Dynamics of Physical Systems
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Lecture - 31
Dynamics of Nonlinear Systems – II

In the last class, we have seen some salient properties of non-linear systems. First we saw that the property of the state space is not the same everywhere, there are some part of the state space in which one property pertains another part of the state space another property. So, that is one very important feature of a non-linear system that it is not the same everywhere.

We have seen that, in the simple pendulum also there was one equilibrium point with circular orbits around it, there was another equilibrium point with saddle character. So, that around these points the behavior would be entirely different, so if you take any initial condition away from these points they will be guided by some property of that local region of the state space. So, you have that as the fundamental shift from your knowledge of linear systems.

Now, apart from that we have seen that in some cases it is possible to work out the behavior of the whole state space, by working out piecemeal starting from each equilibrium point. So, this the standard procedure would be, that look at each equilibrium point in a non-linear system, there can be many equilibrium points and for each one you work out the local linear neighborhood and the behavior around it.

And then from there it is possible to sort of extrapolate the vector fields to the rest of this state space, now this procedure also fails under certain conditions, we will see that. But, one thing I did in the last class, that was to point out that the exactly imaginary Eigen values where somewhat unnatural condition, because in that case the real part has to be identically 0. And you pointed out, at on that day that the particular example that we had taken, there what was the shape of the Jacobian matrix.

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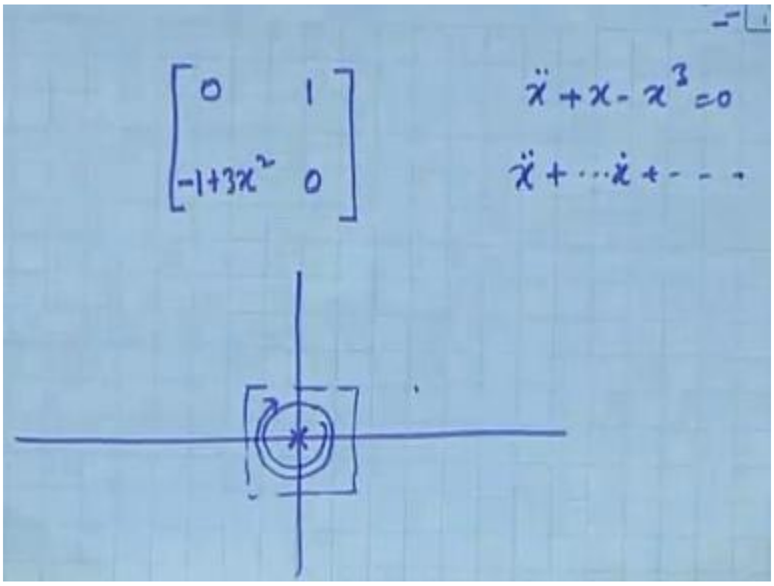

The Jacobian was let me see, what it was the x double dot plus x minus x cube that was the system yes, so what was the Jacobian 0 1 minus 1 plus 3, minus 1 plus 3 x square and 0. Now, because these two terms are 0, so whatever be the x it will always turn up into purely imaginary Eigen values, but the point is not that point is here, what are we doing we are locating this equilibrium point and then arguing that this would represent the behaviour around it.

Now, this would represent the behaviour around that equilibrium point away from the equilibrium point say another point here, then the same logic will not occur. So, simply by substituting this particular x value here, will not give you the proper Jacobian matrix here. So, that is why the right way of working out the real character of the state space would be, to start from an equilibrium point say here from an initial condition say here.

And then, work out by Runge-Kutta method that way you will be able to plot the evolution of it. Now, if that comes back to itself you know something that it is really complex conjugate, but around it the local linear approximation that yields a purely imaginary Eigen values is correct.

But, it can also be like this, then you would you would infer that no it was not really correct very approximately correct, it is correct only at that equilibrium point elsewhere it becomes somewhat wrong, so that was the logic. In general if you have a term plus something x dot plus something, then that term will lead to that real part and then, it could be whatever that means, in a system like this you are actually assuming that this part is identically 0 that is why it was yielding purely imaginary Eigen values.

Anyway, so the point is that in some very simple cases it is possible to work by work piece mean by looking at the local linear neighbourhoods of each equilibrium point, but under certain condition that has to be taken with a pinch of salt. But, there are situations where it cannot be done, and we had analyzed one situation in the last day…

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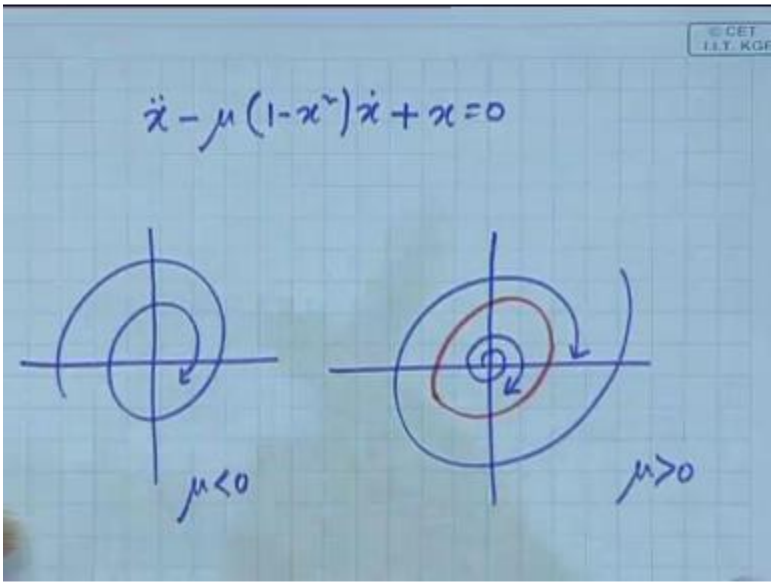

That was x double dot minus mu 1 minus x square x dot plus x is equal to 0 this equation we had handled in the last class, we had decided, we had concluded that for mu less than 0, it will be an orbit something like this for mu less than 0. And for mu greater than 0 it would be locally like this, but that is a linear approximation which might not work might, not with there is no reason to believe that it will also work elsewhere. So, there it will still be an incoming orbit. So, in between there will be an orbit which is maybe something like this, where if you take an initial condition inside it will converge on to that what?

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Both are..

Student: ((Refer Time: 07:15))

No, no is it t was it clockwise or counter clockwise.

Student: ((Refer Time: 07:24))

Check that once I mean both have to be in the same direction, it cannot be the inner one is the counter clockwise the outer one clockwise that is not possible, why not possible. Because, as you come very close to this if you take a slight perturbation this way, and slight perturbation that way they cannot lead to opposite vectors, that is why they have to have the same kind of rotation fine.

So, this is the example of a limit cycle now, one very common misconception let me first clarify that you have come across situations with such closed orbit. For example, if you have purely imaginary Eigen values in a linear system then what do you have?

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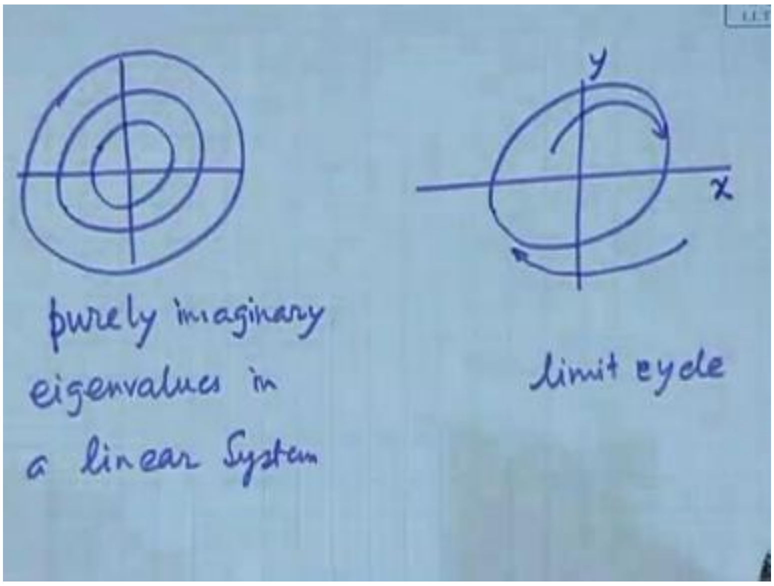

You will still have like this, we have seen that and we have seen, so this is purely imaginary Eigen values in a linear system and second is the limit cycle, which also looks similar, so is there any difference between them, can these be called a limit cycle, no, note the word limit. If you start with a different initial condition in a case with purely imaginary Eigen values say, you start from here it will go into another loop a different loop not the same loop.

Start from another initial condition it will look go like this, a different loop not the same loop, while in this case if you start from another initial condition it will converge on to that, start from another initial condition it will converge on to this. This is the fundamental restriction that means, starting from different initial conditions this one leads to different orbits, I will orbit with the same frequency that you have seen already.

But, here they all converge on to the same orbit that is why this is not a limit cycle, so this is not a limit cycle, this is limit a cycle. So, whenever you see a cyclic orbit do not call it a limit cycle, one of the very common mistakes the student do, this is not a limit cycle, this is a limit cycle, because in the limit it reaches this while in the limit there is nothing yet that it converges on, so that is one important property of a limit cycle. Secondly, you would notice that if you have an orbit like this, and if you it somehow it gets perturbed then it comes back to this one, while if you have an orbit like this somehow it perturbed it goes into another orbit.

So, it is in those respects different and, it is not difficult to realize that wherever, we need some kind of a oscillatory motion in a physical system or an engineering system. We really have to do this why because, there are always some perturbations there are always something to disturb a particular motion in nature or in engineering systems.

And in all systems we would like it to be stable whatever, we want to be a stable behaviour and you can see that, this is a stable behavior, so where do we need a oscillatory motion in engineering any oscillator, all the oscillators that you heard off. You want that to oscillate with a definite frequency with a definite amplitude and that is possible, only if you have only if you design something like this not like that.

So, none of the oscillators are really linear systems with purely imaginary Eigen values, all oscillators that you have studied in say electronics they are all harbouring limit cycles. Now, with that knowledge with that concept, you can go back to your books and in electronics, where you studied about oscillators, the simple oscillators that you probably studied, what are the oscillators you studied then…

Student: ((Refer Time: 11:50))

Student: ((Refer Time: 11:51))

And.

Student: ((Refer Time: 11:53))

Colpitt's Hartley yes, so all that you can revisit and try to find out, where is a non-linearity and how is that non-linearity being productively used in order to create a limit cycle. And in nature also wherever there is a necessity of a cyclic motion that is needed to be stable you have limit cycles, a very common example is the human heart, so a human hearts pounding the motion is a oscillatory motion obviously, and that oscillatory motion obviously, needs to be stable.

Suppose imagine that your heart goes by this principle what will happen, suppose your heart is something with a linear system purely imaginary Eigen values, so it will have the same kind of cyclic motion you will. So, what will happen what is the problem?

Student: ((Refer Time: 12:51))

Supposing, there is a cracker bursting here immediately it is starts pounding faster and for the rest of your life it go it goes on pounding faster, so obviously that is not allowed. Go for a jogging, you have got a greater heart beat and for rest of life you are locked with it, so that is obviously, undesirable and that does not happen, so your heart is a limit cycle.

Similarly, everything that you see around yourself, where there is some kind of a cyclic motion that is always a limit cycle, I am not talking about this motion of the moon around the heart are something like that. These are not the cases, I am talking about the naturally occurring rhythms, where you having a rhythm which in the biological world or in the natural world, you find there is a rhythm there it is must be this.

So, we have understood that in a non-linear system there can be a limit cycle, notice that a limit cycle is not a local behavior, because locally it is just a outgoing orbit by starting the local area, you would conclude that it would be unstable. But, it is not unstable it is a global behavior, so it is a global behaviour of the whole of the state space and that is why it cannot be studied in any means, if you only concentrate on the local neighbourhoods, so these are some typical features of limit cycles.

Now, if you have a limit cycle like this, what is these and what is that what are the axis. The two states say x and y, now you could also plot the x against time and y against time what will you get?

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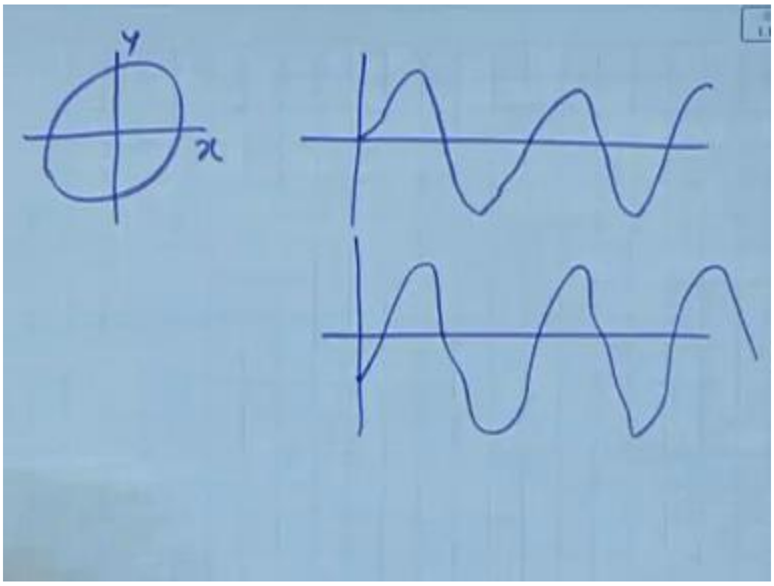

If you plot say an orbit like this, and if you plot x against time, so you will get a you will get what?

Student: ((Refer Time: 14:53))

No, it is not a sinusoidal motion that is, what I wanted to point out it is not a sinusoidal motion, but a periodic motion, so something like this may be, but it is not exactly a sinusoid why, in order for it to be exactly sinusoid you will need really this ((Refer Time: 15:11)), if you have a purely imaginary Eigen value, then we have seen that that its sinusoid. So, if you do not have then depending on the character of the state space it will be not exactly a circle bent, here bent there pressed here squeezed there whatever, it is it will be some kind of a orbit, but not really circle or ellipse exactly.

So, if it is not then it will not be purely a sinusoid bit it is a periodic wave form, now let us ask in what way can, we have what will be the y waveform, shifted in phase, but the waveform would look the same correct. So, it will be shifted in phase, but it will be more or less the same.

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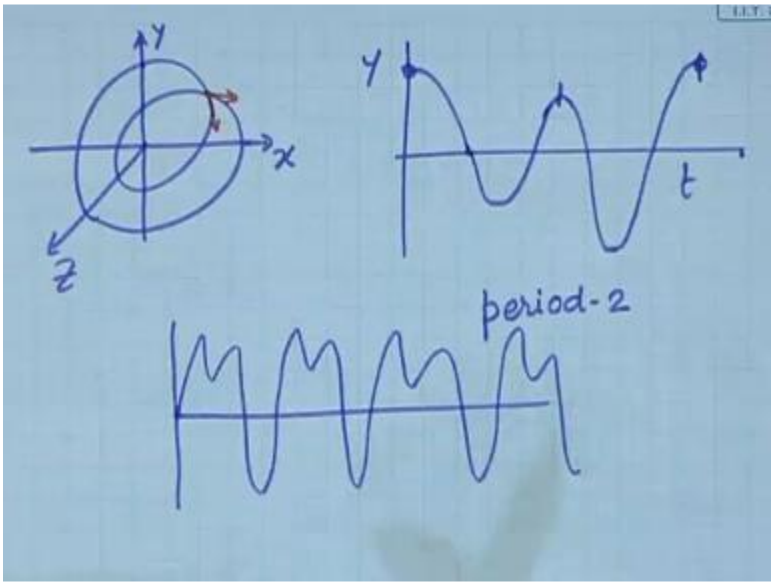

Now, if I ask is it possible to have something like this, is it possible to have something like this, no, because what is this point at this point the orbit is intersecting itself which means, that if you exactly are here then where is the vector, once since it is like this and, since is like that. Obviously, it is not possible because, for every point the vector is unique, so this is impossible to happen physically. So, such an orbit would normally not occur but, then ((Refer Time: 17:09)) why did we say it will not occur.

Because, in order for a orbit like this to occur there has to be an intersection and intersection is a prohibited, but then suppose it is not a 2 d system. But, a 3 d system in that case what you are seeing as, the intersection could actually be orbits that are separated along the z direction. So, in that case there will be no intersection, that will be allowed is that clear.

So, such an orbit will not be allowed in a two dimensional system, but such an orbit will be allowed in a 3 dimensional system. Like you have seen a sinusoidal oscillation is not possible in a one dimensional linear system, but it is possible in a two dimensional linear system. Similarly, an orbit like this is not possible in a two dimensional non-linear system, but is possible in a 3 dimensional and above non-linear system.

If you have an, orbit like this these are actually real they happen, now if you have an orbit like this then, what would be the evolution of the x coordinate against time look like and the y coordinate against time will look like. Say if I am plotting here the y coordinate against time, see I start from here after that it goes down after that it goes down and after that it goes up. So, after that it goes up, then it goes down further then it goes down further then it will go up to the same value right is that clear, so actually you have to develop the ability of translating a state space behaviour into a time domain behaviour by arguing like that.

So, at this point it goes when it turns around, you should not thing that in time also it turns around because in time it cannot go back, it always goes forward in time, so it goes down up and so on, and so forth. And this orbit will repeat, now notice that in this one you have started from here, you have come here, so starting from here ending here is one one cycle, peak to peak again from here to the peak again another cycle.

So, there are two cycles and after that only it comes to the same value, there are two periods one period and two period, if you are thinking of the 0 crossing start from here, one period, two period. And then only it comes to the same value, so it actually has two periods inside it is complete cyclic behavior, that is why it is called a period two limit cycle.

So, a period two behaviour in general, if you want to see such a thing on the CRO screen it would normally be something like this, where the same behaviour is repeating after two cycles two periods. Now, the immediate conclusion about this is that such an waveform is not possible to see in a two dimensional system. That is, what we have concluded from this, so such a waveform you will never see in a two dimensional system linear non-linear or whatever.

It is not possible to see theoretically impossible, so if you have only two storage elements in a system you know that in a, when we talked about bond graphs we learned that the number of storage elements would be equal to the number of state variables. So, if you have only two storage elements, you will never see something like this clear, so is the period two concept clear, that is what a period two is called…

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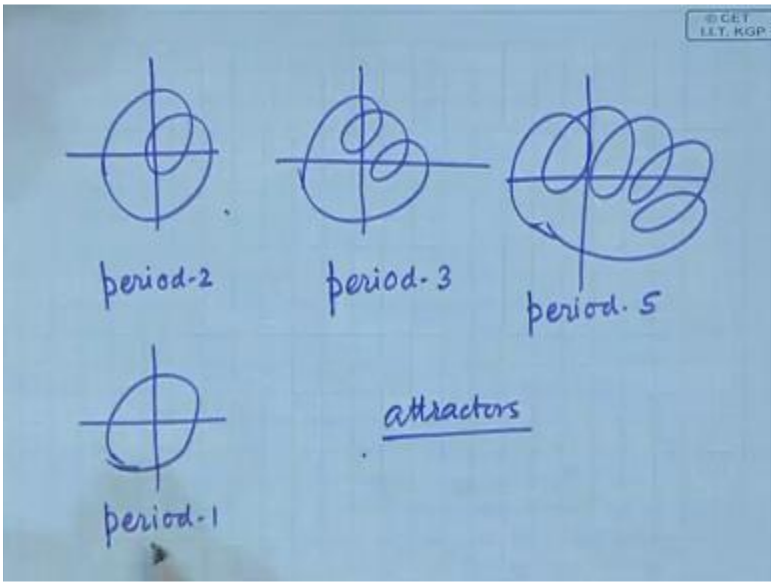

Now, if you can have an orbit like this why not an orbit like this, That is, also possible then what is the difference between them, this one has 2 loops, and this one has 3 loops. In this case, you can easily see we have seen that the same state comes back after two periods two cycles. In this case the same state will come back after 3 cycles, so this is period 3, so this is period 2, what about that?

Student: ((Refer Time: 22:33))

Possible, well at least you will not be able to argue that mathematically, that it is not possible. And there, is a nice saying that if something you cannot prove as not possible must be occurring in nature. So, this fellow also occur such rather complicated orbits are also possible in nature, so this would be 1, 2, 3, 4, 5, so these are perfectly possible. So, the point is that in a linear system none of that could happen, but, the moment you shake of the possibility of that nice, linearity character all possible behaviours are then possible.

Now, notice that here we had initially started with a periodic orbit period 1, orbit first establish that such a thing is possible then this is period 1, then we said the period 2 is possible, then we said the period 3 is possible. And notice that, all the time how are we computing this simply by starting from an initial condition and then, going on by some kind of a numerical routine, you have already learned the in Runge-Kutta numerical routine.

So, all these will have to be done that way remember, it cannot be done by any of the linear means, so starting from any initial condition you go on and, but you start from another initial condition here what will happen. Then, if it is a limit cycle, then it will converge on to that it will slowly progressively go closer and closer to that limit cycle. And finally, it will converge on to that, in that sense this is also a limit cycle and we have said in the last class, that these are sort of gravitating bodies in the state space in the sense that, if you start from another initial condition quite far of it is gets attracted to this.

So, these are also called attractors, so you would see in books reference to say period 1 attractor, period 2 attractor, period 3 attractor, period 5 attractor which means that, these are attracting in the state space clear.

Student: ((Refer Time: 25:07))

Yes, I will come to that, what he is saying is that…

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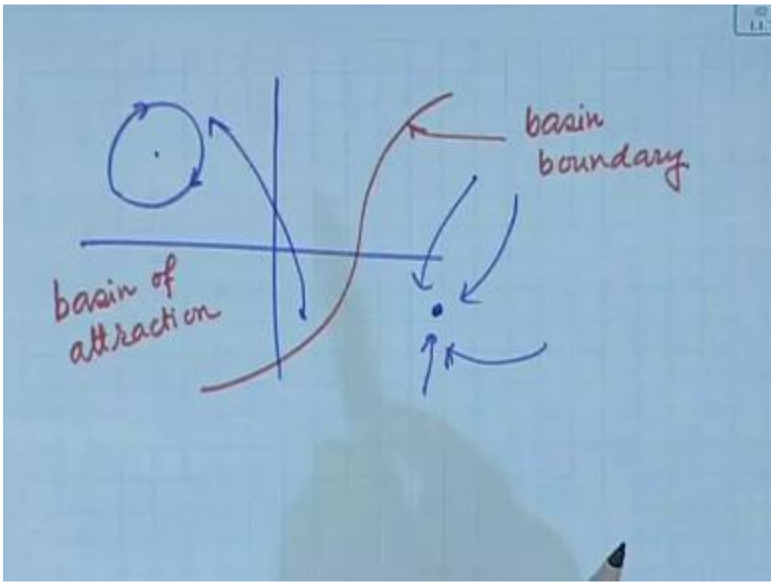

Supposing in the state space, there is somewhere an attractor like this, does it always mean that all the state space will be attracted to this one. There is no reason to believe that, because we have the first point that we noted in a non-linear system was that the state space, state space could have different behaviour in the different parts. So, there is no reason to believe that there is somewhere it will also converge on to that, it could be converging on to another say an equilibrium point here.

So, from here it goes, but from here it goes which means that there is one attractor which is attracting a part of the state space, there is another attractor which is attracting another part of the state space. It is possible in a non-linear system, in that case it is also possible that, not only possible it must happen that, there should be some kind of a dividing line, so that this side goes to this one.

And that side goes to this one, like your football fields one hostel plays there the other hostel plays there in between there is a line, so you cannot cross the line. So, you have, if we, if you are in this side you always go to this attractor in this side this the, so then the region of the state space that is attracted to one attractor will be called the basin of attraction.

So, there is one basin of attraction here, there is another basin of attraction here, and the dividing line would be called the basin boundary.

Student: ((Refer Time: 27:20))

No, the his question is that, how many such basins of attraction can there, be is there any upper limit, no there is no upper limit. In a non-linear system there, could be very large number of such attractors, which are existing at the same time often called co existing attractors. So, you have different attractors in different parts of the state space and there, is no upper limit, but one thing is sure that, the number of attractors are also related to the number of equilibrium points.

For example here, there was an equilibrium point inside, which had become stable and so, this fellow is now the attractor here is I can see there, if it is a point then it is a point attractor which means, ((Refer Time: 28:25)) also an equilibrium point. So, similarly I am not going into the mathematical details of it, but at least it does sound intuitively possible that the number of attractors will be related to the number of equilibrium points. That is why, it cannot really be infinity.

No, it can be infinity for example, the pendulum itself has infinite number of this, it could be, but it is definitely related to the number of equilibrium, equilibrium points that is all, such details I will come to a little later, but then if you can have orbits like this ((Refer Time: 29:11)). Period 1, period 2, period 3, period 5 why not period 16, period 28, period 27, period 135 is there anything to stop it. No, at least not mathematically, so if you extrapolate that argument, then you evidently reach the conclusion that there is no limit to the periodicity of ((Refer Time: 29:40)) orbit.

It could be a million it could be infinity, now what is the meaning of a period infinity it is it just means that, it is an orbit that has no periodicity, it does not come back to itself that is all, yes, it is possible to have an orbit that never comes back to itself. So, that as it goes around it always traverse as a new path, and still it is bounded that means, you can have an oscillation like that, let us understand that slowly by working with one example system.

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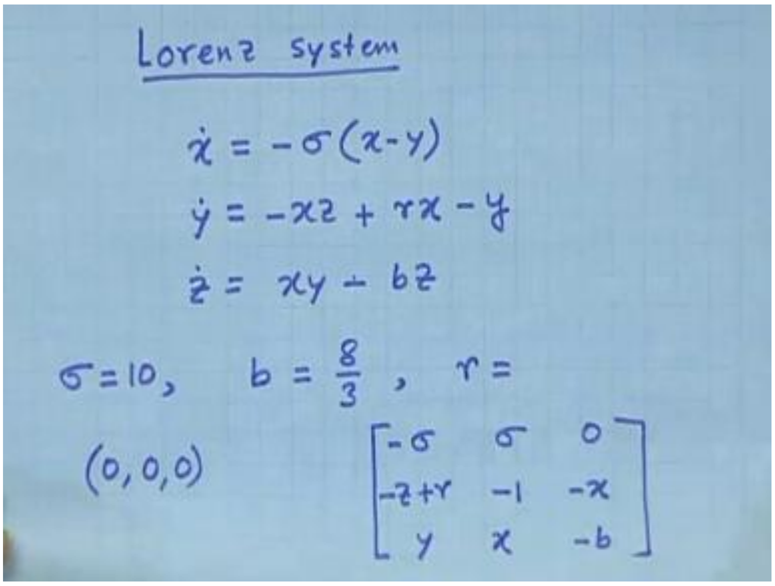

Let us take this system it is called the Lorenz system, x dot is equal to minus sigma x minus y this is a linear equation, y dot is equal to minus x, z is the non-linear term plus some r x minus y and z dot is equal to x y is another non-linear term minus b z. So, in this system, there are a few parameters I can see x y z, z are the variables Parameters are I can see sigma is a parameter I can see b is a parameter, I can see r is a parameter.

In such a system obviously, we will not be able to do the simulation by simply working analytically, but let us try as far as possible as far as, we can do for that we need to assume some parameter values. So, let us say sigma is say 10 b say 8 by 3 and r we will be able to arrive at some conclusions depending on that firstly, where are the equilibrium points of this.

Student: ((Refer Time: 31:01))

No, 0 0 0 is 1, if you set the left hand side to 0 and 0 0 0 yes, so it is an equilibrium point, so equilibrium point would be 0 0 0 anything else. There should be, because these terms are there, so can you just work out what are the other equilibrium points, 0 0 0 is definitely one equilibrium point. The other thing is, what is the Jacobian matrix of this minus sigma this is plus, so sigma 0 this is minus z plus, r the second term is times y minus 1, third term is minus x then, this is y x and minus b. So, this is the Jacobian matrix, if you put 0 0 0 you get this is 0, so r remains fine, you substitute this and find out, when will this equilibrium point be stable.

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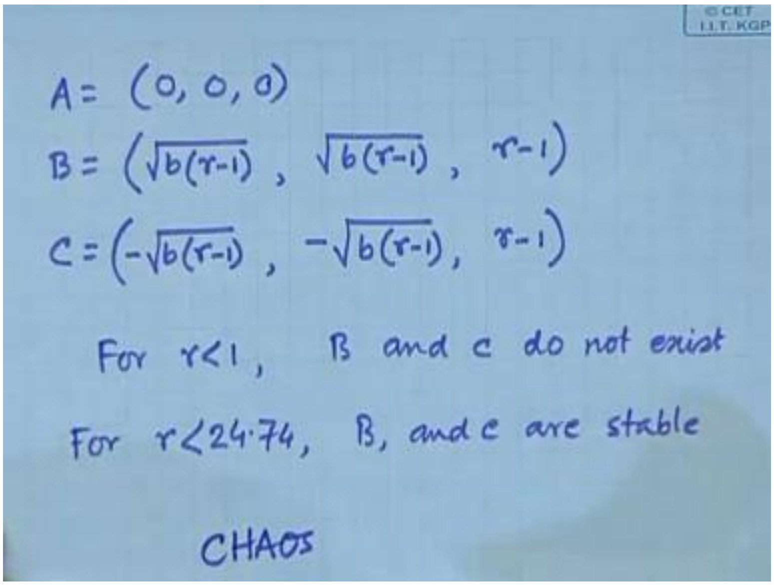

Firstly, what are the other equilibrium points, the other equilibrium there are three equilibrium points, A is 0 0 0, the other equilibrium points are, you have notice that it is rather simple to work out, but in order to save your time let me write them. Root over b into r minus 1 root over b r minus 1 and r minus 1. And C is mirror image of it minus root over b r minus 1 minus root over b r minus 1 and r minus 1, now these immediately tells that.

So, long as r is less than 1, this term will become imaginary and it is a real state space in, which we are talking about the location of an equilibrium point and that cannot be complex that cannot be imaginary. So, whenever it yields imaginary numbers it would only mean that, the equilibrium point does not exists, so if this means that for r less than 1, B and C do not exist. And when that is, ((Refer Time: 35:49)) so then what is the status of this point, A 0 0 0.

Is stable Just put this, and put these values, you will find that when these fellows do not exists A fellow is stable. So, what happens is that there is one equilibrium point and all the initial conditions will converge on to that fine, but as you move the parameter r through 1 as it becomes greater than 1, these two equilibrium points start to exist before that it was not there.

So, notice another typical feature of a non-linear system that as you change a parameter, new equilibrium points may start to exist which were not existing earlier, again a situation that you never come across in a linear system. So, the number of equilibrium points are not really fixed, they also depend on the parameter values and how to decide when they exist or not exist. That is, by whether or not you are getting real numbers, when you actually try to solve this if you get real numbers, they are existing if you do not get real numbers they are not existing fine.

Next step is, to enquire whether they will be stable how to enquire that, by looking at the Jacobian matrix and its Eigen values. ((Refer Time: 37:07)) So, just substitute this we have everything right the position of the equilibrium point is there, the values are there position of these equilibrium points are there values are there just substitute, you would find that for r less than say 24.74, B and C are stable.

You would have, do that exercise later, but you have understood how to do that exercise that you will simply put it here find the position, B and C and then you put the values here obtain the Eigen values in place of this x and z and y, you will put these numbers. So, that you can exactly obtain this matrix and it is Eigen values trivial to do by ((Refer Time: 38:10)).

So, just in five minutes you can do that, and then you will see that for this condition, these Eigen values, the Eigen values of B and C would be, would have real part negative. And then, at that point it become real part exactly 0 and beyond that it becomes real point positive, so what do you expect the behaviour to be then. The behaviour to be, behaviour is below r is equal to 1, only one fixed points is there existing the others are not there.

When r becomes greater than 1, this fixed point becomes unstable and there are two fixed points, now that have been bond and they are now stable, but they have complex conjugate Eigen values. So, it will spiral around 3 d space state space, in a 3 d state space we have already understood the local linear behaviour around the equilibrium point, what the behaviour will be, there it is 3 d. So, locally it will be a 3 d linear system harbouring at least a pair of complex conjugate Eigen values, the other Eigen value must be stable must be real the other Eigen value must be real.

So, in this case the other Eigen value will have a corresponding Eigen vector along which, it will converge on to that and it will converge on to the Eigen plane associated with the complex conjugate Eigen values, how to obtain that. See this is where whatever, we learnt in terms of linear system would be actually, applicable in the sense that you start from here there will be a plane, it will converge on to that plane first and then it will do whatever, it can do depending on the Eigen values.

Whether, the real part is negative or positive, if it is negative then it will converge along that plane, if it is positive then it will diverge along that plane, but along that plane where is that plane. So, we have learnt that for that, we have to divide the corresponding solutions in to the real part and the imaginary part and the imaginary part itself is a real number times j. So, the real part will give one vector the imaginary part will give another vector and the plane passing through, these two will give the plane corresponding to the complex conjugate Eigen values right. All these are somewhat difficult to visualize right, so let us try to visualize that if you really solve it what happens, let us try to visualize with a example.

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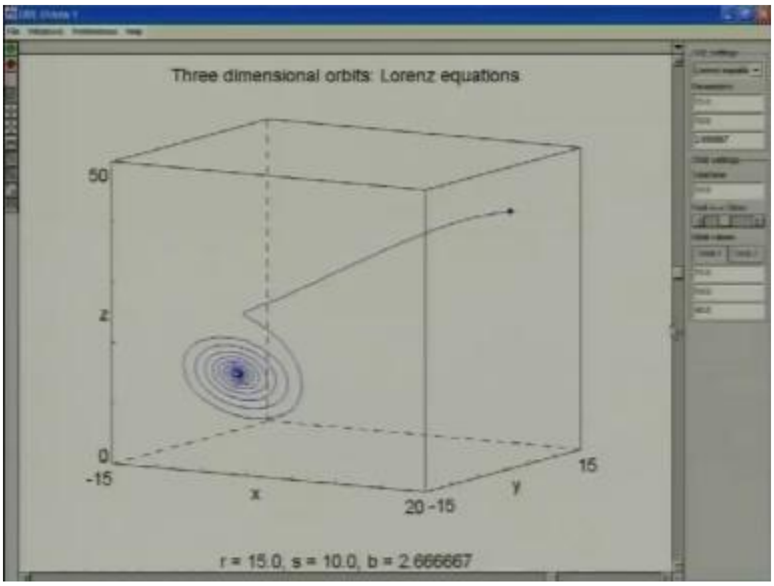

Well, so this is the equation for the Lorenz system that has been put in and r, I have set as fifteen rest of the things are as they are, so start from here see the evaluation. It first approached, it approached it rather fast why, because it is corresponding Eigen value was very small or negative large number, sorry, not small a negative large number. So, it approached very fast and then, it was converging on to that equilibrium point I can see it is here along a plane can you see the plane now, it always does, so along the plane.

But, this is one of the equilibrium points there, must be another equilibrium point also, so let us change the initial condition to, I will put a minus sign here. So, that I can start from some part which is here, this is x is equal to 0, so this is x positive i want to start from x negative somewhere here. So I will just put a minus sign and then I will…

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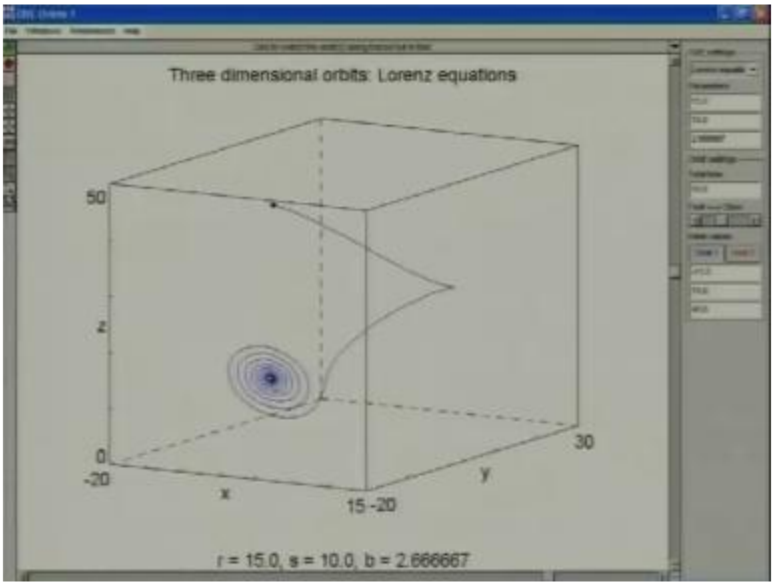

See it is behavior, no it goes to the same one then, let me change it further maybe I will put minus here also, yeah.

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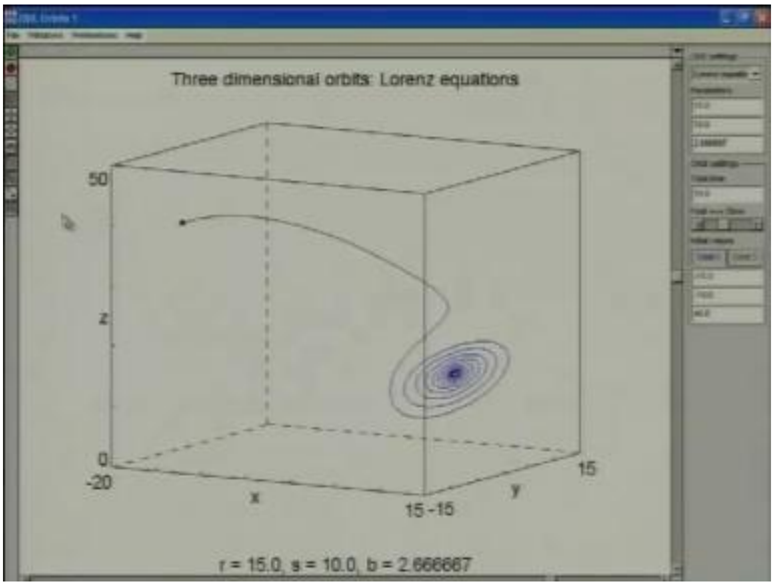

See different initial condition, it goes to a different orbit. If you see its plane, so there are two equilibrium points, the two equilibrium points both of them have the same they are actually, mirror image of each other. And if you substitute them, you get the Eigen values also a mirror image of each other, so you have more or less the same convergence property of the time 2. Now, probably you would like to see them, saved them together right, so let me do it for orbit 1 and orbit 2, 15, 10, 35 I will give the numbers 15 10.

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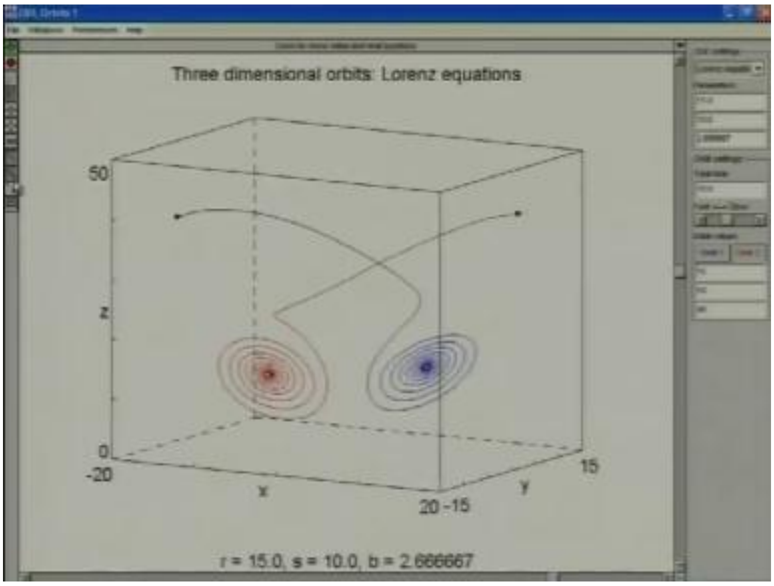

Happy, starting from two initial conditions, it is going around and converging on to two different equilibrium points that is, one of the properties of a linear system. Now, tell me what will happen, if I now progressively increase the r what will happen is that, ((Refer Time: 44:17)) if you see the then you will see that, so long as r is less than this, they are stable which means, the real part is negative.

Negative and then, it goes through real part 0 into the positive side at this value, so if I now go closer for example, 22 what do you expect physically, what do you expect the real part would be small which means, the decay rate will be small. Which means, that it will it converge on to the equilibrium point all right, but it would be very slow in doing, so let us do that is what, our theory would say let me say put 22 here.

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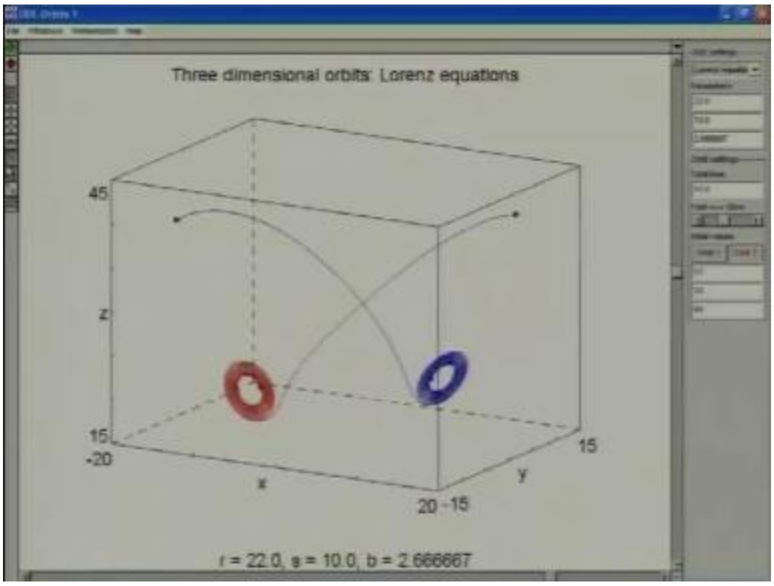

And, Very slow right, but still you can see there, is a e to the power lambda t kind of term. Can you see the e to the power lambda t yes, you have to be see you have to be able to see the e to the power lambda theta. ((Refer Time: 45:26)) Now, if I increase it through the 24.74 may be say, I make it 26 then what will happen.

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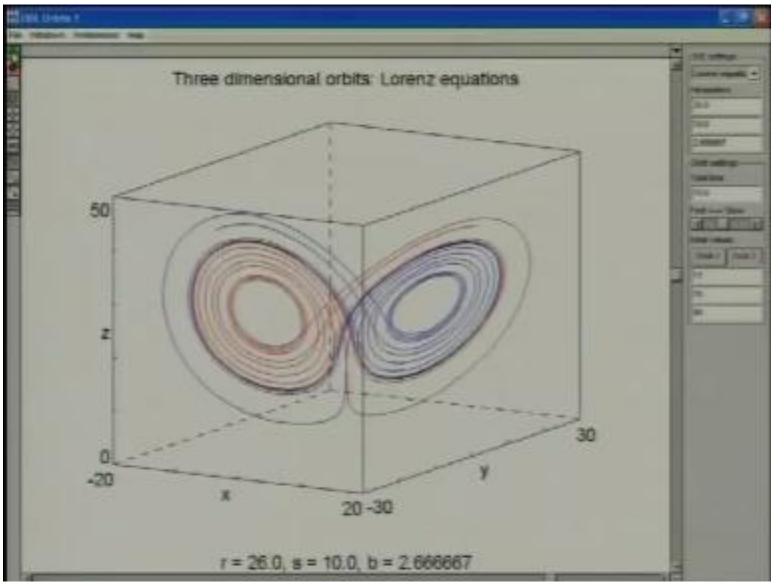

Then see what happens, they are going out right, but then after sometime they now what has happened, there are two planes corresponding to two different equilibrium points, those two planes are not the same planes. So, there should be intersection somewhere and moreover along one the rotation is clockwise, along the other the rotation is counter clockwise see, so what happens is after sometime this fellow gets thrown into this one and then, it rotates clockwise. Here, it is counter clockwise and here it is clockwise and then, after sometime it is get thrown into the other one, so it cannot escape it gets locked.

But, then both are unstable and so, it goes on in that, so this is one way in which, an orbit can be created, where you see it does not repeat the same state, if it repeated then you would see in the same place it is going on and on, and on. It is not that, and if i do it for a larger time say, I will make it 100 seconds and I will make it larger.

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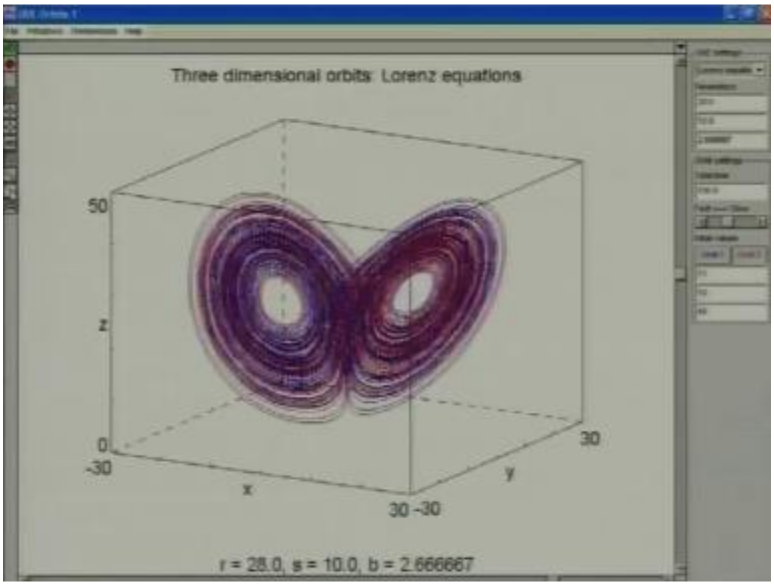

Now if I, so this is an orbit in which, nothing is really being repeated can you see they are always going through a different route, different path. None of these paths are actually intersecting, because it is actually 3 d system. And it is possible to rotate it then, you see the positions, for example if I rotate it, I will make it faster.

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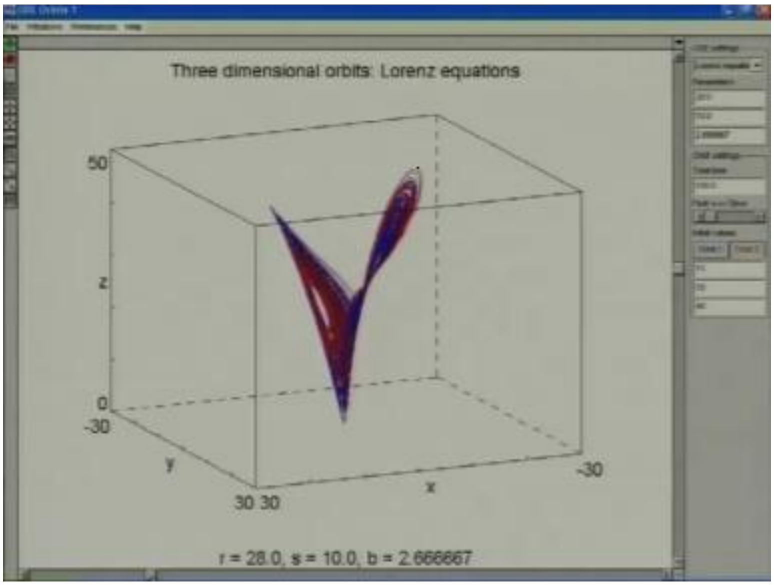

Now, if I see from different angles you can see the whole thing happening.

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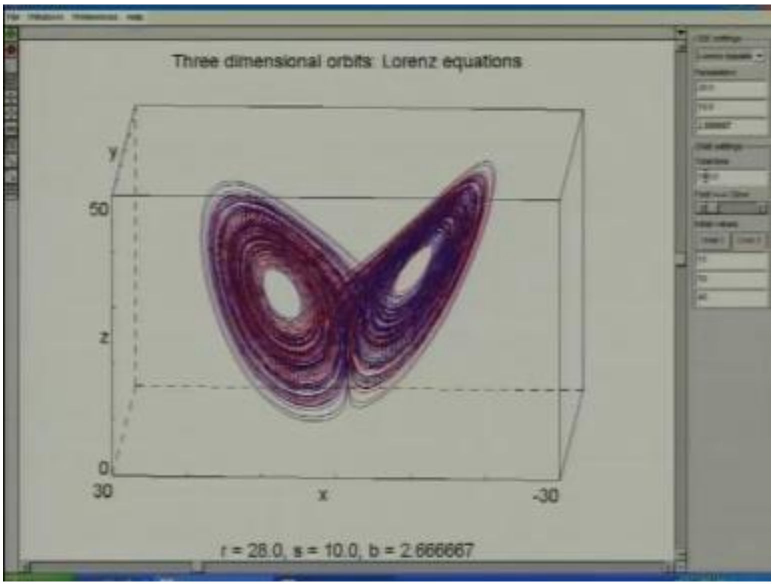

Now, if I change the positions.

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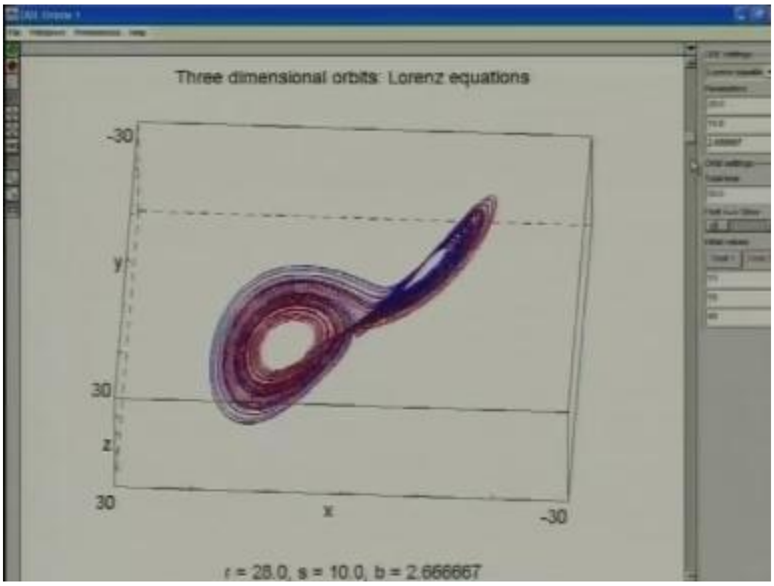

From different angles you can see this, and you can see the planes right two different planes. Normally it is convenient to see from this angle.

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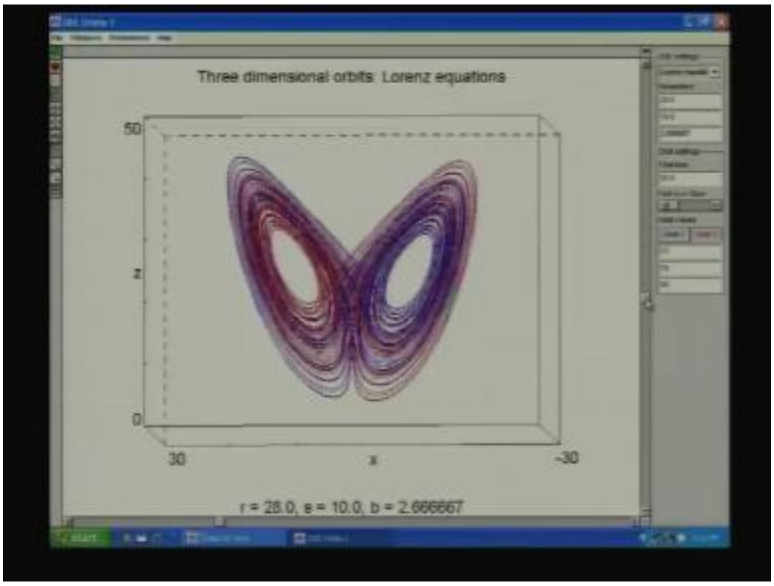

But, you can also see from other angles, where it looks.

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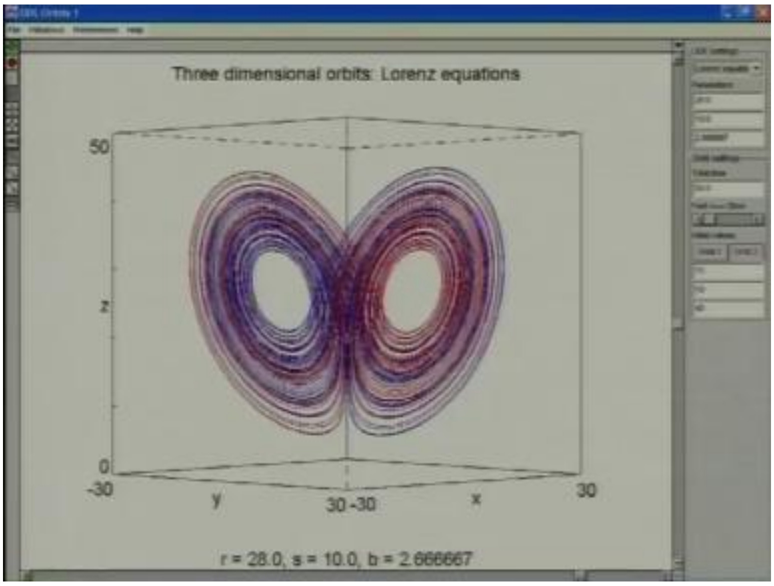

Quite different in from different angles, but all the time you are seeing actual projections.

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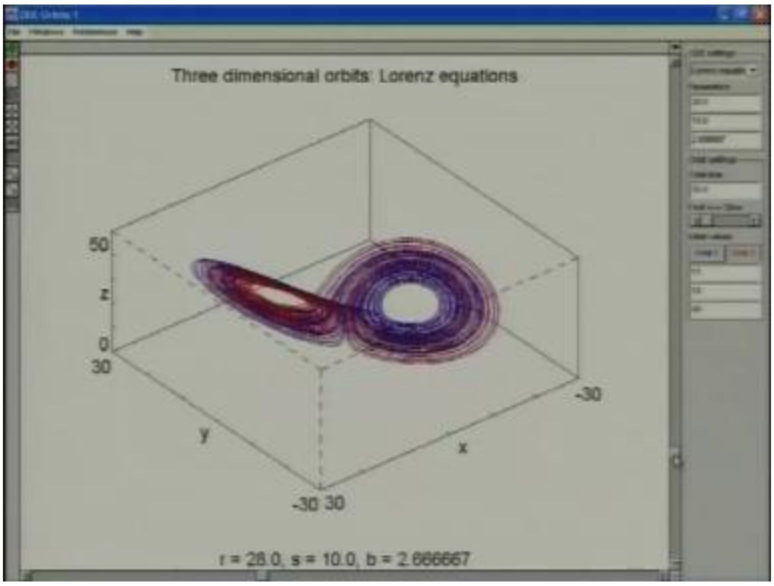

So, you can see that there is one cycle like this, there is another cycle like that and from here, it gets thrown in to other cycle here and from here, it just thrown in the this cycle here and they are not really intersecting lines they are separate. And that is what can create an orbit, which is also an attractor in the sense that if you start from initial condition, somewhere here it will converge on to that attractor. In that sense it is also an attractor and it is still a completely aperiodic behavior, there is no periodicity in it.

If you see in time domain, it will be completely aperiodic behavior, now that is what is known as chaos, so chaos is nothing but a bounded aperiodic behaviour in a dynamical system. So, such behaviour can also be in a non-linear dynamical system, so this is called chaos, let me write down the definition.

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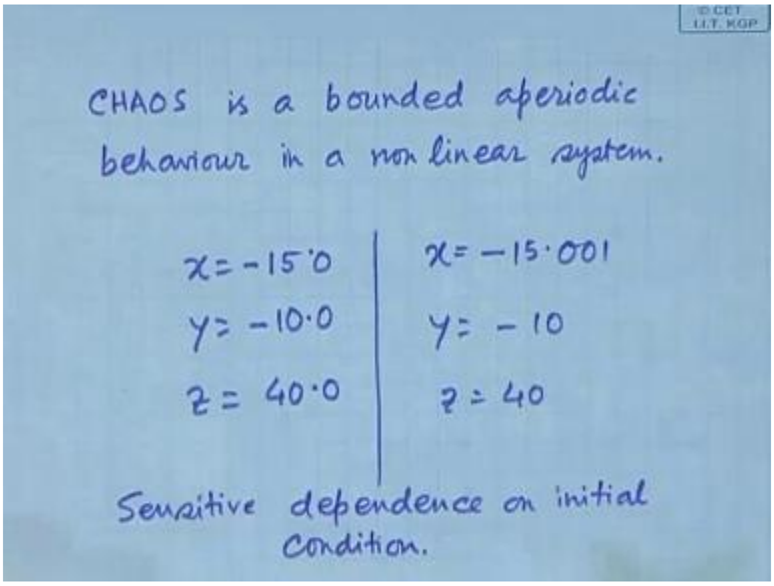

So, it is pretty simple really, All that is happening is that we have found that there, can be period 1 attractor, there can be period 2 attractor, there can be period 3 attractor, there can be period n attractor and why not. Then take n to the extreme limit, which is infinity in that case it is completely aperiodic behaviour and that is, exactly what we were talking about that is what is chaos.

There, are other things which, I will come to little later, but there is another important issue here. The issue is that suppose you start from this initial condition, which I have given probably it is too small for you to read, so let me write down the initial condition, that I have given x is minus 15, y is minus 10 and z is 40.

Now, if I start from a very, very close initial condition, another initial condition, say x is equal to minus 15. 001, and y is equal to minus 10 and z is equal to 40. We can start from those initial conditions, ((Refer Time: 52:26)) so this is orbit 1 minus 15 minus 10, no I will put minus 15.001 a very tiny difference minus 10 40, now if I start I will make it slow.

((Refer Time: 52:52)) You can see, one line why both are being calculated, but both are absolutely going together that is, why you do not see them they are actually two points. That is, what you normally would expect that it would give more or less the same way, but now, they are started to separate out can you see that, now see they have gone in completely different directions right.

Which means, that if you start from different initial conditions however, minute that difference be they all evolve in different directions, they will never follow each other. That is one important property of a non-linear of a chaotic system, which is known as sensitive dependence on initial condition. So, if you make that, if you might argue that make it 0.000001.

So, what I have made it the difference after, at the 5th decimal place, so if I now copied and ((Refer Time: 54:32)) I will make it faster, you have got wasting time. it is still going together, still going together, still going together. Starting to separate out now, they have gone in different directions can you see that, so that will always happen. The character is of the system is, such the two ultimately arbitrarily nearby initial conditions will always evolve in different directions. What the technical implications of that is I will come to that in the next class fine.

Thank you for today.