… it is it is theoretically possible, if the what does it say it says that, the if this signal has a frequency spectrum, which is limited to let us say, some F capital F, that is this signal spectrum this signal has no other frequency component more than F, then you sample it at greater than or equal to twice a frequency, then you can theoretically reconstruct it from the samples, using some reconstruction functions which are called sink functions. That is but remember that, you cannot practically do it you can you can be approximately correct, but you cannot practically do it because, the even the reconstruction process involves a infinite number of terms to be added. So it is not it is not practical, but the importance of the result is that, if you sample at lower than two F then you cannot even theoretically do it, it is not possible, right.

So so so the first thing is that; if you want to sample we have to ensure two things, first thing you have to ensure is that, the signal that we are sampling is dam limited. So it have a bound low in frequency, if it is if its frequency.. content goes on and on and on, you cannot sample it, at whatever rate you sample it, you are going to have trouble. That is why, we put what is known as an anti-aliasing filter. We put an anti-aliasing filter just to ensure that, the signal that I am sampling is bounded in its spectrum, its its its bandwidth is bounded, right. Second thing that, you have to do is after that, so now one when you have put an anti-aliasing filter, you know that its bandwidth is restricted. Now you can sample at twice the bandwidth or more actually practically speaking, you should do it at five to ten times the bandwidth but theoretically speaking, you should at least do it twice. So, that the samples that you are getting is a faithful reproduction.

See what is the idea? the idea is that suppose you have got the samples so so. What tells you that, this that the that in between two samples, the curve is going to follow this and not this… How can you be sure, so you can be sure because, you have put a put a frequency restriction on the signal, so the if between two samples which are twice the bandwidth frequency. If it has to have a component like this, then the frequency component of the original signal must be higher. But you have limited its frequency component, so so such widely varying signals are not possible. That is the.. that is the basic logic, okay. So so that is a so we have to remember these things, all though in our course. We will never encounter it we will we will we will we will not even mention capital T, this capital T we will always talk about x n. But we must remember, that is that that if we do not keep this in mind, then we are going to have serious trouble. What is the trouble? See the trouble is that suppose, this signal has a bandwidth of this, that is spectrum of frequency, spectrum of this that is f t and.., and this is say mod F j omega the phase plot I have not done, this is omega, okay.

Now we are now; coming back to the original question, that is if what is the what is the relationship between the discreet Fourier transform of the discreet sequence and the original Fourier transform of the continuous sequence? What is their difference? So this is the continuous signal, this is the this is f j omega, that is the continuous Fourier transform of a f t. It turns out that, if you.. if if you take the discreet Fourier transform of this sampled signal, then the that Fourier transform will look like this. That is it will be this band only, but it will get repeated. It will have it will become, it will go up to infinity, even if this is band limited this is not going to be band limited, this will continue. It is actually very easy to see, why it will continue it it actually continues because of the because of the because of the basic property that, after all x e to the power j omega, which is the Fourier transform is sigma x n e to the power minus j omega n, no this is the

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It will be no no no no no no this is it will be sigma k is equal to minus infinity to infinity x n e to the power minus j omega, x k e to the power minus j omega k, right. Now you see this function is necessarily periodic in omega, because because this function itself is periodic in omega so so therefore, the the value of x e to the power of j omega must repeat from whatever value you get zero to twice pi. You will get the same value again from twice pi four pi, again four pi to six pi that will happen. So so this must repeat, now the problem is what is the problem, if it repeats? Our question is that, how I can get back, if I can get back this one from this one, its okay. So so the question, how can you get back these two? How can you get back this from this? Very simple use a filter, cut off the other parts, tick, have only one loop.

Now the question is when can you cut it off when they are separated? If if if these if these centre frequency F… which is basically the sampling frequency, if it becomes less than, actually this is not F, this is actually this will be,. this is the sampling frequency. See this is this is the centre, so if this is F max that is the maximum frequency component, then this is going to be twice F max. That is if it if this this is right, so if F is greater than twice F max, can you see this? If F is greater than twice F max, then they will be separated, otherwise they will not be separated. Otherwise; they are going to be overlapped, that is it will look like this.

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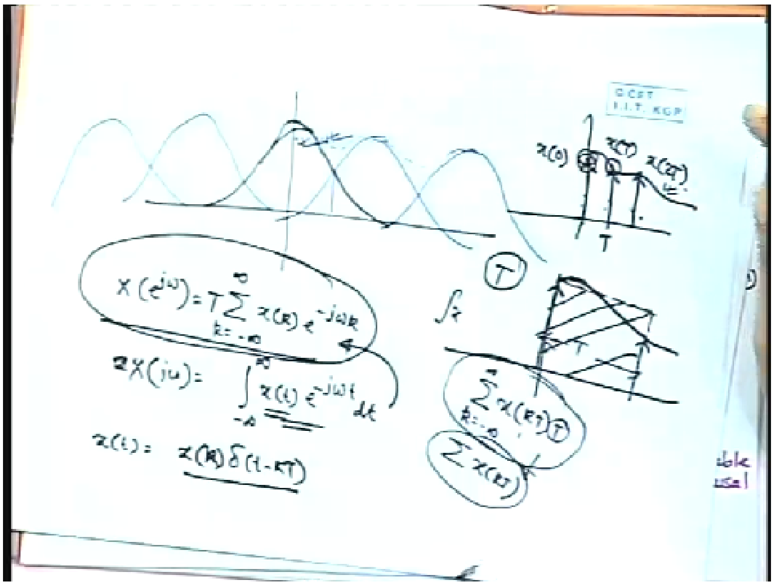

If you have F less than twice F max, then after sampling, they will one curve will be say, this is one F j omega another F j omega will be this one, they will they will actually overlap. So the overall frequency response will actually be you have to add them,.. so you have to add this index and this and this so, it will become actually twice.

So you see that, the frequency response of the actual signal will get totally distorted and there is no way of getting, once they are mixed up, there is no way of getting this load. That is why you should have a sampling time of sampling frequency of more than twice F max, roughly speaking. So this is a this is this is a fundamental result, which you which which we should remember. Similarly there is also another important thing that, we should mention is that because, these things are not always mentioned in books and sometimes they are just they are just glossed over. And when they are taken as, if they are very natural but they create some serous confusion. For example, just imagine that you are defining that x e to the power j omega is equal to sigma k is equal to minus infinity to infinity x k e to the power minus j omega k, this is how you are defining the d f t while, the x j omega you are defining as minus infinity to infinity x t e to the power minus j omega t d t. Now suppose, I have suppose I have x t is equal to x n delta rather x k delta t minus KT. That is I have this signal at every say at zero, I have a impulse at t equal to t I have an impulse at t is equal to two t, I have an impulse, the strength of this impulse is x zero, the strength of this impulse is x t, the strength of this impulse is x two t and so on. Then if you now put substitute this here, what will you get? You will get this.

So this is why the process of so, you can say that this discreet Fourier transform is nothing but, the original Fourier transform of a signal, which can be imagined as an impulse train, a train of impulses one here, one here, one here. The strength of each impulse will be corresponding to the value of the sample. So a signal is now replicated or rather substituted by an impulse train, okay. This is all very fine, there is no problem. There is one excepting that that that, there is a big problem. What is the problem? The the main problem is that the energy of the signal has been multiplied, by many many times through this process. What is the energy of the impulse signal? That is the that is the area under the curve, the area under the curve of the impulse strain and the area under the curve of the of the original signal are roughly related by t. That is the continuous signal, that is integral x simply because, imagine that samples are here just just I am. So so what is the what is roughly the area of this signal? Roughly the area of this signal, if these two are close I am just drawing it, area of this curve is roughly equal, to this this rectangle. What is this? This is the sampling time t, so it is roughly x, say K T into T, sigma, k is equal to minus infinity, to plus infinity.

This is the area under the curve, because each each curve you just replace by the area of this rectangle. If these are close enough, then it matches .But but but what is the, what is the area of the impulsion? Area of the impulsion for each impulse, the area under the curve is one, so it is only sigma x KT, because at each point you have one impulses. So you see that, you have this was the area under the curve of the continuous signal and this is the area on another curve of the discreet signal. So you have amplified this T is generally a very small quantity, because you sample fast so the time period is small. So you have through, this just to sort of that is why I, some some people though this is the this is the most commonly found definition of the d f t. Remember that the area under the curves will not match, you have already made an made an area amplification, by a factor of T. That is why some people say that, it is it is better to put a T here. Though this is not the usual definition, if you put a T here then it will match, right. So we must remember this because, in the in in the whole of the d s p codes this will be referred, to without this T, okay. So this is how the discreet sequence and the continuous signals are related.

Now let us see how they how they get filtered, because that is our prime concern, okay. So we are talking about linear discreet time system. So now what we what we are trying to do is, we are trying to construct a model, which will.. which will use with the discreet time sequence and get out other discreet time sequences and they will match realities, that is our, that is our objective. So we will work in terms of sequences, because that is simpler. That is done that has to be done in in many cases or in in in all of digital filtering, that is what has to be done, so we know this results again. We know that y n will be by a convolution of x n to h n, which is defined like this.

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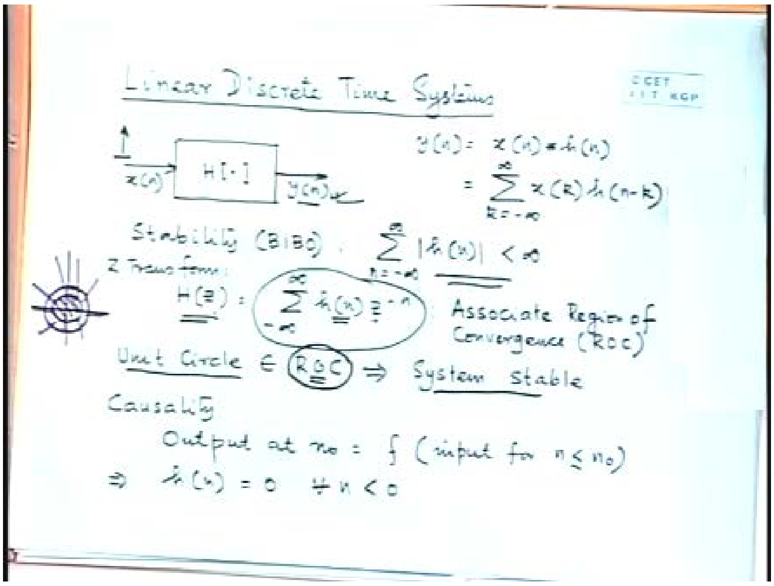

Now obviously we are going to.. the concerned, about things like stability, because something goes unbounded its generally of no use, either it is dangerous or it is it is of no use.

So so when when when will.. when shall we say that; this filter is BIBO means bounded input, bounded output stable that means, if you apply a signal all of whose values are bounded, then you will get output. You will out and sequence all of whose values will be bounded. So if you feed bounded input, you will get output. This is called this is the most general definition of stability; it is called the BIBO stability. And it turns out that remember that, we are talking about only talking about linear systems because, unless your linear system; this definition does not hold, the convolution integral is basically derived, based on the principle of super position. So we have we have we have already made an assumption that, we are talking about linear systems, okay.

So if we have such a system that, it will be bounded. It can be shown that, if this condition hold, this is called the the I mean what is called the absolute boundedness impulse response sequence, mathematically speaking. So if the impulse response sequence, what is the impulse response sequence? Impulse response sequence is that, if you had fed an impulse to the system and then you would, that is if you feed an impulse here in other words, if we feed a discreet time impulse and we talk about impulse, once we have come down from the to the to the to the to the discreet domain. We always talk about discreet time impulse, discreet time impulse, means one of them is a one and other is a zero. Then in general you will get a sequence of y, suddenly output will rise and it will rise, then it will slowly fall, generally speaking. If it is of y, if you give one, once the output will rise to some well and then fall, this is what generally happens. So in general we will get sequences of y n, that sequence is called its impulse response sequence, okay.

So if its impulse response sequence, satisfies this mathematical condition, then this system is bounded input bounded input and and visa verse. If it has to be bounded input, bounded output stable then its impulse response sequence is must satisfy this, so it is if an only if, right. This proof is rather simple and is okay. Now we generally, we deal with a transform, here you see you must always remember, the transforms are just like you know just like mirrors. You are you are you are you have a signal, actually you have a time signal. You are just seeing the time signal in a mirror, because of certain properties so in rather than studying the studying the signal as a function of time, you are studying as a function of omega because, that helps you purely for.. I mean convenience, okay.

So once we have given this impulse response, we would like to define a transform so we will now study it in terms of a function of z, because that is a that gives us insight. So we define the, what is known as a z transform, you can very easily understand that if you put z is equal to e to the power j omega, you get the Fourier transform, but in general z is not e to the power z be or may not be the e power j omega, z can be z is a complex quantity it can have a real part also. Now there are certain things that, which we do not study generally, we sort of gloss over in our in our courses is that, when is this defined, is it defined for for all values of z always? Obviously not, for for some values of z, it can this summation is an is an infinite summation, it can always blow out. So I mean if it blows up you cannot... if if this.. if this summation does not converge to a value then, then there is no use of writing an H z.

So associated with every z transform, there is a region of convergence for certain values of z. It converges and it is defined for certain values of z, it is not defined. We generally do not pay attention to these things, in our especially in our early courses. We are always you know talking of tables and you know inverse transforms and things like that, but we must remember these. Now these are some mathematical results, which are basically based in complex analysis theory. So you have to understand, if you read complex analysis these these results can be proved, we are not proving them is that clear?

For a system to be stable, the region of convergence that is in in the the region in which z should take values, such that this is is has a meaning must contain the unit circle in the z plane. There is a unit circle which we are which we deal with stability, that unit circle must be must belong to the z region of convergence. For example, if if this is the unit circle and if you say that the region of convergence is here, then it is not a stable sequence. Then that system is not stable, but if you say that the that the region of convergence is this.., then it is stable because, it contains the unit circle, right. So why we why we are talking about these things, will be will be clear very soon. Then we are also you know; so so in general, why we are talking about stability? We are always concerned about stability because, stability is an useful thing. If rather I said if something is unstable, it it generally serves no purpose, when it has to be stabilised in that case, which is a prime concerned of control.

Another thing that we are talking about is causality, your thinks causal means, what causal means that output at that is nothing can be nothing can be effected by the future, in short.

Things can be affected only by the past. What so the so the output at any time n zero must can must be a function of inputs, for n less than or equal to n zero. It cannot be a function of an input which is which has not yet occurred, you are you are standing at time T. How can you how can your anything be be be effected by an input, at T ten plus, because it has not yet come, right. So it is a very intuitively meaningful requirement, but this model it means certain things, it means that when will this system be causal? The system in general is not causal, because this look at this, what does it mean? What is h n minus k? h n minus k is h n minus k is actually an impulse its impulse response for an impulse occurring at k.

So this output y n merely, says that is the summation of all such impulses occurring at, so so x k is an impulse occurring at k, that will a response h s k into h of n minus k. Now you add up all this responses, then you will get the total response. That is what this this summation is saying. Now it will turn out that, what now what is this h? suppose this index becomes negative, that is h of n is not zero for n negative. Now n negative means what? means the means the impulses occurring at, [Conversation between Student and Professor – Not audible (42:57)means, that the impulse is occurring, that is that is the impulse response of a signal which has not yet occurred, because x k is here suppose x k is positive but h n minus k is negative, so the which means that, you are getting responses due to future inputs, right.

So so if this happens this in this formula there is nothing, which says that it cannot happen. So if you additionally impose this condition, additionally impose this condition, that h n equal to zero for all n less than zero, then you get a causal impulse response sequence, okay? And we will actually, we have to when we have to derive certain results, why why I am saying is that we will always assume implicitly. We we we will we may not always say, but sometimes it happens that you cannot, that is you can choose for example, suppose you are given it it will I will presently show that, given an H z you can have various impulse response sequences, which will give the same H z, but some of them are going to be causal and stable and some of them are going to be non-causal and unstable. In this course we will implicitly, always choose the causal and stable sequences, all though mathematically speaking, you can get anyone of them, okay. So this is a this is a very prime condition which is which is related to this concept, that is invert ability and minimum phase systems.

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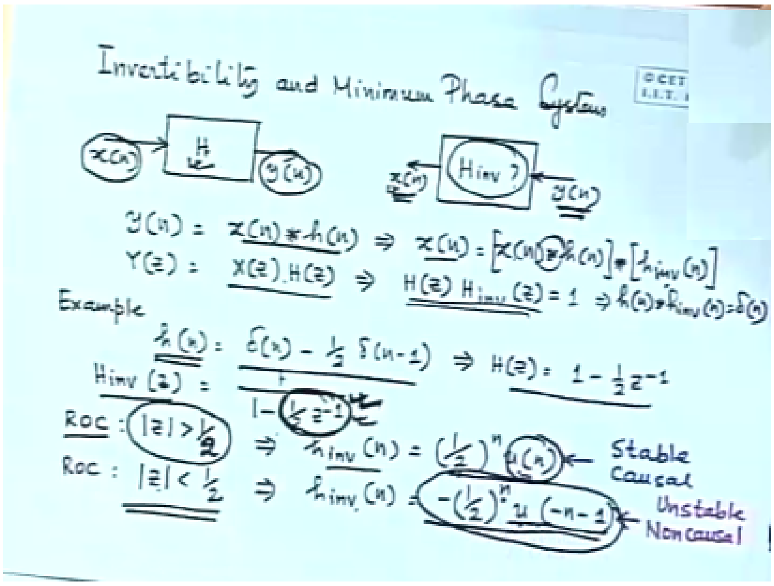

You know this invert ability is very much used by sometimes used by geologists, who actually want to understand they will they will will measure, let us say ground vibrations and then they will try to infer, what happened two kilometres under the surface, right. So you can imagine that the that the earth behave, somewhat like a filter. So in say two kilometres below the earth there was an there was an explosion, so something occurred inside, right. Some signal was generated, that signal actually travelled through earth and you are measuring it, at on the earth surface and you want to know what happened down below, right. So what happens is you are what you are measuring is y n, this is the earth and something happened down below. Now you want to understand from this, you want to understand what happened here. This is a standard geo-physical signal processing requirement, right. They will sometimes call it, de-convolution, so y n is obtained from x n by convolution, and you want to de-convolve y n to get back x n. This is a this is a signal processing requirement.

So the question is when is it possible, is it always possible? It is not always, I mean it is not always uniquely possible, unless you make assumptions and assumptions about stability and causality. If you make an assumption about stability and causal, then you can uniquely determine there x n not otherwise, right. For example say, so it it turns out that so so the question being asked is what is this h inwards? Can we think of an of an equivalent system, to which if we feed y n we will get back x n? If we could find it, then we can know what to what happened down below. So it turns out that, now y n is this x n convolution h n and if it exists then, x n is y n convolution h inverse n. Again again under the we are we are implicitly assuming that, if h is linear h inverse, will always will also be linear, that we are assuming it so happens that, if h is linear and if h inverse exists, it is going to be linear also, right. So now it is rather than you know, this double convolution is a bit because, I mean it is a bit confusing because, this star is a actually a misleading I mean it actually means, an integral, okay.

So though a though it looks very simple it is not a it is not a product, right. But it becomes a product in the frequency domain. The moment we take z, it becomes a product that we know that in if we take z transform, then convolution means product. So now obviously what is going to be, so if you just put this it is going to be x z H z h inverse z which means, that H z into h inverse must be equal to one. So in the z domain we can very simply find out h inverse z, it is it is nothing but one by H z, right. Now the question is, if it is one by H z, what is I want to finally find out x n, so I have to convolve.

For example, if it h n is suppose this, delta n minus half delta n minus one then H z is this. So so what is h inverse z is one by this. When the point is at given that, h inverse z is equal to this can you find out what is h inverse n? It depends on what is the region of convergence, you are assuming. if you assume that the convergence is greater than half; that is the that is this sum should converge for z greater than half, which we which we generally assume always in our standard z transforms, then it turns out that h inverse n is this, u n means a u n means an unit step function. U n is equal to one, for n greater or equal to zero and the zero for n less than zero.

So now what does it mean; first of all see that it is half to the power n so, as n progress it comes down, so it is stable. Second thing it means that, so it is stable means, if you if you if you evaluate that sigma mod h n up to infinity, this sum will converge. So the so the system is stable, and secondly because of this u n you are having that, h n equal to zero for only all n less than zero because, u n is the unit step function, so for all n less than zero u n is zero, so then h n is zero a rather h inverse n is zero which means that, h inverse n is causal. We just cloned that if h any impulse response sequence has to be causal, it must be zero for n less than zero. But you can find out that this exercise you can do yourself that, if you if you assume that the region of convergence is this, then for then you can find that this this h n will give this H z, this h inverse will give this will give this h inverse z that you can find out easily because for.

What does it mean? This means that actually you know, if mod z is less than half then actually this term becomes greater than one, that is the problem. One by one minus x is one plus, x plus, x square plus, x cube plus, dot dot dot .Now this x one plus x plus x square converges, when x is less than one; otherwise one plus, x plus, x square, cannot be written as one by one minus x, right. So this so this term must be less than one that is the problem. So whatever summation you are doing, it should be such a summation that, over which the one plus, x plus, x square series is is decreasing. So it will so happen that, if you take n is equal to negative then, this series will converge to this value. You can you can just work it out; it is a matter of working out the series, okay.

But it turns out that that this is clearly first of all unstable, why it is unstable, because n is negative, right. So so it will it will kind of grow, second thing is that, it is non-causal because, this is the because this is a negative index. So you see that it is not it is not always possible, unless you make that assumption, what I am trying to say is that, unless you make that assumption it is not always possible to get, a unique h n. So we will make that assumption very often, because we are only interested in finding solutions which are causal and stable. We are not interested in finding other solutions,

Okay.

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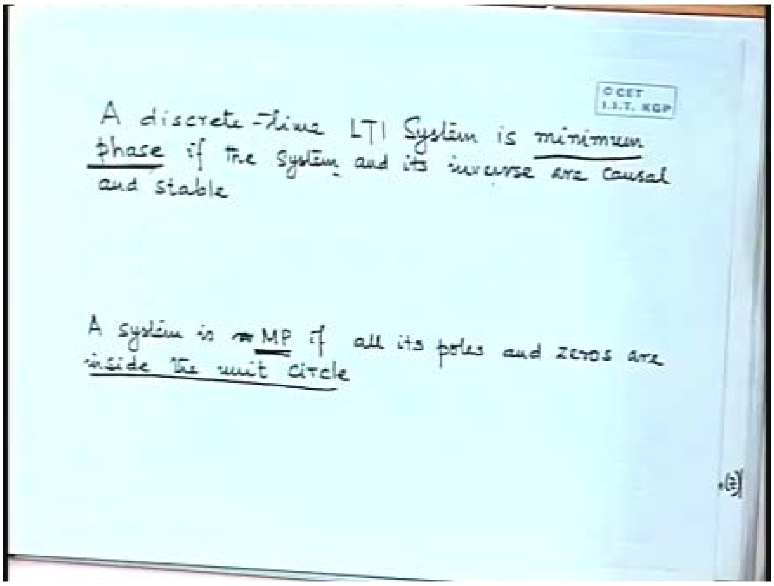

So now it says that, a discreet time system is minimum phase, if the system and its inverse are both causal and stable. That is a very nice result. It says that, if you go from this way to that way, you have a causal operator causal stable operator; you come back this way to that way, you have a causal operator, that is the kind of solution that we are looking for, they are useful to us, right. And so this is the definition of having a minimum phase, many of you will come across this minimum phase, non-minimum phase business even in control, they are actually similar. And it turns out that, if a system is minimum phase, if it's all its poles and zeros are inside the unit circle, and you can easily realise that, if its poles and zeros are poles and zeros are both inside the unit circle; so even if we invert it, if you from H z, if you go to one by H z, zeros will become poles and poles will become zeros, but if all are within the unit circle, then the inverse is also stable, right. So this is why we are we are interested in minimum phase solutions, okay, because we are always going to get, stable causal things.

This merely says that, I will not do it, we are I will just say that, you know there are some all pass filters.

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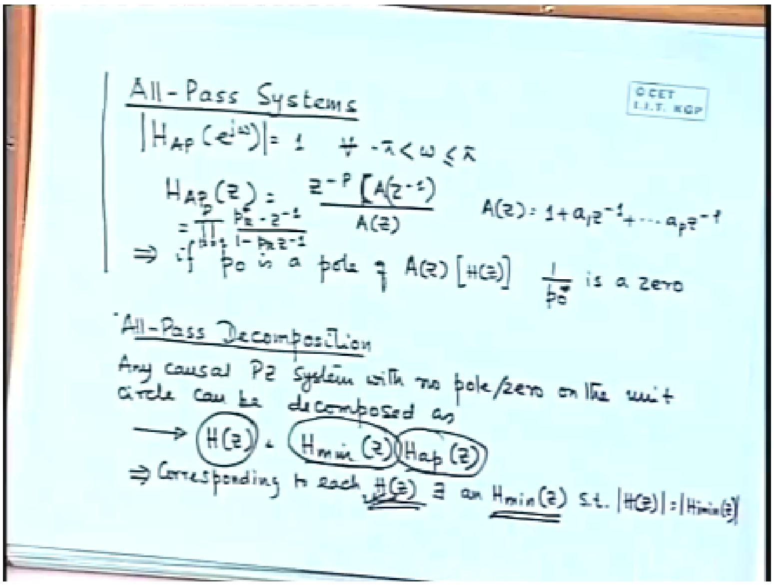

All pass filers they are they are magnitude response is one, at all frequencies, only their phase response changes, right. So for example, Z minus A by Z plus A, magnitude response is one, but phase response will change with omega, such filters are called all pass filters, because they pass all frequencies with equal magnitude, right. Now it says that this this result says that, you you take any H z, it can always be be decomposed into two other H z’s, one of them is going to be minimum phase, the other is going to be a all pass filter.

Any H z whether it is minimum phase or not, can always be broken up in this form That is very simple to see because; if you have any any pole which is outside the unit circle, you have to reflect it back inside the unit circle. This is so now, why I am mentioning this result to you because, this says now, what is what is going to be the magnitude of H z? It is going to be the magnitude of this into the magnitude of this; this is the complex quantity, this is the complex quantity. So if C is equal to A B and and and A B C are complex then, mod C equal to mod A into mod B, right. Which means that, they are corresponding to every H z, there is always a a minimum phase system, whose amplitude response is this, is the same as H z, but whose phase response will be different, but magnitude response will match. So you can always find out an H z a a minimum phase system, which will be whose amplitude response will be equivalent to the amplitude response of H z. Why is that important?

That is important because of the fact that, with I will I will skip this one, now.

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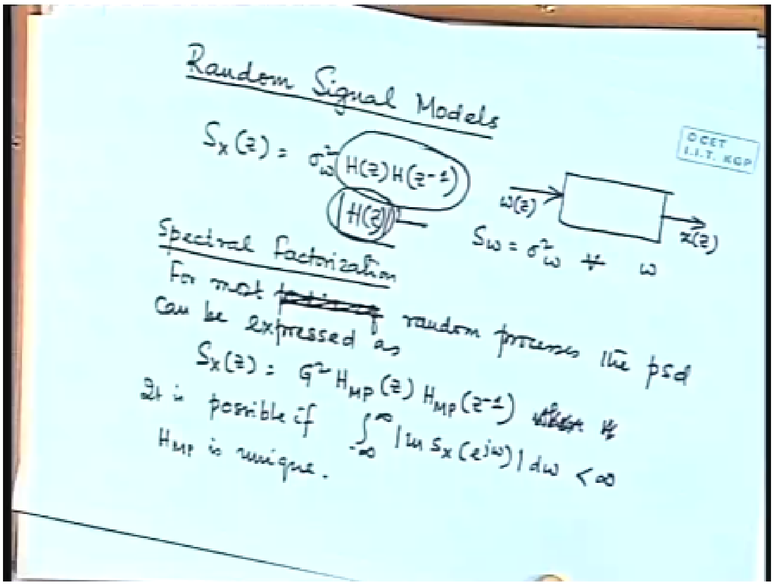

That is important because of the fact that, remember that when we talked about, especially when we talk about random signals, which is of our primary interest. What what do you we want? What do know? We know what power spectral density is. We do not know values, we cannot calculate just Fourier transforms, we can calculate power spectral densities and power spectral densities are always related by H z, H z inwards or rather mod of H z whole square.

If you remember that the power spectral density, if you feed a random signal input through a filter H z and get a power output, then the power spectral density of the output is related to the power spectral density of the input, by this factor; magnitude of h e to the power of j omega whole square. So so so the filter that you must use, if you want to match power spectral densities, you only need to match its amplitude response. Phase response can be anything, if you want to match the power spectral densities, then you have to match the amplitude response that is required. So you can choose phase response arbitrarily; which means that, you can always choose a minimum phase response, because there is always a minimum phase transfer function, which will match any amplitude response. So we will always choose that is why, we will always choose minimum phase transfer function, because they are causal stable and inverse stable, okay.

So you have to you have to you have to read these things, there is we will there is also a very interesting result, called the spectral factorisation theorem; which says that, most power spectral densities can be factorised like this, in terms of minimum phase systems. That is why, more power spectrum densities that you get in practice can be expressed like this, right. This this can be done under the under some condition, which is which is called the polyvinyl condition, mathematically speaking. But that is that is that is generally satisfied, it is not a hard to satisfy condition.

So which means that, most real power spectral densities, if you if you take a random variable and if you take a power spectral densities, you can find out a spectral decomposition of that power spectral density. This is these things are important, because we will when we will actually what we are going to do? We will get a random signal and we have to construct a model. How do we construct a model? Unless we can characterise its power spectral density, in terms of a model we cannot compute its model, so these results are going to be very useful in doing that. So we will stop here today, we have and we we will start with, we still have a little bit left. So we will start with the various kinds of models that, we use and come to estimation very soon. Let us see hopefully next in next class, I am promising but I am not coming to the estimation problem but the point is that, the moment I come to estimation problems all these results are going to be used. So, unless you at least know them, hear of them we cannot, so that is all. Thank you very much.