Dynamics of Physical Systems
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Lecture - 33
Discrete-time Dynamical Systems – I

Before we start the next chapter, let us give you some problems on the non-linear systems area.

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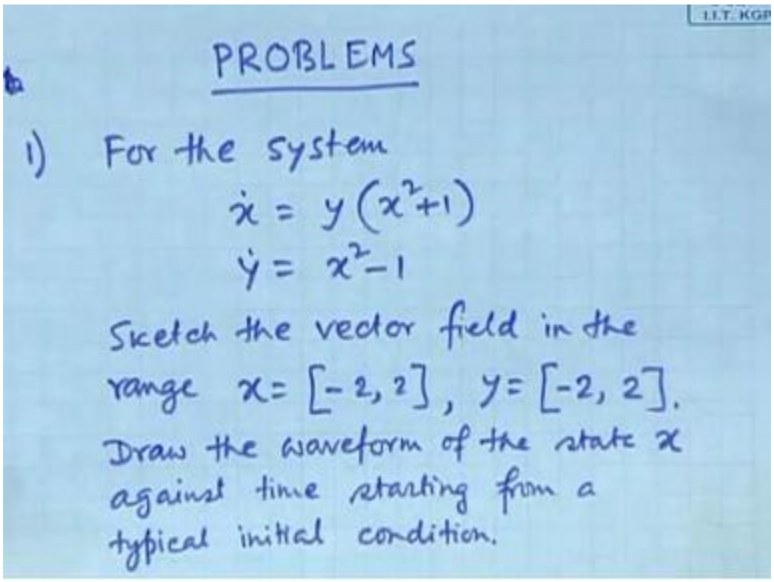

The first problem is for the system given as this, where the right hand side you can see as non-linear, you have to sketch the vector field and I am giving a range in which you will sketch the vector field in addition to that you also draw x, so you have to also draw the waveform of the state x; that means not only the vector field, but also the waveform.

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2) $\dot{x} = y, \quad \dot{y} = -x - x^2$

3) $\dot{x} = 2xy, \quad \dot{y} = y^2 - x^2$

4) $\dot{x} = x^3 - 2xy^2, \quad \dot{y} = 2x^2y - y^3$

5) $\dot{x} = x - y, \quad \dot{y} = 1 - xy$

6) $\ddot{x} + x - x^3 = 0$

7) $\ddot{x} + x + x^3 = 0$

8) $\ddot{x} - a(1 - x^2)\dot{x} + bx + ax^3 = 0$

So, I am giving the problems that were not recorded earlier next is a large number of differential equations, non-linear differential equations given, where you have to essentially do the same thing that is to draw the vector field again in all these problems, the essential method is that you will have to obtain the locations of the equilibrium points, then locally linearize around the equilibrium points. And then finally, on that basis you will have to draw the vector fields around the equilibrium points, and then extrapolate it to logically obtain the vector field elsewhere. In these problems, you will have to first obtain the first order representation, because these are in the second order, and then you apply the same procedure, so you note down these problems.

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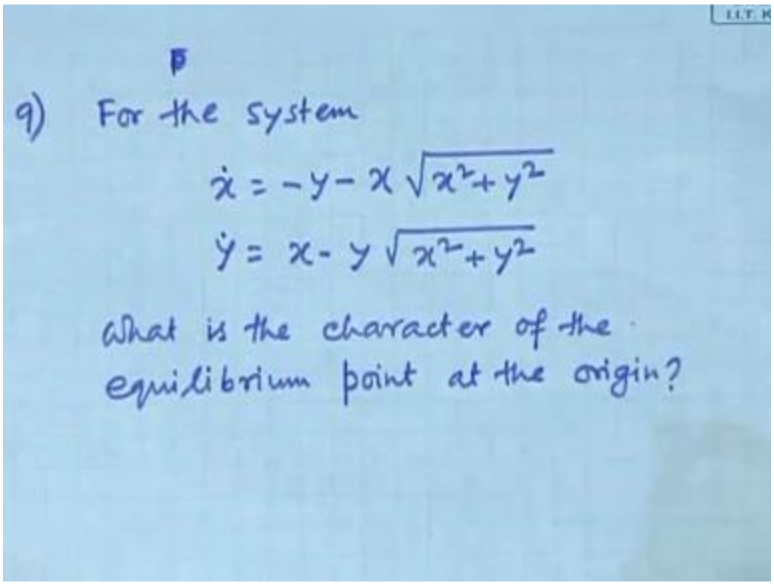

Now, here in this problem again it is a non-linear set of differential equation and you can see that x equal to 0, y is equal to 0 is definitely an equilibrium point, so there is definitely an equilibrium point at the origin. Now, the point is that you have to obtain the character of that equilibrium point on this which means, around that obtain the Jacobian matrix obtain the Eigen values and from there infer the character.

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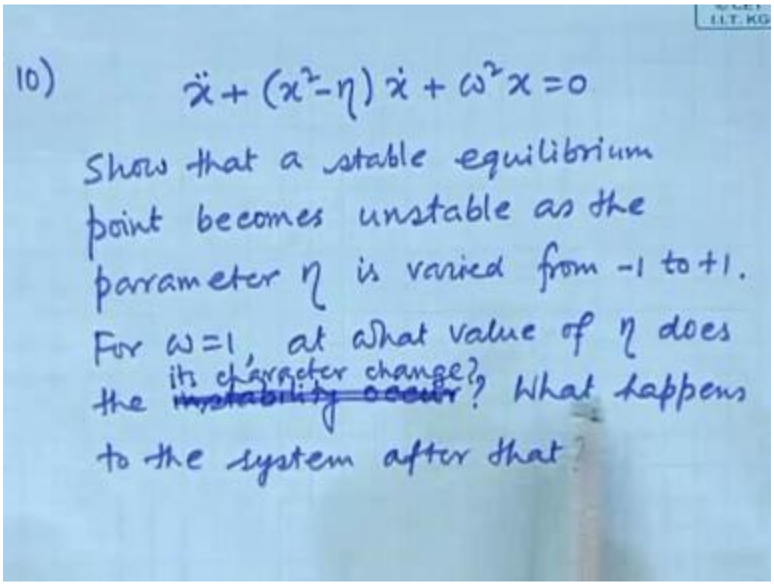

Now, in this problem there is a second order differential equation given obviously, you will have to obtain the first order form in the first step, and then you will have to investigate the character of the equilibrium point. So, here to show that a stable equilibrium point becomes unstable as the parameter eta is varied from, minus 1 to plus 1. Now, in this case again 0, 0 will be an equilibrium point that is pretty clear from here, and then you will have to obtain the Eigen values at that and then on that basis you will be able to infer that this particular statement is correct.

Now, set the value of omega equal to 1, then at what value of eta does its character change that means, you know that there are various type of equilibria, it could be having real Eigen values, could be having complex conjugate Eigen values. Or it could be stable or unstable, so these mark the changes on the type of the character of the equilibrium point. So, you have to infer at what value of eta does this character change. And, then physically you will have to predict what happens to the system after that character change.

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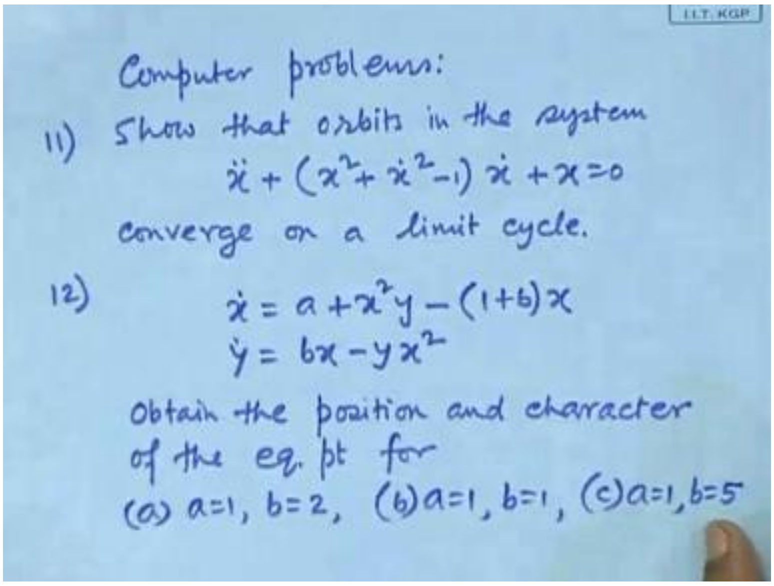

Now, we will also do some problems with computers where for example, this set of equations have given, this cannot be solved by hand, because here is a system that converges on to a limit cycle and as we have already seen a limit cycle cannot be predicted through local approximations. So, you solve this with a with a help of a computer; that means, you start from typical initial conditions any given initial condition not at the origin and then allow it to evolve by Runge Kutta method and see where it converges start from another initial condition see how it evolves.

So, in that way you start from many initial conditions, and then plot the vector field. In this case you have a set of equations again non-linear differential equations, now position you know already how to do it, character of the equilibrium points means the character of the Eigen values, but then you will have to find it for different values of the parameters a and b. So, these are the three possibilities. So, these are the problems that we will have to do for the last chapter on the non-linear systems. But today we are dealing with a different problem, we have seen when dealing with the non-linear systems that there are situations in which a limit cycle takes place.

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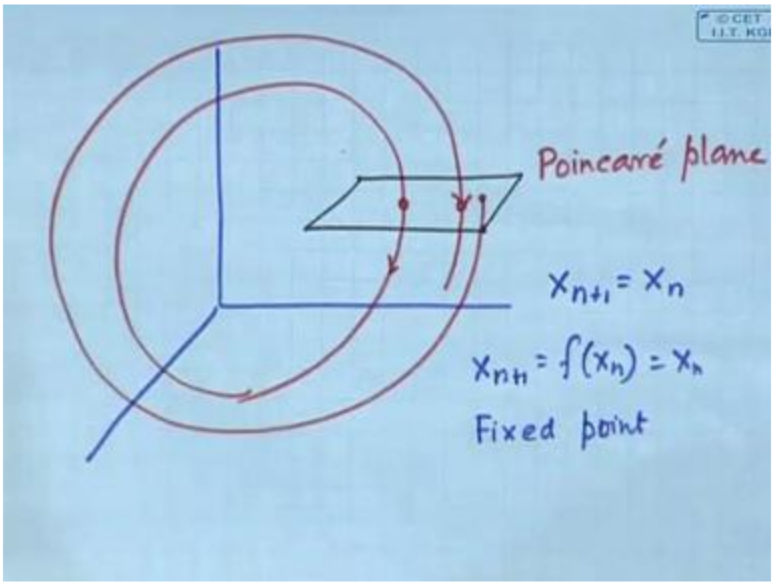

Something like this, its character is that if you start from another initial condition it converges on to that but then we have also seen that such limit cycles are very important in both nature as well as in engineering, in nature all the stable oscillatory behaviors are limit cycles, in engineering wherever you want to obtain a stable oscillatory behavior you have to create a limit cycle. So, the stability of such limit cycles are matters of importance.

That means, when we say that if you start from one initial condition then it converges on that that may or may not be true that depends on the character of the limit cycle after sometime or for some parameters that stability might be loss. So, it is our concern to find out the stability of this limit cycle, under what conditions will initial condition starting away from that limit cycle will converge on to that limit cycle, but how to do that.

The problem is that the method that we were adopting for understanding the stability of equilibrium points what was it, at the equilibrium points we obtain the local linearization obtain the Eigen values and if the Eigen values were in the negative half plane, left half plane then it was stable, that approach cannot be applied here why because it is not a point, it is a set of points.

Therefore, where would we apply you cannot really go on applying in all these points because each of this point will turn out to be unstable, had it been stable what will happen all the points will converge, all the initial condition converge on to that point that does not happen. If you start from here it goes away which means that this points is not stable point, so just by working out this individual points and working out their stability will not help, so we have to do a completely different approach in order to understand the stability of the whole cycle.

Fortunately such an approach exists and you will see that we will be ultimately be able to use the same methodology by that different approach, the approach was this suppose, you have got a limit cycle like this then suppose, you sort of place a plane something like this a section, in that case this orbit pierces that section it goes through that section. So, it will be piercing the section at that point and then it will be going through and again it will come back and pierce it.

So, on that plane what will you see you will see a point only a point, but not always suppose, you take a general what should I say any initial condition here, you have started with the initial condition on the limit cycle that is why it came back to that point. But you start from a initial condition here, what will happen is that from here, it will come go on and then it will pierce somewhere else and then it will go on; which means, that if you start from an initial condition on this plane it will normally land in another point in that same plane.

So, what are you doing, we are now imagine this was the original orbit, this was the original orbit. Now, suppose you have place the Poincaré this is called the Poincaré plane or Poincaré section, after the scientist who invented this procedure Henry Poincaré a French mathematician. So, imagine that now you forgot about the whole orbit and you are concentrating only on the plane what happens on that plane, what will you see, you will see a point and then after sometime you see another point, after sometime you see another point and so on and so forth in general clear.

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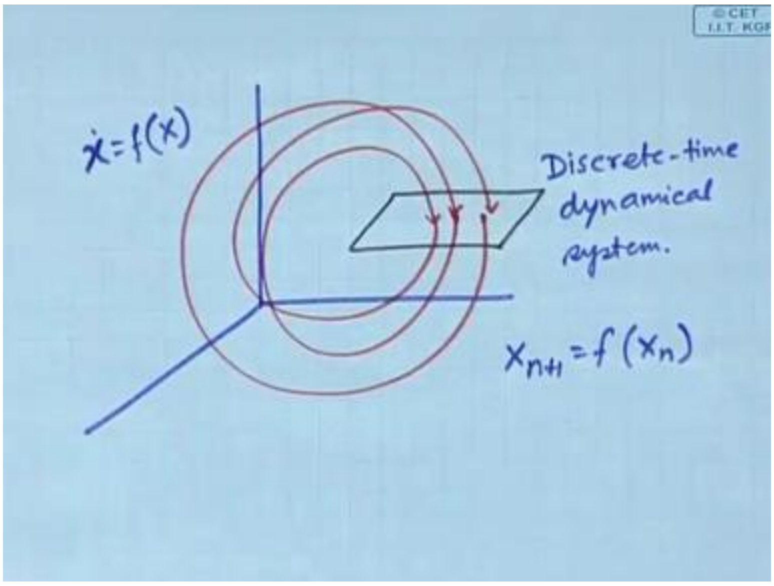

Let us do that once again. This is the three dimensional space imagine and suppose you have the Poincaré plane place like this, then if you start from a point normally it will land on another point, then it will land on another point, then it will land on another point so on and so forth. So, if you concentrate your attention on what is happening on the plane what will you see, a point mapping to the point, that is mapping to another point, that is mapping to another point, a point going to another point.

And what defines how it goes from one point to the other the actual set of differential equations, because that is what is ultimately followed in order to land at the next point, but from here to here it is on the plane and as if it has jumped discretely it is not that it has moved from here to here, it has jumped discretely from this point to this point. So, what have we effectively done is to reduce a continuous time dynamical system to a discrete time dynamical system because, this is discrete time, time is discrete.

First piercing, then the second piercing, then the third piercing, then the fourth piercing, discretely it is moving, so on this you will see a discrete time dynamical system. Now, the discrete time dynamical system how that be represented, the continuous time dynamical system was represent by x dot is equal to capital X dot is equal to some f of

X. The discrete time dynamical system on this plane how would that be represented, that would be represented by X n plus 1 is equal to some function of X n.

So, this is the position at the n th instant and it will be given by a function of the position at the n th instant to give the position at the n plus one th instant, so that is a discrete and dynamical systems representation, X n plus 1 is a function of X n, so notice the two different things. This was the original things that we had, but we had obtained this, how we obtain that we will come to that later, but what is the result of that what is the advantage of that.

The advantage is that firstly, a periodic orbit or limit cycle like this will be seen just as a dot a point of which character, it is the point for which this is true or I will write it this way X n plus 1 which was actually f of X n is equal to X n. Now, the point that satisfies this equation is called the fixed point, notice the nomenclature it is somewhat equivalent to the equilibrium point that you have come across, but in discrete time.

It is similar to the equilibrium point in the sense that if you start there it will always remain there, but it could be stable or unstable depending on in the same way that we saw the equilibrium point. So, the nomenclature is important for continuous time dynamical systems, we call the let me separately write.

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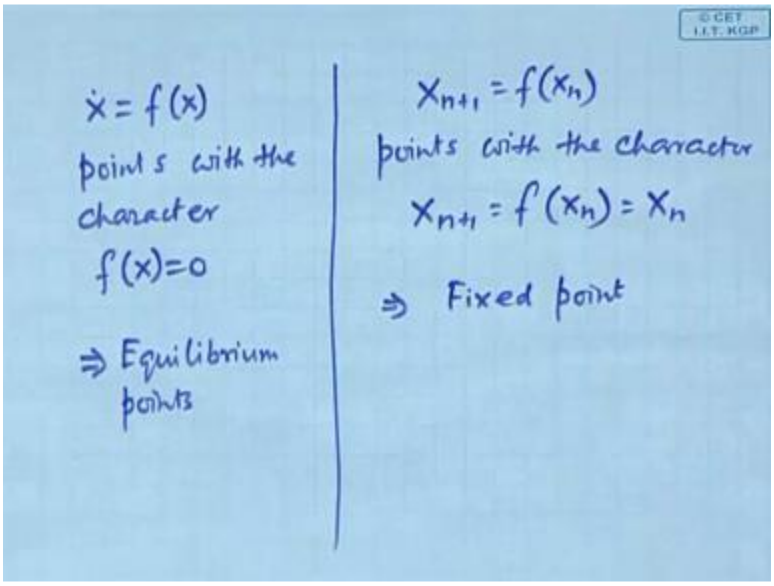

For continuous time dynamical systems points with the character f X equal to 0 are equilibrium points, for discrete time dynamical system points with the character, what character X n plus 1 is equal to f X n are fixed points, so remember this nomenclature. The point is that a limit cycle will be visible on this Poincaré plane as a fixed point is that clear because if you start from there it will land there itself, if start from somewhere else it will not land there because it is not the fixed point, it will land somewhere else. But then you can imagine that you will go on seeing this orbit and how it pierces, how will it pierce if this fixed point is stable.

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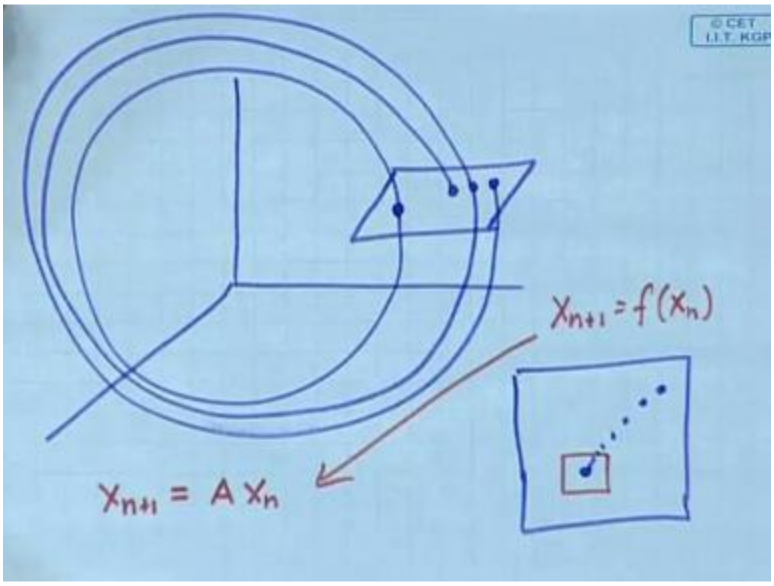

Imagine, you will have a Poincaré plane in which you have started from a point and suppose this is the ultimate limit cycle, but you have started from this point. If it is a stable limit cycle then what will happen, ultimately starting from any initial condition it will get to a closer, closer and closer to this equilibrium this limit cycle, as a result of which you will see piercings that go closer and closer to that or in other words you will see a sequence of points on this plane that converge on to this fixed point.

So, on to this plane what will you see, on this plane you will see a sequence of points that comes closer and closer and closer ultimately converge on to the equilibrium point or to the fixed point. So, this character that starting from other initial conditions, the orbit is slowly homing on to that limit cycle can be seen as a character of the iterations as it comes, the iterations ultimately converging on to the fixed point.

If that is so, and suppose this is known this function is known, how that function will be found out I will come to that later, but suppose this is known, what function is this is known. Then, you can locally linearize that function that will be normally be a non-linear function, but you can locally linearize that by the same way the jacobian. If you can locally linearize that then around that you get a local linear representation, you get a local linear representation like, so X n plus 1 is equal to some matrix times X n.

We are proceeding exactly the same way, we first said that our old approach of locally linearizing at some point in the states space does not work for the limit cycles, because of this problem. But, now we have defined another discrete time state space in which it should work, why because the whole limit cycle is nothing but a point and therefore, you can locally linearize on that point, which we will yield this from here and then this A matrix will yield Eigen values you can apply the same procedure for that.

So, the whole thing will again fall in place, the whole thing that we learnt about linear systems, then in the non-linear systems local linearization around the points, all these will now be again applicable, but in discrete domain. Let us see, so we have landed up in this neighborhood, in this neighborhood we have landed up with a, I will come to that a little later, because before going in to the matrix and its Eigen values and Eigen vectors I need to talk about other things also.

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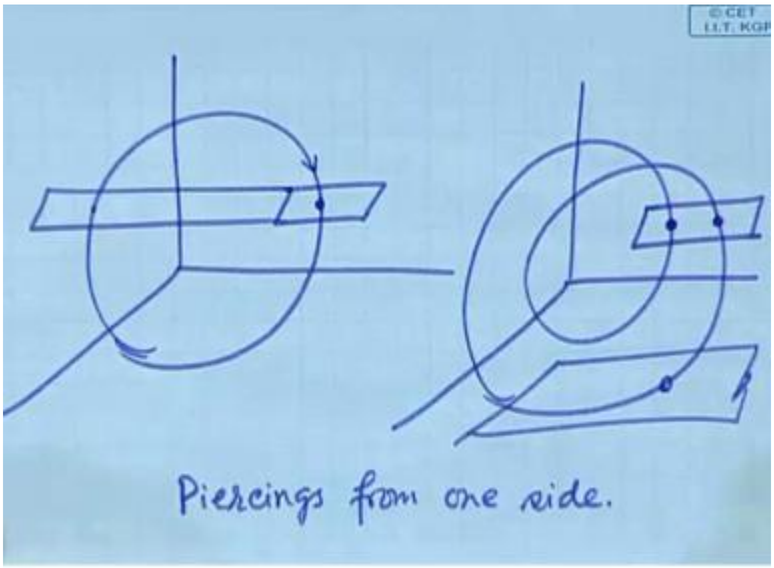

So, suppose you have a single limit cycle like this a steady state and you have placed a Poincaré section what will you see a point. Suppose, you have a period two limit cycle and you have placed a Poincaré section, what will you see two points, supposing you have got a period five limit cycle and you place a Poincaré section what will you see five points. So, this means that the character of the orbit is completely captured on that Poincaré plane by looking at what is happening in that Poincaré plane in steady state, you can infer the character of the whole orbit.

Remember for that you have to understand this, that it cannot extended like this and this also becomes a piercing, why because in that case the periodic orbit will have two piercings and the whole idea that I was trying to put forward will fall, will be invalid. So, in order to save that picture, all we need to do is to observe the piercing from one side not from both the sides, so piercings from one side.

The second point is that this section, where will you place it you cannot just place it anywhere for example, if you place it here then you see just one piercing, so you cannot just place it anywhere this is a period two orbit, but if you wrongly place it then you will see just one piercing. So, you had to place it, you have to first observe it and then place it in a way, so that all the loops intersect that Poincaré plane, so that is another pointer you have to keep in mind.

But once you keep this in mind; that means, firstly, you are observing piercings from one side only not from both the sides and secondly, you have placed it in such a way that all the loops in that orbit are intersected by that Poincaré plane, this thing is absolutely correct that whatever is the periodicity of the orbit will be exactly seen as the number of points in steady state on that Poincaré plane. Now, we have also studied a few different types of limit cycles. For example we have studied what was known as quasi periodicity, which was orbit on a torus.

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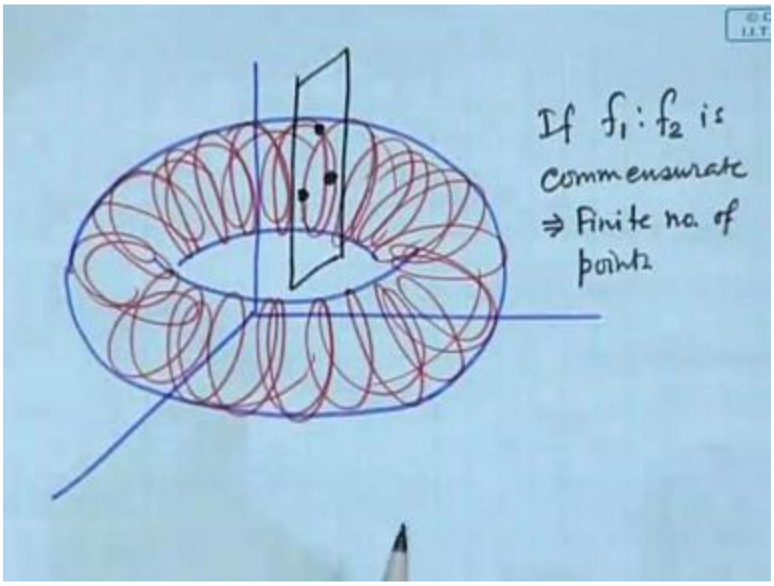

Suppose, this is a torus and you have an orbit something like this. Now, we have seen that there are two possibilities, possibility one is that there is a frequency component along this direction, there is another frequency component along that direction and these two frequencies are commensurate. In which case this orbit will ultimately become a periodic orbit because, the same state will repeat, in that case if you now place a Poincaré section like this what will you see, you will see a finite number of points, you will see the periodicity there exactly the same periodicity.

So, if f 1 to f 2 is commensurate; which means, the ratio is a rational number then you have a finite number of points. It is like this that you will see a point say one piercing here and then it will come back and then again it will come back somewhere else here, you will see another piercing here, again it will go around and then it will again land up somewhere say here, but again it will go around and come back to this point.

So, you will see a finite number of points on this Poincaré plane, which reflects that the orbit is really periodic, is that point clear. So, here also that idea is applicable that you have a finite number of points on that Poincaré plane representing the periodicity of that orbit. But what happens if this ratio is incommensurate.

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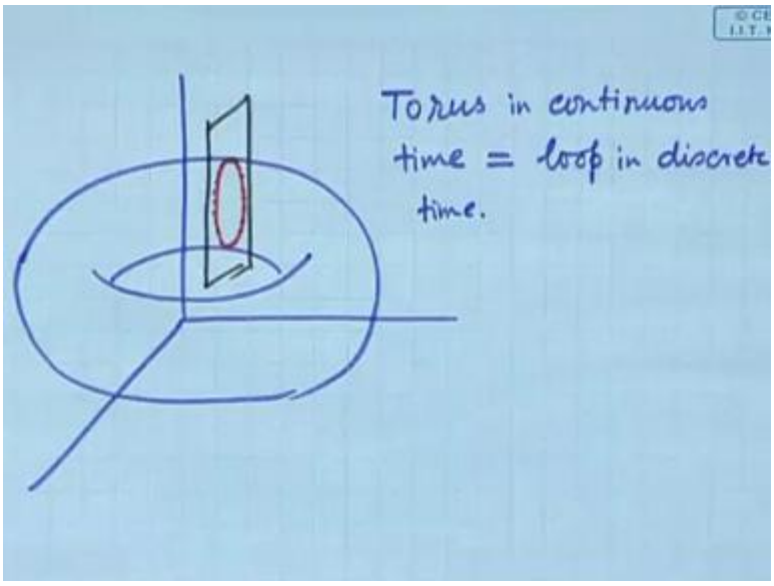

Suppose, you have a torus and then you have placed a Poincaré section and you are trying to observe how will it intersect, what will happen if you start from here and then as it comes back it will land at a different point, again it goes around and comes and lands at a different point, again it goes around comes at different point. But because these two frequencies are incommensurate their ratio is a irrational number they will never land on each other.

Which means, that the way the plane intersects the torus whatever you have seen on the intersection there will be infinite number of points on that intersection, how will that look a closed loop and on that there will be actually infinite number of points on this closed loop. So, if there is a infinite number of points on a closed loop in a discrete time dynamical system, it represents that in the equivalent continuous time dynamical system it is a quasi periodic orbit, it is a quasi periodic orbit. So, here also we see the advantage of placing the Poincaré section it is a simpler representation, is a simpler one, but at the same time it captures the essential of that orbit, so in general the conclusion is that torus in continuous time is same as a loop in discrete time.

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Yes, only when it is quasi periodic, if it is not quasi periodic; that means, if it is a mode locked periodic orbit two frequencies are commensurate then also then you will see a finite number of points on this, but a finite number of points on what on a loop, a finite number of points will always be placed on that loop. And therefore, even if you do not see the loop, mathematically you can visualize that there is a loop and then how that happens there are theories for that you can actually trace out the loop that was actually there even though you can actually see a finite number of points.

In fact, if the two frequencies are the same what will you see, if f 1 is equal to f 2 just one point, so there will be just one point, but that one point also will be placed on somewhere on the loop, depending on how this plane intersects that torus. So, we have understood that a period one orbit will be seen as a single point on the Poincaré section a period two orbit will be seen as two points a period five orbit will be seen as five points. How about chaotic orbit, if there is a chaotic orbit and you place a Poincaré section what will you see.

An infinite number of points why, because the orbit actually winds around without repeating without intersecting itself and therefore, it has in each of the loops it traverses a new path, so if you place a Poincaré section, in each loop it will create a new point; which means, there will actually be infinite number of points on the Poincaré section clear. So, on the Poincaré section there will be infinite number of points, so infinite number of points drawn in a piece of paper, what is it, a picture.

So, in case of chaotic orbits, you will see a picture emerging which represents the structure of that orbit. That picture I will show you some of the pictures may be after sometime, but that will represent sort of the internal structure of the chaotic orbit, I will show you some of them later, let me show you now if I have it here. Poincaré sections, so you see we have seen the Rossler attractor, we have seen the one that was going like this.

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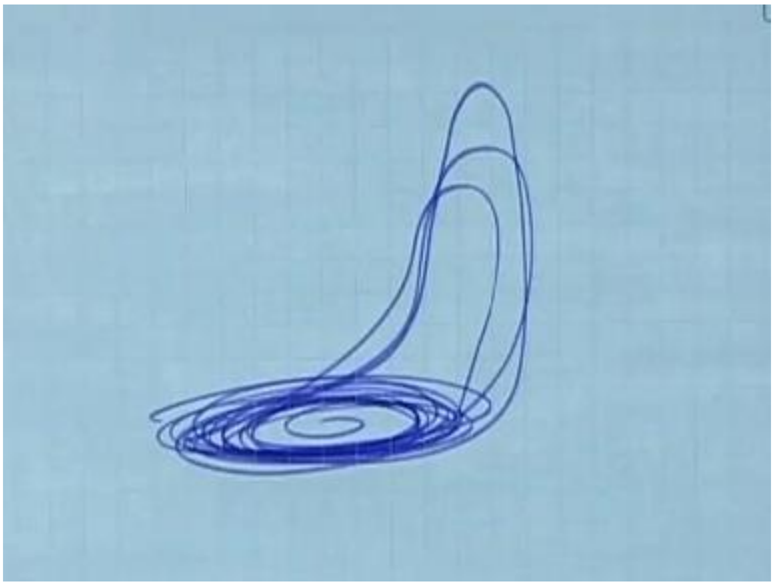

Then, then it goes, goes out and then goes do you remember that if you now place a Poincaré section to that orbit say here then you will see something like this, I will increase the number of points, I will make it larger can you see, can you see this can you see the computer, I will come back to the computer a little later, in any case, then no it has come it is good fine.

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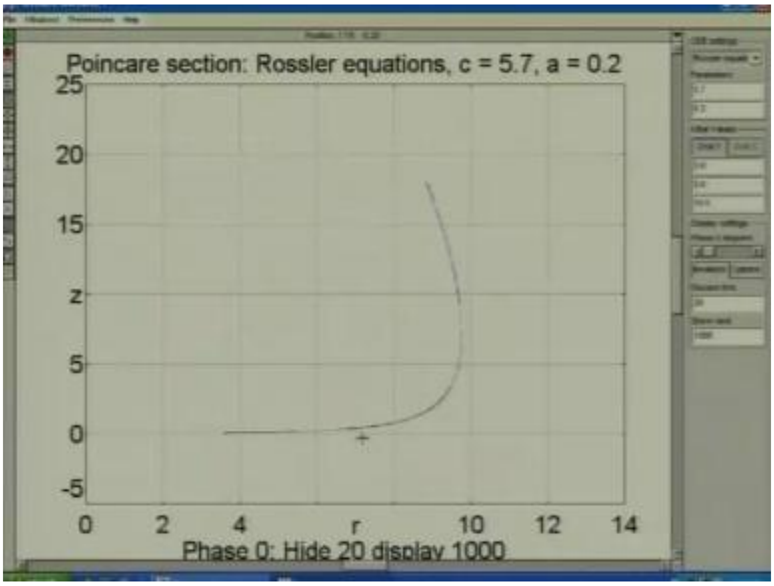

So, if you place the Poincaré section, it shows a very definite structure can you see that it shows a very definite structure not just any which way it has a very concrete structure and there are actually an infinite number of points placed on this. So, that is the structure of that orbit as seen on the Poincaré plane, there are infinite number of points there. If you take a different equation say another system, I will make larger number of points to make it clearer.

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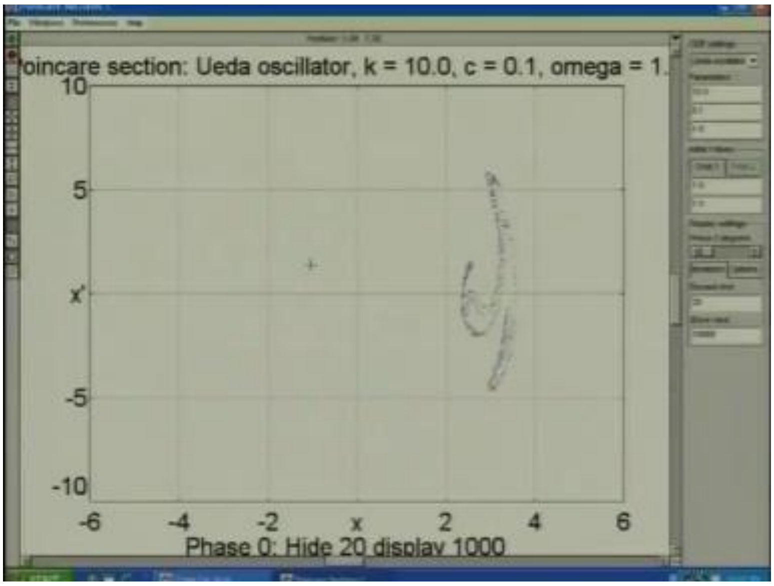

See a specific structure emerges, I will make it even large number of points, so that it becomes a little more clearer, it will take some time to compute that because what it is doing and how you should do it, I will come to that but first see what the character of the orbit is…

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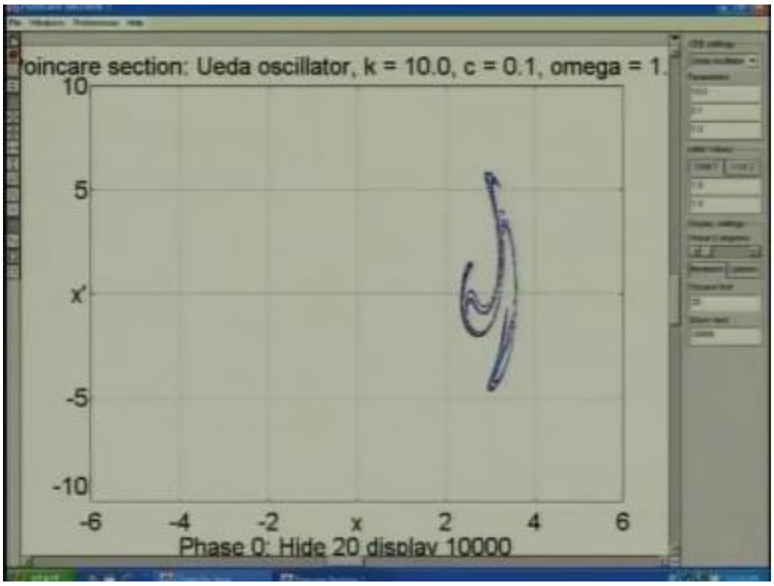

So, you see a specific structure has emerged, the points did not fall just any way, a specific structure is there, now that structure is very important because the point is that in a chaotic system therefore, the points does not fall, points do not fall just anywhere they fall in a specific well defined structure that is something that is not clear unless you really place the Poincaré section and see how it pierces.

So, in this case what has happened each of these points are really piercings of that orbit on a suitably placed Poincaré section, so it has it has pierced and then it has gone around and then it pierced again it has again, gone around and then pierced again and all these piercings have been plotted and that ultimately makes this picture. If a system where a periodic orbit there will be a finite number of points, but since it is a chaotic orbit it has an infinite number of points ultimately making out the picture.

So, that is then the attracter in discrete time, if you start from anywhere else in this state's space, it will converge on to that attractor and then move inside that attractor, so that is the chaotic attractor in discrete time. So, we have understood that when we have a period one point there is a just one point on the Poincaré section a period five orbit five points on the Poincaré section, chaotic orbit infinite number of points are on a Poincaré section, quasi periodic orbit also infinite number of points on the Poincaré section.

But, what is the difference in a quasi periodic orbit it will be placed exactly on a loop, while in a chaotic system it will not be placed exactly on a loop, it will still be a picture pretty picture though, but it will not be placed on loop, so that is the essential difference. Now, you might ask how actually to obtain this.

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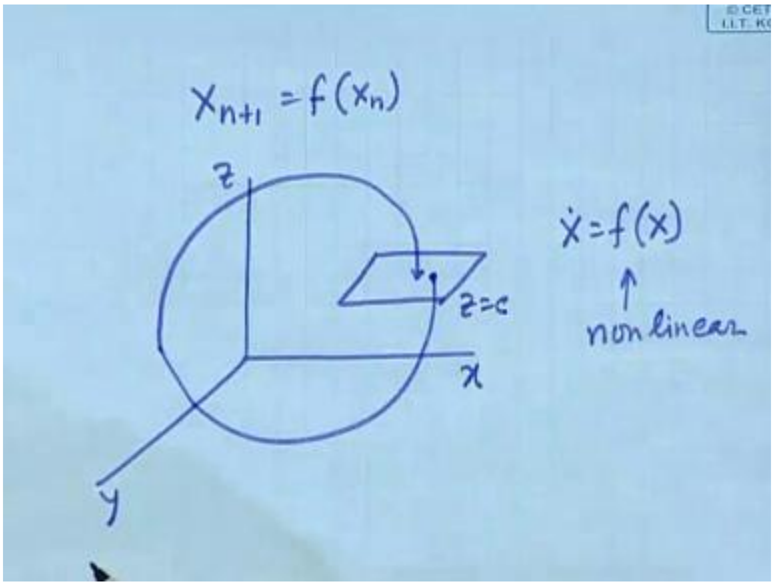

How would we obtain that now obtaining that may be a will normally be a computational affair, why because what are you actually doing, you are starting from you are you are placing the Poincaré section, starting from a point on the Poincaré section allow it to be evolved and then finding out where it pierces next. And then you would say if this is my X n this is my X n plus 1, so obviously you will need to evolve it from this point to the next piercing.

Now, this system of equation that actually does the evolving at is X dot is equal to some function of X and that function is normally a non-linear function, if it is a linear function then all these will not happen, so the whole business of making a Poincaré section and other things will be pretty unnecessary, why because there in linear system cannot arbitrary closed loop like this, there cannot be a limit cycle. So, normally this will be a non-linear function.

And if it is a non-linear function, you cannot obtain a closed form solution representing this, you will have to do it by Runge-Kutta method by computer by means of a numerical routine. So, you will take small steps by the Runge-Kutta method and finally, find out finally, go around it find out when it again satisfies the condition for that particular plane, it is normally convenient if this is x coordinate, this is y coordinate and this is z coordinate to place the Poincaré section either at x is equal to some constant c or at y is equal to some constant c or at z is equal to some constant c.

In this case I have pictured it as z is equal to some number, so the process of obtaining this is actually a subroutine in your code, what does a subroutine do, it starts from a point any given point on the Poincaré section and it only calculates till the next piercings; that means, go on solving it till you have satisfies the condition for this particular Poincaré section, from the same direction remember that from the same direction; that means, this will not be the same you will have to do it for the for the same direction; that means, not only the condition for satisfying this, but also the condition of the direction in which it is being satisfied have to be met then only you get the point here.

So, start from this point end with this point, in the subroutine you have the argument at which the initial point is given the output of the subroutine is the final point that whole thing, whole chunk of code is the map, have you understood that how to obtain that.

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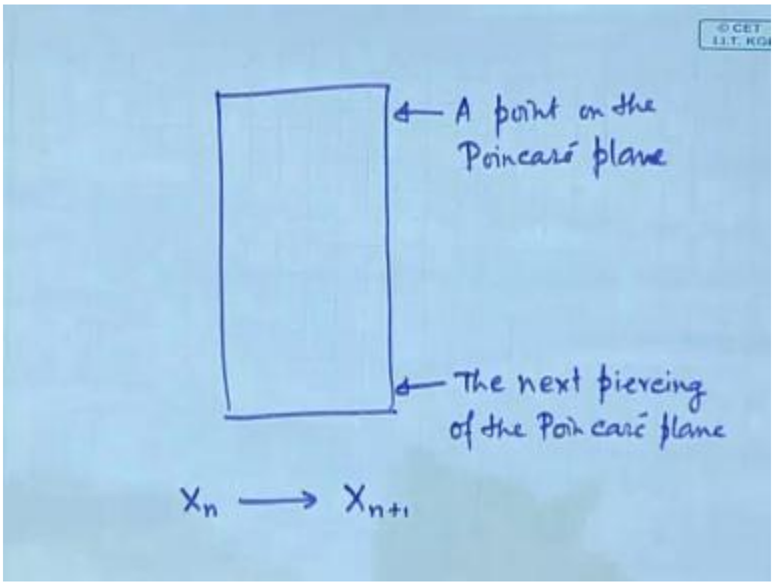

So that will actually be a chunk of code, what is there before that what is there after that I am not going to that it is a particular portion of your computer code in which the first line should give a point on the Poincaré plane and what should it end with, the next piercing. Therefore it has started with X n and it has given X n plus 1, where X is a vector, so in normal cases you will have to do it like that.

And I have given you some equations like, I have given you the equation for the Rossler system, I have given you the equation for the Lorenz system, for that you can yourself write this code pretty trivial code really because you have already written down the code for the solution of differential equation by the Runge–Kutta method. Therefore, finding this is very, very trivial problem, all that we have to write is to write properly, the if statements, so that this condition is satisfied. So, write that problem write the problem, so whatever I showed on the computer you should also be able to compute that.

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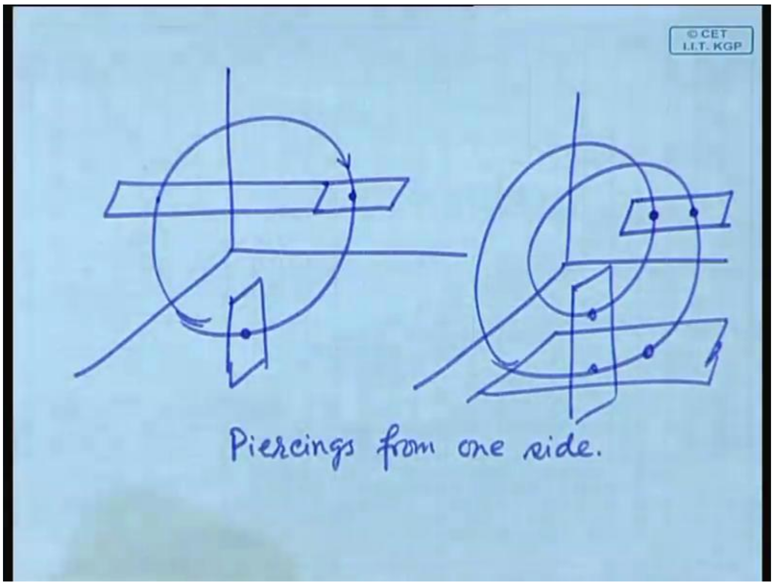

Now, notice one thing that if you have a situation like this or this, then your original system was 3 d, but your Poincaré plane is 2 d. So, this procedure has resulted in reduction in the dimension of the system that is one advantage, because 2 d is easier to handle than 3 d, 1 d is easier to handle than 2 d.

Student: ((Refer Time: 44:56))

It is dependent on the plane we choose yes, but the character of the point does not depend on the plane we choose why, because supposing we choose instead of this, we choose this plane then also we will see just one point. In this case suppose we choose this plane then also we will see these points, these two points. So, the character whether it is a period one orbit or a period two orbit or a period five orbit or a chaotic orbit is clearly visible from the Poincaré plane wherever you place it, provided you place it with that condition satisfied that in every loop it should intersect the plane, if you do not satisfied it then it is it is not true.

Student: ((Refer Time: 45:50))

Yes; that means, when we do that essentially we first obtain the limit cycle take a look at it, if you place it here then it has the piercing then that we place it is somewhat trial and error game. First you have to see the limit cycle and then only you can logically place it, but this is a problem with systems that have no external periodic input, but you have seen there are large number of non-linear system there is some kind of external periodic input say the pendulum with oscillating support, the electrical circuit with a sinusoidal input and all that.

These are systems with some kind of external periodic input, in those cases there is a unambiguous procedure of obtaining the discrete time dynamical system what is that. What is happening you have some kind of a external forcing; that means, that imposes some kind of periodicity on the system in time, there the procedure of discretizing it simply to observe it in synchronizing with that external forcing. Imagine, in this way that the external forcing is a sinusoid.

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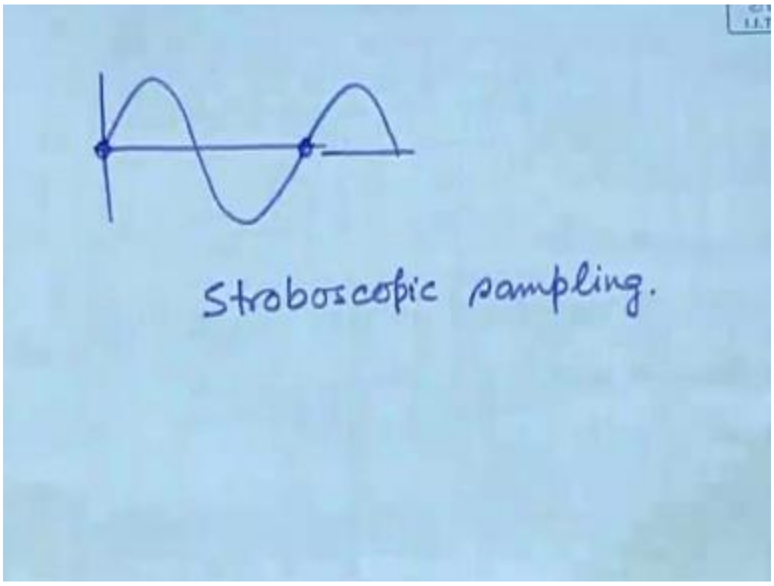

Suppose it a sinusoid, and then you can observe the state discretely without placing the Poincaré section like that you can observe the state at every positive edge 0 crossing of the sinusoid. Then also it will lead to the same result that the state, which was actually continuous state has been discretized, now you can earlier you had to talk about the continuous evolution, now you can talk about the discrete evolution, now you can talk about X n to X n plus 1, all that is true.

Moreover, now there is no ambiguity, you can always place it like that this is also similar to placing a Poincaré section I will come to that later, but notice one thing what are you doing, there is a intuitively appealing way of looking at it suppose, this room is darkened and this room is actually not the room physical room with x y z coordinates this is actually the state space imagine that which means if you sit like this you will be able to see the orbit going like this in the state space.

And then observing it like this means you are sitting here with a stroboscope, which fires every time the 0 crossing occurs and wherever the state is it is lit, so once you see the state here, then you see the state there, then you see the state here, then you see the state there, discrete move motion, that is why this kind of observation is called also stroboscopic sampling. Now, remember one thing those who have done some DSP course may have come across, if you want to disctertize a system how frequent this discretization should be come across, but here we are not talking about that.

There the idea is that if you want to have a some kind of a wave form and if you want to represent that waveform through discrete observation as in an analog to digital converter then what should be the frequency of that converter, so that you can reliably reproduce that waveform that was the question. But here the problem is not that here you are discertizing the system as observed in synchronism with that external periodic input and therefore, if this point and that point are separated by one second you will observe it one second later no problem.

So, the point is that here we are not talking about that here we are talking about discertizing the system in order to observe it on the Poincaré plane and so in case of autonomous systems we have to do this Poincaré plane business, in case of non-autonomous system we will normally do the stroboscopic sampling. This is also like placing a Poincaré section, why?

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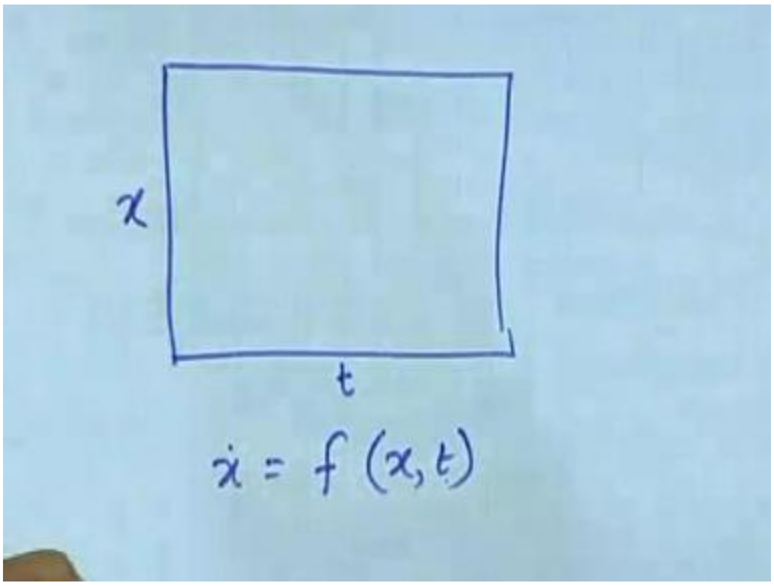

Because normally you have the state like this, state space like this and imagine a system with x dot is equal to f of x and t, this t comes because of the external forcing. Then the function depends on both x and t. Therefore, you can imagine the state space comprising of x and t and this t variable is actually repetitive, why because the external forcing comes back to the same state after some time. So, this t variable is repetitive and so the x coordinate here, t coordinate here, the value of the external periodic input whatever is here is also here after sometime it has come back to the same state.

This is sort of equivalent to imagine that this is like a sheet, imagine that this is like a sheet in which this is there, this point and that point are the same, in the sense that the external periodic input whatever the value of the external periodic input here is also the same here what does it mean you are essentially rolling it into a cylinder. It is like rolling it into a cylinder, why because these two ends actually represent the same state in time, so it we are rolling it into a cylinder.

So, the state space of a non-autonomous system can be visualized as the motion of the orbit on a cylinder and the Poincaré section is nothing but a section in the time axis and that is what we are talking about. When we are doing a stroboscopic sampling we are placing a section on the time axis, like this and that is exactly why the stroboscopic sampling as I just told you is also a Poincaré section.

It is a different type of Poincaré section because in a non-autonomous system t also becomes a like a state variable and we are observing by placing a section on that variable. So, we have understood the different ways of obtaining, you can visualize in this way.

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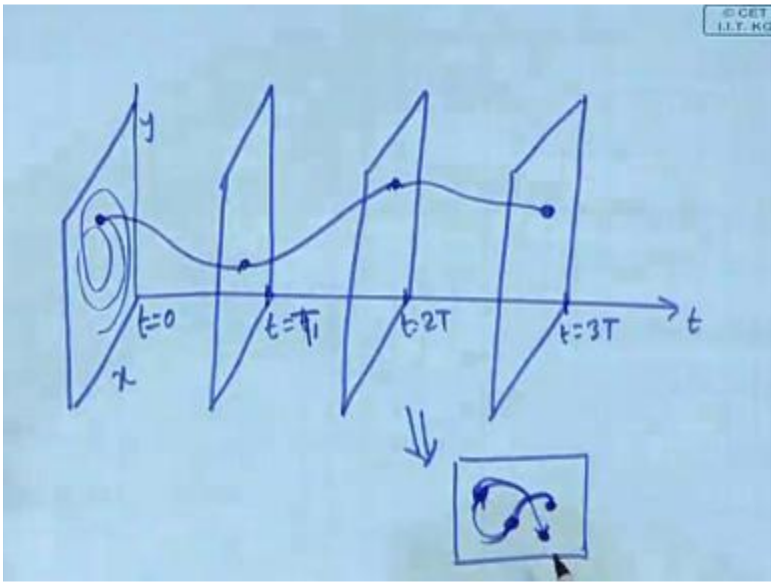

Suppose this is your t axis and suppose you have x and y as the coordinates, so there is a t and the t axis is repetitive after this times, repetitive means this external forcing comes back to same value after these times. So, here you can imagine a plane, so that at t is equal to is equal to 0 that the state was somewhere, here also you can imagine a plane like this, so t is equal to say t 1 and at this time the state is here, so in between it has come like this, here also you can imagine that the state is somewhere say it has come like that.

At this time also you can imagine, so this is the state's x y s state place, this is the time axis in which the time is repetitive every say the capital T seconds, so t is equal to T, t is equal to twice T, t is equal to thrice T and so on and so forth. So, even though, if you look at the x y plane you will see it something like this happening. But the t is proceeding in this direction and you are observing at equal intervals of t, so that as it proceeds you will see it like this and you will observe it discretely here.

So, on that x y plane what will be there ultimately the whole result is that you will see, a point mapping to a point, mapping to a point, mapping to a point and so on and so forth, that is the picture of the Poincaré section in a non-autonomous system. So, even though in this plane if you plot the orbit you will see it going like this, but actually if you discretely sample it in synchronism with the time axis or whatever is the periodicity in the time axis you will see discrete points a point mapping to another point, mapping to another point, mapping to another point and so on and so forth. So, that is what you wanted to achieve, let us stop here now and we will continue in the next class.