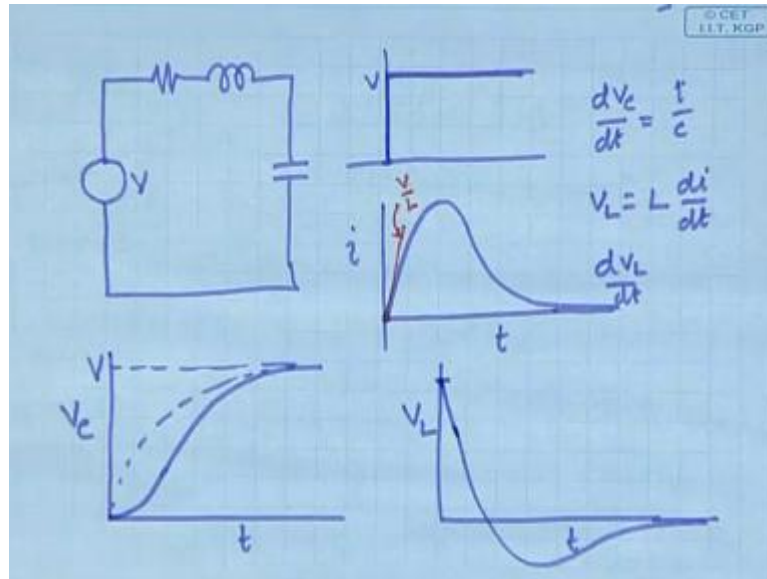


Dynamics of Physical Systems
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Lecture - 28
Linear Systems with External Input – II

(Refer Slide Time: 00:59)



In the last I had asked you a question related to the RLC series circuit, where the circuit was like this. And you were asked to predict the waveforms and I could see that, in this case you are somewhat confused, so let us revisit this issue. In this case your applied voltage waveform was like a step, so it was step input like this up to some voltage V . So, the question was what would be the waveform for the V_R , V_L and V_c .

So, how would you argue with it, how argue about it, we had decided that the current waveform that we did yesterday. The current waveform should be i over t how was it, turn the page, it should rise and then it should decay like this. And when it rises, there was a slope here that is also what we decided, what was the slope.

Student: ((Refer Time: 02:21))

So, this slope is V by L , so if this is, so then your V_R should be exactly proportional to this. Because, V_R is R times i no problem about it, but the question is what would be the waveform for V_L and V_c that is, where I had asked you to work it out at home draw

and show me. So, let us see what you have done Teja is smiling, but never the less let me try to figure out how it will look [FL].

So, what is your logical conclusion about it, so we are trying to work out V_C against t and V_L against t . Notice, when you try to work it out, first talk about the extreme condition, what will be the value when t is equal to 0 and what will be the value when t is equal to infinity first work this out. So, what is the value when for V_C , what is the value for t is equal to 0, obviously 0 because it is not starting with anything. So, it is start from here where does it end.

Student: V .

V good, so you will have the value V here and then it should reach like this. But, you can easily see there are two possibilities of doing like this, one is like that will it go like this let us consider, exponential rise up to this value is it correct. So, here the ((Refer Time: 04:15)) question is dV_C/dt ; that means, the slope here. So, what is dV_C/dt .

Student: ((Refer Time: 04:22))

Yes. So, I will write here dV_C/dt is equal to i/C , i at the initial condition is 0 and therefore, i/C is 0. So, dV_C/dt must start at 0, so it cannot be like this, it must be like. Now, come to V_L what should be the wave form like, it should be going down and up, how would you decide about this one, ultimately you have to talk about when it starts what is the value of V_L .

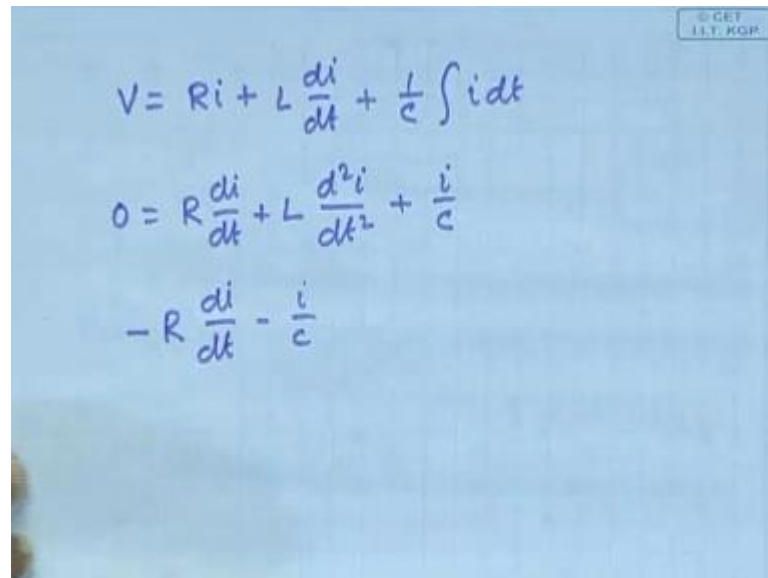
Student: ((Refer Time: 05:16))

No, what is the value of V_L , V_L is $L di/dt$, so always V_L is equal to $L di/dt$. So, V_L 's value will be the starting value of; obviously. Now, you see initially current is 0 therefore, the voltage across this is 0, voltage across this is 0 therefore, the whole voltage appears here. So, starting point has to be V , so it is starts from here, after that after an infinite amount of time what happens, the current again goes to 0, if current goes to 0 then di/dt also goes to 0 and therefore, $L di/dt$ goes to 0 it has to be 0.

So, starts from here ends here, it could be like this, it could be like that, it could it has many possibilities. So, what is the real thing the point is that it has to always follow the derivative of the current function, current functions derivative initially starts with V , V

by L and times I it will be V . So, it starts with the slope, starts with V/L starts with what value V , but then what will be the, so it ends at infinity at this point what will be the slope, which requires dV/dt . So, we will be then interested in dV/dt , what is it that will be obtainable from the equation itself. Look at the equation, yesterday we did it look at the equation what was the equation last time.

(Refer Slide Time: 07:12)



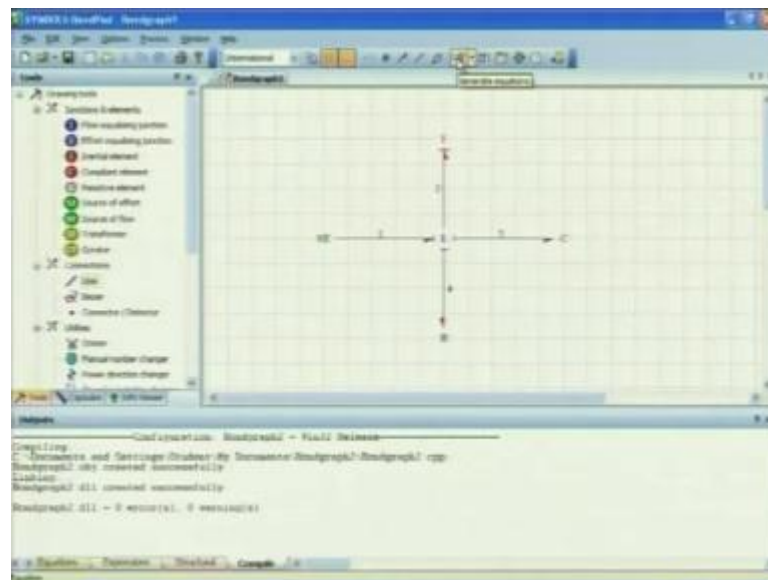
$$V = Ri + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

$$0 = R \frac{di}{dt} + L \frac{d^2i}{dt^2} + \frac{i}{C}$$

$$-R \frac{di}{dt} - \frac{i}{C}$$

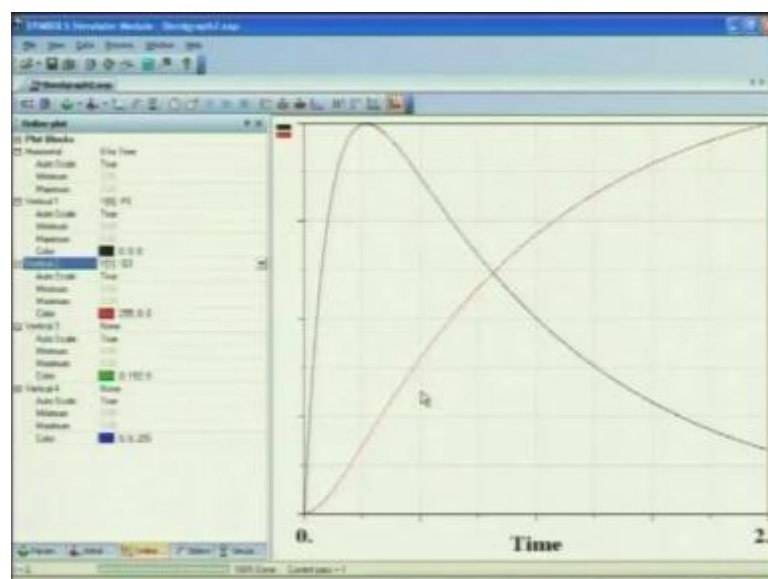
We had written the equation as V is equal to Ri plus it will be $L di/dt$ plus $1/C$ integral $i dt$. And we had differentiated it to get 0 is equal to $R di/dt$ plus $L d^2i/dt^2$ here it is plus i/C , so if you are talk about the this term, we are essentially talking about this term, it will be minus $R di/dt$ minus i/C , i initially 0 , but $d i/dt$ is not 0 . So, it will have a negative slope, ((Refer Time: 08:08)) it will drop at a negative slope like this. But, will it go like this no, because the slope reduces and finally, goes to 0 and then goes negative and so it has to go negative and then it has to go like this. So, does it logically sound, just quickly.

(Refer Slide Time: 08:51)



Let us check it on the computer we had I will just go to the bond pad. Now, here we can see that I had simulated earlier, the L C F circuit with a source of effort, we had already given the numbers, we had already given the power directions and if I view without the circles, we had also have the causalities. So, the next step would be to generate equations, so we can generate equations here. The next step would be to generate the simulation code, which we had saved yeah good.

(Refer Slide Time: 09:37)



And then we have to compile it, now we have to enter the parameters and then we have to simulate good. So, let us say a $C = 1$ just one I need to put in values for which it will be exponential, let me should not go into oscillation mode. So, what will be the R I will put somewhat large value of $R = 10$ and this would be somewhat large value check that, check if for these values; that means, a C not necessary $R = 10$ or that is the value of L is 1 and $K = 3$, which is the capacitance is 10, just check if it is giving the correct result or is it giving any way we can check here also.

So, we will give the time duration say may be 2 seconds and then we simulate oh we have not given the plot blocks. So, 0 the horizontal is time vertical 1 I will give p_2 and vertical 2 I will give q_3 , so that is what you expected notice that q_3 is the voltage across the capacitor it starts with 0 slope. That is, what we predicted it goes up and tries to reach the final value V the current goes up and down like this that is what we predicted.

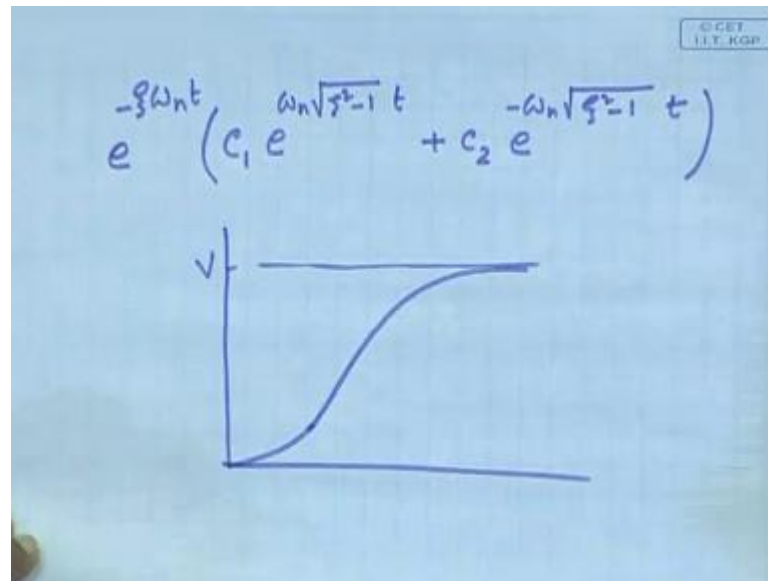
So, what we predicted logically were actually correct. Now, the question is I had asked you this question and I was surprised that nobody objected, what would be a time constant in such a system. See, I had in passing said that if you have a two dimensional system and if the parameters are such that your exponential response, then you can work the same way to obtain the.. But, now notice this and let you try to argue out, what will be the time constant or the concept of time constant in this case.

Notice, here it is rising like this ((Refer Time: 12:05)) here it is like this, this is actually a combination of...

Student: ((Refer Time: 12:10))

Yes, not only to time constant the exponential functions, a combination of exponential functions.

(Refer Slide Time: 12:29)

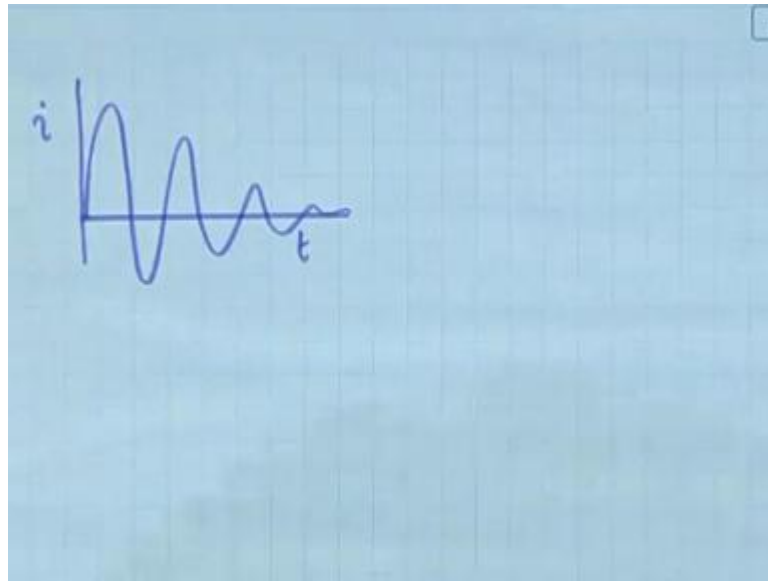


And we had earlier seen, that the expression of this is the expression will be something like E to the power minus $\zeta \omega_n t$ $c_1 e$ to the power $\omega_n \sqrt{\zeta^2 - 1} t$ plus $c_2 e$ to the power minus $\omega_n \sqrt{\zeta^2 - 1} t$. You notice that, this is an exponentially rising function and this is an exponentially decaying function and the whole thing is a combination of these two.

So, that is why it rises and then falls, for some time this dominates and for some times that dominates. And then, the whole thing is multiplied by this term, which is an exponentially decaying term, this essentially ensures that for very large values of t this would be going to 0. But, now there are three different time constants here, so that will have to be taken into account in this case, so there is no unique time constant in this case.

Look at this wave form of the voltage, it was ultimately reaching this value V , but it was going like this. What was the logic in working out the time constant, we said that if the starting slope continued for some time at what time will it reach V . So, at what time will it reach V , it will not reach because it is 0, the other point was that yes you can argue that, you can say at what time does it reach 0.63 of that, but essentially that came from this logic that if it continues it will reach. So, in this case it would be somewhat meaningless to define the time constant, by the way how will the waveforms look, if the parameters are, so chosen that you have sinusoidal response.

(Refer Slide Time: 14:38)



First the current, if you have an exponential response it is like this, if you have sinusoidal response how will it look.

Student: ((Refer Time: 14:55))

Yes there would be a sinusoid with decaying amplitude. So, there will be a sinusoid with a decaying amplitude like this, in this case also you cannot define the time constant in the same way, but there is an exponential term. So, instead of time constant we can say that there are certain characteristic times in the system, one characteristic time relates to the time constant as obtained from that exponential.

Another characteristic time relates to the time or the characteristic time on the sinusoid period of the sinusoid. So, in this system therefore, there are two characteristic times, so instead of time constant we will try to think in terms of characteristic times and in more complicated systems, where you have higher dimensions, four dimension, five dimensions it is not difficult to see that there would be many such characteristics times.

There will be some exponential terms, there will be some terms, if there is a five dimensional system there could be one pair of complex conjugate Eigen value another one pair of complex conjugate Eigen values. They, individually giving rise to different characteristics times of the sinusoid omega different. So, a system can have many characteristics times with in itself and if you are having to solve it by numerical means,

then different characteristics time means one says, suppose one is very short time; that means, a fast time constant.

The other is a slow one; that means, characteristics time is rather large, what does it mean, it means that it has two sinusoids in it one has a very long time duration, the other has a very short time duration. A situation that would happen for example, when you have the motor with a shaft between the motor and the load and we said that in order to make it integrally causal, you have to assume some kind of a stiffness on the shaft.

But, that stiffness of the shaft it will be very stiff. So, if it at all vibrates it will vibrate vibrating very fast, much faster than the characteristics time of the other rotations. The result will be that the problem will be very difficult to solve numerically, why because all the time in the numerical routines, you are trying to set the h the step length depending on the difference between, what you are calculating by say third order Runge Kutta method and fourth order Runge Kutta method. And all the time it will try to reduce it, because all the time it will see differences.

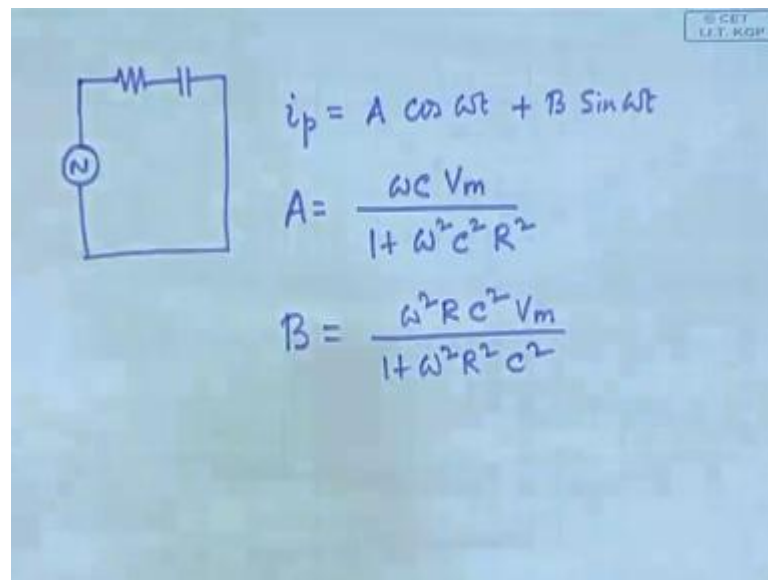
If you have such very sharp oscillations in it, such systems are called stiff systems where it is difficult to solve it. So, in a system if there are different characteristics times which are very different, then it becomes rather difficult to solve, technically it becomes rather difficult to solve. And in reality also such systems are relatively difficult to handle, because there will be ringing there will be vibrations at higher frequencies. So, we would normally try to avoid such systems.

Now, let us go ahead with what where we were last time, we said when we broke of the last day we were, what we were doing, we were doing a sinusoidal excitation on a.

Student: R C circuit.

R C circuit.

(Refer Slide Time: 18:55)



The image shows a handwritten diagram of an RC circuit on the left, consisting of a voltage source, a resistor, and a capacitor in series. To the right of the circuit, the current i_p is given by the equation $i_p = A \cos \omega t + B \sin \omega t$. Below this, the coefficients A and B are defined as $A = \frac{\omega C V_m}{1 + \omega^2 C^2 R^2}$ and $B = \frac{\omega^2 R C^2 V_m}{1 + \omega^2 R^2 C^2}$. A small logo in the top right corner reads "© CET I.I.T. KGP".

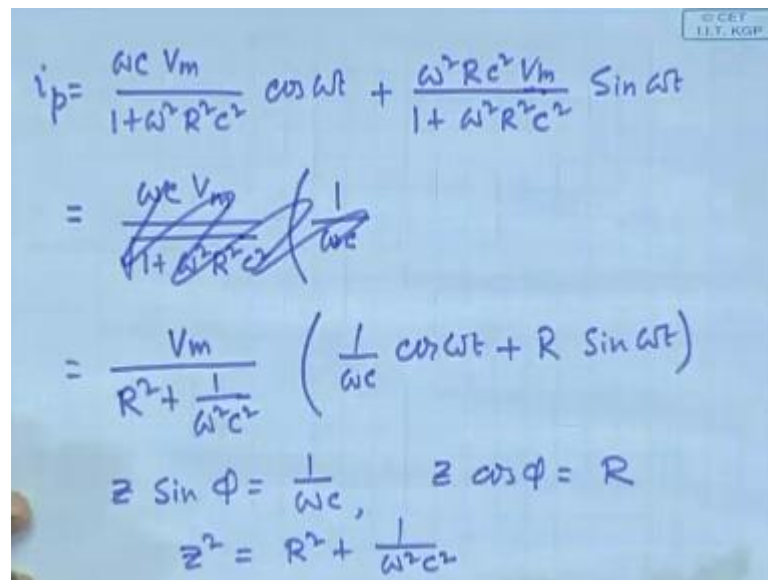
$$i_p = A \cos \omega t + B \sin \omega t$$
$$A = \frac{\omega C V_m}{1 + \omega^2 C^2 R^2}$$
$$B = \frac{\omega^2 R C^2 V_m}{1 + \omega^2 R^2 C^2}$$

So, you had a R C circuit with a sinusoidal excitation and we had decided that it would be given by an equation of the form i_p would be given by an equation of this form $A \cos \omega t + B \sin \omega t$. And then on the basis of the logic that the solution actually tries to follow the change or shifting fixed point. So, wherever the fixed point goes accordingly the solution also tries to follow based on that logic we had guessed, that this solution will also be a sinusoid.

Notice, that somewhat like this logic you have also come across in mathematics classes. Because, how do you actually solve, you try to fit into some trial solutions you remember that. Now, often when you learnt that, how is this trial solution obtained or in a general case, general situation how would you guess a trial solution. That is, often not very clear to students that is why I went by that logic.

The reason the whole logic rests on the fact that the solution also tries to follow the fixed point. And therefore, it should have the same functional form as a fixed point as the motion on the fixed point and that is why we had guessed it. And we had arrived at the expression from there A is equal to just recall $\omega C V_m$ divided by $1 + \omega^2 C^2 R^2$. And your B was $\omega^2 R C^2 V_m$ divided by $1 + \omega^2 R^2 C^2$. So, this we had obtained in the last class, so we will plug it in and obtain the total solution.

(Refer Slide Time: 21:25)



$$i_p = \frac{\omega C V_m}{1 + \omega^2 R^2 C^2} \cos \omega t + \frac{\omega^2 R C^2 V_m}{1 + \omega^2 R^2 C^2} \sin \omega t$$

$$= \frac{\omega C V_m}{1 + \omega^2 R^2 C^2} \left(\frac{1}{\omega C} \cos \omega t + R \sin \omega t \right)$$

$$= \frac{V_m}{R^2 + \frac{1}{\omega^2 C^2}} \left(\frac{1}{\omega C} \cos \omega t + R \sin \omega t \right)$$

$$Z \sin \phi = \frac{1}{\omega C}, \quad Z \cos \phi = R$$

$$Z^2 = R^2 + \frac{1}{\omega^2 C^2}$$

So, your solution would be i_p is equal to $\omega C V_m$ divided by $1 + \omega^2 R^2 C^2$ plus $\omega^2 R C^2 V_m$ divided by $1 + \omega^2 R^2 C^2$ times $\cos \omega t$ plus $\omega^2 R C^2 V_m$ divided by $1 + \omega^2 R^2 C^2$ times $\sin \omega t$. Now, let us try to express it, let us try to take some common and let us try to express it in terms of something $\sin \omega t$. So, that we can express it as $\sin \omega t$ plus θ , how would you go by that logic, how would you do the algebra.

Student: ((Refer Time: 22:17))

Yes, but if you keep just like that it does not come in that form. So, we need to get something out, so the point is that it should have to be something $\cos \omega t$ and something $\sin \omega t$. And those something's should be expressible in the form of $\cos \theta \cos \omega t + \sin \theta \sin \omega t$, then it gets in that form. So, try to express in that form.

Student: ((Refer Time: 22:53))

What do you take out.

$\omega C V_m$ by.

Student: root over.

Root over.

Student: One plus omega square R square C square.

And then what remains.

Student: ((Refer Time: 23:12))

Wait, suppose something $\cos \omega t$ now if you take the whole thing common it remains just $\cos \omega t$.

Student: ((Refer Time: 23:32))

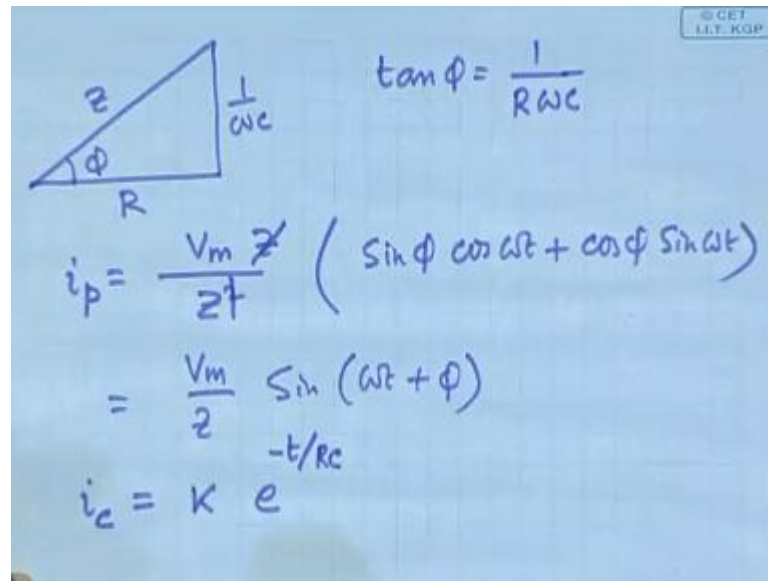
How does it help.

Student: ((Refer Time: 23:42))

Wait, it will be far more easier if we keep this or I will write it separately, if you keep 1 by $\omega c \cos \omega t$ plus $R \sin \omega t$, you will see how what I am driving at. So, in order to do that, because R was here R was not here, this is $\omega^2 C^2$ here is ωc . So, you can appropriately take common, what comes common it is V_m divided by $1 + \omega^2 R^2 C^2$ divided by that.

So, divided by $\omega^2 C^2$ it becomes $R^2 + 1$ by $\omega^2 C^2$ square check. So, this is here is $1 + \omega^2 R^2 C^2$ $\omega^2 R^2 C^2$ goes above, so that is what we have taken common. So, this remains, the advantage is that whatever appears here also appears here, that is what we wanted. Then we say that, now $Z \sin \phi$ is $1/\omega C$ $Z \cos \phi$ is R , which means Z^2 is equal to $R^2 + 1/\omega^2 C^2$.

(Refer Slide Time: 25:44)



$$\tan \phi = \frac{1}{R \omega C}$$

$$i_p = \frac{V_m}{Z} \left(\sin \phi \cos \omega t + \cos \phi \sin \omega t \right)$$

$$= \frac{V_m}{Z} \sin(\omega t + \phi)$$

$$i_c = K e^{-t/RC}$$

This means, Z^2 equal to R^2 plus $\omega^2 C^2$, you can easily see what I am driving at, this means that if you have R here, ωC here this is Z/R by ωC this is Z . And then, this means exactly that by doing this manipulation, what we have actually obtained is, in that case your $\tan \phi$ is $1/R\omega C$. In terms of these this equation becomes, then this equation becomes i_p is equal to V_m it was by this. So, it is, this is Z^2 is it, this is Z^2 in the denominator.

So, and this is equal to $\sin \phi \cos \omega t$ plus $\cos \phi \sin \omega t$.

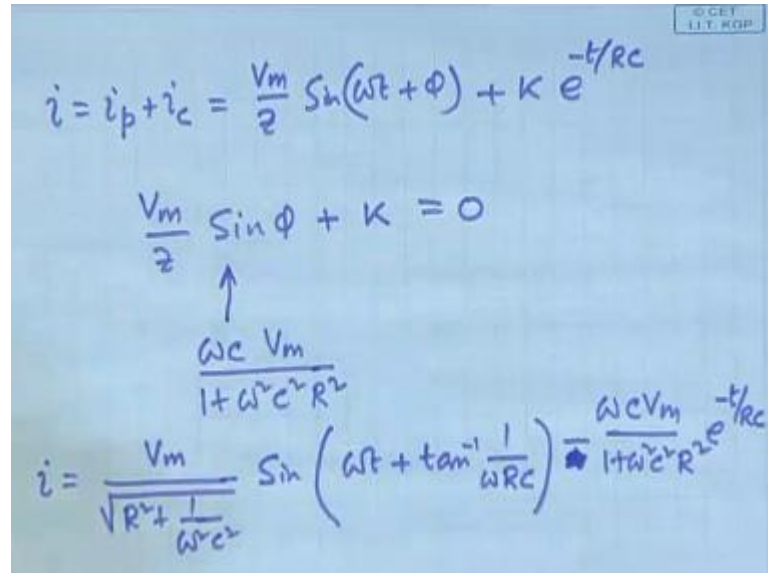
Student: ((Refer Time: 27:25))

This is equal to V_m by Z into $\sin \omega t$ plus. So, simple thing we started with a big thing, but actually it is a simple thing and that is what we actually know. That is, what we did in the first year actually, when we started with the LCR circuits and sinusoidal voltage. But, then why did we write p , because it is a particular this is the steady state solution, we had learned only the steady state solution and this is exactly what we obtain.

So, far it matches with what we intuitively understand, but there will be also an transient part. And the transient part, in this case will be a something that is damped something that is exponential, so we can guess that the complementary part will be some K times e to the power minus t by RC . Where, do you get that K from K has to be obtained from

the initial condition. But, now notice that the initial condition has to be initial condition of the total current which is a addition of these two, not just obtained from here.

(Refer Slide Time: 29:06)



$$i = i_p + i_c = \frac{V_m}{Z} \sin(\omega t + \phi) + K e^{-t/RC}$$

$$\frac{V_m}{Z} \sin \phi + K = 0$$

$$\uparrow$$

$$\frac{\omega C V_m}{1 + \omega^2 C^2 R^2}$$

$$i = \frac{V_m}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \sin\left(\omega t + \tan^{-1} \frac{1}{\omega RC}\right) + \frac{\omega C V_m}{1 + \omega^2 C^2 R^2} e^{-t/RC}$$

So, your total current is i is equal to i_p plus i_c , your particular integral and the complementary function, there to taken together you have V_m by Z sin omega t plus phi plus $K e$ to the power minus t by $R C$. Now, we have to obtain this from the initial condition, suppose the initial condition is where ((Refer Time: 29:50)) the charge stored in the capacitor is 0, that is a reasonable initial condition, charge stored in the capacitor is 0.

So, what will be the initial current applied voltage, whatever is the applied voltage you have to no notice I am telling something that is important applied voltage is a sinusoid. So, what is the initial condition there applied voltage depends on.

Student: ((Refer Time: 30:16))

The phase, you cannot say that something is applied voltage, the applied voltage could be 0. If it passes through 0 at the instant of switching on, if the applied voltage initially is 0 then your current through is also 0, if it is not 0 then current through is not 0. So, you see supposing let us start with the condition, that the initial current is 0 the applied voltage has been switched on at the moment of 0 crossing.

In that case you are the current is 0. So, this should be at t is equal to 0 yield 0 in this side, so what is the this side at t is equal to 0.

Student: ((Refer Time: 31:00))

What V_m by Z .

Student: $\sin \phi$.

$\sin \phi$.

Student: Plus.

Plus.

Student: K .

K , so you know what $\sin \phi$ is, we had already written down substitute that here or we can do it somewhat easily ((Refer Time: 31:44)). The particular integral term was this at t is equal to 0, this should be 0, the whole thing would be 0, this would be 1 and therefore, this should be the remaining term. So, you can say this term should be $\omega C V_m$ divided by $1 + \omega^2 C^2 R^2$.

So, you have the K minus this and therefore, the actual current the total current will be i is equal to i_p plus i_c , which is V_m divided by your Z which is $R^2 + 1/\omega^2 C^2$ $\sin \omega t + \phi$ which is $\tan^{-1} 1/\omega R C$. This is the i_p part it will become minus, because K is minus of this minus $\omega C V_m$ by $1 + \omega^2 C^2 R^2$ yes that is enough, then.

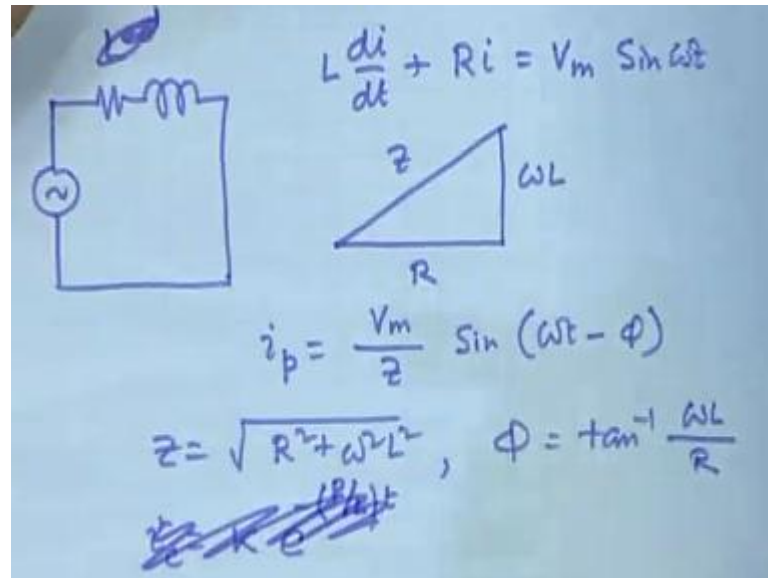
Student: ((Refer Time: 33:15))

e to the power is visible e to the power minus t by $R C$, so that is the total solution. So, this is how you obtain the total solution for a case, where a sinusoidal voltage is applied at some instant. But, this will not be the total solution, if you switch it on at a different time, if you switch it on at a different time there would be, this will yield something else at the initial instant.

And you will have to obtain the K from that. So, in general there would be one term, which is the exponentially decaying term, which will represent the transient. And this

will be the steady state term, this will be the steady state term and this will be the transient term. Now, there is a condition under which, there will be no transient can you find it out, there will be a condition under which there will be no transient.

(Refer Slide Time: 34:44)



Your differential equation was we will come back to that, just think about this particular problem that here we have different results of this. If you start the sinusoid at different points, different phases, we will we will come back to this particular issue.

Student: ((Refer Time: 35:08))

When, K is 0 you are right.

Student: ((Refer Time: 35:13))

Yes, so under what condition is V_m by $Z \sin \phi$.

Student: Zero.

Zero, so that would be the question then. So, it is important because there are certain condition under which you would like to avoid the transient, so it is necessary in a dynamical sense to understand, under what condition will there be no transient at all. We will come back to that. But, first let us try to see what will happen in case of R L circuit apply a sinusoid across a R L circuit.

In that case simple you have the equation as $L \frac{di}{dt} + Ri = V_m \sin \omega t$ no problem here. So, we proceed exactly along similar line as we did the last time and then we can conclude, what we can conclude, we will conclude exactly the way that we even learnt in the elementary electrical engineering classes. That, the solution will be the steady state solution in particular one will be this sinusoid and this seen as a Z which is ωL and R .

So, we will construct a triangle like this, this is ωL , this is R , this is Z and the solution would be i_p would be $V_m / Z \sin \omega t - \phi$. So, let us keep all these algebra that we did, because it follows exactly the same route, it is not necessary to repeat all that again we will arrive at this. And what about the in this case what is Z and ϕ is, so there is no difficulty. The complementary function will be i_c again will be a similar thing $K e^{-t/\tau}$ to the power minus, what is the time constant now.

Student: ((Refer Time: 37:57))

No, in something times t .

Student: ((Refer Time: 38:03))

R by L .

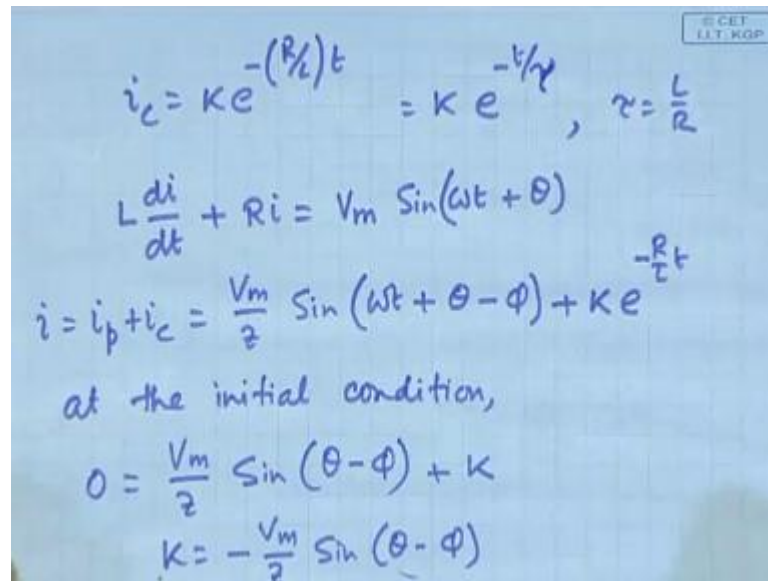
Student: ((Refer Time: 38:10))

What.

Student: ((Refer Time: 38:12))

Oh yeah, let us write it like this is correct, but it this time constant is L by R .

(Refer Slide Time: 38:30)


$$i_c = K e^{-(R/L)t} = K e^{-t/\tau}, \quad \tau = \frac{L}{R}$$
$$L \frac{di}{dt} + Ri = V_m \sin(\omega t + \theta)$$
$$i = i_p + i_c = \frac{V_m}{Z} \sin(\omega t + \theta - \phi) + K e^{-\frac{R}{L}t}$$

at the initial condition,

$$0 = \frac{V_m}{Z} \sin(\theta - \phi) + K$$
$$K = -\frac{V_m}{Z} \sin(\theta - \phi)$$

So, i_c is $K e$ to the power minus.

Student: ((Refer Time: 38:34))

I am not writing as t by τ I am simply writing as R by L t , which is $K e$ to the power minus t by τ , τ is equal to...

Student: ((Refer Time: 39:01))

It is right or wrong?

Student: Right.

So, the complementary function will be of this form again we will have to find it the same way, depending on the initial condition. So, now suppose the sinusoid starts with some phase, if the sinusoid starts with some phase it will be $L \frac{di}{dt} + Ri$ is equal to then we will have to write $V_m \sin \omega t$ plus some θ . So, this will be the original equation whose solution we are trying to obtain and the solution will be your i is equal to i_p plus i_c is equal to i_p is V_m by $Z \sin$.

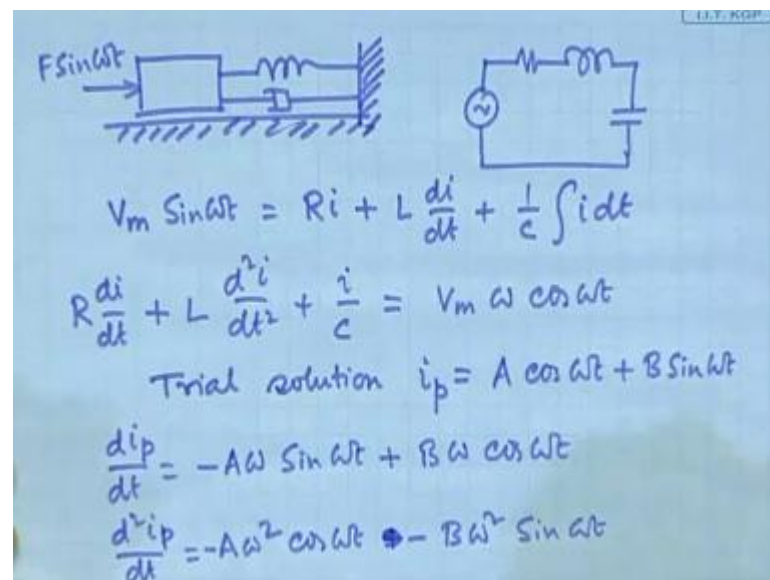
Now, in place of ωt we had the original one like this, in case of ωt we will have to write ωt plus θ minus ϕ plus this term $K e$ to the power this 1 minus R by L t . Now, at the initial condition this is 0 this is 1 , so you have at the initial condition

the current is 0, because it is inductive circuit, you cannot have the current changing instantaneously.

So, 0 is equal to $V_m \sin(\theta - \phi) + K$ or K is equal to minus this, so K is equal to $-V_m \sin(\theta - \phi)$. Now, you would notice that if θ is equal to ϕ , then this term is 0, therefore K is equal to 0, if the K is equal to 0 then the transient is 0. So, what is the condition that the transient will be 0, that if the system has a phase angle ϕ , then if you start the sinusoid exactly at that amount of delay then the transient will be 0.

And this the decision will the conclusion will same for the R C circuit, which I asked, you have done it for the R L circuit, but the R C circuit the conclusion would be the same. So, here we have more or less done the situations for first order systems excited by the sinusoidal excitation is there any question on this kind of systems more or less I have done it exhaustively. Now, comes the question, if you have a second order system and excited by a sinusoidal excitation what will be the situation.

(Refer Slide Time: 43:20)



$$V_m \sin \omega t = Ri + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

$$R \frac{di}{dt} + L \frac{d^2 i}{dt^2} + \frac{i}{C} = V_m \omega \cos \omega t$$

Trial solution $i_p = A \cos \omega t + B \sin \omega t$

$$\frac{di_p}{dt} = -A\omega \sin \omega t + B\omega \cos \omega t$$

$$\frac{d^2 i_p}{dt^2} = -A\omega^2 \cos \omega t - B\omega^2 \sin \omega t$$

Second order system means an RLC circuit or a mechanical system something like this all the time we have to keep in mind that we are talking about a situation also like this, a mass spring and damper and there is a forcing function which is your $F \sin \omega t$. So, you have got a mechanical system whose applied force is a sinusoidal force $F \sin \omega t$

t and there is a mass spring and damper, that is a equivalent of a circuit something like this.

So, their dynamics will actually be the same, so in that case how would you proceed, first write down the equation of this circuit. Let us proceed with the circuit example and then we know that it is equivalent to the mechanical example, so you have $V_m \sin \omega t$ is equal to $R i$ plus $L \frac{di}{dt}$ plus $\frac{1}{C} \int i dt$. We differentiate it through we have $R \frac{di}{dt}$ I am writing in the right hand side this side plus $L \frac{d^2 i}{dt^2}$ plus i by C is equal to $V_m \omega \cos \omega t$.

What would the solution be, this is the equation a second order differential equation, what would the solution be. Again go by the same logic, it is a sinusoidal excitation and therefore, the equilibrium point is also moving sinusoidally. And therefore, the actual state will try to follow it and therefore, it is logical that it would also be some kind of general sinusoid.

Again, we will write the solution a trial solution as here we are talking about the steady state solution. So, i_p is $A \cos \omega t$ plus $B \sin \omega t$, how would you obtain these values A and B simply by putting it here. So, we will first obtain from i_p , i_p will be substituted here you have to first obtain $\frac{di_p}{dt}$ which will be substituted here $\frac{d^2 i_p}{dt^2}$ will be substituted here.

So, write down these two and substitute it here and there by you will obtain the result, so your $\frac{di_p}{dt}$ is $-A \omega \sin \omega t$ plus $B \omega \cos \omega t$ and $\frac{d^2 i_p}{dt^2}$ is equal to $-A \omega^2 \cos \omega t$ plus it is come minus $B \omega^2 \sin \omega t$. So, we have the first thing is different derivative is double derivative, so just substitute it here plug it in here you have first R times this.

(Refer Slide Time: 47:47)

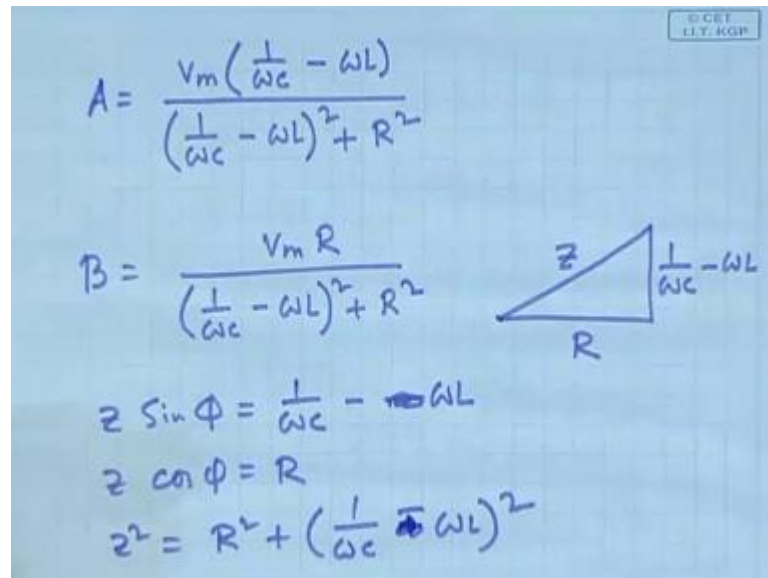
$$\begin{aligned}
 & -L A \omega^2 \cos \omega t - B L \omega^2 \sin \omega t - A R \omega \sin \omega t \\
 & + B R \omega \cos \omega t + \frac{A}{C} \cos \omega t + \frac{B}{C} \sin \omega t \\
 & = V_m \omega \cos \omega t \\
 & \left(-L A \omega^2 + B R \omega + \frac{A}{C} \right) \cos \omega t + \left(-B L \omega^2 - A R \omega + \frac{B}{C} \right) \sin \omega t = V_m \omega \cos \omega t \\
 & -L A \omega^2 + B R \omega + \frac{A}{C} = V_m \omega \\
 & -B L \omega^2 - A R \omega + \frac{B}{C} = 0
 \end{aligned}$$

So, it will be first we will take this one L times this it is $L A \omega^2 \cos \omega t$ minus $B L \omega^2 \sin \omega t$ minus. Now, the first derivative term $A R \omega \sin \omega t$ plus $B R \omega \cos \omega t$ plus the original term $\frac{A}{C} \cos \omega t$ plus $\frac{B}{C} \sin \omega t$ is equal to the forcing function $V_m \omega \cos \omega t$. What was the next step, this will hold through if the coefficients of the cosine terms are 0 and the coefficients of the sine terms are also 0.

This will give, this term first let us put together the cosine terms minus $L A \omega^2$ plus $B R \omega$ plus $\frac{A}{C} \cos \omega t$ plus minus $B L \omega^2$ minus $A R \omega$ plus $\frac{B}{C} \sin \omega t$ is equal to $V_m \omega \cos \omega t$. This immediately yields this is equal to this and this equal to 0. So, write this and then obtain A and B from there two equations, two unknowns you can do it minus $L A \omega^2$ plus $B R \omega$ plus $\frac{A}{C}$ is equal to $V_m \omega$ that is one equation, the other is minus $B L \omega^2$ minus $A R \omega$ plus $\frac{B}{C}$ is equal to 0, from here you will obtain.

See, though the algebra looks a bit involved ultimately the things are conceptually rather simple. So, you have to obtain A and B from there, just do a bit of practice, because if I ask you in the exams; otherwise you will get messed up.

(Refer Slide Time: 50:52)



The image shows handwritten mathematical derivations and a phasor diagram on a blue background. In the top right corner, there is a small logo that reads "© CEE IIT KGP".

$$A = \frac{V_m \left(\frac{1}{\omega C} - \omega L \right)}{\left(\frac{1}{\omega C} - \omega L \right)^2 + R^2}$$
$$B = \frac{V_m R}{\left(\frac{1}{\omega C} - \omega L \right)^2 + R^2}$$

Below the equations is a right-angled triangle representing a phasor diagram. The horizontal base is labeled R , the vertical side is labeled $\frac{1}{\omega C} - \omega L$, and the hypotenuse is labeled Z .

$$Z \sin \phi = \frac{1}{\omega C} - \omega L$$
$$Z \cos \phi = R$$
$$Z^2 = R^2 + \left(\frac{1}{\omega C} - \omega L \right)^2$$

So, from here you will have to obtain A and B, A comes out to be I will skip all these algebra which you can easily do yourself, but I will just write the result V_m . Now, I will also arrange it in a bit more intuitively understandable form, I will write $\frac{1}{\omega C}$ here minus ωL here and this is $\left(\frac{1}{\omega C} - \omega L \right)^2$ plus R^2 square. B comes out to be $V_m R$, here it is $\left(\frac{1}{\omega C} - \omega L \right)^2$ plus R^2 square.

So, immediately things fall in place, because then this will be the component that you draw like this, here it is R , here it is $\frac{1}{\omega C} - \omega L$ here it is Z . Which means, your $Z \sin \phi$ is $\frac{1}{\omega C} - \omega L$ and $Z \cos \phi$ is equal to R and you have Z^2 is equal to R^2 plus $\left(\frac{1}{\omega C} - \omega L \right)^2$.

Student: ((Refer Time 52:31))

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$$i_p = \frac{V_m}{Z} \left(\sin \omega_0 t \cos \phi + \cos \omega_0 t \sin \phi \right)$$

$$= \frac{V_m}{Z} \sin (\omega_0 t + \phi)$$

$$X_c(t) = c_1 e^{\lambda_1 t} V_1 + c_2 e^{\lambda_2 t} V_2$$

For complex eigenvalues,

$$X_c(t) = c_1 e^{\sigma t} \begin{pmatrix} -\sin \omega_1 t \\ \cos \omega_1 t \end{pmatrix} + c_2 e^{\sigma t} \begin{pmatrix} \cos \omega_1 t \\ \sin \omega_1 t \end{pmatrix}$$

So, everything now falls in place your final solution is i_p is equal to well, it will be again V_m by Z . Then, we will write it is $\sin \omega t \cos \phi + \cos \omega t \sin \phi$ is equal to V_m by Z , you have $\sin \omega t$ plus ϕ , ϕ is now obtainable from here. So, this is the particular integral, what about the complementary function.

See, obtaining the steady state solution is not difficult, because you already know that if you have a R L C circuit, the way we do in electrical engineering is that R and then $j \omega L + 1/j \omega C$ add them obtain it you get the same thing. So, far I was doing writing a bit faster, because this is something that is already known, what about the complementary function. That might give you trouble, so that might given you trouble. So, what will be the complementary function in this case.

Student: ((Refer Time: 54:26))

Yes, wait a minute.

Student: ((Refer Time: 54:38))

It will be where you have not applied the sinusoid, you are considering the left hand side of the equation, where was the equation. You are considering the left hand side of the equation, ((Refer Time: 54:57)) which is a second order equation and if I mean there are two possibilities, if you obtain the Eigen values it could be real, it could be complex.

Suppose, if they are real then also 2 Eigen values, suppose they are λ_1 and λ_2 along the directions V_1 and V_2 .

So, if this were not there, then the solution would be the I will write it as not as $i c$, but rather the full vector $X c t$ as we know e to the power $c_1 e$ to the power $\lambda_1 t$ times V_1 is a Eigen vector plus $c_2 e$ to the power $\lambda_2 t$ times V_2 that Eigen vector. In terms of the first order system, this writing out the complementary function was rather trivial. But, in case of the second order system, it would be that unforced system it is response will give you how the thing will decay and it will be same exactly like that. And then, if it is complex conjugate then what will be the complementary function, if it is complex conjugate what will be the complementary function.

Student: ((Refer Time: 56:23))

No.

Student: ((Refer Time: 56:27))

Yes, there will be sinusoid yes we have already learned that and that will be the complementary function. So, if for complex Eigen values $X c$ will be $C_1 e$ to the power σt times remember, we had obtained it and this will be the complementary function. So, for real Eigen values this will be the complementary function, for complex Eigen values, this will be the complementary function.

And ultimately the total will be where is it, this plus either this or that and then from the initial condition you will have to obtain the...

Student: C_1 and C_2 .

C_1 and C_2 clear.

You will not be clear unless you do problems, which I will give and you will have to do in the tutorial class. But, this will not be clear unless you actually do some problems like this, but conceptually it should be clear that the unforced systems solution it is Eigen values and Eigen vectors should give the transient response and the steady state response will be given by that.

Now, notice that here there is a problem, because this ω was...

Student: ((Refer Time: 58:31))

That was the forcing functions omega.

Student: ((Refer Time: 58:35))

Yes, you have to keep in mind that and this omega is...

Student: Natural frequency.

Natural frequency, so it will be say let me separate them out, otherwise you will have confusion you will mess them up. Notice that, here there are two characteristic times of the system, one it is own if you let it oscillate by itself it is own omega 1 and the one that is applied on it. So, the actual dynamics will be a combination of these two frequencies, two different characteristic times of this system. And I hope you understand how to proceed we will see and we will do problems later, that is all for today.