Estimation of Signals and Systems

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Lecture - 01
Introduction

Welcome to this semester's course entitled 'estimation of signals and systems' that is how we have named it here. This course could have been given some other names as well. For example, this course could have been named as 'statistical signal processing'. So they are I mean… so basically we will be talking about estimation of signals and systems. So before we get into the course we will…… let us see what we……… so this is…….. today's lecture is titled introduction and what we indent to do is to basically give two things introductions of two kinds: firstly we want to give introduction to the area a very brief introduction to the area because many of us may not have had any exposure prior exposure to estimation and secondly to give an introduction to the course; what are we going to cover in this course, what are we not going to cover in this course, how are we going going to go about it etc. and from the next class we will actually go into the course.

So it is relevant to ask, probably relevant to begin our course, you know it is always before we……. very soon we will get into a thick of mathematics and so while you are beginning it is good to take a bird's eye view and ask some other fundamental questions. So it probably relevant to begin from this question, what is estimation?

I, whenever I have to define something it is my habit and I have generally found it rewarding that you should go to a dictionary. So yesterday evening while I was trying to write this, while I was trying to write what is estimation I checked up a dictionary and the and the dictionary mentions two things: I mean it it describes estimation using two phrases that is what I found.

So it says….. First first it says it is an approximation calculation. By the way can you see this clearly alright? And the second thing it says it is that it is it is a kind of judgement based on available evidence, that was very interesting because because it really covers a lot of things which are typically covered under I mean under estimation theory; you know I mean people who write dictionary they are really careful about what they are writing and they really get the meaning out very nicely; especially if you take good dictionaries.

So so basically estimation is somehow these two, how? So let us let us let's look a bit close on these definitions. First of all it is judgement. Judgement is a sort of a non-technical word so what do we mean by judgement? Actually we do various kinds of things which can come under judgement. For example, we can do we can do a kind of prediction, we can do detection that is whether something is there or not it is a kind of judgement, we can do classification whether the object that we are seeing whether it falls in this class or that class, we can we can try to judge a judge its value and we can also try to predict something in the future and all these, so these are so these are actually very well-defined you know well-defined kinds of estimation problems.

So when we talk about estimation in this course we will in in in various course of discussion we will go through these kinds of problems, so we will consider prediction problems, we will consider detection problems so all these come under estimation theory and it is judgement based on available evidence that means that we do not really know it for certain. We we try to kind of you know we try to kind of guess it in a way; it may be it may be our educated guess but but it is guessed nevertheless based on incomplete information that is why it is written as available evidence. So so there is some unknown uncertain element in this process that is what makes this makes this problem of estimation nontrivial right; that is why it is it is so interesting to study it and that is why people have studied it for more than hundred years and I have taken two on very high level of sophistication as we will at least some of it we will see in the course.

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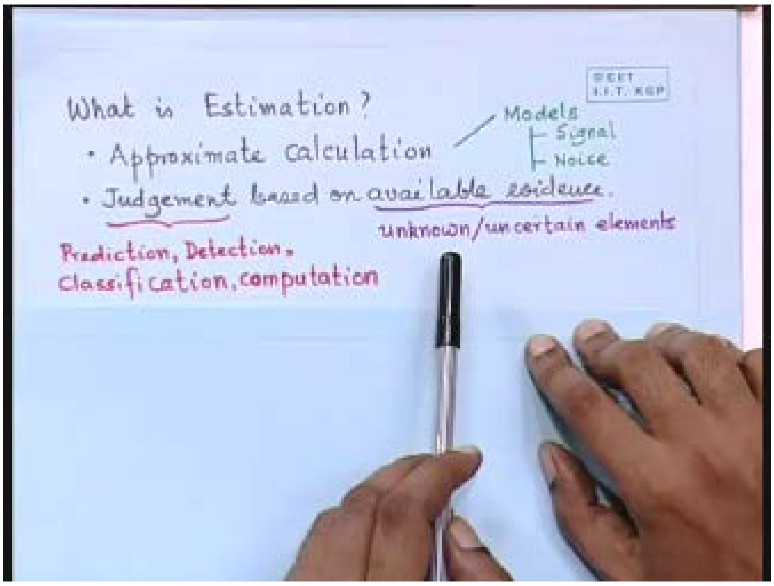

So so it is basically it is a kind of computation which is made in the in a in a back drop of under certainty. there are certain things which which we know there are certain things which we do not know and we have to we have to we have to compute we have to make very good guesses about some other things about which uncertainty exist, this is the basic nature of the problem right; and naturally for doing that I mean the kinds of computations we are doing they will become approximate as we will see this this issue will also come in because……. actually whenever we we we will try to predict something or detect something we have to we have to create or create an use what we call models right? That that is somehow we have to we have to……. using some mathematical structures we have to we have to describe or capture a kind of generalised behaviour of the object that we are……..

You know, we have to say that okay if we if I observe this then I have seen that for the for the last one month I have been observing and if I observed this generally with good certainty that is likely to happen. So you know this is the kind of behaviour. So these kind of behaviour we will have to capture using using mathematical structures which we call models and it also turns out that that that that real physical phenomena that we……… you know everything in this world is actually a cause and effect thing. So what we are seeing is actually caused by definitely caused by some phenomenon or the other, only thing is that these phenomena, I mean some them are predominant and we will try to explain them.

There will be hundreds of other things which will be which will be so I mean which will be so complex, they may be so complex that we will not be able to describe them. They also may not be so important because they will eventually contribute less to the overall phenomenon right?

I mean I I typically give this give this example that suppose….. and and to what level you will model also depends on your purpose. For example, all of us have known about digital electronics. You see suppose when you are when you are you have already studied about digital electronics, so when you when you when you look at a counter you think that a I mean you think that a counter is made of flip-flops and you have a state transition you know what is the state transition table of flip-flops and you make you construct the behaviour of the counter in terms of the state transition table of the flip-flop and there you stop, is it not? But but just imagine that this flip-flop is actually made up of gates. All these gates are actually made up of transistors and all these transistors are actually nonlinear dynamical systems.

That is if you see a small signal model of a transistor or or even a large signal model of a transistor it is actually a dynamical thing. So you now imagine that if you started modelling that counter from this transistor or even had gone to poles and electrons then what would have been the model of the counter; it would have been intractable, you cannot do anything about it, you cannot handle it, you cannot describe it, you cannot compute it, you cannot do anything. So we instead so so we are going to we are not finally at the I mean down everything is made up of electrons and then may be etc. so that does not mean that we are go to model everything at this at the level of electrons right. So so so we will model it at a certain abstraction level and and why we will model at that abstraction level; you see the the characteristic of the transistor actually comes out when when this counter will be changing states that is how fast it changes states, what is its rise time, what is its fall time only at that time this this particular dynamical behaviour will be of interest if you really what to look at that very closely.

But but otherwise for for in in the counter’s lifecycle. 99 percent of the time either it is staying at 1 or it is staying at 0 and and only 1 percent of the time it is it is I mean not even 1 percent it is actually much less than 1 percent, it is changing state so we can ignore that. Why take so much complexity just to model that. So, that may be suitable for your purpose but remember that the that the person who actually designs that gate or or designs that flip-flop I mean the actual VLSI designer for him that edge which is of paramount importance so he always tries to… one who designs it always looks at that edge and then sees it as a transistor and then tries to optimise its rise time etc. So where you will model it depends on your need right. So so here also we will we will… especially I mean random phenomena depends on n number of factors.

For example, suppose what is the if he if an if an aircraft is moving then what is the kind of force that the that the atmosphere is actually exerting on the aircraft. That depends on so many things. That depends on the that day's particular weather, it depends on wind, it depends on the aircraft's velocity and… so are you going to model all that? So you will probably not model it. So you will think so you will actually… that is how does these uncertain things come. I mean it comes because you you you you do not want to consider you do not want to explain some phenomena and you say that these things are random. So you use the word random because you do not want to explain it random for your purpose, just want to know whether it is… in general it it goes positive or negative so so you are you you are you are satisfied at that level. So it is in on an average it goes positive and negative it it is it is its mean is 0, I do not want to explain more than that.

So so it is for these reasons that these that these calculations in the in the strict sense are approximate because they use certain approximate descriptions of the actual physical reality which takes place and that is basically the origin of randomness because certain phenomena are either not explainable or not measurable or it is not desirable to try to explain or measure them right, so this must be borne in mind okay.

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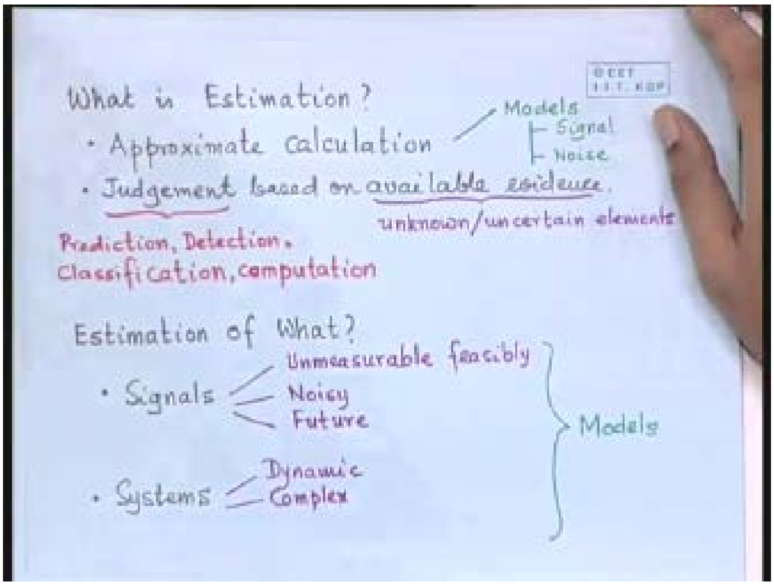

Next is next question is estimation of what what do you want to estimate. See in this course we are talking about estimation of signals, why do you want to estimate them because they may be unmeasurable right; now for example, unmeasurable feasibly. Sometimes if you if you see if you really spend a lot of money many things may be measureable. But I mean we will see some examples where… but it may be unmeasurable for a for a given application either because you cannot spend that much of time at that point of time to measure it or because you do not what to spend that much amount of money so you so so you treat it as unmeasurable. So it may be unmeasurable or it may be measurable. But estimation is needed typically when things are unmeasurable; even if they are measurable they could be noisy.

These are some examples (Refer Slide Time: 13:19) where you need to estimate. So actually the signal that you are getting is actually mixed up with another signal which you which you do not know or it could be…. sometimes it could be that that you want to know a signal in the future so anyway you do not have it right so so you want to know that what is going to be the stock market value two hours later, if you could know it it will be really fine I mean if if people could predict that but people cannot; if you really predict that you I mean you will become richer than Bill Gates but but you cannot do that that is because there are why why you cannot do that because there is a lot of factors.

For example, if a government falls tomorrow it is going to have an effect on the stock market. So there are so many unmodelled factors that it brings so much randomness into the problem that you cannot predict it right. So you see randomness directly comes from unmodelled from from from factors which are very difficult to modelling in this case social, political, physical, natural various kinds of phenomena. If you have an earthquake tomorrow in Tokyo the stock market is going to fall right?

And we are going to typically talk of systems which are dynamic that is we are not going to talk of static systems which are simpler but we do not have time to so we will directly go to dynamic systems which are somewhat complex and may be single input single output, various kinds of… they are basically dynamic systems we will consider.

And again… so for for both signals and systems we will have to… whenever we say estimation actually we are we we we will represent those objects as models and then always pose the estimation problem in terms of that model. So the model is a paramount thing right. So so this is what we are going to estimate.

Estimation… actually we are we are we are mainly students of control. In terms of control estimation is gradually gaining a… I mean a lot of importance because estimation are of two kinds. There there may be something is called online estimation and something is called offline estimation.

What do you mean what is meant by online and offline?
Offline means that [Conversation between Students and Professor ((16:01 min))] that is right that is no, I would not say that but I say is that suppose you have collected some data then you have when you when you are estimating the model that is the collection of the data during which the system is functioning and the time at which you are you are estimating the model they are actually unrelated. So you could do that… you could just collect the data today and then probably estimate the model two days later, this is offline.

Online means when you are estimating the model the system is working. May be you are getting the data one by one. As you are estimating the model you are the system is working, you are getting data through your channel and you are continuously simultaneously estimating this is roughly speaking online. There are fine distinctions between terms like online and real time in in computer science context. But we will not we will use online and real time here as similar ones okay.

Now, offline estimation I mean the kinds of algorithms that we will use here mainly we will look at online estimation so that is more challenging and more difficult. And… now online estimation is becoming more and more important in a in a in a in a especially in control applications basically because of the fact that in control, yes because estimation is now becoming an enabling technology. See you people have generally solved the problem of fixed control more or less well in the sense that offline, getting the model, designing a designing a controller for fixed controller and then letting it work this problem is being tried for for a long time and it has reached some stage and except for very very complicated processes like biological processes which we understand very I mean do not understand properly the control problem is reasonably solved. But now what people are trying to do is that they are they are gain higher and higher level of performance so how they are going to do that? So they want to make… now the modern control systems that they are trying to make they are trying to make it adaptive so it should change according to situations.

So if you know if a if a ship… suppose your designing a controller for a ship now what is the what is the main job of the controller for a ship it is to maintain direction of of motion of the ship right. Now what is calling problems why cannot the ship once it is set why cannot it move that way because there are waves which are always causing disturbance so… and remember that this that the that the nature of these waves in a sea can drastically change depending on weather. If you have a calm sea both amplitude and frequency of the sea waves hitting the ship will be totally different from that which will occur on a choppy day I mean which will occur choppy sea or during during storm; the amplitudes will be more, frequency will also be more. So so controller which has been designed for this one will not work so well for that one right. So this is the typical case where where you may like to adapt your controller and depending on what kind of disturbance you are you will like to use a different controller that will give you I mean quite good performance quite better than having a fixed controller for all weather right. So this is a typical case where adaptive control is to be used.

Similarly, people want to have want to build systems which are which are you know which are which are autonomous that is they can they do not need supervision they do not need manual intervention, they can work by themselves right. People talk about intelligent systems; systems which can gain information from its environment, assess it, take decisions about and systems which are which are fault tolerant that is if even if something fall something fails after all it is a machine, even if it fails it is going to give may be may be performance will degrade but it will degrade gracefully it will not suddenly come apart right. So, if you… so now after after solving the basic problems in control and automation people are now trying to build systems which are having this kind of characteristics and this kind of characteristic immediately will imply that you have you you have you you must have a system which continuously monitors what is happening in the system and its environment. So it must continuously acquire signals, it must continuously assess whether the signals are okay and it it must classify, compute and find out what is the situation and then make make appropriate decisions, change control strategies so all these things basically will require two things: one is online system monitoring so you need a technology for that and you need a technology for what is known as supervisory control.

Actually much of what you will learn in this M. Tech programme here is for a level of control which is called automatic control right. We have a… there is just higher to that you have you have another layer of control which is called supervisory control I mean typically speaking you must have… even in your under graduate you must have learnt about this control loop that is here you have a controller, here you have a plan, here you have a feedback this is the typical… so this is c controller, this is plan p and this we call reference input r, this is plus, this is minus, this is the error e, this is this is what we know as the control input.

Now where does this comes from we do not bother about it generally. But these come from a very sophisticated… I mean to to be able to know when to apply what reference actually requires another level of very sophisticated computation and this comes from supervisory control.

For example, if you take an aircraft example. They clearly distinguish between these two levels of control. If you say a controller to an to an to an aerospace engineer he will understand these, this is this is the control loop. If you ask where where this comes from he will say it comes from guidance. There is another loop which decides what command to give to the controller that is is called guidance. So basically guidance is a kind of supervisory control.

So you need a… for for achieving these you need a lot of functions of this supervisory control. For example, changing a controller, now use this controller now use that controller who does that that is to be done at the supervisory level right. So basically this on-line estimation you can think that it is like a… just like just like a sensor just like here it is a sensor which gives feedback to the automatic controller, you can say that the that the online estimator just acts like a sensor for the…it just acts like a sensor for the supervisory controller.

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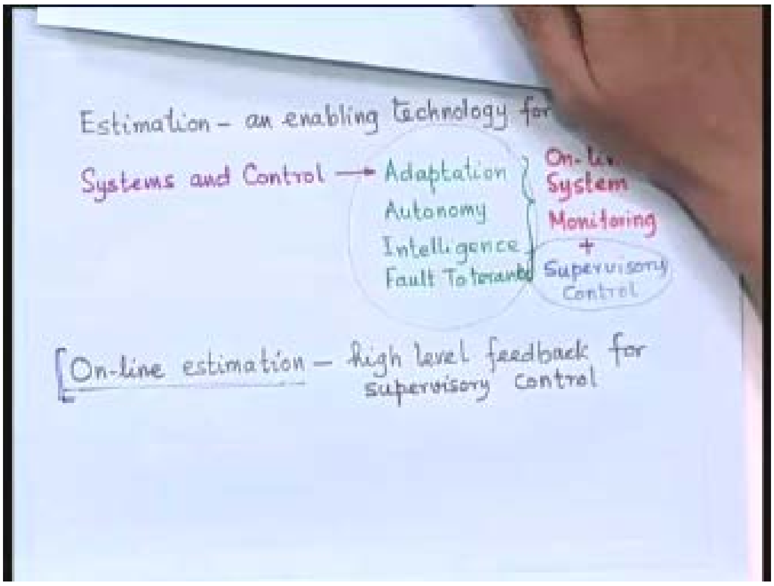

After all a sensor gives the information which it needs. So, all the supervisory control information is provided by an online estimator to the supervisory controller which can give accordingly gives set points to the automatic controller, right. So I am just trying to explain that in that in a in a typical control situation where does this estimation come.

So, having seen so many abstract things let us let us see some let us see some concrete examples right concrete example of application.

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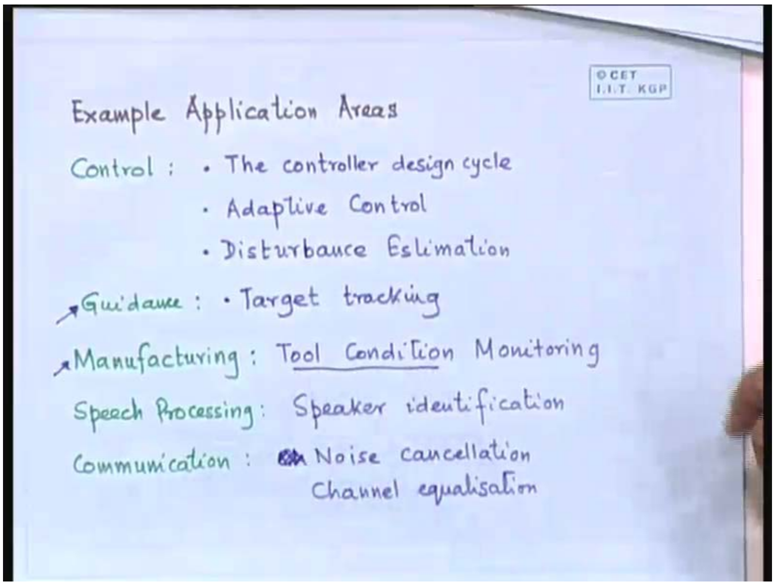

For example, we have we have already talked about guidance. So you have seen during the during Wars that one missile is going another missile hits it so the missile has a control, who tells it where to go, which way to turn; that comes from guidance. how does a guidance decide to I mean what should be the turning, it know its own position and it tracks the target, there is another missile which it wants to hit so it tracks the target so so it so how does it track the target? It tries to get… it tries to estimate the position velocity acceleration of the target so it says that okay if if if if if this is the position velocity acceleration of the target then after 0.1 second it is going to be there so now I am oriented like this I should turn like this. So you so you need estimation right that is why target tracking.

For example, in manufacturing when you are doing very precision manufacturing the quality of manufacturing depends on the condition of the tool how sharp it is, whether it is gone blunt, if it is gone blunt then you are going to get bad surface finish, you are going to get bad dimensional accuracy.

Now the point is that every every one hour if we have to stop the machine, open the tool, take it to a microscope, measure its dimensions, that is not feasible then you are going to interrupt production like anything. So can you monitor the condition of the tool online may be may be putting a force sensor on the tool which will sense what is the what is the cutting force in the y direction. So I tis from there can you estimate, so you are actually looking at the cutting force and you are trying to estimate what are the dimensions of the tool without measuring it right and you are trying to judge whether the tool needs to be changed so you are judging based on available evidence right.

So, for example your speech processing: so I am speaking, now, for example you know now a days people are going… I mean I was I was rather surprised to find that in the United states if you are going to the bank too many times you are going to be charged they will cut money from your account because their logic is that if we would like to give very good very good service to my customer then if too many customers are coming to my office then I need to have a bigger office I need to have bigger staff, so by coming to the bank too many times you are actually putting financial pressure on me so you should pay. But but you can do all business without coming to the bank.

For example, if you want to know your bank account you want to do something you can do everything and and you can do it over phone. So over phone you can ask for your bank account balance. Now point is that anybody can do that, anybody should not be told what is your bank account balance. So the system will should have will have to recognise who is speaking, whether it is the account holder speaking or whether it is somebody else speaking, so how will the how will the system know, so the system has to model your speech and estimate some parameters of your model and then store that. So when you are speaking online into the into your bank server it will take the speech signal, it will extract parameter, it will estimate whether it matches with your with those stored coefficients and then it will allow you to know whether the what is the what is the balance in a given account, this requires estimation right? So this is speech processing.

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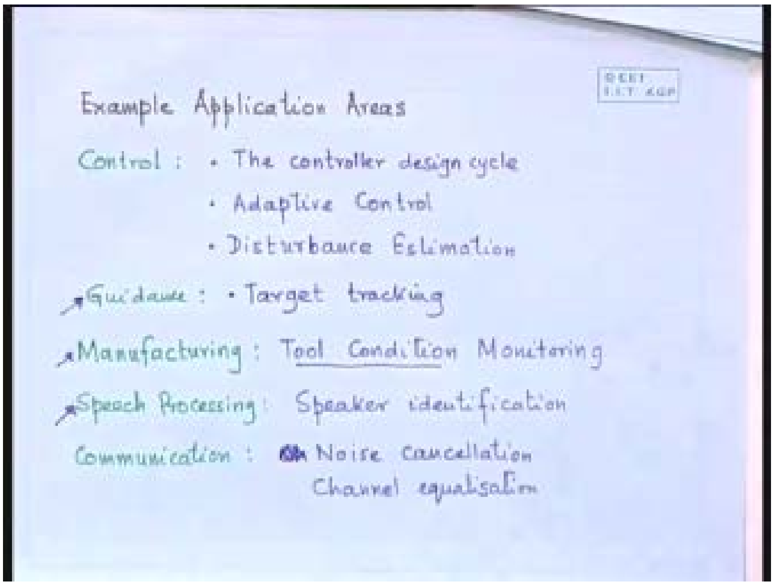

In fact, in speech processing nobody you know speech signals, image signals they are they are they are very dense samples and they take a lot of bandwidths for communication. So generally when they are transmitted nobody transmits the signal values sample by sample. actually what they transmit is a set of model coefficients which are … so you estimate the model at this end, send only the model coefficients to the other end and then reconstruct the signal using that model at the other end, you do not send the actual speech signal sample by sample that is that will take too much of network bandwidth right. So you have speech processing.

There is huge application in communication. One of the biggest applications of estimation is in communication; various kinds of problem, noise cancellation, channel equalisation, you know noise cancellation have you seen that there is a there is an area called active noise cancellation that is you create a noise which will be in which will be in anti-phase with the other noise which you want to kill kill. So, if you can predict that my next noise sample is going to be minus 1.5 volt so you apply you create a noise which is going to be plus 1.5 volt and they will cancel because it waves so there is going be destructive interference and the noise will go, this is called active noise cancellation. Now if you want to do that then you have to predict what is the next noise sample going to be, how are you going to do that? You do not have the sample in your hand, you are going to in advance produce a sample which in which with very good probability it is going to cancel the sample which is actually going to occur, that requires estimation right?

So it is for these kind... so it comes from a large number of wide areas from communication to manufacturing and and all advanced applications are now starting to use estimations. So this ... I mean I think our subject is going to be pretty interesting and we will we will also try to put in some application though this subject is somewhat mathematical and theoretical in nature but we will see right we have 10 minutes time.

So let us quickly take a look at the course organisation right; may not make too much sense to you right now, before that let me discuss what is the what I call the course flavour that is the broad things which I thought to be there.

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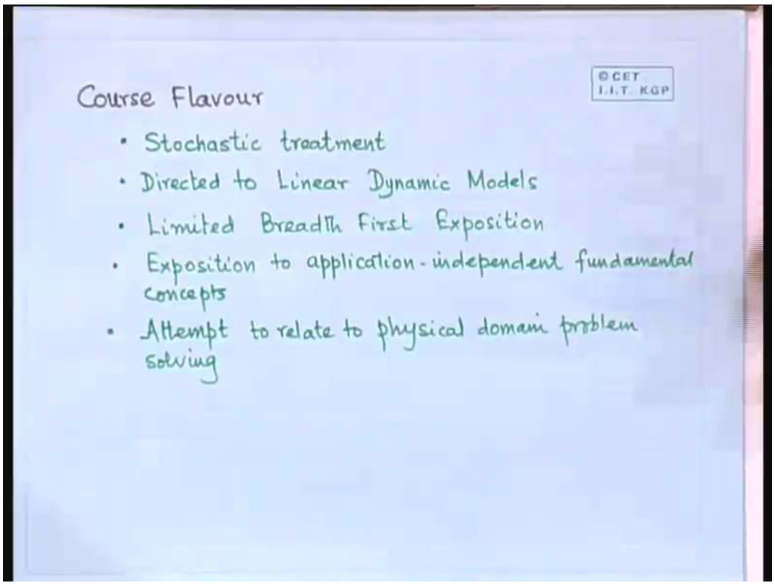

First of all we are going to talk about a stochastic treatment; stochastic means probability theoretic, random. There is a that is use probability theoretic method, there are other methods also, we are going to stick to them. Typically we will we will direct our study towards linear dynamic models because they are simpler and what I mean by limited breadth first exposition is that I am not going to take one problem let us say Kalman filtering and then go deep into Kalman filtering. I think as a as a first level course it is good to know that the basic principle of estimation how they they can be used in a number of applications right. So we are going to take a breadth first exposition. We will cover each to to certain depths okay; we are not going to go too deep into any one of them. Because there were various kinds of applications it turns out that there are actually there are some fundamental concepts.

For example, maximum likelihood estimation that is a concept that can be applied to various kinds of problems. So it is good to first of all know what is the concept itself without getting I mean without always learning it in a in a given context because then you I mean tend to think that that method is actually only applicable for that problem that is not the case, that method can be applied for many problems okay.

So we will try to have this application independent fundamental concept of estimation separately distilled out and then show that the that how in given context they can be applied. That is going to be our approach. and we will always, I feel that at this level you should always keep one feet on the ground; even if the other feet is not on the ground at least one should be on the ground otherwise you have a tendency to fall okay. So we will always attempt to relate to physical domain problems. So we will fall numerically, we will try to we will try to do assignments, we will try to write codes.

Actually you know in this subject unless you write some code this subject is highly numerical. So unless you get a feel of the numbers you do not really get a feel of the method. So it is very nice and to code some of these methods and actually run them and then I mean feed them with various kinds of noise and see how it actually performs what is the what it is really doing that gives you a different kind of feel which theoretical notations do not give you, so we are going to also try to do that okay. So now let us look at the course organisation.

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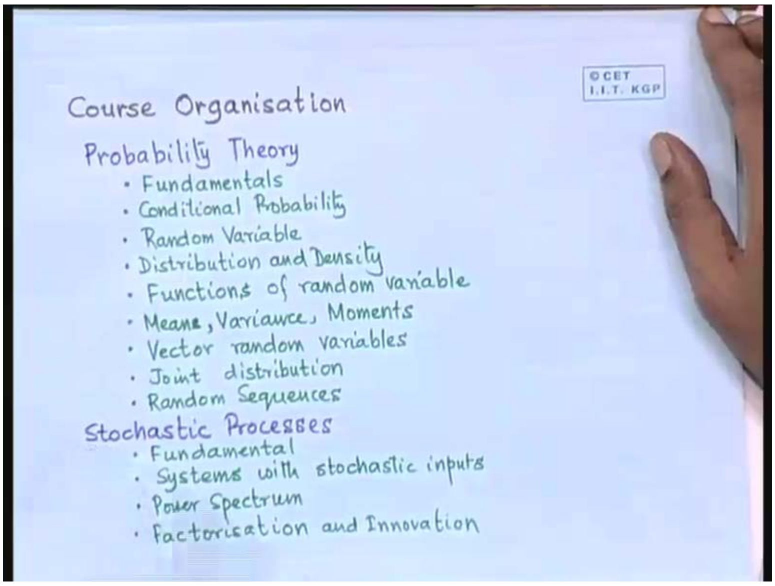

Initially we will review probability theory and more or less this will do a bit fast although we will start from fairly basics. So you have to read it read this up okay and then we will come to stochastic processes. This is our main object of study. We want to study this. But to come to that we will have to first review some concepts of random variables, before that we cannot really discuss this. So then we will come to stochastic processes this gives me the basic mathematical tool of looking at the problems which are supposed to be handled in estimation.

I will give you the... you do not have to copy it from the screen because I will I will give you all these notes so there is no need to try to desperately copy from the screen, you cannot; there is too much material here okay.

Then after having learned about the basic concepts of estimation we will... the basic properties of random variables we will see some basic concepts of estimation, basic approaches; for example, the Bayesian approach: what does it mean, what does it say, how does it try to use available information before trying to solve a problem okay that is the that is that is the hallmark of the Bayesian approach then it says what do you know about it anyway. So try to use that to to get better estimates right, so it is a it is it is actually a philosophy so we will have to look at that.

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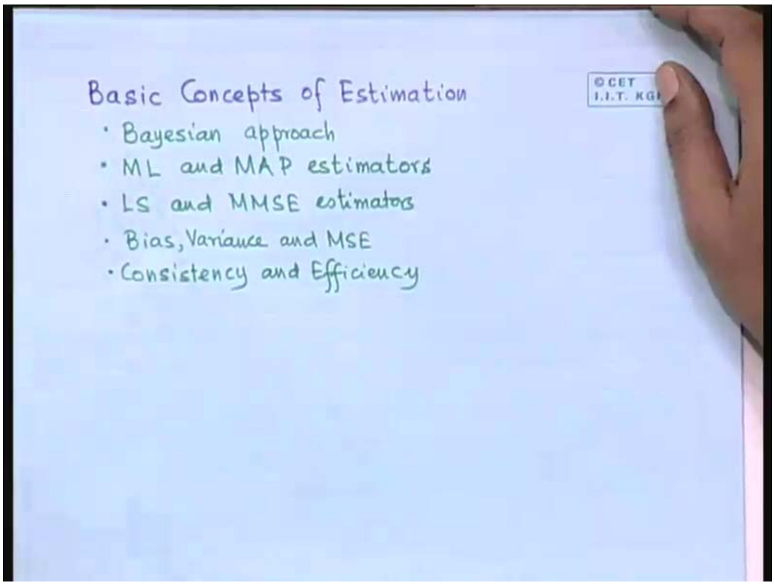

Then we will look at two basic kinds of estimators called maximum likelihood and maximum aposteriori estimators they are also I mean approaches which can be applied to various areas.

Then we look at what is known the least square and the minimum minimum mean square estimates mean square error estimators, they are very convenient widely used. Then we will have to look at various property that how do you know whether an estimator is working fine, what are its performance characteristics. So you have to know of various things like bias, variance, mean square error, consistency, efficiency; there are certain basic properties which which will apply to any stochastic estimator. So we will see what they are, how they are to be evaluated right. So, by by doing this we will learn the the some of the fundamental concepts in an application independent wave first okay. Then we will come to estimation of signals with linear dynamic models.

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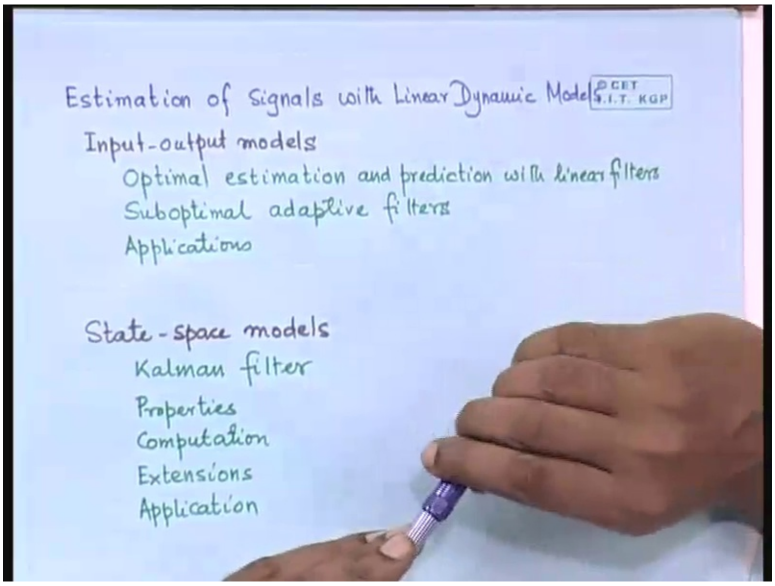

So again the model is very important and so we will use we will see estimation in two kinds of models. for example, we will we see the... if you see the signal processing literature then you find that they will always begin initially with input output models you know: FIR filters, IIR filters that is the way they learn signal processing right so we will also learn it that way initially because because input output models are simpler. So we will learn what is the what is the optimal estimation and prediction with linear filters, they are very nice simple results available which tell you that what you could achieve what is the best that you could achieve theoretically speaking.

So we will we will see what is best achievable and then we will see that that obviously we will not be able to achieve that why because because it will require certain I mean knowledge of certain things which we will not have in practice. So it will say that do you know the covariance matrix of dash quantity, if you know it then you can construct an optimal estimator but we will not know it in practice. So, under the given limitations what we can do so that we can reach very good suboptimal performance and how we can keep on learning more with the data, adapting our filters so that we gradually approach the optimal performance that we have to study because because that is the practical approach, this is the theoretical result this says that this is the bound which is... this is the best that you can do and this is how you can practically try to achieve that best, come closer and closer and then we will say some applications and there we will come to the state space model and then basically look at the celebrated Kalman filter okay.

So we will see the Kalman filter its its basic derivation, what are its basic property, orthogonal properties, we will see how to compute it and we will see some extensions again because the basic Kalman filter we will have to you know stretch and stretch in various directions to take care of realistic situations so you have you know extended Kalman filters, adaptive Kalman filters so we will see some extensions which are required then again we will some see see see some applications.

So this says basically covers signal estimation right and then we will study an area called system identification. We said estimation of signals and systems, so now the signal part is over now we are talking about system identification which means that how to get the model of a system. so obviously to get the model of a system you have to apply some inputs, get some outputs and then see what is the relationship between the input and the output and then try to try to compute that relationship so that is called system identification.

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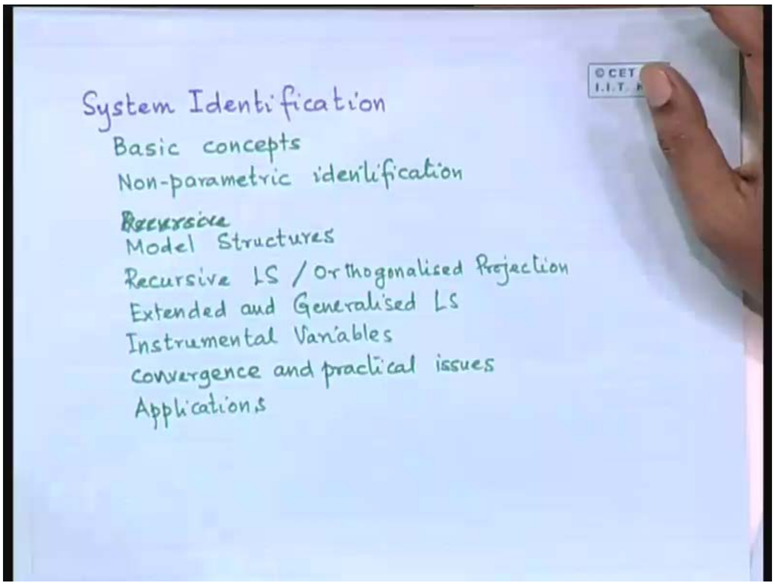

So we will again see basic concepts. There are two kinds of models. There are some models which are called non-parametric and there are some models which are called parametric okay. So we will briefly see non parametric methods. Non-parametric methods are not so much important because they are not so much useful. What is more useful is the parametric methods. So basically parametric methods assume that there is already a structure. So, for example second-order, you you can assume that the width the this system is going to be a second-order system, second-order transfer function, then you find out what is going to be a 0 a 1 a 2 b 0 b 1 b 2 so this structure is assumed and you only want to estimate the parameter in that structure okay this is what people usually do.

So what kind of model structures can be used and there are some most widely used algorithms, recursive least square, orthogonalised projection, extended and generalised least square which are used for you know other kinds of noise models and instrumental variables which is a very useful approach and then finally convergence and practical issues.

After all you want that that that... finally the model parameters if it always keeps on changing then you are in trouble. So it finally should converge into some value which you can use later, so under what conditions does it converge and to make it converge in a in a practical situation what are the precautions that you should take, so we will see them and then finally see an application. So this is the basic body of our course. We possibly we will be left with a few lectures so I have not formalised this part instead this we will we will have to cover but what is left that we can we will we will fill up with different other kinds of estimation problems.

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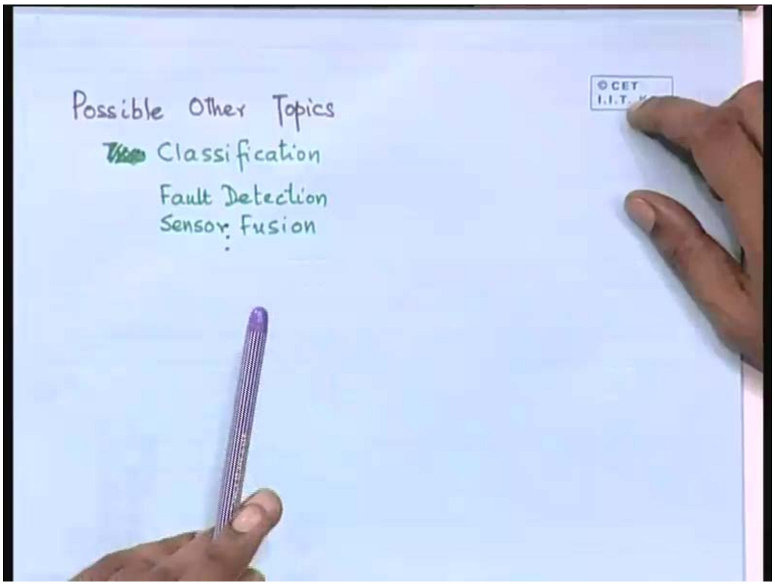

For example, there are various very nice classification problems, then there are... which are which are especially used widely in image processing in areas like robotics that is how does a how does an autonomous moving robot after getting into a room or or looking at a situation how does it understand whether what kind of situation exists, what it should do, what mode it should work so that is basically a scene classification problem right. So you can have various kinds of things like classification, you can have fault detection, you can have a problem of sensor fusion that is that is if you if you say suppose you are trying to get the motion of an aircraft now if you use one radar what kind of accuracies you will get, if you use radars will you get better will you get a better accuracy of its real position. So it turns out that if you use multiple number of sensors you can actually construct a better better estimate. So what is what are the things relate to that. These are these are these are highly practical things and and people do it.

So, depending on the kind of time that we have we will try to take a look at some of them. So this is our plan in this semester right; any questions? Sometimes I tend to go a little faster, for example I think today I have spoken rather fast because I if I if you if you if you have any difficulty just do not bother about this paraphernalia if you have a... you are the most important person. So if you have a difficulty you are going to raise your hand and say I did not understand this please help me, no problem, do not be I mean intimidated by these things okay. This is a normal class.

So our next class is on tomorrow at 4 hour not 3 hour right? Fine.