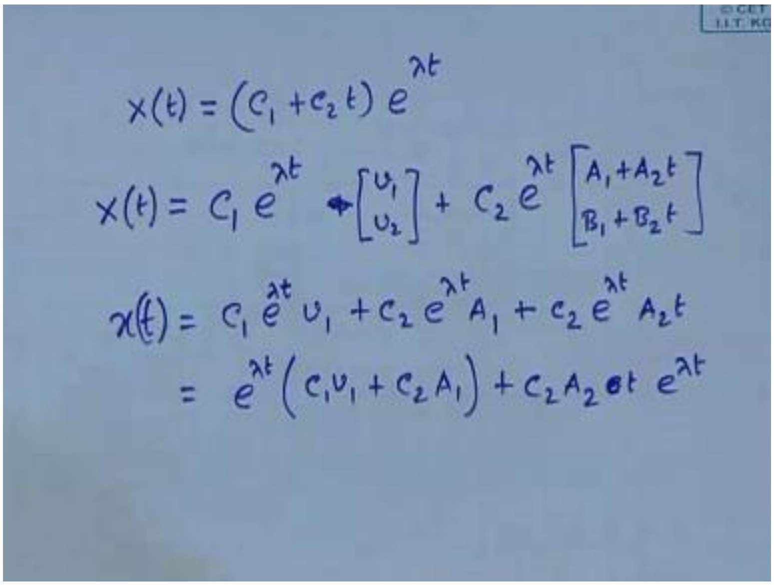
Dynamics of Physical Systems

Prof. S. Banerjee
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 26
Higher Dimensional Linear Systems

In the last class, there was a question by him. That was when we were dealing with the equal Eigen values.

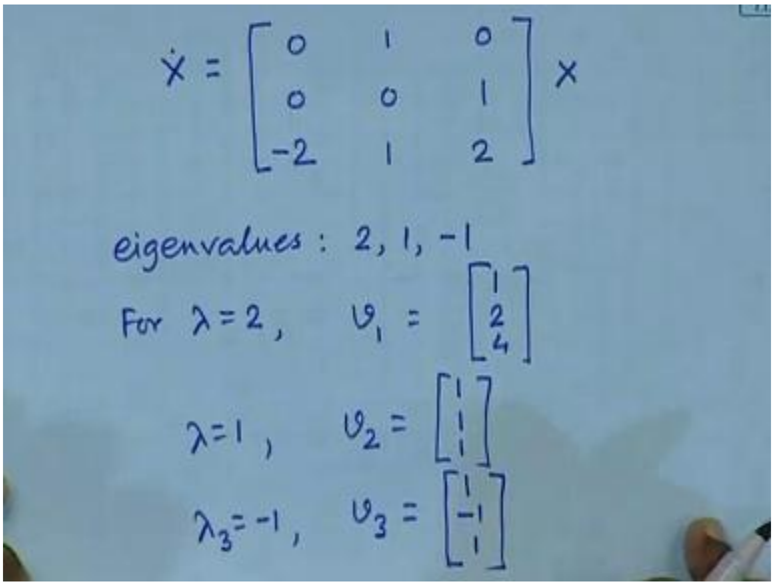
(Refer Slide Time: 01:04)



Then we had said, that the solution would be X of t will be C 1 plus C 2 t, e to the power that is what we said. But, when we dealt within the in the earlier classes, then we said that it is actually a combination of C 1 e to the power lambda t plus, it would be v 1 v 2 plus C 2 e to the power lambda t and then you had A 1 plus A 2 t and B 1 plus B 2 t. Now, the reason that I abbreviated this was, suppose you are talking about the first line, which is X t and the second line is Y t.

So, your x t is C 1 e to the power lambda t, v 1 plus C 2 e to the power lambda t, A 1 plus C 2 e to the power lambda t, A 2 t. Now, we can club them as it is e to the power lambda t times. So, you see there is a one term, one term with e to the power lambda t, which is with a coefficient that is a constant. There is another term with t e to the power lambda t with another coefficient constant that is what we have written here. So, it is just an abbreviation of that. We have clubbed all these together as the new C 1 and all these together as new C 2; that is all. So, in the last class, we had given a problem and asked you to approach it.

(Refer Slide Time: 03:30)



The problem was that the X vector is 3 d and now that 3 d equations are 0 1 0, 0 0 1 minus 2 1 2 with X, so in this case, how would you proceed. First, we will go the same way; that means, we will try to identify the Eigen values and Eigen vectors. We will write the independent equations along the Eigen vectors and then proceed. So, what are the Eigen values of this?

Student: ((Refer Time: 04:10))

In this case, the Eigen values you have already calculated. So, Eigen values are

Student: ((Refer Time: 04:23))

2, 1 minus 1, the next question is to obtain the Eigen vectors, for each of this Eigen values. So, what are the Eigen vector for 2, what the Eigen vector is for 1 and what is the Eigen vector for minus 1.

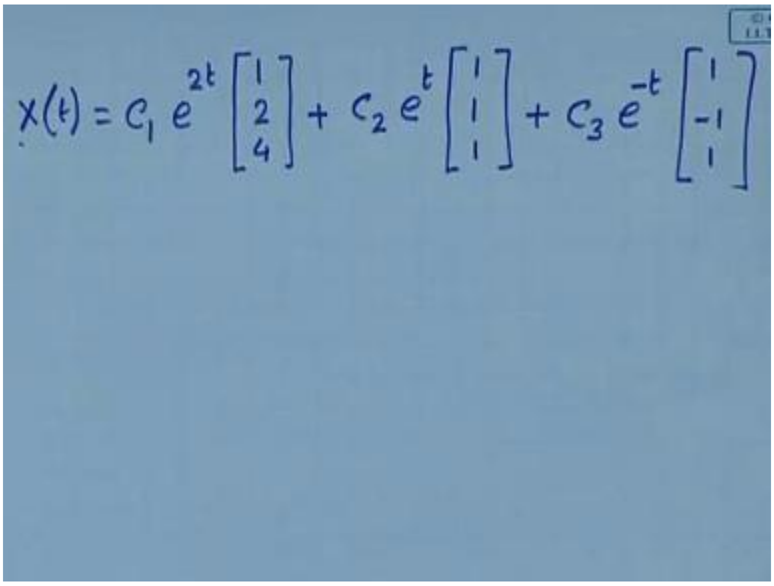
Student: ((Refer Time: 04:44))

For lambda is equal to 2

Student: ((Refer Time: 04:49))

V 1 is 1, 2, 4, is that, have you fellows done it. For lambda is equal to 1, v 2 is 1, 1, 1 and for lambda is equal to minus 1, v 3 is 1 minus 1 and 1.

(Refer Slide Time: 05:36)



So, now we can compose the general equation as X of t will be C 1 e to the power twice t 1, 2, 4 plus C 2 e to the power t, 1 t actually 1,1,1 plus C 3 e to the power minus t 1 minus 11 which means the first 1 X, second 1 Y, third 1 Z. X of t will be C 1 e to the power twice t plus C 2 e to the power t plus C 3 e to the power minus t. Y t will be C 1 2 e to the power twice t plus C 2 e to the power t minus C 3 e to the power t so on and so forth and this C 1, C 2, C 3 will have to be obtained from the initial conditions.

Now, as we have always done in this class, we will try to develop a geometrical understanding of what is happening here. You had notice, that there are two Eigen values which are positive and one Eigen value that is negative which means that, there is this Eigenvector along which it is contractive; that means, it comes to the equilibrium point. And there are two other Eigenvectors, along which it is expansive; that means it goes out.

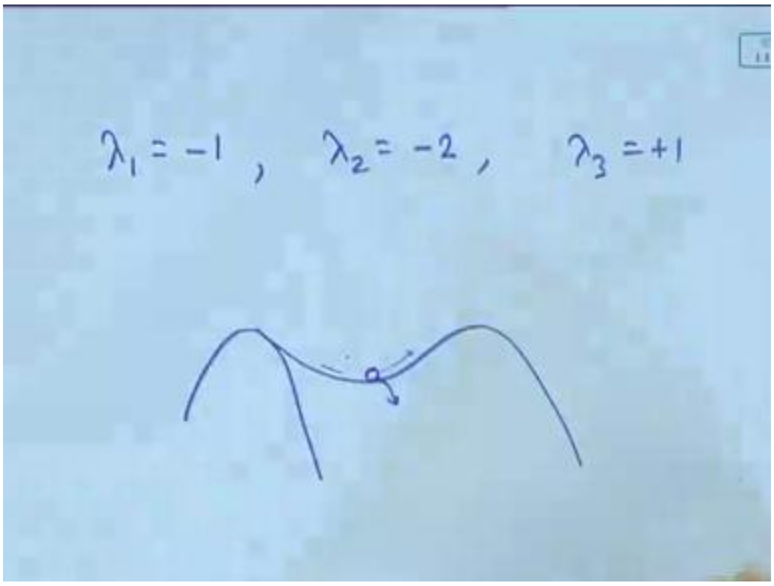
What is it mean; you can imagine then a plane passing through those two Eigenvectors, which have positive Eigen values. Imagine, these are two Eigenvectors along which it expands. Now, imagine a plane passing through these two Eigen Eigenvectors, then the system will be unstable along that Eigen plane and it will be stable in the other Eigen plane, so these are subspaces of the 3 d space.

So, we can in-general conclude that it will be stable along this subspace. Along and it will be unstable, along the 2 d subspace spanning these two Eigen vectors. So, depending on the Eigen values and Eigen vectors, you will have conditions, where even though this is the space is rather large and on the face of it is somewhat difficult to visualize. You will be still able to concentrate on subspaces that are either stable or unstable given by the Eigen values.

If an initial condition is along, that particular subspace which is this, then there is no component along these two and therefore, it will converge directly on to the equilibrium point. If there is an initial condition along exactly this, then there is no component along these two and it will expand as e to the power t. But, if there is an initial condition in the plane that is spanned by these two Eigen vectors, then there is no component along this. Therefore it will not contract at all and it will expand along that as if it is a 2 d system.

Exactly, the way it would behave in A 2 d system, if you have any initial condition in that plane, it would behave in exactly the same manner and it will go out, but going out will be dependent on these two exponents. How fast, it will always converge them into one of the Eigen vectors and then go out to infinity. That we have already learnt and that is what will happen in this case. So, you see, you have the situation, if you have three real Eigen values depicted like this.

(Refer Slide Time: 10:27)



Supposing, you have a different situation, where the lambda 1 is say minus 1, lambda 2 is say minus 2 and the lambda 3 is say plus 1. What will be the behavior like, there will be associated with them, Eigen direction Eigen vectors. But, in this case, you can see there will be an Eigen plane that will be stable. Associated with this, there will be an Eigen vector, the plane that passes through these two Eigen vectors will be a stable plane, even though the system is globally unstable.

Now, globally unstable means, there is an expanding direction. Any deviation along that will be expanding will be going out, so why is that important, it is important, because if you have to control it, if you want to keep it within that you do not really need to pay attention to the these two. You have to pay attention to this fellow and you have to do something, some control to push it back along this direction and it is not going to go out along these directions.

And that is why, in general it is necessary to understand, what the expanding directions are and what the contracting directions are. In this case, there will be a subspace that will be contractive subspace. Well, start at the origin they meet and there are two lines and these two lines will define exactly one plane. In a 3 d space imagine, they are not just any lines, they are meeting at the origin. At the origin, there is one line here another line here, it defines exactly one plane.

In a linear system, it will be a plane in a non-linear system that will be some kind of a general surface. But, never the less you can imagine, you can identify, see always what I am trying to drive at is, that you should be able to sort of close your eyes and visualize that it the behavior will be like this of the system. But, then if the system is has real Eigen values, you can have the four possibilities, either one all the Eigen values negative in which case, it will be contracting. Globally contractive; that means all initial condition will converge on to the equilibrium point.

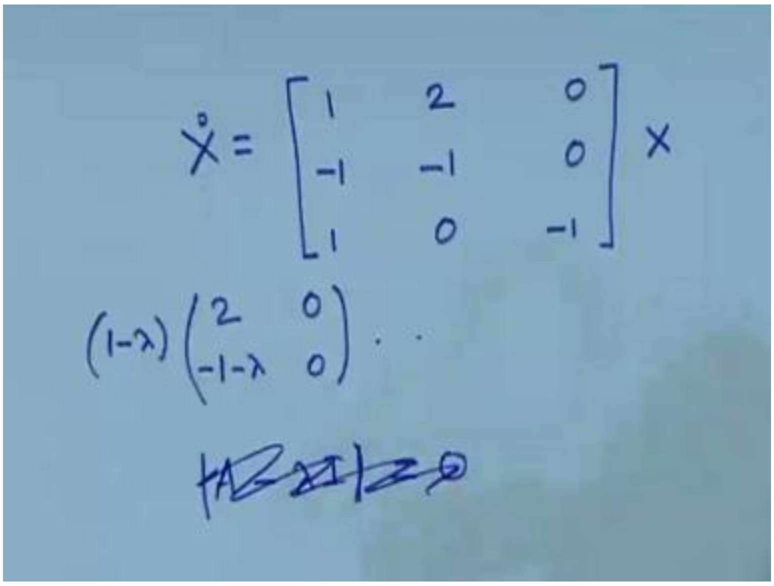
And why does so, it should always come closer to first an Eigen plane and then a particular Eigenvector and along that it will converge to the fixed point the equilibrium point. If there is one Eigen value, which is negative, two Eigen values positive and then we have already seen, two Eigen values negative, one positive we have already seen. If all the Eigen values are positive, it will again be globally expansive.

It will be unstable in all directions, but then it will always converge on to one of the Eigen directions and then go to infinity, not just any visual. What are the other possibilities, all negative, yes all negative is the case, where you have all converging. In which case, that point will be almost attracting all initial conditions from anywhere in the space, such a point is an attractor.

Such a point attracting equilibrium point, which attracts all initial condition towards itself as if it is a gravitative body, it behaves like a gravitative body. If you have this kind of situation, in general it is a saddle. Our imagination of the saddle was a 2 d concept, because we had imagined the saddle something like this. Where, if you have a marble here, then it will be stable in this direction, any perturbation in this direction will again bring it back to that. But, any perturbation in this direction will make it fall.

So, it still have the same property, in this case, that was one of the Eigen directions and this was another Eigen direction. In this case, you have the Eigen directions three of them, not only three of them, I am also introducing the concept that two, we will introduce a plane which is a stable Eigen plane or a unstable Eigen plane digested. If you have digested this, then let us go on to the situation where there could be complex conjugate Eigen values.

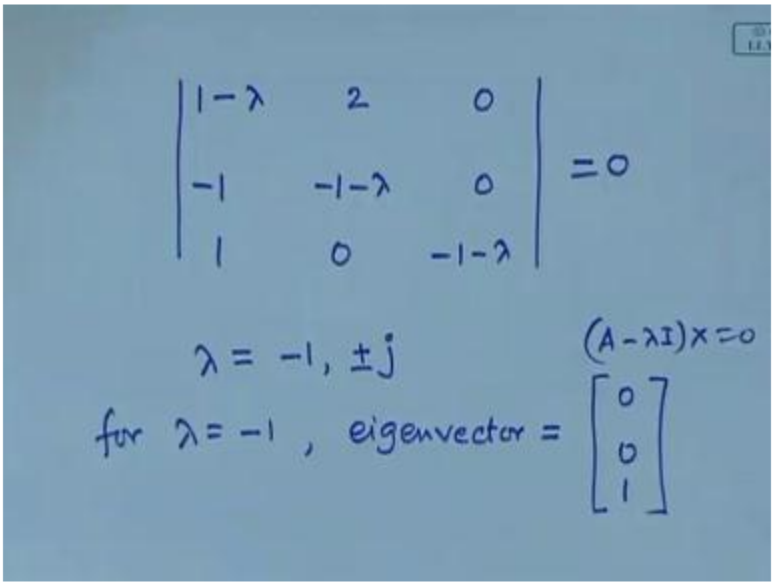
(Refer Slide Time: 15:44)



First, let us deal with a situation, where there are purely imaginary Eigen values. consider a system with X dot is equal to 1 2 0 minus 1 minus 1 0 and 1 0 minus 1 X.

What will be the Eigen values, quickly do this, it will be 1 minus lambda. So, it will be times 2 0 minus 1 minus lambda 0 and all that just write it correctly. So, it will be A minus lambda I, It is equal to 0; that is the concept, lambda I will be here and here. So, you have to obtain the determinant of this 3 by 3 matrix. That is what, we started doing and this term will become 1 minus lambda. This term will become minus 1 minus lambda.

(Refer Slide Time: 17:24)



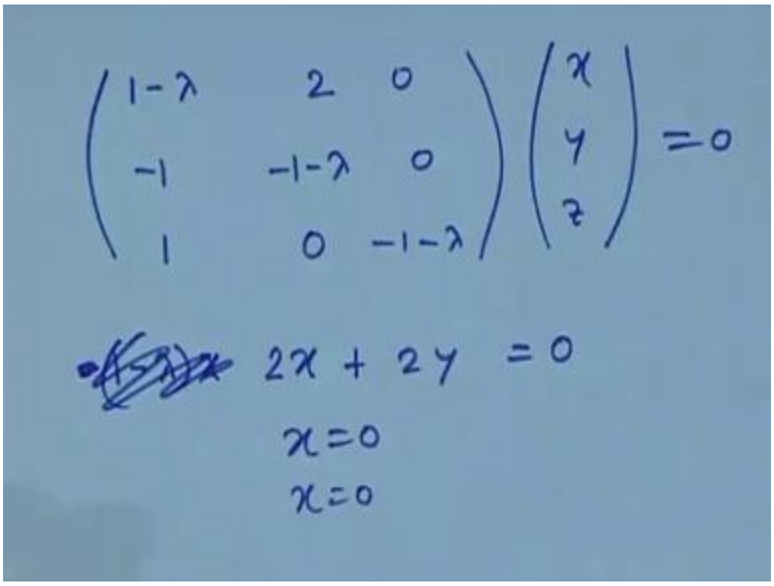
So, whose determinant it will be 1 minus lambda 2 0 minus 1 minus 1 minus lambda 0 1 0 minus 1 minus lambda. So, what has it yielded?

Student: ((Refer Time: 17:44))

So, if you solve this you were getting lambda is equal to minus 1 plus minus j. So, there will be an Eigenvector associated with minus 1, there will be an Eigen vector associated with plus j. And as we have done already we will not bother about the minus j component, because that gives no further information, no new information So, just go ahead, exactly in the same way.

First what is the Eigen vector associated with minus 1, A minus lambda I X equal to 0. From here, what do we getting, well let me do it, since you seem to be a bit confused.

(Refer Slide Time: 19:24)

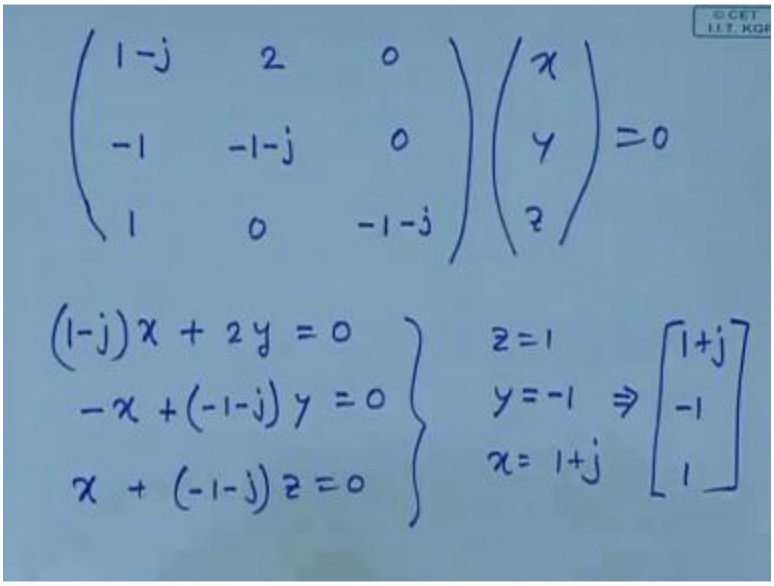


Let me do it, A minus lambda I will be 1 minus lambda 2, 0 minus 1 minus 1 minus lambda 0, 1, 0, minus 1 minus lambda, x y z. The first one says 1 minus lambda, x lambda is in this case minus 1, 2. So, twice x plus twice y is equal to 0, second one is this is minus 1, 0 0, so x equal to 0. So, what will be 0 0.

Student: ((Refer Time: 20:30))

So, just assume anything as Z, it will give it, say 1, so x is 0, this is say, y is 0 and z could be anything. Therefore, it is actually in that direction, so z could be anything, say it is 1, so in this case, it gives 0 0 1.

(Refer Slide Time: 21:10)



Similarly, you do this for say plus j, if you take it as plus j, let me let me do it, because it might be a bit confusing 1 minus j, 2, 0 minus 1, minus 1 minus j 0, 1, 0 minus 1 minus j, x y z is equal to 0, so you have 1 minus j x plus twice y equal to 0, first equation. Second equation is minus x plus minus 1 minus j y equal to 0 and third equation is x plus minus 1 minus j z is equal to 0. What will satisfy these, you cannot say 0 0 0, obviously that is not a good result.

So, this is

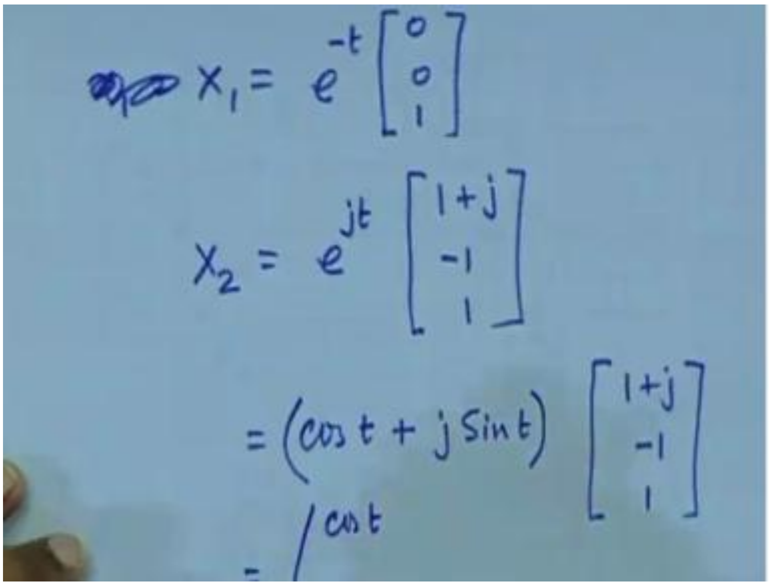
Student: ((Refer Time: 22:33))

Z equal to 1, his suggestion is let us say z equal to 1, z equal to 1 means, it will be…

Student: ((Refer Time: 22:47))

Y equal to minus 1 and x equal to, so your Eigen vector is 1 plus j y is equal to minus 1 and z is equal to 1, is the concept clear, how to obtain it. So, you have this and you know that it has two parts 1 minus 1, 1 and j 0 0. Then, there are two parts, one real part and another imaginary part. So, let us now proceed with this, if you proceed with this, you have the solution, as for this one, it was known for this one, it is that.

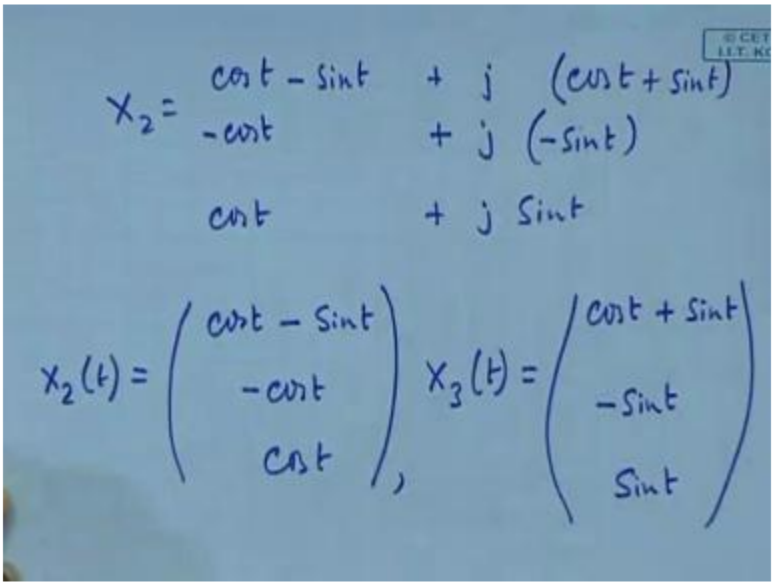
(Refer Slide Time: 23:52)



So, there are three solutions, I will write capital X 1 equal to for this Eigen value and this Eigen vector, it is e to the power minus t 0, 0, 1. Second one, X 2 now that we have already done, if you have these as the Eigen vector and plus j as the Eigen value, It will start you by writing e to the power j t, then this 1 plus j minus 1, 1 and then, we will follow exactly the same procedure. That we did earlier, this will yield cos plus cosine plus j sin omega t and then, we will separate the real part and the imaginary part.

After have it done so, we will say the real part gives one possible solution, the imaginary part gives another possible solution. Just obtain the two solution, then cos t plus j sin t, 1 plus j minus 1, 1. This yields cos t, I will write separately.

(Refer Slide Time: 26:02)



X 2 is equal to there are two parts, cos t times 1 plus j cos t plus j cos t plus j sin t minus sin t. The first line, so I have just separated them out, second line is cos t minus 1, so minus cos t plus.

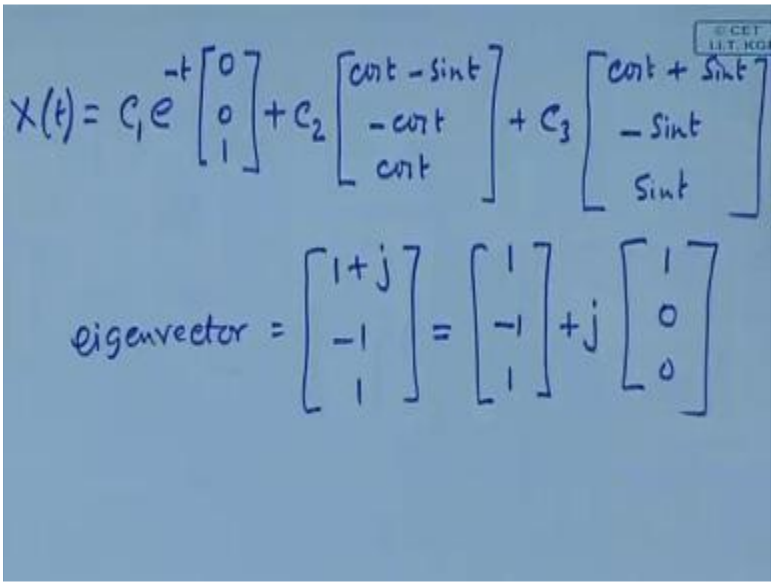
Student: ((Refer Time: 27:02))

Minus j, so j minus sin t, no then I have to talk about the

Student: ((Refer Time: 27:14))

Third one is cos t plus j sin t. Now, we separate them out, we will say that X 2 t is this part, which is cos t minus sin t minus cos t cos t and X 3 t equal to this part cos t plus sin t minus sin t and sin t, done. So, we have X 1 and we have X 2 and X 3, X 1 was this and then we have X 2 and X 3, now just write down the final solution.

(Refer Slide Time: 28:28)



So, your final solution X t is equal to C 1 e to the power minus t, 0, 0, 1 plus C 2 cos t minus sin t minus cos t, cos t plus C 3, cos t plus sin t minus sin t. So, that is the solution, happy, this is how it is to be obtained. See, we have produced a method and any problem can be approached by that, any given problem. So, your X t is C 1 times this, which is constant in all that. You notice that, it gives a sinusoidal solution, but it decays along Z direction, can you see that from this.

Now, let us figure out what will be the behavior in the 3 d state space. Yes now it is, it is interesting, why because there is an Eigen vector, which is associated with a negative Eigen value. A negative Eigen value means contractive, so there is an Eigen direction Z, Z direction along which it is contractive. So, any deviation along that will be exponentially decaying.

But, if there is a deviation along that, what will happen to that, it will not decay, no, wait slowly; I am saying that here is a direction, along which it is contractive So, I have given some deviation along that direction, which I know will decay. But, I have also given a deviation along the orthogonal direction to it. That means, it is away from that Eigen direction, what will happen to that component, that component will not decay, but that component will cause a with a oscillation motion like this. So, this component will decay, so it will be going like this.

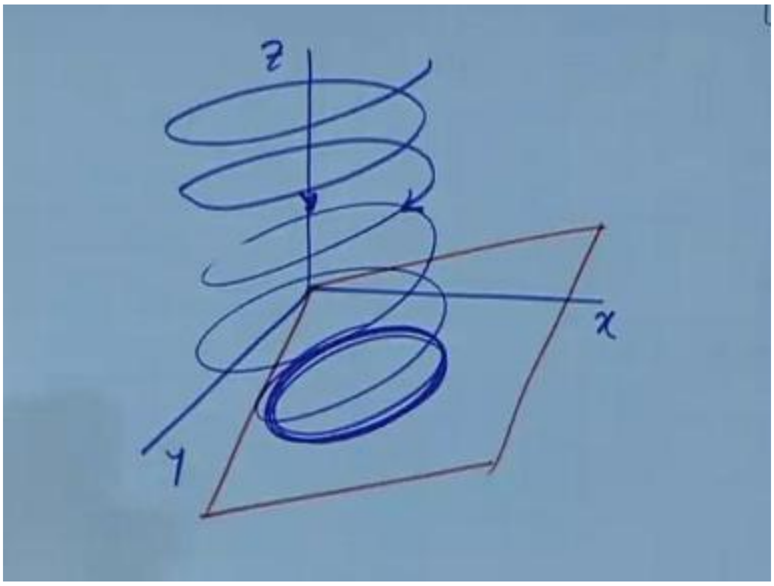
Obviously, it will converge on to something, because exponential decay, that that part will converge on to something, on to what. Where it is even Eigen vector that is to be real, so see a plane in the 3 d space, has to be real plane. It should be able to visualize, where that plane is and if you have complex plane, you cannot visualize that.

Now, that is essentially the plane formed by the two real part and the imaginary part of the Eigen vector, that we started with. See, what do we did, we took this, we use that to write down the equation and then separate the real part and the imaginary part, saying that the real part and the imaginary part separately contain the information. So, these are the two things, two components of the vector real and the imaginary.

So, the Eigen vector was 1 plus j minus 1, 1 which is 1 minus 1, 1 plus j 1, 0, 0, you are right. This is a real vector and this is another real vector and we have composed the final solution in terms of these two real vectors, really. So, there will be a plane passing through these two real vectors, the system will ultimately converge on to that plane, so that is the Eigen plane.

In case of purely imaginary Eigen values, the Eigen plane associated, there is an always an Eigen plane associated with purely imaginary Eigen values and that Eigen plane, real plane. A plane that can be drawn visualized, real plane has to be identified, from the real part and the imaginary part of the corresponding Eigenvector. So, in this case, the behavior will be something like this.

(Refer Slide Time: 33:33)

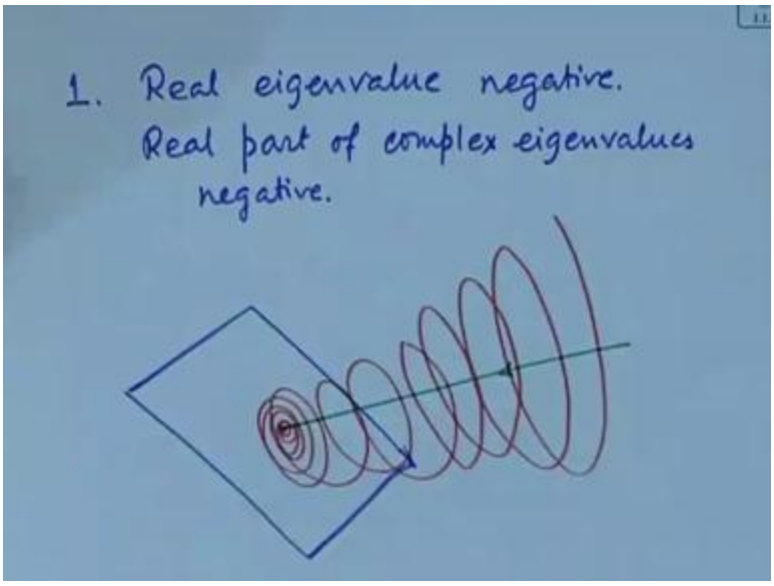


Say, it was x y z, in the z direction, it happen to be contractive, like this, that just happens to be one of the Eigen directions. So, if you start from here, it will just converge here, but if you start from here, it will go like this. But, it is natural intuition of many students, that it will converge on to the x y plane, it will not. It will not converge on to the x y plane rather it will converge on to the plane that is spanned by these two vectors. This is the x coordinate and this is not, it will converge actually on to an inclined plane.

And then, in that inclined plane what is the behavior, a circle. So, imagine that, if this is the inclined plane, then finally, it goes like this and is that digested. Now, if I slightly change the values, this was a purely imaginary case. If I slightly change the values, say I make say 1.2, 1.1 something like that, what will happen, it will no longer be remain purely imaginary, it will become complex conjugate.

So, instead of taking each value, let us try to visualize, what will happen, if you have complex conjugate Eigen values. Two can be complex conjugate; one will always be real, so what are the possibilities, possibility 1; that real Eigen value is positive, possibility 2; the real Eigen value is negative. Regarding the complex conjugate, there are two possibilities, the real part of the complex conjugate is positive and the real part is negative, there are four possibilities. So, let us write down the four possibilities and let us figure out, what will be the behavior without solving equations. Now, we can close our eyes and figure out the behavior from the general geometrical description.

(Refer Slide Time: 36:24)



1. Real Eigen value negative and real part of complex Eigen values. In that case, what will be the behavior, in both the directions, it will be decaying. But, remember that there is an Eigen direction; Eigen plane associated with the complex conjugate Eigen values. And along the other Eigen direction, it will exponentially decay and converge on to that plane. And then in that plane, because of the real part of the complex conjugate Eigen values, it will spiral inwards; it will form a shape like a screw.

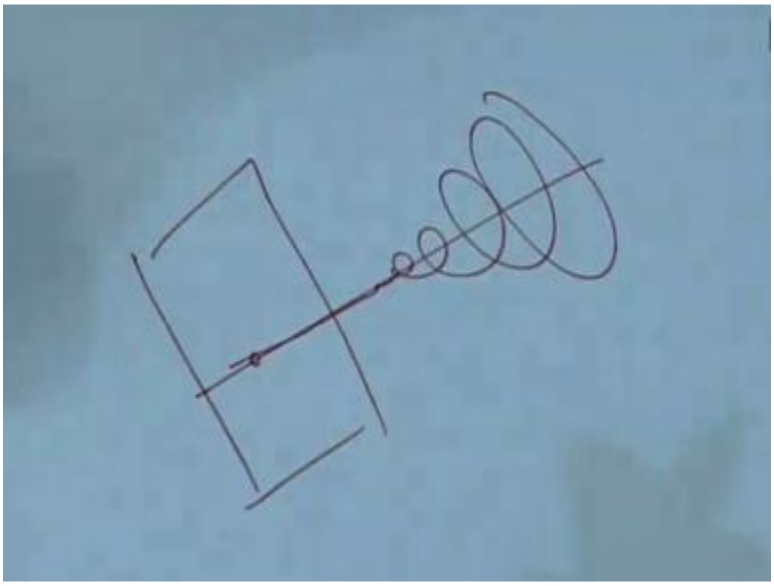
It will be, well screw, when we think…

Student: ((Refer Time: 37:33))

Screw, when we think, it is not exactly like this. Suppose, you have this plane, as the Eigen plane, evaluated the way, I just shown. Eigen value associated with the complex conjugate Eigen values and suppose, this is the real Eigen direction. Then, the behavior will be say starting from here, it will, yes like a screw. But, it will not be just like that, remember ultimately it will converge and then, on this plane, it will converge on to this plane and then on that plane it will finally and home on to the equilibrium point.

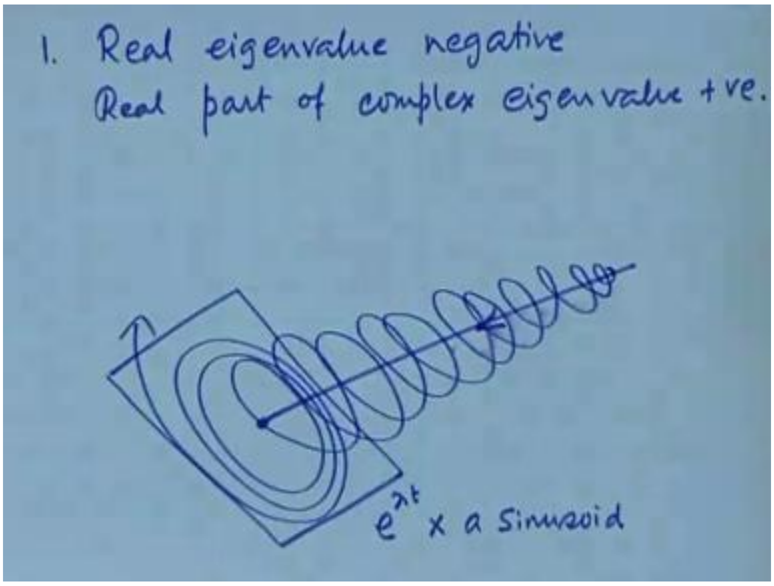
So, see I did not solve any equation. Just from the concept, we can infer that this will be true. The equations will give the details, but it is the concept, geometrical concept that ultimately matters for us, that is what we are trying to develop. So, that will be the situation for, whether or not this decay is faster or that decay is faster depends on that the real part and this real part. If this real part is more larger, that mean, larger negative, then what will happen, it will first converge on to this line and then it will…

(Refer Slide Time: 39:26)



So, whether this picture will true or this picture will true, it will be dictated by the magnitude of the real parts of the Eigen values. Obviously, there is an oscillation here, that oscillatory component will be given by the imaginary part by the imaginary part. So, the oscillatory component sin omega t omega will be given by the imaginary part. In this case, we had all these sin t s, because the imaginary part was 1, that is why otherwise there will be values there, done.

(Refer Slide Time: 40:19)

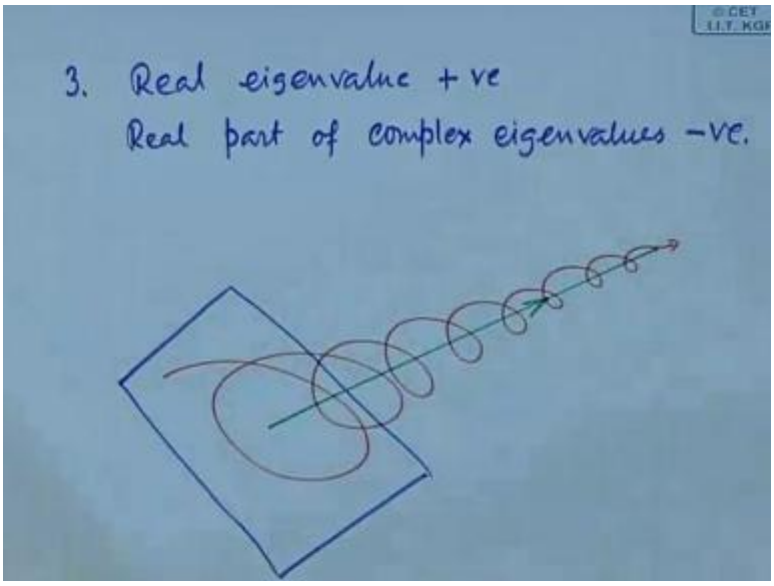


Let us come to the next situation, real Eigen value, negative and real part of complex Eigen value, positive. Then, what happens, again there will be a plane associated with that complex Eigen value and there will be an Eigen direction associated with the negative Eigen value. This is negative, so it is contracting and this is the 0, 0, 0. This is the origin or in this case the equilibrium point.

This plane is then an unstable Eigen plane, start from here, it will exactly go there. Start from a close neighborhood of it, it will decay in this direction; it will expand in this direction. And due to the influence of this Eigen direction, from here itself you too see the component expanding. So, on this, it will be e to the power lambda t, which is positive, times a sinusoid and in this direction, it will be just decay.

Again, in this case, which one will be dominating depends on the magnitudes of the Eigen values, it may be so that it will very fast converge on to this plane. And then, will go around or it will be slow. And finally, assume a large value, even before reaching this plane, that is possible that depends on the magnitude of the Eigen values.

(Refer Slide Time: 42:42)

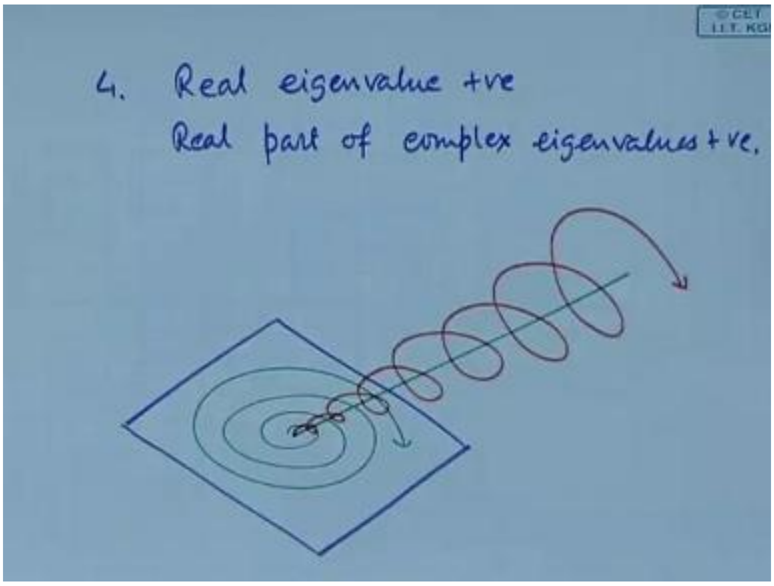


Third case, real Eigen value positive and real part of complex Eigen values negative. In that case, what will happen, there will be a plane associated with this and there will be a direction associated with that, then what. It will go away and in this direction, so this is this has an outgoing vector direction, while this is negative. So, start from very close to here, it will it will go like this.

Now, these orientations will be given by the exact differential equations, where this will be, where that will be, you should be able to see that or if you now that you have written down the program for solving any differential equation, take any given 3 d system. Set the parameters such that you have this kind of combination and then see the result in 3 d. If you are solving by mat lab, it has a possibility of plotting in 3 d or you can use any other program, for example, I use Unit Plot; it has a 3 d plotting program.

But the moment you do that it will be able to see these planes, planes not drawn as planes. But, you see that it is converging on to that plane and then it is going all that something like that and now the final one.

(Refer Slide Time: 44:53)



Fourth: Real Eigen value positive and real part of complex Eigen values positive. In that case, again there will be some kind of a plane and this is unstable, that is unstable, in both directions, it will go out. But, then that is why, when we start to depict it, we will have to take a point very close to the equilibrium point. But, then whether it will expand faster along this direction or it will expand faster in that direction will depend on the Eigen values, magnitudes of the real part of the Eigen values.

So, in one possibility, what will happen is that, it will expand faster here, but also go that that way. In other case, it will be expanding, faster that way and slower this way So, let us depict that, the behavior will be like that.

Student: ((Refer Time: 46:30))

If it is starting from that Eigen plane, unstable Eigen plane and there is no disturbance in this direction, no fluctuation or perturbation in this direction. Then, it will simply go like this, so this is the general picture that can happen in a 3 d system. So, at the face of it, the 3 d system looks somewhat difficult, but that is how. Remember, here we are talking about only the local linear neighborhood of the equilibrium point; you are not talking about elsewhere.

Because, in general we have seen when we were dealing with obtaining the differential equation, we saw that most of the cases it will be non-linear. So, we argued that we will first take a look at the local linear neighborhood. So, all these pertain to local linear neighborhood, what if the system itself is 4 d, 5 d, 6th dimensional, then, do not get worked above about it.

Because, all that will happen, see all these concepts are slowly built up, because that should not be difficult for you to visualize. For example, a 4 d system, what can happen, after all, there will be Eigen spaces, sub spaces. There could be a 3 d sub space, that is contractive and one sub space that is expansive. There are could be 2 d subspace, that is contractive and the 2 d subspace that is expansive, these are the possibilities.

And in those 2 d subspaces, there could be a complex conjugate Eigen values or real Eigen values, so these are the possibilities. But, now once we are equipped with this, it is not difficult to visualize. That now, we know that these are all that can happen, it can either converge on to that or expand on to that and if it does expand, then it will be along the this directions, that is all that what will happen, nothing very big about it.

So, the general idea is that, for any given system, whatever the dimension, you can always identify some subspaces that will be contractive, some subspaces that will be expansive. Some subspaces, where there is no oscillation, some subspaces there will be oscillation. So, any system you will be able to identify such subspaces and this is very important in terms of the general geometrical concept to be able to identify those subspaces.

And in general these are not difficult, all that will happen, the thing that will really give you trouble is to write down a 4 dimensional or 5 dimensional matrix and obtain the

Eigen values. Beyond 3 d, I will not ask you to do that, just give the task to mat lab or some other program. You can it will give you the Eigen vectors also, so no problem, you can always play with that.

So, from here onwards, you will not do it by hand, you will do it by some kind of computer program. But, always that the objective will be to obtain the Eigen values, Eigen vectors and visualize the behavior of the system in terms of that. Now, some or many of you may have done some course in control theory, have you done. Some people have done, some people have not done, does not really matter, because we are going completely parallel do it. Those, you have done for them let me, tell you something.

That the almost the whole of control theory pertains to the local linear neighborhood of equilibrium points. That is why, in a control theory we generally try to operate something at the equilibrium point and whenever there is a deviation from the equilibrium point, we try to control it back. That is why; we mostly assume the deviation to be small, so that the local linear approximation is valid. And that is why, almost the 99 percent of the control theory concerns linear models.

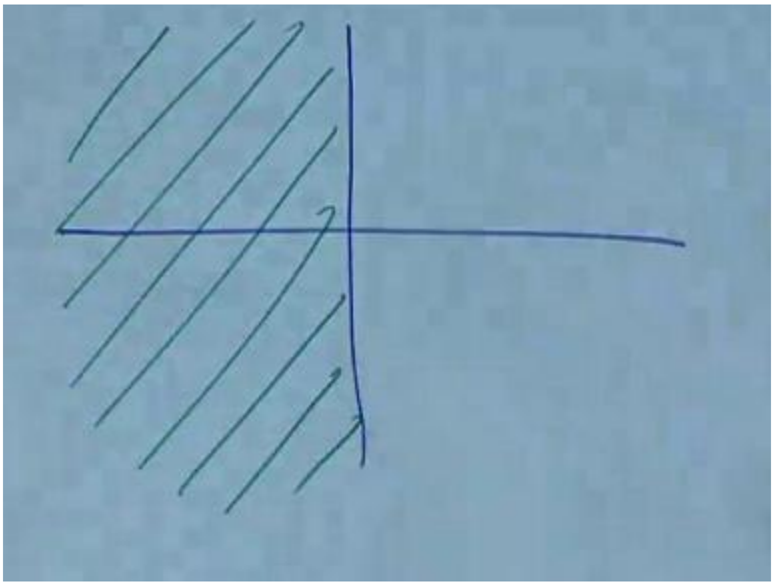
Means that, we dealt with but in control theory we take a certain slightly different route in solving the differential equations; we take the Laplace transform route. While, studying the Laplace transform, we have learned, that that offers a nice way of solving differential equations. So, that is how, we solved differential equations and that then, use that to obtain controllers and stuff like that.

But, here we have taken a slightly different route. Those who have done a control theory should be able to relate that 2, in what way. In control theory, you do like the transfer function, and then we talked about the roots of the transfer function. And the moment, you look at the similarity you will find the routes of the transfer function are nothing but the Eigen values of the matrix that we have obtained.

So, whatever we have concluded in terms of the Eigen values of the matrices, you conclude the exactly the same things in terms of the roots of the transfer functions. So, when you study those things or those you have not studied control theory yet some day you will and then just keep that in mind. That what we are studying now, were studied in a different perspective, different way, where in this course, we are mainly dealing with the time domain behavior.

How does the system evolve with time, here we are concerned mainly with that. While, in a control theory course, they are mainly concerned with the stability and stuff like that, but here we have understood. When will a system be stable, when either the Eigen values are real and negative or complex conjugate and the real part is negative.

(Refer Slide Time: 52:40)



So, in general, this is the Eigen plane, the negative side represents the stable time values But this has also given us the idea, that if a system is unstable, do not try to control it in a blind way rather give more attention to the unstable direction try to push it back in that, it is possible

Student: ((Refer Time: 53:06))

Yes, if it is exactly on this, it will be sustained oscillation, exactly complex conjugate Eigen the imaginary Eigen values, so it will be exactly sinusoidal oscillation, un decaying oscillation

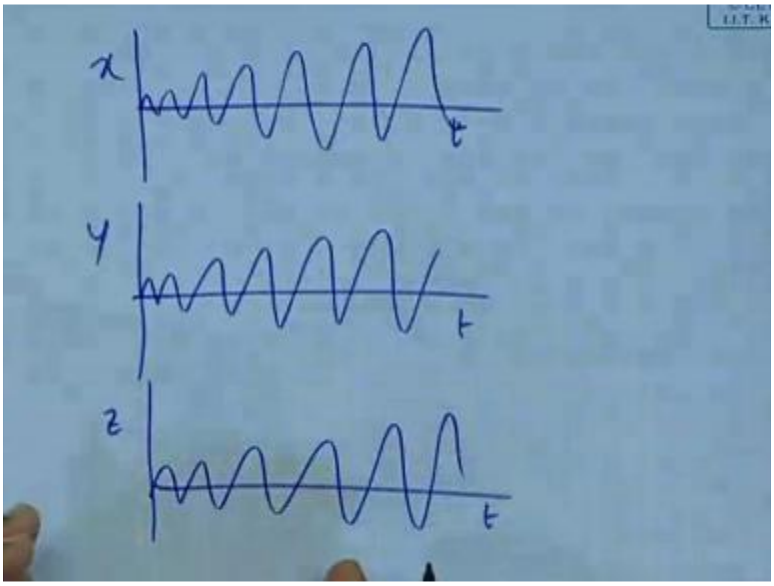
Student: ((Refer Time: 53:25))

In it is equal to marginally stable in a different sense. See…

Student: ((Refer Time: 53:40))

Supposing, you have something like this, this plot is where you were plotting it in the state space, where the x, y, z are the coordinates.

(Refer Slide Time: 54:22)



While he asked if you have the individual plot, says X versus t, y versus t and z versus t, then what additional information does it give? No, actually these and that contain the same information, but this is a visually appealing, if you look at the waveforms here, how it will look. For example, a waveform like this, how will it look here and here?

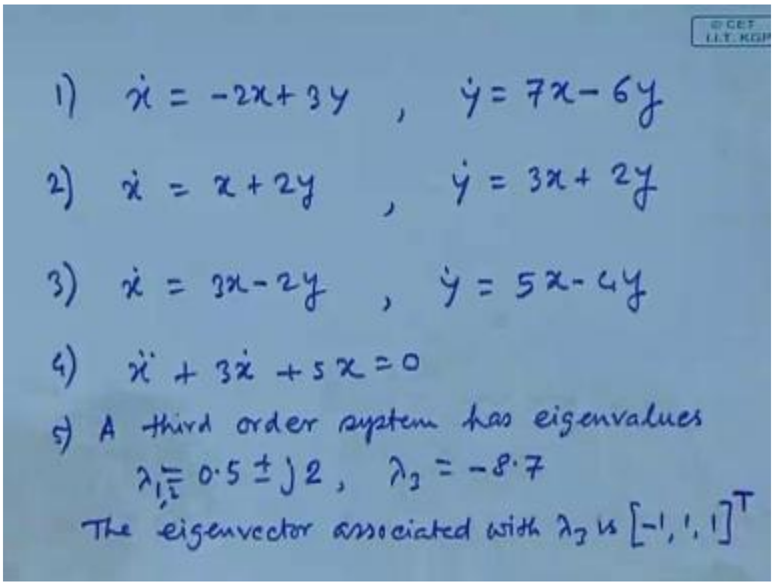
Student: ((Refer Time: 54:51))

And here and here, do you see the structure, no, it is just conceptual advantage. But, ultimately what happens, what are you interested in, how this varies with time, what that varies with time, what does this vary with time. Ultimately, you are interested in this, but this does not give the conceptual clarity, which is offered by this. Not only that, this will be invaluable when we deal with non-linear systems.

When, we expand our views to say, so far we were considering, only the linear neighborhood, now we talk about elsewhere. Then, this simply does not help and then you have to look at, what is happening elsewhere. Around this particular representation is fine, go away from this, then we do no longer can count on your linear description. Then, you need to look at it this way that is the only way, you can look at it then. That is why, I am slowly building up.

So, far as only the linear system is concerned, this is fine. But, see we are dealing with dynamics in a general sense. You first looked at the small part and then, we will expand that, in order to do that we need this. Now, I will give you some problems, so that you can do in the tutorial.

(Refer Slide Time: 56:32)



So, in all of these cases, you have to find out, what is the behavior. A third order system has Eigen values, 1, 2 is this and 3 is, so in this case describe the behavior, starting from any initial condition close to the origin enough, that is all.

Thank you very much.