Dynamics of Physical Systems
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Lecture - 25
Vector Field around Equilibrium points – IV

In the last class, we were or the last few classes, we studied, how to understand the Vector Fields around Equilibrium Points. But, there we started with the first order definition of the differential equations.

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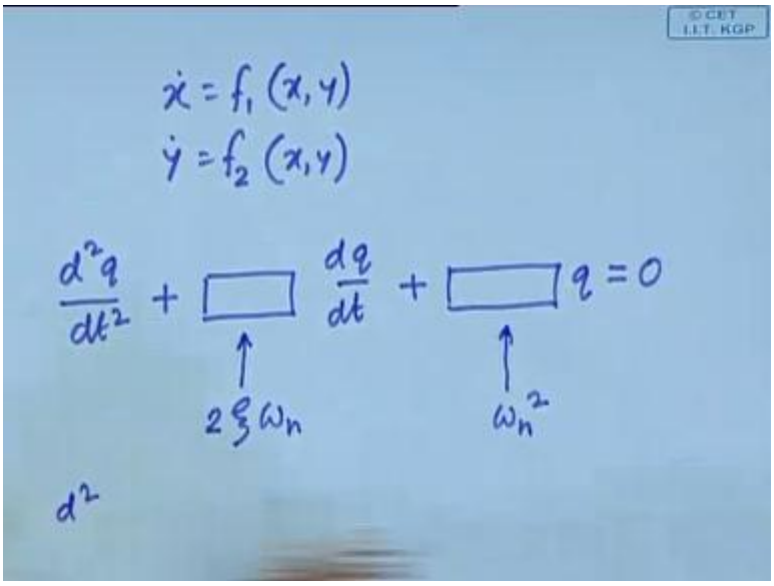

That means you started with x dot is equal to some function of x and y and y dot is equal to another function of x and y. But, earlier we have seen that this is initially obtained in form of second order equations by the Lagrangian method and then we reduce it to first order. So, the natural question is, if you first obtain it in second order form, then why not try to analyze it in the second order itself and we can do that on the basis of what you have already obtained in first order, so that is what we will try to start with today.

So, normally the equation will be given in the form, if q is a state variable, second order equation would be d 2 q d t 2 plus something times d q, d t plus something q is equal to 0. We normally have equations given in the this form, why because, suppose you have a simple pendulum, initially when you obtain the equation, it would be in this form and then, we understood, how to obtain in the first order form. Now, in many cases, it would be simpler to obtain in this form. For example, if you have a circuit, say there is an inductor, there is a capacitor.

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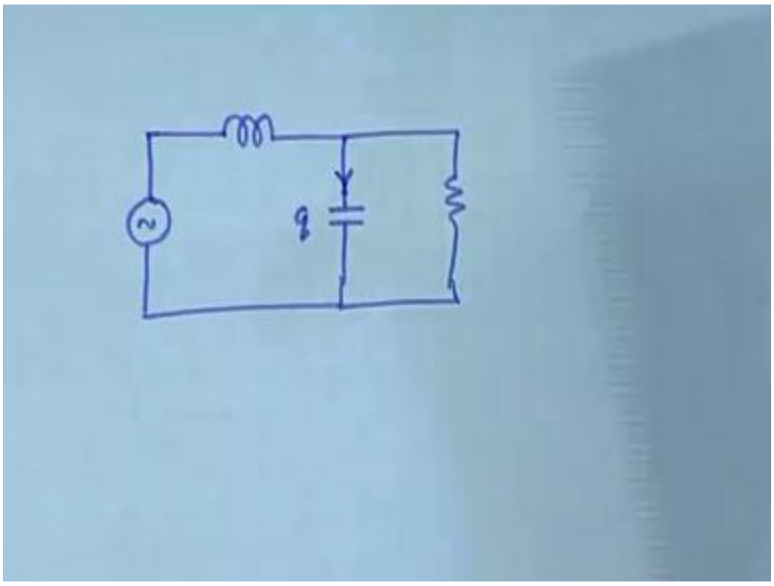

And there is a resistor, how will you do it, well there are 100's of ways of doing it, it is very trivial, very simple, but ultimately you will apply the Kirchhoff's laws. And when you apply the Kirchhoff's law, you will find that you either obtain it in form of the capacitor voltage or in form of the inductor current. If you obtain in form of the capacitor voltage, it comes in this form.

So, suppose the charge here is q, then at this point you have the current flowing is c d v d t or q dot, at this point, it is the voltage divided by the resistance and at this point, it is the i, which is also state variable, so you can easily write it in this form. I am not going into this, because it is trivial. The point is often, you will find that it is easier to obtain, it in this form.

So, on the basis of what you have already understood in first order form, can we understand the dynamics, understand the behavior, if it is given in this form. Now, in considering that question, it has become customary, to express this term as twice zeta omega n and this term as omega n square. Why, I will come to, so that the equation becomes d 2, I will write in the other page.

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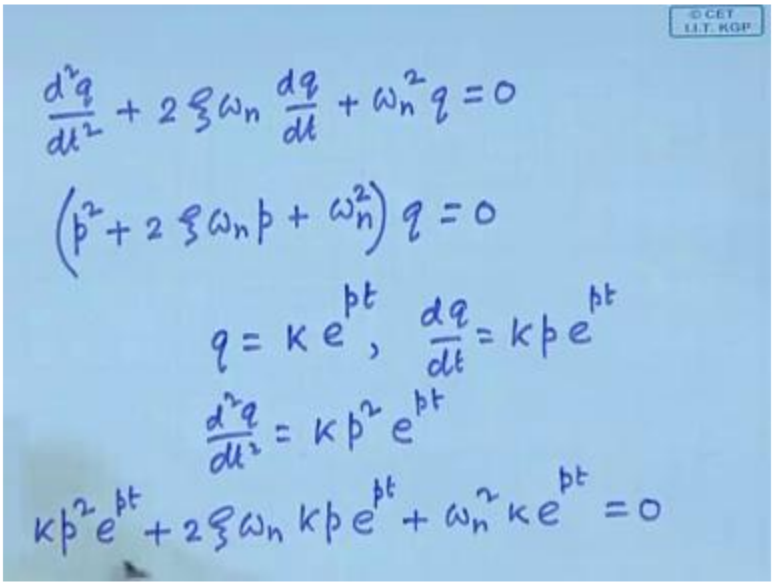

D 2 q d t 2 plus twice zeta omega n, d q d t plus omega n square q is equal to 0. So, normally these are numbers, but whatever the numbers are, we express it in this form that is the reason for it, I will illustrate a little later. But, firstly how do you proceed to solve this kind of a differential equation, you have learned in mathematics, what we do is a trick. The trick is to write the d d t as p and then write an equation for that, you have learned that. So, it would be p square plus twice zeta omega n, it is p plus, it is omega n p square q, which one

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This is p square, p square of q, I am writing it this way.

Student: ((Refer Time: 06:05))

What, square, yes, now this you have probably learned in in mathematics class, that you can solve it this way, that means d d t simply written as something p and write an algebraic equation like this. Do you know, why it works, to me looks like a mathematical ((Refer Time: 06: 25)) So, where does it work After all d d t is not something like a number or p, but still it works, why?

Student: ((Refer Time: 06:36))

Yeah, why, the reason is that after all you are trying to solve an equation like this. Notice that, it is a combination of q, the derivative of q and the double derivative of q. So, something, what that something is we do not know yet, something whose derivative plus the double derivative with some coefficients should yield 0. And therefore, these and that must have the same form, otherwise this cannot happen.

And what is the function that you know, which has this property, that it is derivative and double derivative have the same form, it is the exponential. So, in general the solution would be some number times e to the power say some kind of exponential. If you use this as the possible solution, then how can you write this, it is if this is the solution, then d q d t is K p e to the power p t and d 2 q d t 2 is K p square e to the power p t.

So, if you substitute it here, then you have K p square e to the power p t plus twice zeta omega n, K p e to the power p t plus omega n square, K e to the power p t is equal to 0. You would immediately notice that, if this is yields this equation, check that. So, these will this cancels off k cancels off, so ultimately you get exactly this equation.

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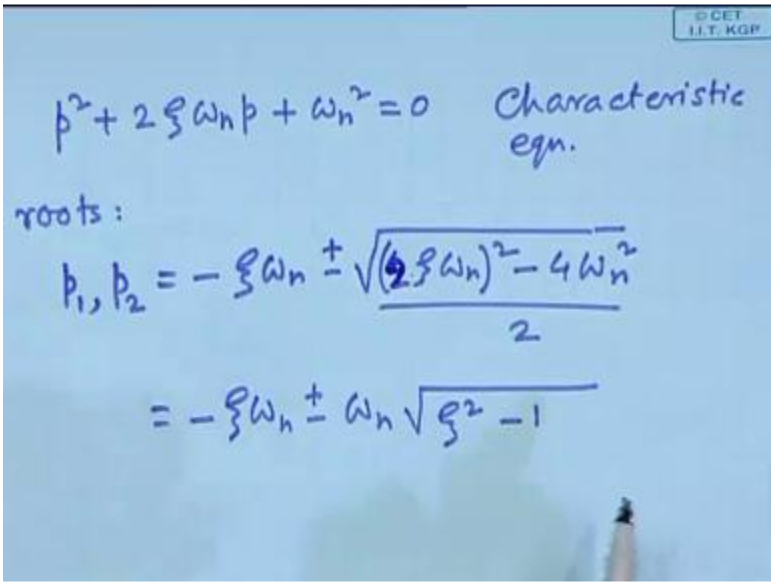

So, what you ultimately yield from this equation is p square plus twice zeta omega n p plus omega n square is equal to 0, so this is the characteristic equation. So, today we are talking about second order differential equations.

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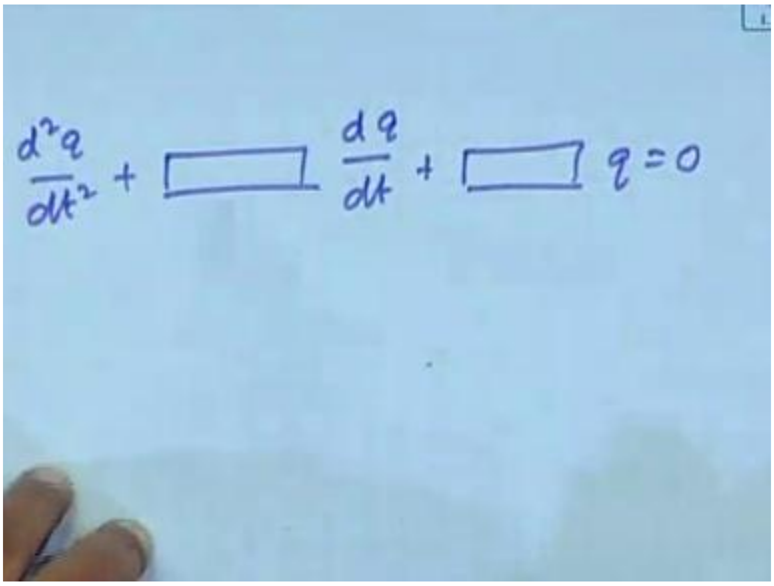

And we had started with something like this d 2 q d t 2 plus something d q d t plus something q is equal to 0, this something's are numbers. Then, we said that we will normally express it in this form. So, that the second one, second coefficient becomes twice zeta omega n and the third one becomes omega n square, why we will come to and then, we said that this yields the characteristic of equation this.

Now, what are the roots of this characteristic equation obtain the roots. Quadratic, so it is easy to obtain p 1 and p 2 are minus zeta omega n plus minus root over. It is 4 zeta omega n 4 zeta omega n square minus 4 omega n square

Student: ((Refer Time: 11:15))

Sorry it is 2, it will ultimately be 4, yes, so this becomes minus zeta omega n plus minus I will now cancel the 2, you have omega n comes to the front omega n root over, it is zeta square minus 1. Now, this is why, we had initially expressed it in this form, why, because it says, we will come to that little later. First, let us try to obtain the solution of this equation in terms of what you will already know.

What you already know, we know how to obtain the solutions for first order differential equations, so we will make the similarity. So, from this from this, let us obtain the equivalent first order definition, because that is what we already know. How do obtain this; that can be obtained by assuming q is equal to x and q dot is equal to y.

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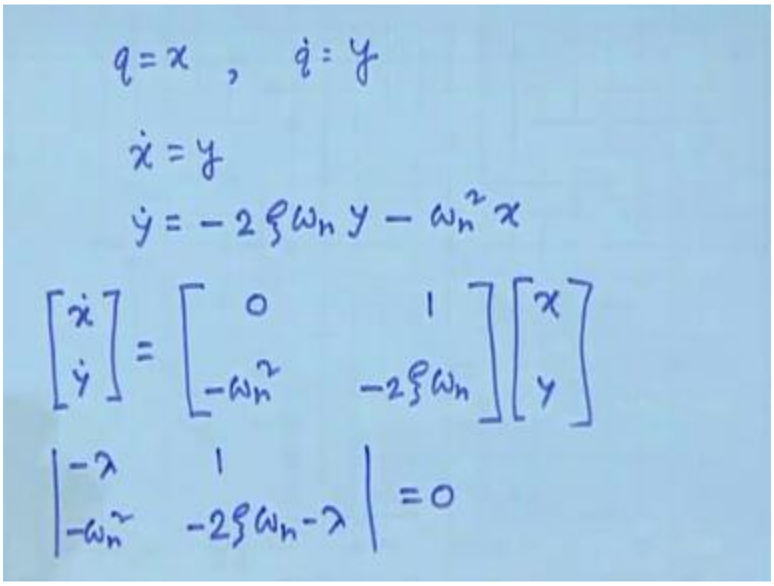

So, if you proceed, you get x dot is equal to y and y dot is equal to that whole thing, y dot becomes this thing taken this side. It is minus twice zeta omega n y minus omega n square x which means, it is x dot y dot is equal to express in the matrix form 0, y is 1, minus this is omega n square and minus twice zeta omega n x and y. So, this is the equivalent first order equation form, whose solution we know.

How do you solve, you obtain the Eigen values of this, obtain the Eigen values of this matrix. The Eigen values of this matrix will be given by this is minus lambda 1, minus omega n square, this is minus twice zeta omega n minus lambda is equal to 0.

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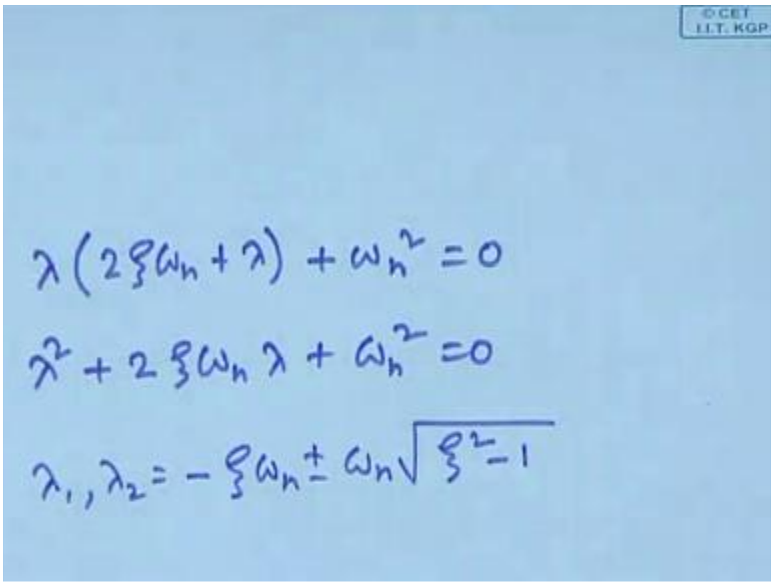

This yields, if I keep it displayed here, this yields lambda times twice zeta omega n plus lambda plus omega n square is equal to 0, lambda square plus twice zeta omega n lambda plus omega n square is equal to 0. Notice that this is exactly the same as the characteristic equation that you had obtained. So, which means that, if you have a 2 dimensional form, then it is characteristics equation is a same as the equation that yields the Eigen values for the first order form.

So, the roots would be the same as the Eigen values, the roots will be exactly the same as the Eigen values. So, in this case what are the Eigen values, we have already obtained. So, lambda 1, lambda 2 are minus zeta omega n plus minus omega n root over zeta square minus 1, these are the 2 Eigen values and we know that such a system. Such a set of 2 first order differential equations, their solution would be given by the character of this Eigen values.

If they are real and this distinct, then there will be one kind of solution. Real and equal there will be other kind of solution and the complex conjugate, there will be another kind of solution. So, under which these 3 possibilities will pertain, tell me. See, firstly when will it be real, when zeta is greater than one, clear.

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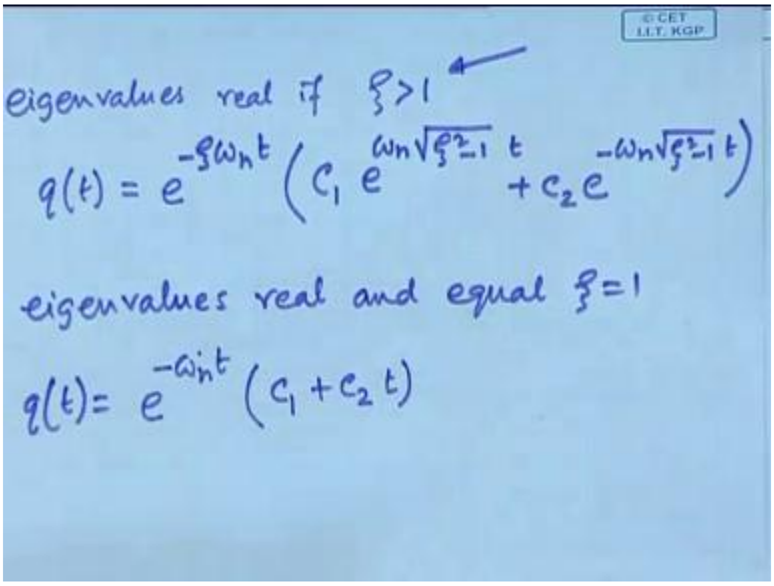

So, Eigen values if zeta greater than 1, clear, zeta less than minus 1, we will consider that issue a little later. What is zeta? We will understand and then we will talk about that possibility, because zeta actually comes from the frictional terms. So, that normally does not get negative, so this value represents the condition for the real Eigen values. If it is exactly equal to 1, then this term is 0, then this term is 0, you have, what

Student: ((Refer Time: 17:17))

Yes, firstly when you have the value as real as and greater than 1, then what is the solution, let us see. The solution, then will be which you already know, q of t is equal to this is the real part and then, not the whole thing is real part, so it is 1 plus another minus. So, accordingly you write, what is the solution of this, there are 2 Eigen values which are both real, so you have e to the power.

So, when it is real take omega n common and what remains minus zeta omega n t, c 1 e to the power omega n root over zeta square minus 1 t plus c 2 e to the power, is it visible, minus omega n root over zeta square minus 1 t visible. So, there is 1 plus another minus, this part is here times this part, which is here, so that will be the ultimate solution. It is clear that depending on the value of this term zeta omega n, you have the exponentially decay.

So, that is the condition of either it being real or complex and that is exactly why, we wanted to express it in this particular form, this specific form, so that this zeta becomes clear. What is the role of omega n, omega n's role will be clear, when you consider the complex conjugate Eigen values. So, when you have complex conjugate Eigen value; that means Eigen values.

Let us consider, second the real and equal, then we will consider the complex conjugate, when that happens, zeta is equal to 1. That is the critical case, then the solution will be, you have already learned that, let us see q t is equal to what will it be, yes, it is no. That, there are there are 2 components, one component is E to the power minus zeta is 1, omega n t times what, c 1 plus c 2 t, so that will be the solution at that case.

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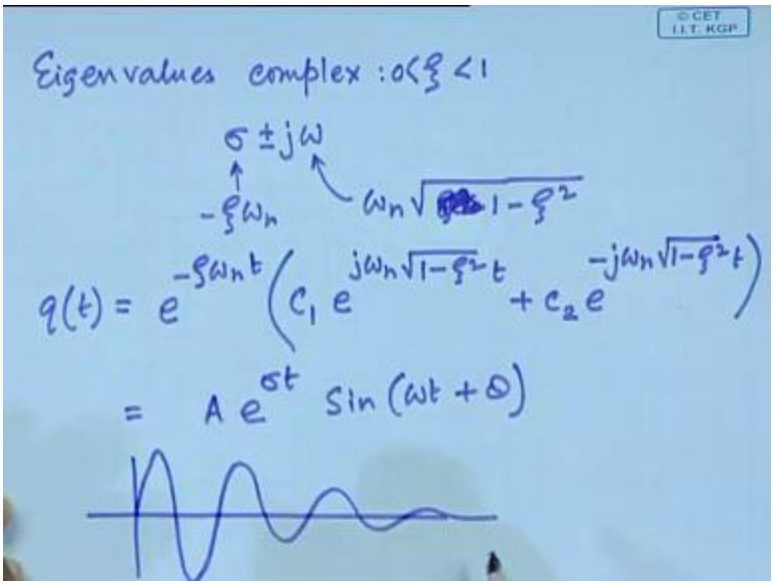

Now, let us come to the third case, where the Eigen values are complex conjugate, when that, does that happen, when is less than 1 or you can say this. Then, what are the Eigen values and what will be the solution, you have already learned. So, write in that form, it is, in that case we have to write it in sigma plus j omega. So, Eigen values complex would be sigma plus minus j omega, where sigma is minus zeta omega n and omega is, omega n root over zeta square minus 1.

Sorry, 1 minus zeta square, because you have to get a negative out of it. So, these 2 are the zeta and the sigma and omega, so in that term, now we can write it yourself. In that cases solution is q t, I am not illustrating the whole thing, why because we have already done it and the solution is exponential. The real part, first it is minus zeta omega n t, c 1 e to the power j omega n root over 1 minus zeta square t plus c 2 e to the power minus j omega n root over 1 minus.

I am now doing the other possible way, which I stated. These are 2 Eigen values, which simply write in terms of that and then, if we simplify that, it will always yield something like. Yes, we can write it as A, then e to the power sigma t times or sin omega t plus, we have already seen that. So, I am not illustrating the point is we start and then we arrive at this point.

So, here you have some amplitude, some sinusoid and some exponential envelop on which it is done and the exponent is in this case minus zeta omega n. Now, let us see in this case, your omega the frequency of oscillation of the sinusoid is not really the undamped natural frequency, it is somewhat modulated by this term, it is slightly different, this is the reduction term, this is the exponential term, here. So, these 2 will ultimately govern the behavior.

But, in order to understand, so what will be the behavior like, if the sigma is negative, then it will be so if the zeta is between 0 and 1, their behavior is normally like this. But, depends on the value of sigma and the sigma is zeta omega n, so that tells us, whether it will be exponentially growing or reducing. Now, notice that when we choose this particular terms, they have the relation that.

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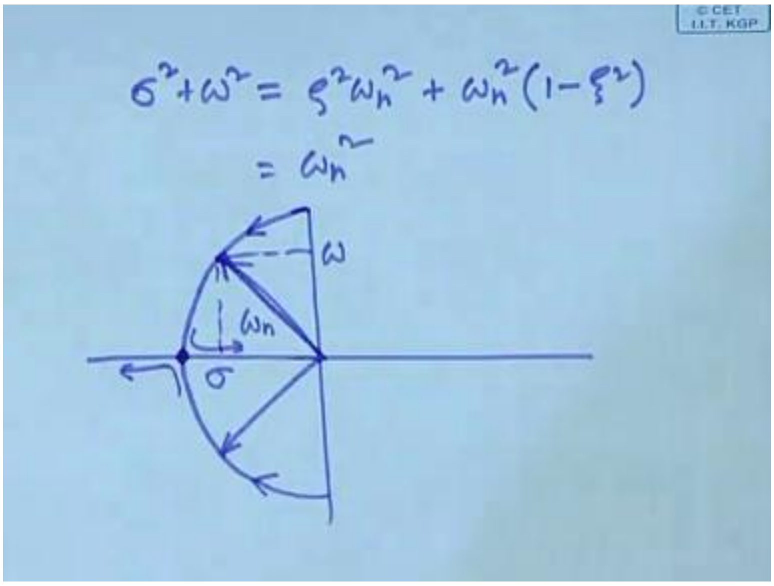

If you write sigma square plus omega square, can you write that, what was sigma square, it was zeta square omega n square plus omega n square into 1 minus zeta square. So, this is another reason, why we do it this way. The point is, that in that case, this and that square is this, which means it is the Pythagoras equation. So, you have, if this is the complex plane and if you are trying to plot this, this is the real part will be plotted along this, this is the imaginary part will be plotted along that.

And then, if zeta satisfies this condition, then the Eigen values would be complex conjugate, say like this, sorry this part will be omega and this part will be sigma. Now, suppose I change this as a result of which this moves, what will happen, this tells us that this will always traverse on a circle. There will be 2 complex conjugate roots or Eigen values complex conjugate and this complex conjugate Eigen values will move like this.

So, if you are changing zeta, keeping the omega n constant, then it will move on a circle with a radius of, what is omega n. Omega n is, you can see, if you look at this, if you put zeta 0, then what has happened, this comes to 0. As a result, you get a purely sinusoidal oscillation with frequency of how much, omega n. So, if the system has no zeta, which means this term is 0.

This term is actually resulting from the friction in the system or the resistance in a circuit. If you go back in your thought to the way we were derived the equation, this term comes from there. So, in that case the natural undamped frequency is omega n, natural frequency of oscillation, say a l c circuit, which you leave to itself without any friction or something like that without any resistance. Then, it will oscillate with a certain frequency and that is that.

So, if you now add a resistance then and if you keep varying the resistance, then it will keep moving like this and then after sometime, say you change the resistance, it was moving like this and after sometime, it will collide. At this point, this equation will be where is it, where did I keep it, that equation we just wrote, which pertains for real and equal Eigen value that will be valid. And after that, what will happen, one will move like this, the other will move like that, so they will go out. And then the first equations that we wrote that will be valid, because of these conceptual advantages, we always express second order equation in this form. Let us do some exercise.

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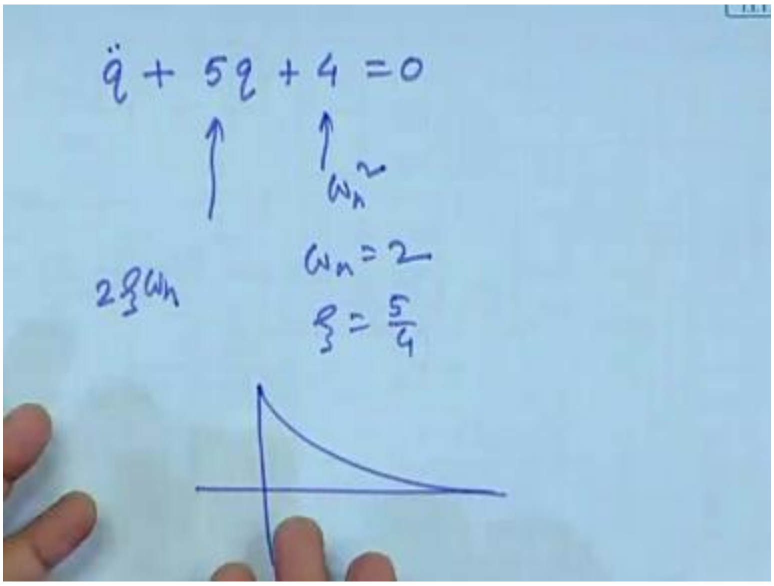

For example, if you have an equation like q double dot plus 5 q plus 4 is equal to 0, what will be their behavior, can you immediately tell, what will I do, we will simply say, this is the omega n square. So, omega n is equal to 2, this thing is twice zeta omega n, 2 and then this is 2, this zeta is 5 by 4, so this gives zeta is equal to 5 by 4, what will be the behavior then, it is greater than one. So, it is real and distinct and therefore, the behavior is going to be behavior is going to be exponentially decaying. So, without solving the equation, we could directly tell. So, that is exactly why we do this, you always try to express in this form.

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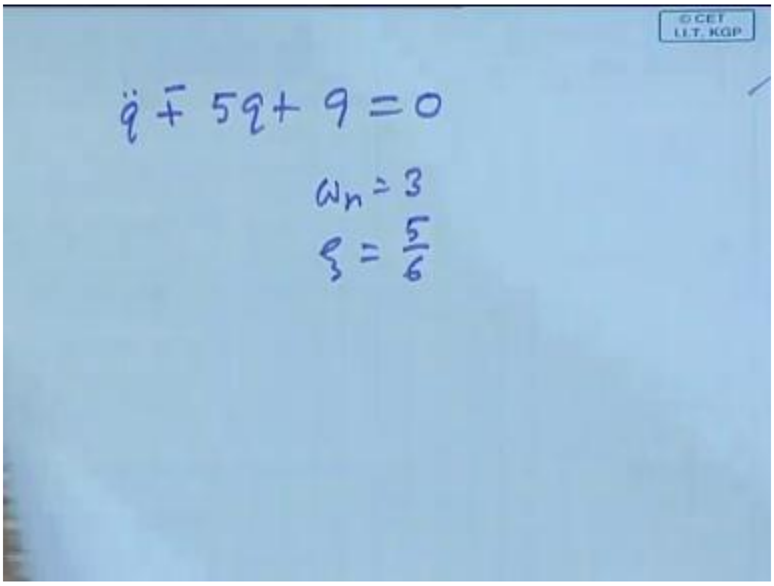

Suppose, this is the equation that we have is something like that q double dot plus say 5 q plus 9, what will be the behavior decaying sinusoid decaying sinusoid. How do you know, in this case your omega n is 3 and zeta is 5 by 6. So, this is less than 1 and therefore, it will be decaying sinusoid, when will it be a growing sinusoid which term is negative.

Student: ((Refer Time: 31:38))

So, this term is negative, yes it will be growing sinusoid. So, can this term be negative, no it is omega n square, so that is how we figure out, how the behavior will be, just by looking at the equation. You do not need to solve really and unless you do this exercise, you will be always be constrained to solve equation and then only say, this will be the behavior. So, at this stage, let us do some simulation with a computer. Let us take a second order system and we will do it by bond graph, what is a good second order system.

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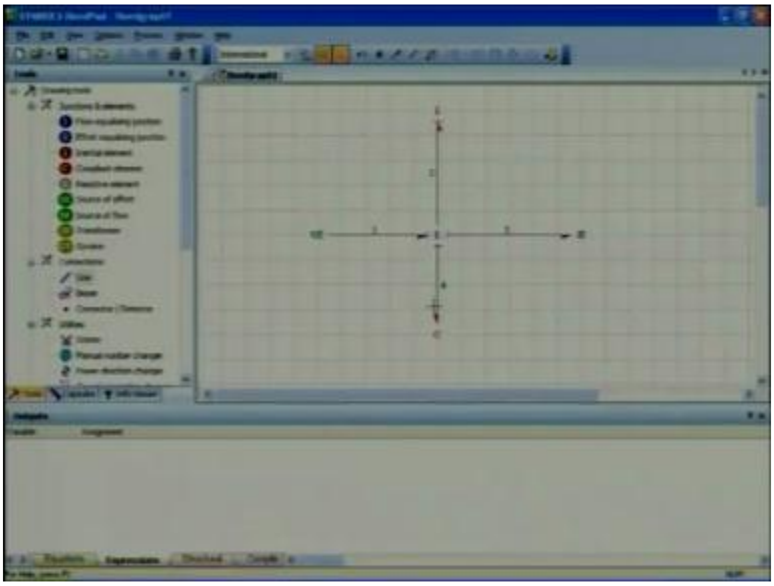

Can you show the computer please, yes, so what is a good second order system. A mechanical system in which you have a mass, spring, damper with some force in. Mass will give one state variable, spring will give another state variable and you have already seen that is nothing but electrical equivalent of a series circuit. Let us do that because, it is second order, we can test.

So, first it will be one junction, then it will be an a c, then it would be an I, then it would be a c, can you see that, then an r. Then, we will connect by the bonds, the lines are somewhat thin is it visible there, good. So, after we have connected, we will give the numbers, we will give the power directions, we will give the causalities. If you cannot see the causalities, because the circles, there we will drop the circles, hide circles, yes.

So, you can see are the causalities is right, a c is receiving the flow information, I is giving the flow information. So, in the one junction, the flow information has come, it is distributing in the other bonds, In any case, that needs to be tested, so check integrity, it says 0 errors 0 warnings, so we are fine. So, then it is to generate equations, I have pressed it and it has already written the equations.

Probably, that too small for you to read is it, I will read it out e 2, which is the second bond here is a c 1 write that minus R 3 times p 2 by m 2 minus k 4 times q 4. This state variable is q here and f 4 is p 2 by M 2. So, these are the two equations, what is expected. Now, we have already generated the equations, now we have to define the maps, what are the state variables q 4 and p 2, we could give a name, but we understand what they are, so let us leave that.

Then, it is to generate the simulation code, I will give it as bond graph 2, same, it overwrites. Now, we have to compile, this is a compilation, so set and compile, let us see what it writes, so it says 0 error 0 warnings. So, we are happy, then we go to the simulation, make it bigger, good.

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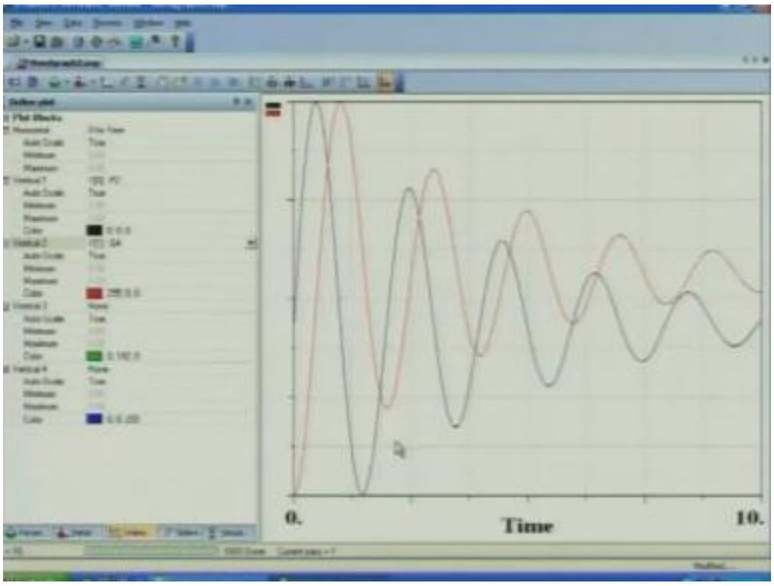

In the left hand side, it shows the 4 parameters in the system and we can set their values say we set a c 1 as 1, M 2 1, say 1 kilogram, K 4 say 10 and r 3, may be a small value. Now, write down these values, M 2, M the mass is 1 k g, k 4 is 10, r 3 is 0.5, in this case the bond graph was, let us come back here, yes.

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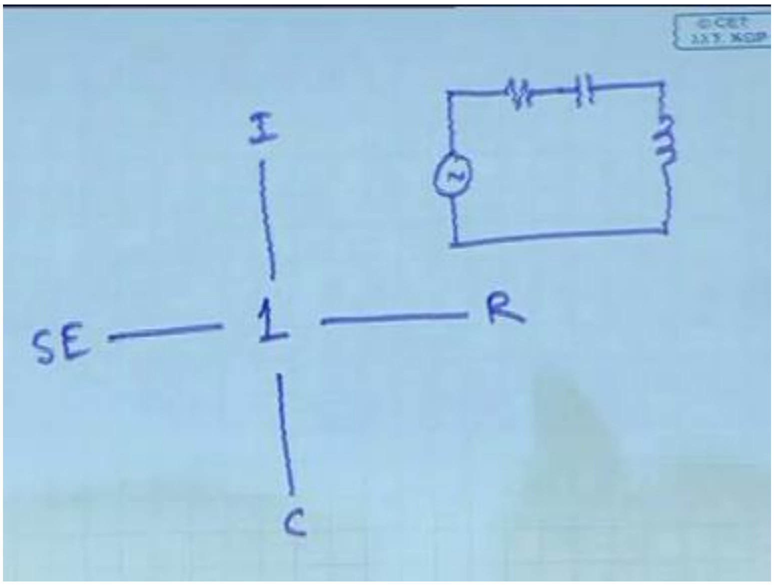

1, S E, I, R, C, now in this case, your circuit is simply, for this you can write down the equation, yourself no problem and it will be a second order equation. Write it down and tell us for this parameter value, what you expect the behavior to be, you understand, what the exercise is. Exercise is for this write down the equation in the second order form and from the parameters, that you have, what you expect the behavior to be.

The circuit is too simple, does it quickly, write down the equation, substitute the parameter values and tell us and how it will behave. Come on do it fast, the simple question, I did not give a tough problem; this is a very simple problem. What will the behavior be, yes Vamsi why are you taking so long, become sinusoid, let tell me the equation.

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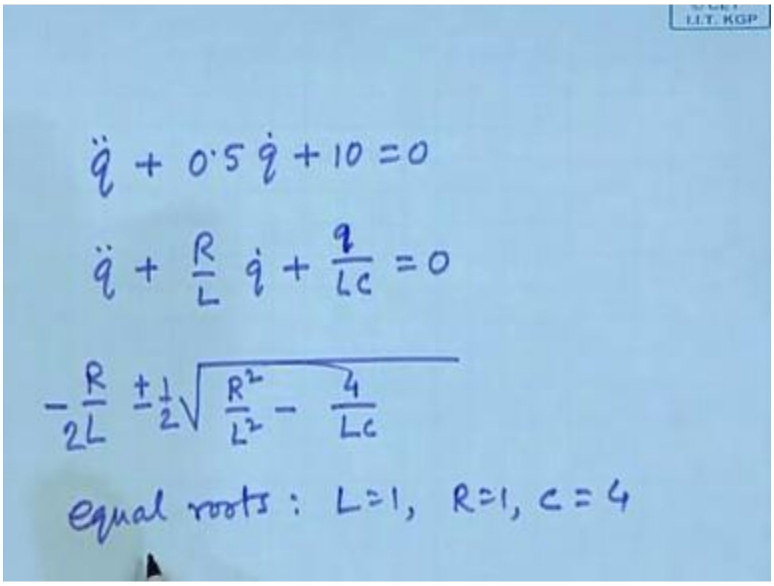

q double dot plus 0.5 q dot 10 is equal to 0, can you in general give this values; that means in terms of the parameters. So, q double dot plus L q dot plus I 1 by L C is equal to 0, q by L C. For this these things you have already found out that it will be a decaying sinusoid. Let us check, we will start the simulation at 0 times and we will end it at 10 seconds and then, I have to set the plot blocks.

The horizontal is time and the vertical 1 is p 2 and the vertical 2 is q 4, now I will simulate. So, it is a decaying sinusoid, can you see that and the 2 are out of phase with each other by some value, what value do you expect to be, check that. So, this and in the state space, it will be incoming spiral. Let us check that, so horizontal will be then p 2.

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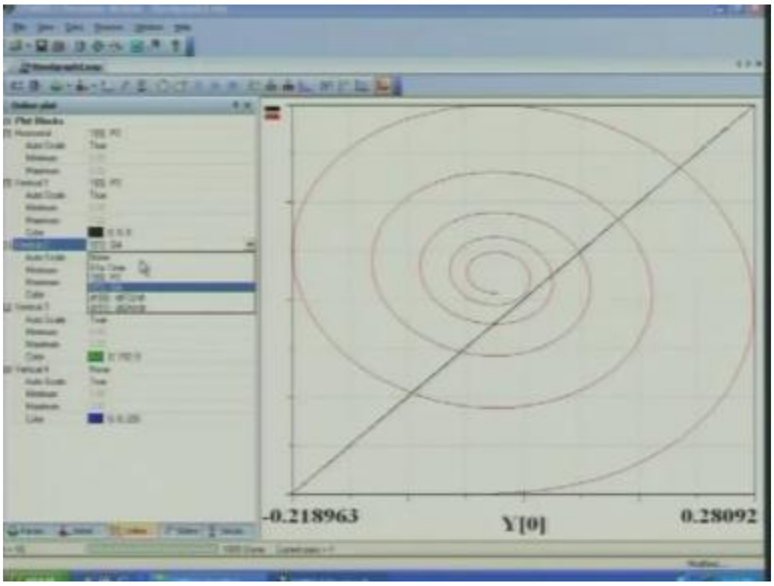

This one, I do not need, so I will none, this one will be q 4.

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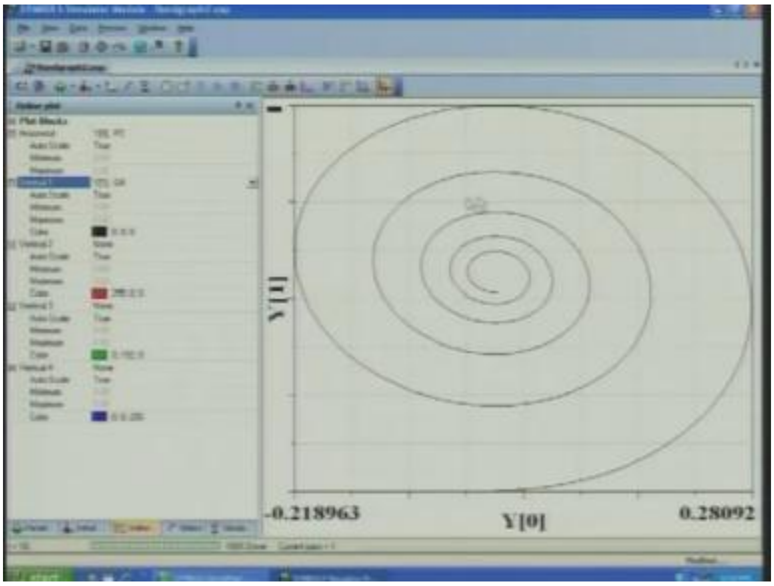

So, the behavior is a decaying spiral, starts from here and goes on, where does it ultimately converge, equilibrium, where at what value, does it ultimately converge. There is an input voltage and therefore, it will not converge at 0 0, it will converge somewhere and it is actually converging there, it is not converging at 0 0. Now, tell me how should I change these parameters, so that I will get exactly a equal real roots.

How I should change, when will you get the real roots exactly equal and what should I do in order to make it real Eigen values. R equal to 2 will give. For that, quickly write down the characteristic equation and the Eigen values for this; quickly write down the Eigen values for this. It will be minus R by L plus minus R by twice L root over is that is 1 by 2. So, when will it be exactly equal, we have to just set this equal, so what value of the parameter.

Say R equal to something, given L C constant, root over 40 bad number, it is difficult to set that…

Student: ((Refer Time: 44:24))

So, equal roots would be, give me good suggestions.

Student: ((Refer Time: 44:36))

L is equal to 1, R is equal to 1, C is equal to 4 and for what condition will it become real and distinct.

Student: ((Refer Time: 44:58))

R less than…

Student: ((Refer Time: 44:59))

R greater than this, let us check. So, we will set the parameters again, M 2 was 1, which is L is 1, K 4 would be.0.25 and R would be1. Let us at this stage, plot it for the horizontal axis should be time.

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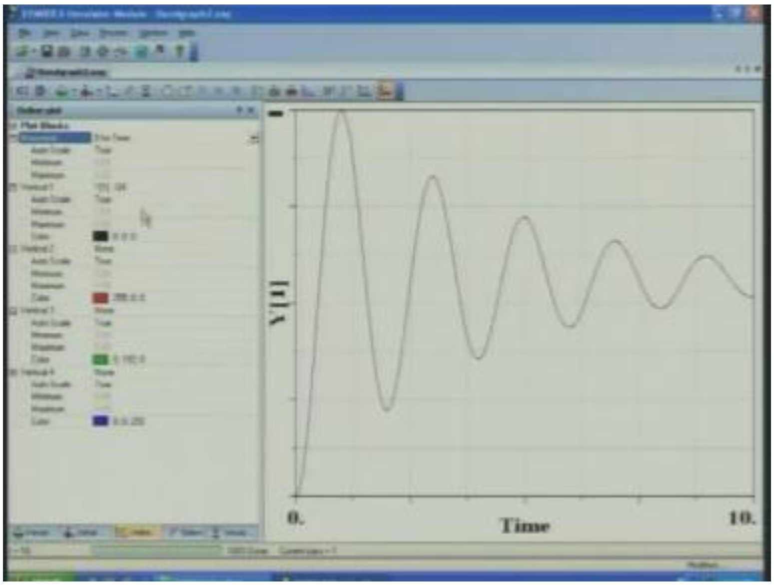

And no, not this, I have not re plot simulated it, vertical axis should be q 4 and y, this looks.

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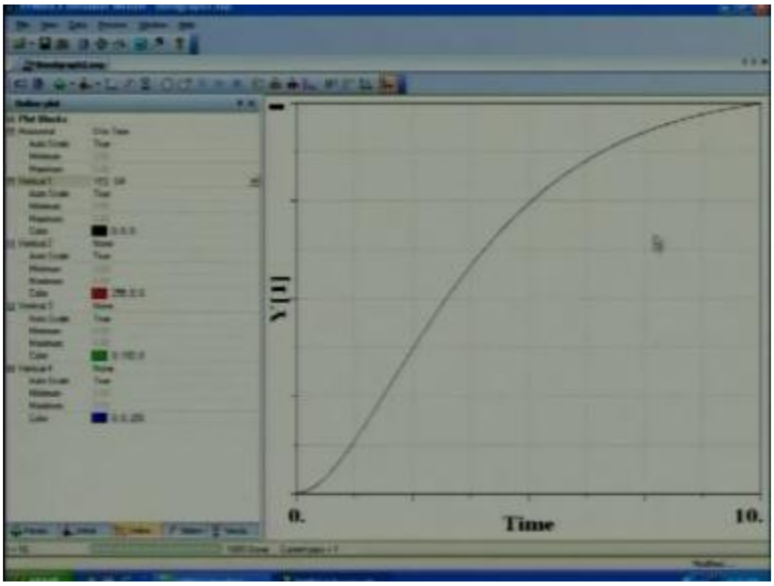

Now, I have to re simulate, so this is the kind of response, you expect from a critically damped system. It is reaching that not yet, you want to simulate further, let us see for a longer time 20 seconds.

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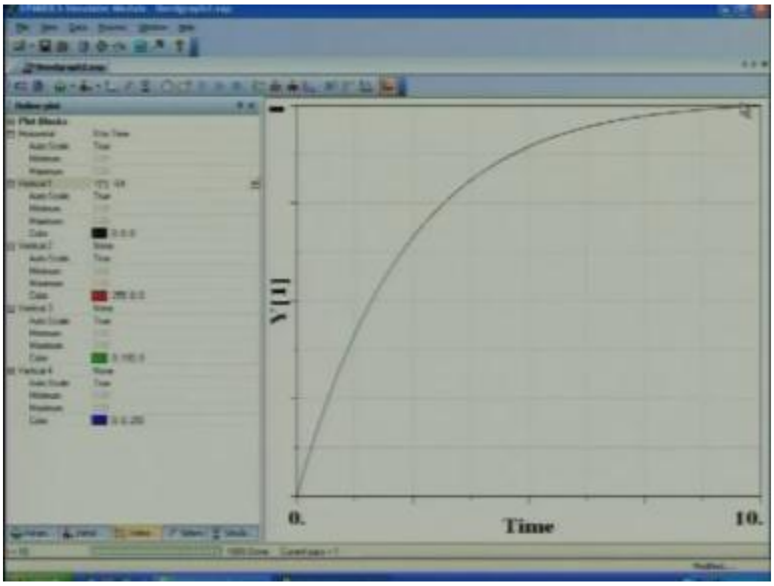

It is reaching that value, but never going above something like that, if you see that in the state space, then I will have to plot these as your p 2.

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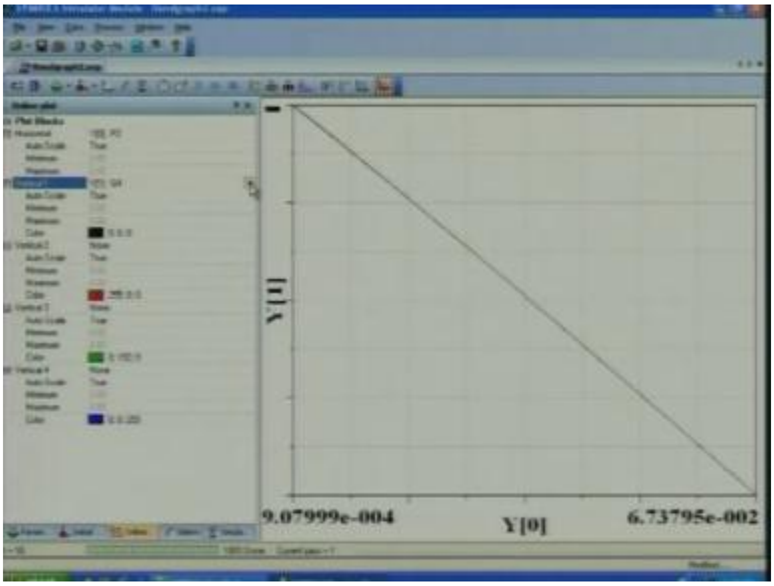

And these as your q 4.

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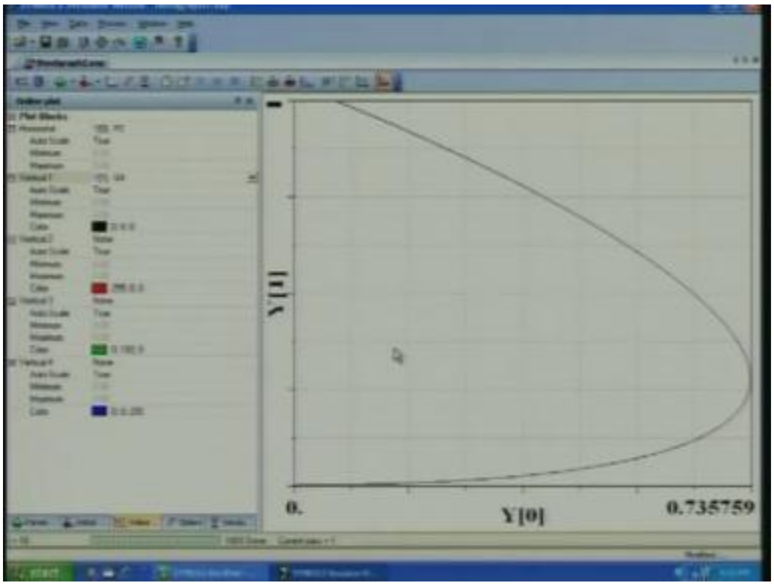

Difficult to see, because in this case, you are not seeing the equilibrium points, but is going equilibrium point, that way. And you say that if you raised are beyond that, then it will be two distinct ones. So, let us set it to happy, let us check.

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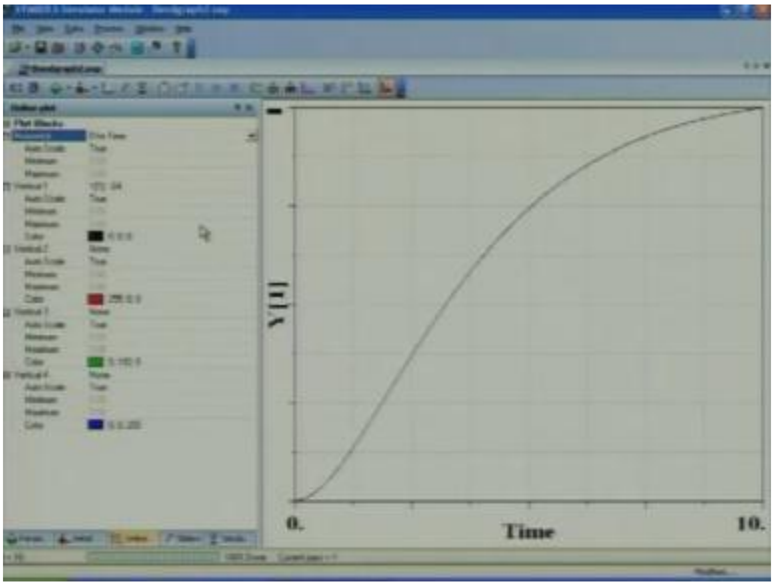

Now, I want to set this since proper, I want it to be time.

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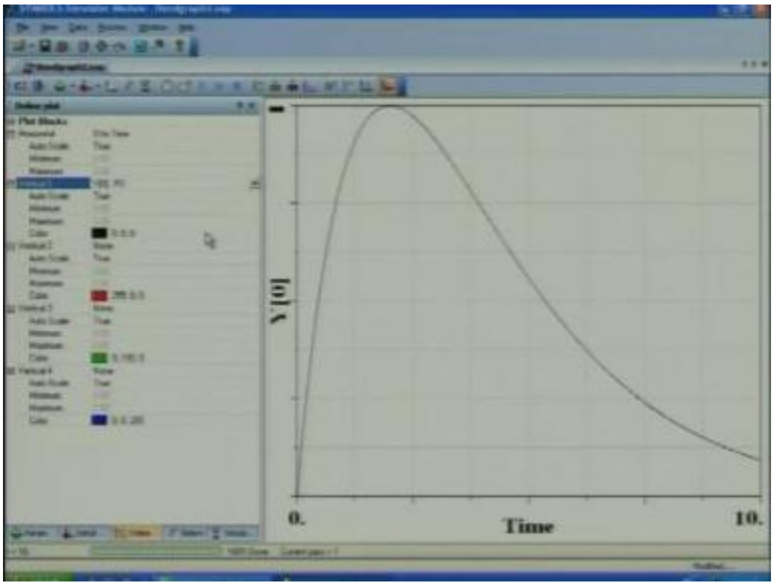

And I want this to be, now I will, let me simulate it once again see. If you want to simulate it for a longer time, let me say 100 seconds, I do not mind that.

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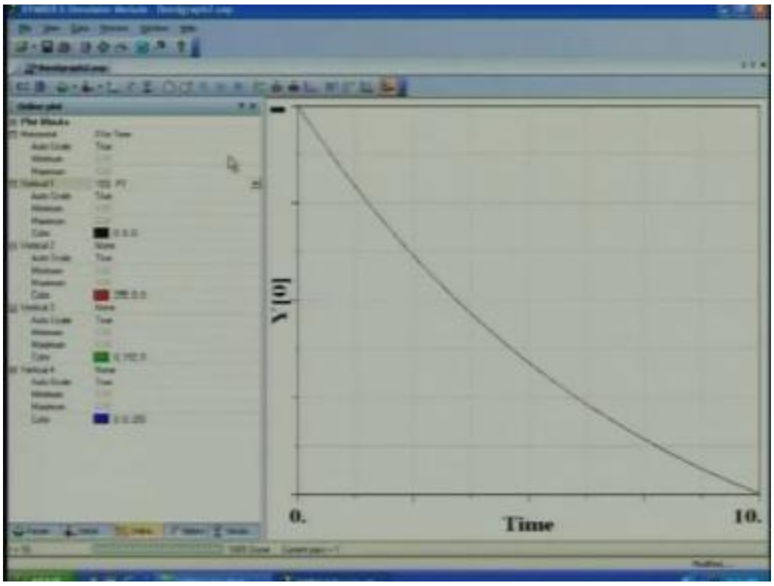

So, from there, it is going or we can set the timings to 0 to 100.

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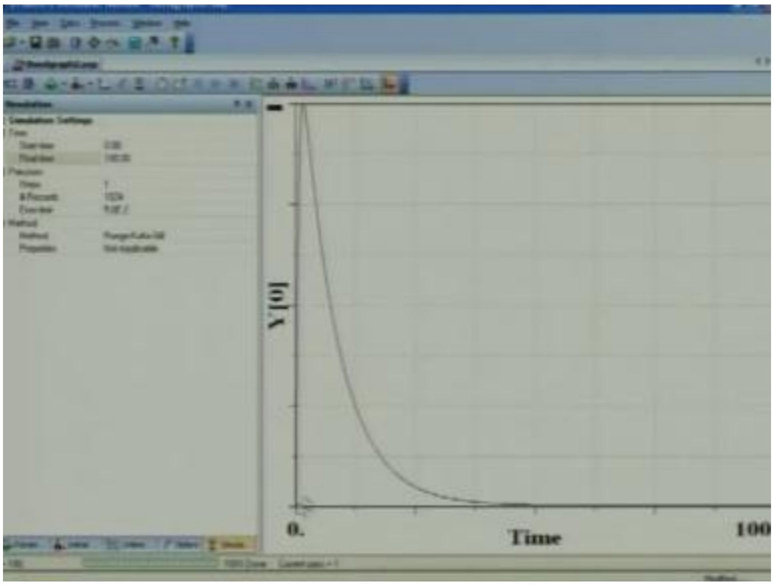

See, it is going and then it is decaying, 100 would be a bad time, so let me change it to 0 to 50 to c.

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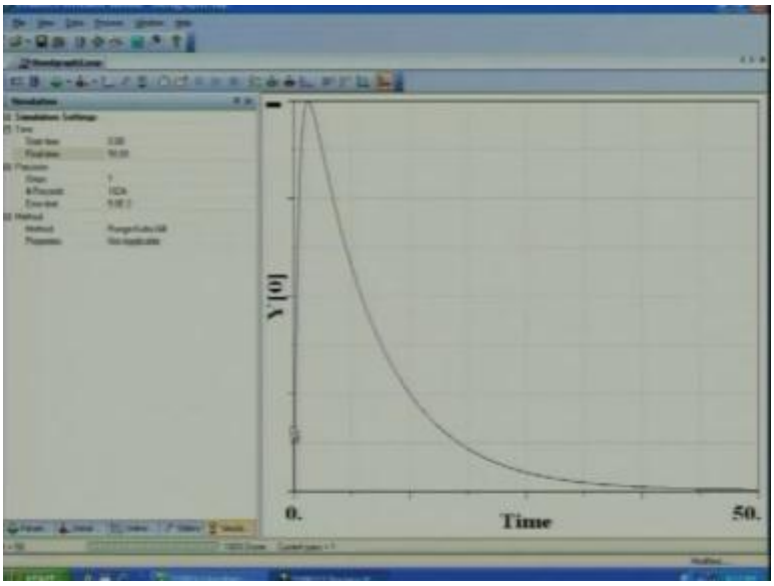

So, it goes and then it, now what did I plot this time, I plotted, no time and p 2, p 2 is the current through the inductor. Notice the circuit, the current through the inductor, because of the capacitor must go to 0. So, it goes up and goes to 0, but what is the capacitor voltage, that is stabilized some other value. So, if I plot something else say q 4, it is stabilize there, but this is a over damped system, it is a over damped system. So, if it is slightly under damped, then what is the behavior, slightly under damped, I will make it say 0.5, no, 0.8 may be one was the critical case.

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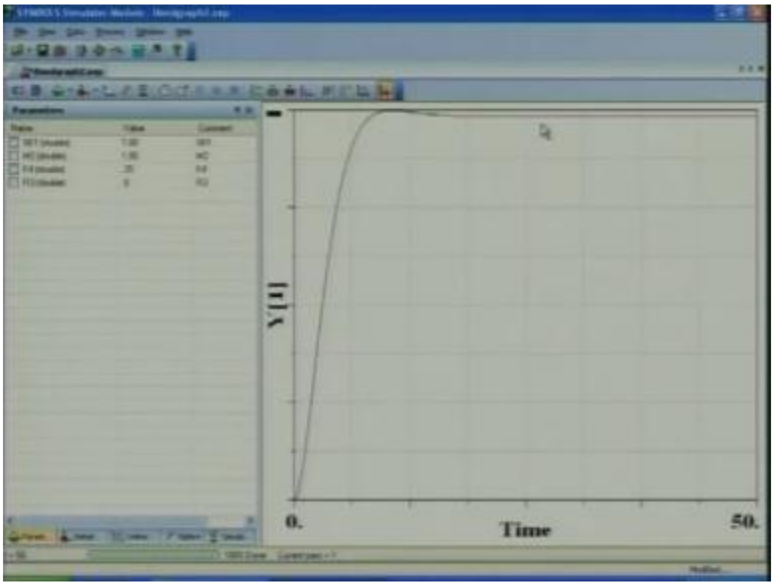

See, it is slightly above and then come back, if you plot it, p 2 versus q 4, see it has gone, but then the large loop and then it is going there under damped. So, it is spiral still, but the spiraling is quite different than what you saw initially. Now, these are the things that you have to understand, I did this deliberately, because when you look at a spiral or if you derive, that it will be a spiral. The spiral may not be that entire nice spiral that comes into view like a spiral galaxy or something like that.

It is often, you have to apply your mind to see that, it is a spiral, it is actually spiraling, in this case it is spiraling into that place, but that is not all that clear. You have to see; you have to apply your mind to see. So, you have understood, what happens for two dimensional systems, what happens for three dimensional systems. See, so far we were doing two dimensional systems, two dimensional systems that can be plotted on the piece of paper.

And we have founded, there are exactly four different types of behaviors possible. One is, no, there actually many, but three distinctly different cases out of which, there can be less than 0 Eigen value or greater than 0 Eigen value, depending on that. It will be either decaying or growing up, but nevertheless, you understood three cases.

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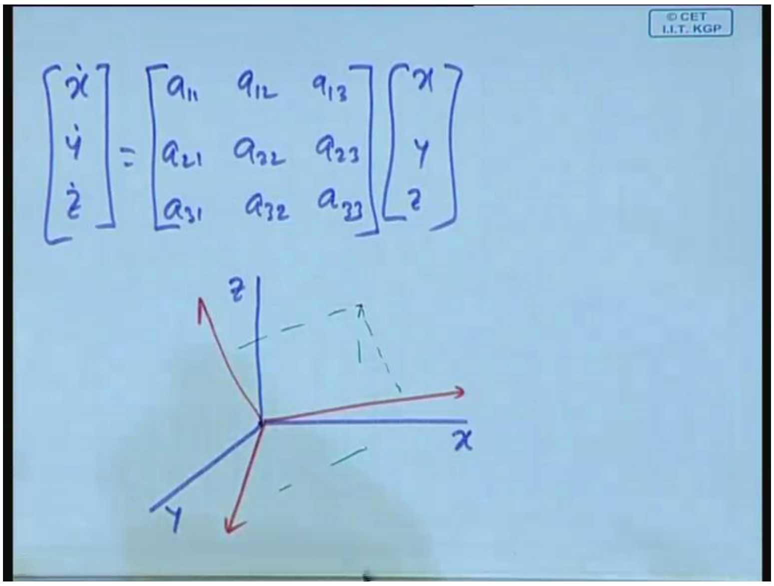

If you have a three dimensional systems, which mean you will get an A matrix which is 3 by 3 matrix. A 3 by 3 matrix means x dot, y dot, z dot will be in the local neighborhood, this would be the equation, which means it will have to be plotted in x, y and z. Now, what will be the meaning, still you will be able to apply the same logic, what was the logic, that in order to solve it. We will write down the independent equations and we will compose the general equations out of that.

How many independent equations will you need, 3, now 3, so you need three equations and you will compose the general solution out of that. So, composing three equations and obtaining general solution out of that, will require you to compute three Eigen values and three Eigen vectors. Now, this will lead to a cubic equation for obtaining the Eigen values.

A cubic equation can have what kind of different possibilities, imagine, earlier we could only say it could be real or complex conjugate. Since, if there is a one complex Eigen value, the other has to the complex conjugate, see that two will always be complex conjugate. So, if this complex, it will be complex conjugate and else it will be real. So, either all three real or one real two complex conjugate, that is the only possibility, nothing more.

So, the possibilities are actually limited not very difficult to figure out. So, if you have this, you obtain the Eigen vectors. Again, say it will be an Eigen vector like this, Eigen vector like that and say another Eigen vector like this. You can obtain three Eigen vectors, arrows may be like that and arrows would be determined by their Eigen values. If the Eigen values are less than 0, it would be inward arrow. So, depending on any starting initial condition, the same logic will apply, that you will first look at their projections on the three like this.

And for this one, you have to it will come down and then you got this way. In any case, you have to look at the projection on the three Eigen vectors and these projections with either grow or decay depending on the corresponding Eigen values. So, the dynamics will have to be visualized in the three dimensional space.

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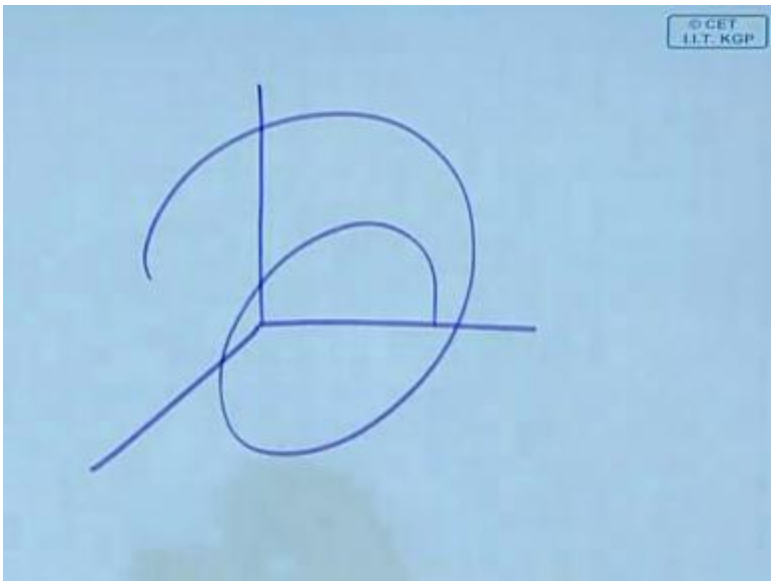

We will have to visualize in the three dimensional space, say something like this will happen, whatever it is, but that will happen at every moment, the behavior will be guided by the directions of the Eigen vectors. If they real, direction of the Eigen vectors and how they either contract or expand, along each Eigen vector. So, obtaining the exact solution is not all that important, understanding the behavior is more important; understanding will come in terms of the values of the Eigen values and the direction of the Eigen vectors. Try to do one more problem, before we come to the next class.

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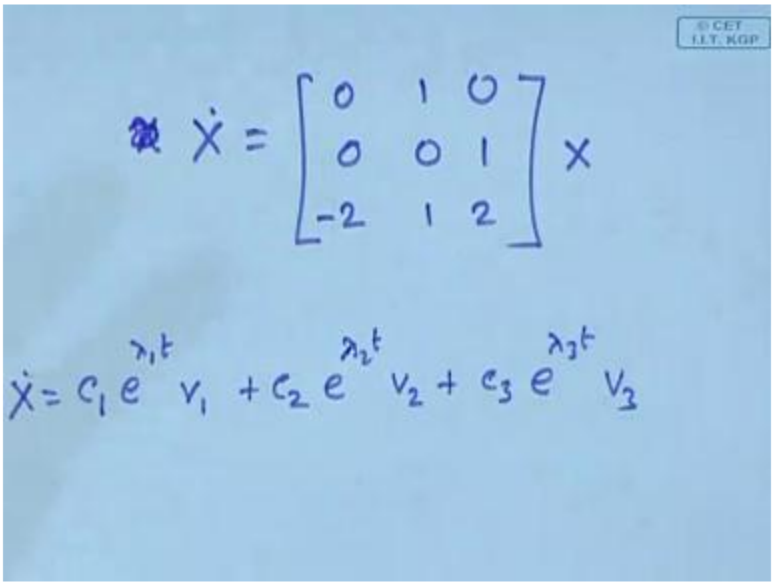

If capital X dot is equal to 0 1 0, 0 0 1 minus 2 1 2 times X. Then, what will be the behavior like, how will you do, from these you will obtain the Eigen vector values, from there you will obtain the Eigen vectors, Eigen vector directions, Eigen direction. And then, you will say that now, I will fix certain coordinate and then obtain the other as a result; we have got a specific Eigen vector. And then we will able to write down the equation, simply as X dot is equal to C 1 e to the power lambda 1 t times V 1 plus C 2 e to the power lambda 2 t times V 2 plus C 3 e to the power lambda 3 t times. That is it pretty trivial, actually no complication, so just do this before coming to the next class, that is all for today.