Dynamics of Physical Systems
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Lecture - 29
Linear Systems with External Input – III

Well, in the last class we were considering the system with external sinusoidal input and the system itself is a RLC circuit.

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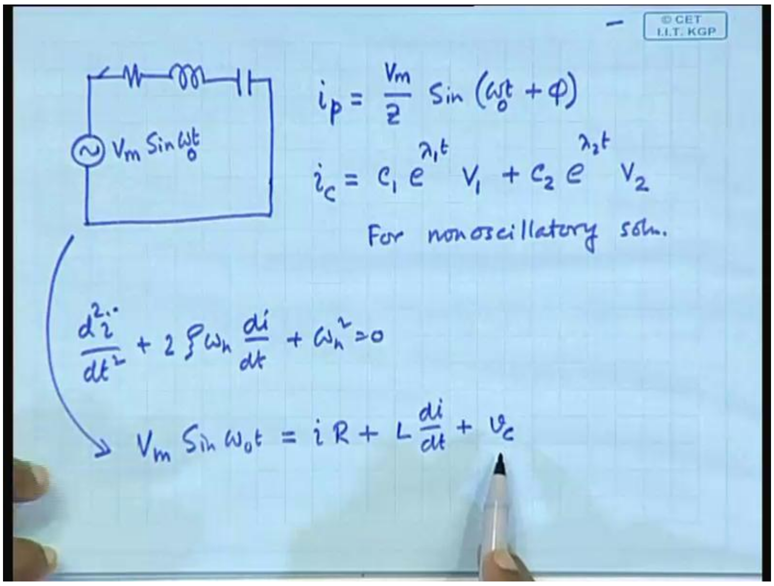

And we had seen that in such a system the solution will have; obviously, two components, the particular integral in the complementary function. For the steady state solution, the matter is rather simple, because steady state solution is what we all know from the basic electrical and mechanical engineering. Where, if this is V m sin omega t and if you assume the phase to be 0, it is V m 0 degree and there will be an equivalent impedance of this whole block we say z.

Then, the current is V m by z and then sin omega t plus or minus zero depending on the relative magnitudes of the inductor and the capacitor. So, that is something we know, so the i p part is V m by z sin omega t plus phi, the i c part the complementary function part would actually be the response of this system, when this is absent. And response of this system is a second order system it is response will be depended on the depended on what?

Student: ((Refer Time: 02:51))

If you reduce that into two first order equations, the Eigen values is not Eigen values it is Eigen value it is a German term g is always (( )). So, it is an Eigen value, the Eigen values are if you write it in the form of the homogeneous, but second order equation. Then, you have expressed it in form of say i twice zeta omega n, in this form and then depending on these magnitudes you will have either a oscillatory solution or non oscillatory solution on that.

Nevertheless, ultimately if you have a non oscillatory solution, then if you consider in terms of the two Eigen values. Then there will two Eigen vectors and you will able to write the complementary function, they the transient part as you have c 1 e to the power lambda 1 t times it is Eigen vector V 1 plus c 2 e to the power lambda 2 t times Eigen vector V 2, this is for non oscillatory solution, where the Eigen values are real.

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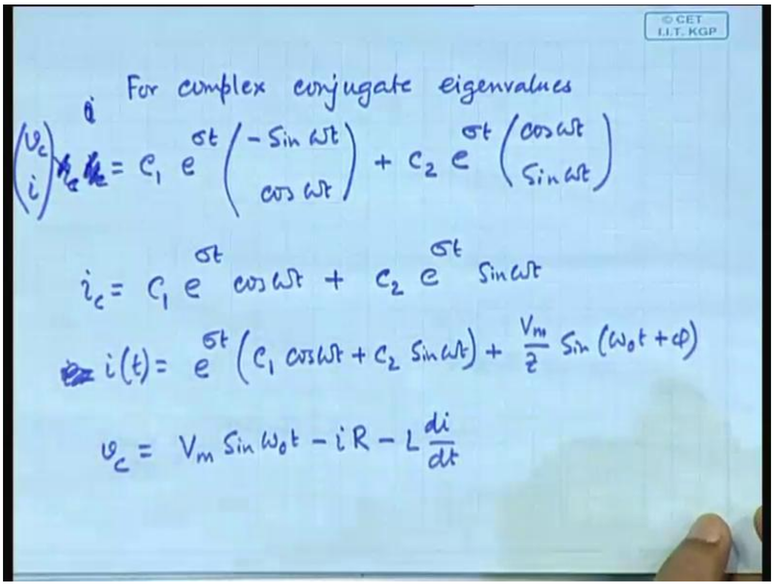

If the Eigen values are complex conjugate, you have i for complex conjugate, you have i c is c 1 e to the power in this case, it would be the real part sigma t times it was minus sin omega t cos omega t and c 2 e to the power sigma t cos omega t sin omega t which is X c in that case, what.

Student: ((Refer Time: 05:24))

Yes that is the point, ((Refer Time: 05:29)) here this omega is what you have applied and this omega is it is natural frequency. Because, when you are considering the complementary solution you are considering the homogeneous equation, homogeneous equation means a ((Refer Time: 05:45)) part is absent. So, there is no influence of the sinusoidal applied sinusoidal, so remember that these two will be different.

So, let us give them names let say this one the applied one is omega naught symbol like that, the point that I am trying to make is a bit different. Because, this is what we discussed yesterday, this is X, X means a vector of voltage and the current, so the current solution i c will be from here the lower line it is c 1 e to the power sigma t cos omega t plus c 2 e to the power sigma t sin omega t there is a current.

So, the particular integral was this of the current and the complementary function is this of the current. So, the total thing the current i is i as a function of t, I will write it that way i as a function of t will be the addition of these two, so first it is e to the power sigma t I will say c 1 cos omega t plus c 2 sin omega t plus V m by z sin omega naught t plus phi.

That with complete solution, the question is how do you obtain c 1 and c 2, in order to obtain the solution you; obviously, have to obtain c 1 and c 2, how would you obtain that from the initial condition good. So, the initial condition of i suppose is 0, because ((Refer Time: 07:52)) here you have a switch and you is turn it on the moment of turn it on the current has to be 0, because the inductor was there.

Inductor current cannot change instantaneously, the current is 0, but that gives you one equation and two unknowns, how would you solve for c 1 and c 2 then...

Student: ((Refer Time: 08:10))

What?

Student: ((Refer Time: 08:12))

Yes, so you have to also worry about the initial capacity of voltage and that together will give you this. But, this is the equation that we have obtained in terms of the current, what is the equation in terms of the capacity of voltage solution of the capacity of voltage equation.

Student: ((Refer Time: 08:29))

No, the because this is only the complementary function part, but what about the solution in the particular integral. There also we have worry about, notice that ultimately your equation was here ((Refer Time: 08:52)) looking at these the equation was the from here, you have this voltage V m sin omega naught t is equal to first this drop, this is i R plus this L d i d t plus fine this was v c.

So, v c equation is actually v c was V m sin omega naught t minus i R minus L d i d t. So, in order to obtain the total solution for the v c, all you need to do is to substitute i this i here. And the derivative of this i here, algebraically is a bit v c, but nevertheless not very difficult thing to do, but the moment you to do it, you can now put the initial condition for v c and then you have two equations and two unknowns got it.

So, how to obtain these the actual solution for a system like this, so do not get worked off, if you do not have two equations write away visible here. You have to obtain the v c equation that can be obtained by two means, first that can be obtained by two means, one the complementary function is already available. The particular integral is available here ((Refer Time: 10:47)) the particular integral would be available for my equation like this, but that will have to be obtained that can be obtained separately.

There are two possible ways as I say said, either you write equation like this substitute it here and then obtain the equation for v c, else you start with an equation like this in terms of v c. That means, you write equation in terms of v c and from there obtain the equation, but it is always to me more advantages to work this way. That means, you obtain a v c equation in terms of i and the substitute i and d i d t that gives you equation for v c right away. So, obtain it obtain.

Student: ((Refer Time: 11:34))

This X c with this actually vector equation, the X c is actually v c i, this is X c X and it is complementary part, X as a vector. Here, we are writing its complementary function part, that is why c I will substitute, but what is that vector, it is a vector of v c and i. And that is what we have written down here, but you do not need that, because once you have written this and substituted that you get a equation for v c straight away, that includes the particular integral as well as the complementary function part.

So, do this I leave it to do for you to do may be later, because it will take a little time and in the mean time it will somewhat uncomfortable for me to sit here. So, do it later, but do it, it is a good practice, so we have seen how to obtain the solution, if the external forcing function is either a fixed value or a piecewise constant function, which is a square wave or a sinusoidal. What else can you think of as a functional form the of the external forcing function.

Student: ((Refer Time: 13:27))

What.

Student: Ramp.

Ramp.

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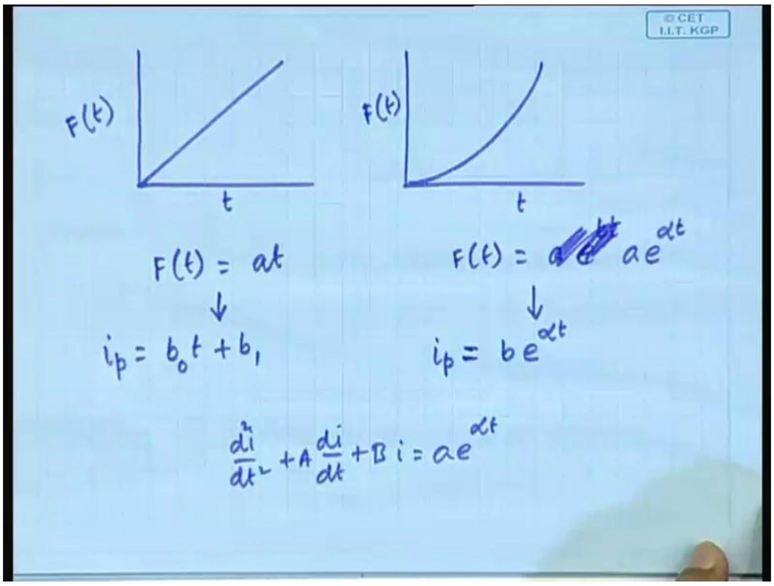

So, it could be a simple ramp function going on increasing yes it is possible, anything else or may be a ramp that go for some time and then goes like this. But, nevertheless the solution will be obtained from here, just one and then that will be repeated. Anything else, may be exponential also I mean we do not normally come across thus such functions, but nevertheless we can imagine, there could be external forcing function that is exponential function.

So, you can also think of an exponential function, so here what is x axis, what is y axis F of t at the y axis t here. So, F external forcing if it is a electrical circuit we will say V external forcing voltage externally applied voltage V of t, what is this t. Now, in such cases how would we go ahead to obtain, notice what about our logic, the logic was that the equation contains of two parts A X plus B U, A X part only says what is the behavior around the equilibrium point and the B U part, says where is the equilibrium point.

So, we first obtain the character on the basis of only A X and there we found that there can be a few different types of characters. And B U part tells where the equilibrium point is and if it is some kind of a function like this or a sinusoidal or a square wave, we say. That it only causes the equilibrium point to move and therefore, the actual state will depending on the character of the behavior around the equilibrium point, it will follow.

If it is stable, then it is always trying to reach that point or while that point is moving. So, that allows as to guess a trial solution, in these cases what will be the trial solution. If say the applied voltage is say in this case a t, in this case it is something a e to the power b t may be, there will be two things, there will be two unknowns. So, in such cases how would you guess the trial solution.

Now, guessing the trial solution is the particular integral remember that we are talking about the steady state solution. In this case what will be the steady state solution, see if the equilibrium point moves away as this, the solution will also follow like that. So, it is not difficult to see that the solution also be a linear function, so imagine a linear function say i p it would be something depended on t and some fixed value, that will be the general linear function. In this case how would do guess a trial solution i p.

Student: ((Refer Time: 17:11))

No, I what I will do is I will in this I put the exponent as a e to the power alpha t, why because they have different characters alpha is the exponent. That is, why I used a Greek later here and the this is not the exponent, so it is a normal in this data. So, in this case what will be the character of this, notice that ultimately it is character to be such. So, that when this is plugged into the equation it will able to solve the equation, satisfy the equation.

And the equation was a differential equation with derivative part with double derivative part and stuff like that. So, what should this be, what should this be, so that it is derivative is double derivative and in one side this, we satisfy see what I am trying to derive it. Normally, what will be the equation like, equation will be that d i d 2 i d t 2 plus d i something say A d i d t plus B i is equal to a e to the power alpha t that is the external forcing function.

Student: ((Refer Time: 18:35))

Yes, so yes there is a point that what should be the nature of i, what can be the nature of i. So, that this equation is satisfied, that is what we are trying to guess as a trial solution, the point is that it will never be solved unless the exponents are the same. And so a trial solution might be b e to the power alpha t is that clear, the line of logic. So, the line of logic is where you are trying to guess the solution based on some logic and the logical structure is clear, that you have the fixed point is moving and so the actual solution will be following.

So, it is logical to guess that this the part it will follow a function it will follow, will be of the same nature as the external forcing good. So, this will be the trial solution in this case, that will be the trial solution in that case, in this case what are the unknowns b naught and b 1, in this case what is the unknown b only. So, how do you obtain these, the method of obtaining these is to plug it into the equation and then solve it, that is how you solve.

In this case what will happen, there will be a term that is independent of t in both sides, there will be one term that is dependent t in both sides, they will individually have to be 0 and that will give you b naught and b 1. In this case how do you obtain b, if you plug it in equation like this, you will see that b is only unknown and you can solve for it.

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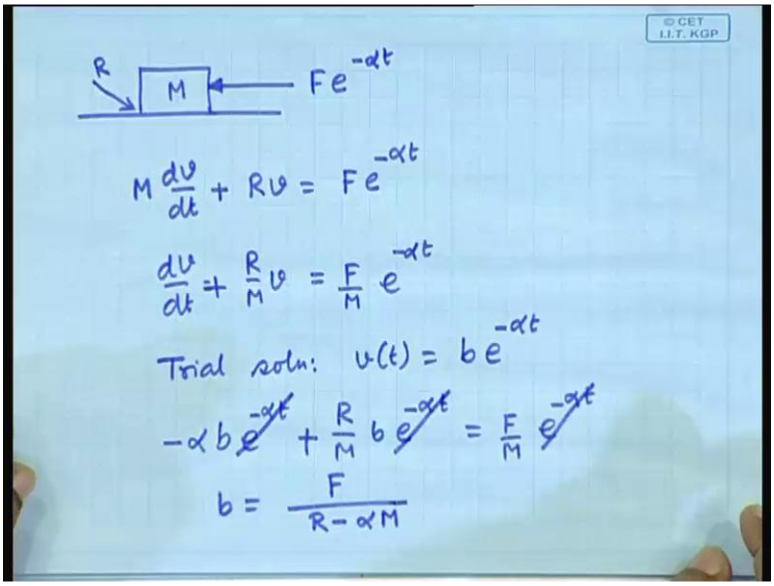

So, let us do one exercise with this kind of a situation, suppose you have got a mass resting on a plane mass of mass M. And suppose it is being acted on by a force, which is F e to the power minus alpha t, which means this force itself is exponentially decaying. And suppose, at this point that there is no spring or anything, but there is a friction, let that be a viscous friction of R.

So, now what will be the dynamical equation or dynamical motion of this mass. That is the question can you do it, do it yourself I will only help, first point is to write down the equation. So, let us this is too simpler system, so we do not need to take all those complicated rules, that we took earlier just balance the forces D' Alembert's principle. So, it is a mass into acceleration M d v d t plus this force R into v must be equal to F e to the power minus alpha t.

So, that is the simple equation for this system or if you write it properly d v d t is equal to or plus R by M v is equal to F by M e to the power minus alpha t. Now, what can be a trial solution, trial solution may be v of t may be b e to the power minus alpha t exponent has to the same. If you put this trial solution into this equation original equation, what do you have b e to the power minus alpha t. So, d v d t will be minus alpha b e to the power minus alpha t

So, minus alpha b e to the power minus alpha t that is this term plus R by M b e to the power minus alpha t is equal to F by M e to the power minus alpha t, this term, this term this, term cancels off. That immediately gives b this b that was the unknown, B is F by b comes common R.

Student: ((Refer Time: 24:06))

R minus alpha M good, once we have found b we can obtain the equation right away. So, the ultimate equation will be by the notice that, this will be true only if R is not equal to alpha M, if these are simple parameters of this equation this could be equal. So, this equation this whole solution will be valid only when R is not equal to alpha M, so once we have obtained this, you have the trial solution is which part, steady state part. What about the transient part, obtain the transient part drop this and what will be the transient solution of this.

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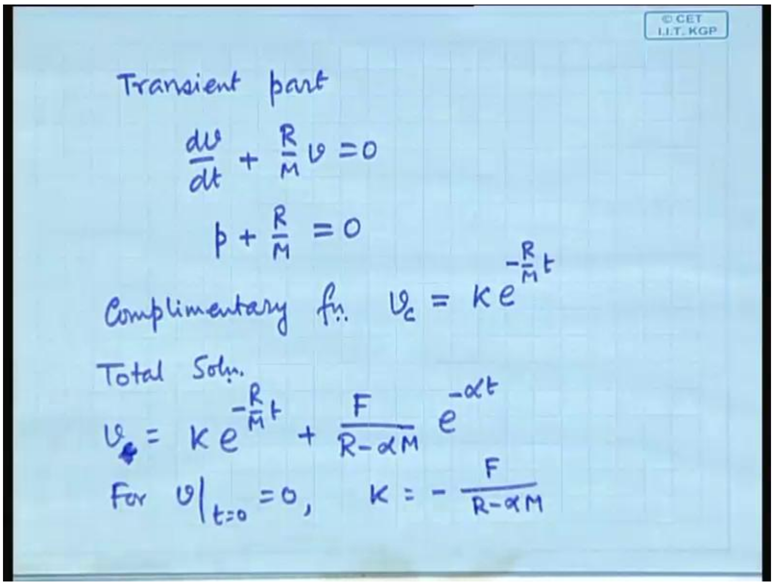

Then, for the transient part you have to take d v d t plus R by M v is equal to 0, what is the characteristic equation, we write this as d v d t as p. So, p plus R by M is equal to 0 that is how we have wrote it, so you have the complimentary function is...

Student: ((Refer Time: 26:03))

Right is v c is some constant e to the power minus. So, we now have the task of finding K, how do you find K, first let write the total solution, the total solution is v c is K e to the power minus R by M t plus b is this F by R minus alpha M times e to the power minus alpha t. And now K has to be obtained from the initial condition only one unknown, so if the initial value of v c is known suppose it starts from rest v c is 0.

In that case K is wait if you put t is 0, then this term is zero. So, here remains F by R minus alpha M, here you put t is equal to 0 this term is 1, so K this is 0, so K becomes...

Student: ((Refer Time: 27:48))

Yes.

Student: ((Refer Time: 27:52))

This is the total v I was writing the complimentary function and that was the particular part. So, your K is for v at t is equal to 0 is 0 K is equal to what it become minus F by R minus alpha M, now just put it here that is solution.

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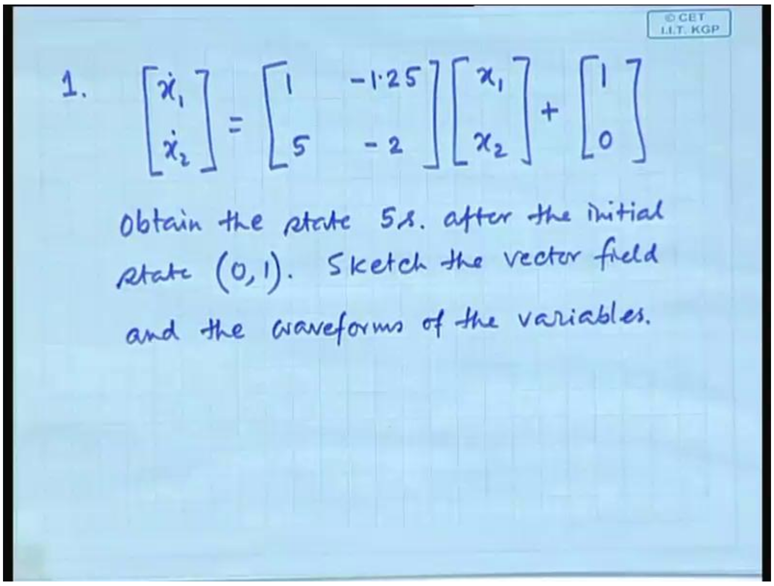

Now, I will give you some problems you solve in the next class some of them I will try to illustrate. One the equation is x 1 dot x 2 dot is equal to 1 minus 1.25 5 and minus 2 x 1 x 2 plus 1 0, obtain the state 5 seconds after the initial state say, in let the initial state be 0 1 not 0 0, where will the state be sketch the vector field and the waveforms noted. So, this problem is state for order the way you have already learnt. So, I will not go into just solve it.

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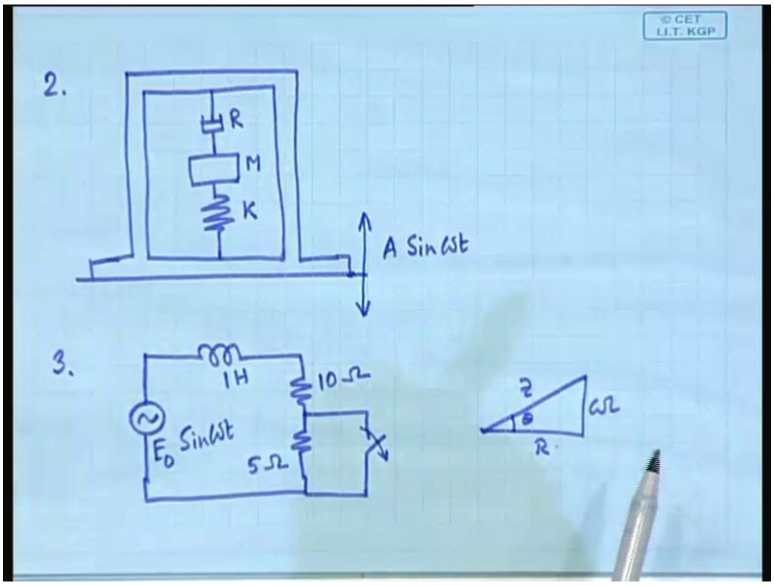

Two, the system is like this, here is R, M, K and this ground is oscillating with A sin omega t. So, obtain the equation for the vibration of this mass, you will see that this arrangement effectively acts as a vibration or isolator. That means, if the this thing is vibrating, then that vibration will not be will be somewhat less in this as seen in the mass.

So, obtain the equation of the vibration of this mass, how will you do it, in this problem how will you approach this problem tell me first.

Student: ((Refer Time: 32:45))

Bound draft, so if you want to draw the bound draft of this, how would you draw I will come back to that, let me give the problems first I will come back to that. So, the third problem is you have got a electrical circuit with an applied voltage, which is sinusoidal here is the inductor and here is a resistor and here is another resistor. But, there is a switch here, this is E naught sin omega t here is a 1 Henry, here is 10 Ohm and here is 5 Ohm.

Suppose, initially the switch is closed and then it is opened at t is equal to 0, it was closed for long time and opened at t is equal to 0. What will be the equation for the current, how will you do it, first when it is closed for a long time, the current at that time was how much. Here is E naught sin omega t divided by...

Student: ((Refer Time: 34:26))

Any is a sinusoidal.

Student: ((Refer Time: 34:33))

No, no what are talking about it is a sinusoidal, so you cannot say by 10 therefore, this will have it is effect know. So, that depends on the omega, suppose the omega is 50 then it would be 2 pi f or omega L, so the omega L will be the impedance of this one and the R and you have to do it this way. So, this is R which is 10, this is omega L here, so this z and then this E naught by Z sin omega t plus phi.

So, that will with the current flow when this was closed and then at a specific time it is opened and then after that what will happen, after that it will be a different R. So, different oscillation all right, but then there are two parts of this question, what will be the equation and then where exactly should you open it. So, that there is no transient get the point, so how do you find it.

You already know, that you have to open it where the sinusoidal is exactly at if the new circuits, here it was 1 Henry it will have a particular omega L. Now, the R is 15 for that circuit whatever is the theta if this one is opened at that phase angle, then only it will have no transient else it will have transient.

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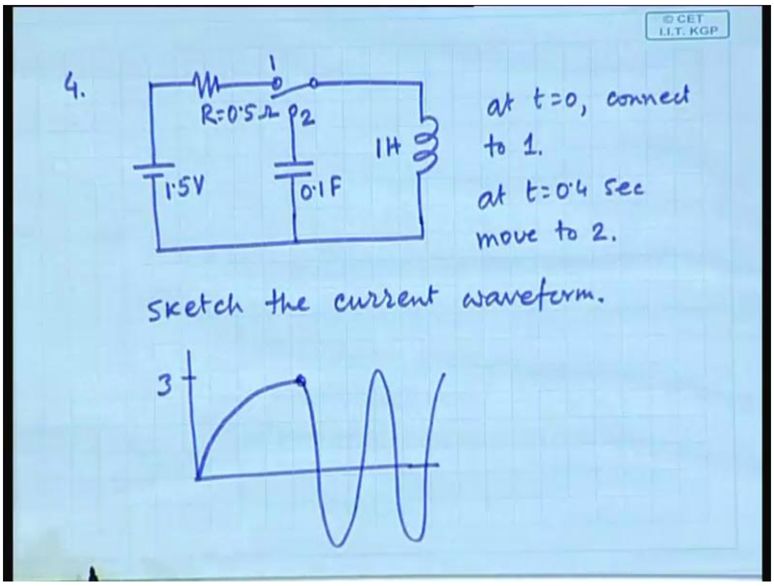

Now 4, the circuit is now here in this problem the initially the switch is opened is not connected to circuit at all. And then, at t is equal to 0 it is taken to position 1 at t is equal to 0 connect to 1 at t is equal to 0.4 second move to 2. So, initially it is opened at 0 it is connected to 1, so that you have this circuit the capacitor not there in the circuit. At 0.4 second it is taken to position 2, in that case it is only is the L C oscillation, why will it oscillate, because there was an initial current flowing through this inductor.

Now, find out that oscillation, the current waveform. So, what will happen, when you have thrown it to 1, you will have the current going like what?

Student: ((Refer Time: 39:05))

It will rise, because initially it was 0, it will rise as if you allow it to remain for infinite time how much will it be, it will be 1.5 divided by 0.53. So, current will actually rise like this, it will tend to reach a value 3, but before it reaches at this time. So, you have to find out where it reach. And then, at this time what will happen.

Student: Oscillation.

Oscillation, but what kind of oscillation like this.

Student: ((Refer Time: 39:43))

Yes, it will oscillate symmetrically about this 0. So, it will start and would go like this it has to, but then what will be the equation of this, where will this point be, what will be the value, what will be the amplitude of this oscillation, what will be the frequency of this oscillation. So, there will be these issues that you need to find out.

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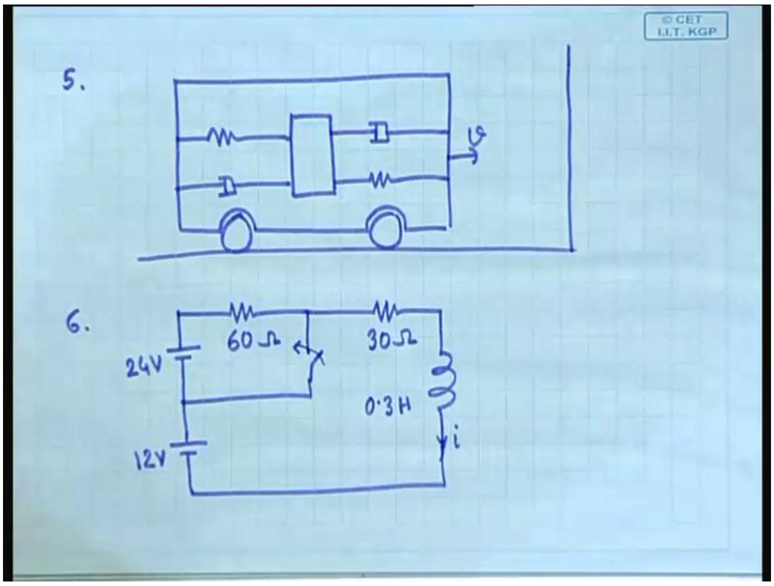

Five, it is a ((Refer Time: 40:27)) stress problem, because you will have to calculate what happens in an accident. Suppose, this is a car moving on a street and inside one of you sitting modeled as a mass and you have the car cushion and stuff like that and you have got a seat belt, which are modeled as say fine in both sides you have this. And then there is a wall and this fellow goes and hits the wall, what will be the motion of this mass.

The car meets with an accident and yet it was moving with a velocity v, when it hit the wall immediately it comes to a stop. It was the whole thing was moving with a velocity v and immediately it comes to a stop, this comes to a stop. Assume the this no recall ; that means, it does not the car itself does not go back stops, then what will be the motion of this mass.

Student: Similar to the last problem.

Similar to the last problem all right and that is what I want you to see that there are similarity between the electrical problems and mechanical problems. But, in this case we are giving the initial condition straight away, in this case we gave the initial condition as having in a particular position for quite some time, reach a particular value not steady state. But, here we are saying it has reached the value v, the speed is v and it is moving at a is definite velocity v.

So, that the in inside it is a at equilibrium position and then it hits, what will happen tell me, what do you see.

Student: ((Refer Time: 42:17))

It will be a damped oscillation; that means, this fellow will move like this and then it will oscillate, but then obtain the equation, engineering is all about obtaining equation. So, obtain the equation and then solve it and then obtain how exactly will this guy feel and this actually done by the way. This is actually done in order to find out how in accidental situation a particular car will behave, these simulations are actually done.

Now, 6 it is again a circuit problem, there are two batteries and as suppose here you have a switch and here is another and this is an inductor it is first order problem, initially the switch is opened. So, at t is equal to 0 it is closed, you have find the waveform for this inductor current i, when it is open it is this circuit, this is 24 Volt, this is 12 Volt, this is 60 Ohm, this is 30 Ohm and this is 0.3 Henry.

Well what will happen, initially when this is opened you have got a, suppose you have obtained a steady state, in that case how much will be the current flowing through here.

Student: ((Refer Time: 44:47))

36 by 90, because it is s d c and then the moment it is open, it is a different circuit and where will the steady state be 12 by 30. So, it is going from one steady state which immediately becomes a transient value initial value to another steady state. So, this initial value of 36 by 90 is a initial condition, which you have to put in order to obtain the constant have you noted.

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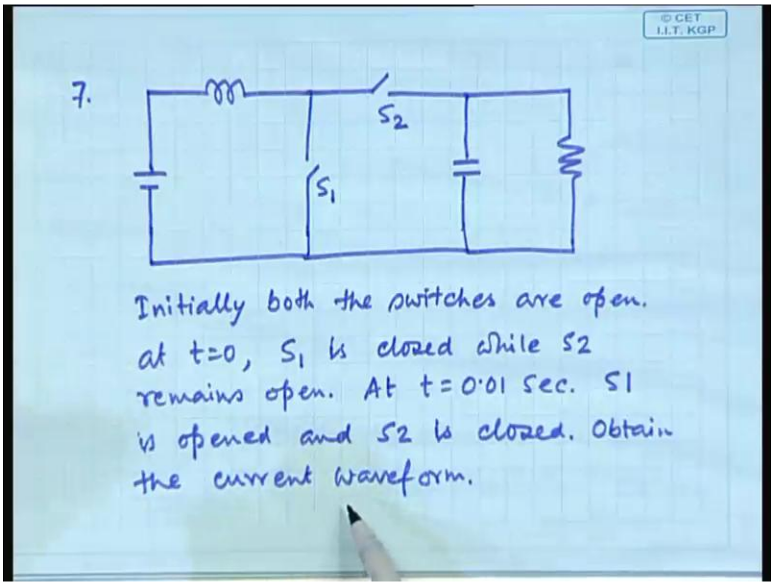

Seven, again a electrical problem you have got a battery, here you have got an inductor, but there is a switch here and there is another switch here. Again initially both the switches are open at t is equal to 0, suppose this is s 1, this is s 2, s 1 is closed while s 2 remains open. At t is equal to 0.01 second s 1 wait, initially s 1 is closed s 2 is opened and then s 1 is opened and s 2 is closed.

Obtain those you have read the beta power electronics would immediately realize that this is nothing but, the boost converter. So, when we talk about s 1 being opened normally this is realized as a by means of a independent switch MOSFET engine well and this is a diode. So, whenever there is a switching on and off, this operates complimentarily, so initially what is the circuit, when s 1 is closed it is only this.

What will be the waveform then, nothing is there here you have V is equal to L d i d t. So, it will rise linearly with the slope of V by L simple, so it will rise linearly with the slope V by L till 0.01 second. So, you have to find out where it reaches and that becomes the initial condition for the next is set of the problem, in the next set of the problem what you do in the next set of the problem, this is opened and this is closed. So, it is this circuit which is actually L, C, R circuit, it will give rise to a second order equation.

And that second order equation solution will depend on the values; that means, it could be a non oscillatory, could be oscillatory and all that. Suppose, it is oscillatory solution, so will give you the numbers, so that it becomes oscillatory solution. And then, you have to solve it with that initial condition that is obtained from the first part and that will give with the waveform.

So, let us have some values just check if it give oscillatory solution, because I want you to do it for oscillatory 1. Suppose this is 20 Volt, this is 0.1 Henry, this is 100 micro Farad and this is 50 Ohm [FL] I am not very sure if I give this numbers it would be oscillatory, but nevertheless do it have you noted. I will take one of these problems and after this class will just do that in front of me.

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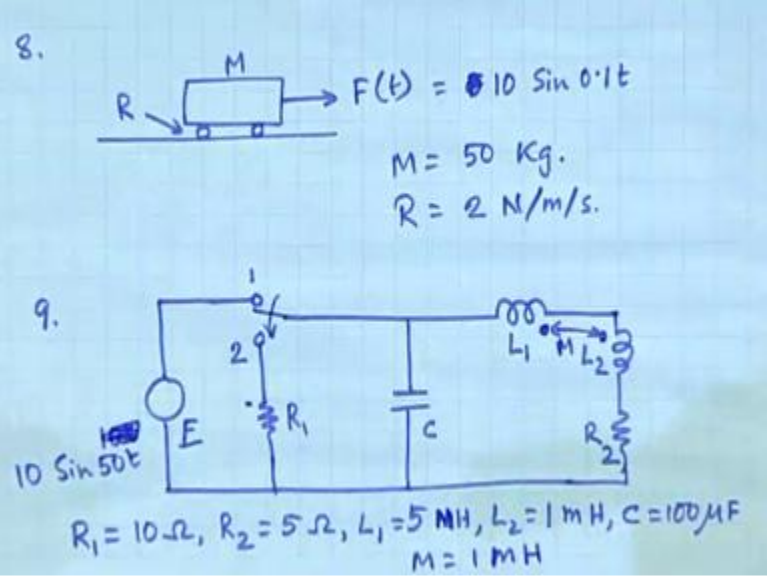

Now, you have a cot which is being pulled with a force F, there is a mass M and here is a friction R. The force is F or I will write number 10 sin 0.1 t. That is, how it is being pulled M is equal to 50 Kg and R is equal to 2 Newton's per meter per second. So, what will happen, here is a sinusoidal forcing, that sinusoidal forcing will make this fellow also sinusoidal move.

But, not exactly in phase it will lag in phase, find out by how much it is motion will lag from the phase of this external sinusoidal function. Then, find out where exactly should you start applying this force, so that there is no transient, it is actually a electrical mechanical equivalent of the electrical problem that you already know. But, give an mechanical problem we do not tend to see, that it is the same kind of theoretical structure.

Last, you have as applied voltage you have got two positions, one and two and this point could either be in one or two and a here there is a capacitor and here there is another inductor and resistor. So, this L 1, L 2 R, C, this is a R 1, this is R 2 and you got either this or that here this E. It was in position 1 for a sufficiently long time and then it is moved to position 2, so it goes like this.

So, at 1 for a sufficiently long time during which what happens, what is the circuit, this become opened, this become shorted. So, it is E by R 2 that current is established, when it comes here this is isolated, you have got a homogeneous equation whose solution you have to obtain with that initial condition, that is it obtain it with numbers R 1 is equal to 10 Ohm R 2 is equal to 5 Ohm L 1 is equal to 5 milli Henry L 2 is equal to 1 milli Henry C is equal to 100 micro Farad.

You will have to obtain the waveform for the current and the voltage across the capacitor after 0.1 second write it, there is no space to write. So, I am not writing, so after 0.1 second what will be the voltage across the capacitor. Once, you have solve this problem do it for a sinusoidal source.

Student: ((Refer Time: 54:56))

E is 10 Volt I just said initially assume it to be d c and solve this problem, but then also do it for 10 sin 50 t, where E is 10 sin 50 t any problem.

Student: ((Refer Time: 55:39))

Why are you giving two inductance, initially I thought there would be. Let me complicate your life M is equal to milli Henry, but this will actually be trivial, because the same current is flowing through them, it is not real if I plus this one here, then it become one unknown trivial, but nevertheless do it. The point is that, the actual problem is where you assume it to be a sinusoidal source, when you have a sinusoidal source.

Then, this one when it is one then what is the current flowing through this it is a R L circuit, R L circuit with this L a bit complicated, but nevertheless that is trivial. So, you have got a R L circuit in which the sinusoidal voltage is applied, so the current flowing will be V by z sin omega t plus some phi. While, that is going on is a sinusoidal thing it is there a long time, means that the transient part the complimentary function part has died down.

Now, it is only the sinusoidal we are talking about and now it comes to 2. And obviously, what happens after that depends on when it comes to 2. Now, if that comes to 2 at the 0 crossing of this, it does not come to 2 at the 0 crossing of the current is that right, because there is a phase, because there is a phase difference. So, the current has some value, when it the switch is put into the position 2 and then you have to find out the transient if that as the initial condition. So, now solve these problems and that will give you reasonably good practice.

Thank you.