**Dynamics of Physical Systems**
**Prof. S. Banerjee**
**Department of Electrical Engineering**
**Indian Institute of Technology, Kharagpur**

**Lecture - 21**
**Dynamics in the State Space**

Once you know how to do that we also have to know how to understand differential equations means, what I mean is that you have a set of differential equations expressed in this form.

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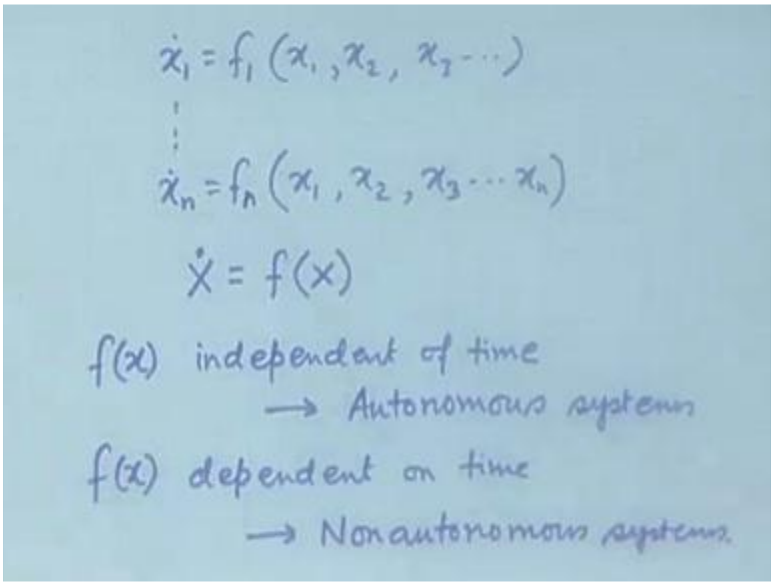


X 1 dot is equal to some f 1 x 1, x 2, x 3 and all that, similarly ultimately have x n dot is equal to some f 1 f n x 1, x 2, x 3. Now, this set of equations would govern the evolution of any starting point. So, you have the equation actually expressed in this form x dot is equal to some function of x, these are vectors then. Now, this means that if I ask you to study the behavior of any given system, that means you have obtained the differential equations and then, your task is to find out how the system will behave.

What do you actually have to do then, then start from an initial condition and then, evolve it see the behavior. Then that particular behavior is only for pertaining to that particular initial condition. If you then change the initial condition there is no reason to believe that it will behave the same way. Then the whole problem becomes enormously big, because in that case you will have to start from infinitely many different initial conditions, see their evolutions by the the method that you have learnt.

And then, after having done all that, after having an infinite number of graphs before on your table, then you say this is what is going to happen. That is the task of studying the behavior in any system, then becomes enormously complicated. A relatively easier task might be to have a qualitative idea about what this system can do, what are the different types of behavior that are possible for the system. And in order to do that, in order to understand that, that means somewhat qualitatively exactly where it will go you will have to solve it.

But, if you want to qualitatively understand that these are the possible behaviors by the system, then we will not go by this numerical routine route. Whether we will try to understand the differential equations what do you mean by that, we have already said before you go in to that we need a bit of nomenclature. Now, this right hand side could contain time varying terms, could also contain time independent terms. That means, for example, the simple pendulum in the right hand side you would have time independent terms.

What was the equation for the simple pendulum x dot is equal to g by l sin theta, so theta only no problem. If you have the point of support oscillated somewhat externally, then that would introduce some externally impressed function in the right hand side, which would be dependent on time. So, whenever you have the right hand side f x, independent of time such systems are called autonomous systems, and where f x dependent on time non-autonomous systems.

So, whenever we will refer to these two terms you have to understand what it means physically. It means that this f x right hand side, is in one case independent of time, the other case dependent of time. And physically if there is a circuit, in which there is a sinusoidal forcing function that would be a non-autonomous system. If there is a circuit in which there is a DC supply, then it would be autonomous system, because that is independent of time.

But, if you switch on and off that DC supply, then it would again become a non-autonomous system, because that will become a independently externally applied time variation. So, if you have this system description, then how do we proceed with it. First I said in the last class, that if you have say let me do it in 2 D. So, that I can explain it by drawing things on the sheet, but you can easily in concept extend it to higher dimensions.

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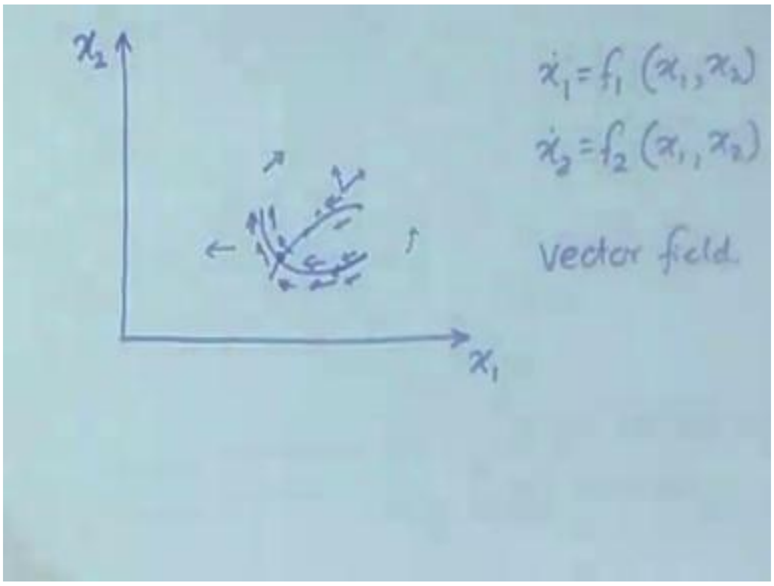


So, your two dimensional system equation where x 1 dot is equal to some f 1 x 1, x 2 and x 2 dot was equal to f 2 x 1, x 2, so we will try to understand this. We have already said that these equations immediately define a vector at every position of the state space. Which means that, if you have the state space drawn, which means the state variables are the axis then every point will have a vector so and so forth.

So, every point in this space having a vector this particular thing is also called the vector field. So, the vector field, the term vector field would mean that a space in which every point has a vector associated with it. Now, it is not difficult to see that, all that this system can do would depend on the nature of the vector field. So, in which direction does the solution lead will be the given by the vector field and naturally everything that system can do will be given by the vector field.

So, instead of starting from all possible initial conditions and trying to work out the evaluation by numerical means, if you simply look at the vector field it will give a better idea. So, one of the things that you would need to understand or do, would be to draw the vector field. Now, most of the modern, higher level computer languages offer the facility of drawing the vector fields. Mat lab has this facility, Maple has this facility, Mathematic has this facility, so learn how to draw the vector field for a given system 2 D system, in 3 D you will would not be able to draw, so it is better to understand in 2 D.

And then, in 3 D you would imagine that this 3 D is a state space x, y, z and then, every point then you will have a vector, a 3 D vector. And then, the state will evolve depending on the vector field, but obviously we will not be able to draw that, because that will become a projection. And if you draw the projection it will lose some of properties, what properties I will come to.

One of the properties is easy to see that, at one point that the vector is unique, you cannot have this type, why because the right hand side is unique. For every point you will get a specific value here, a specific value here meaning that this vector is unique, which means that this vector, this is cannot intersect. You can also draw lines going through, means if you have vectors going like this, starting from any initial condition you can also draw lines going through like this can you see.

If you have lines going through like this and from another initial condition if you have a line going through like that, so you have the vectors like this. It is not possible for them to intersect why, because at this point then the vector would be two different directions that is not possible, that is not allowed. So, the vector field lines cannot cross So, these are some very qualitative character, properties of this vector field that imposes some restriction of on what it can do.

That way the character of this vector field, this lines they sought of match the properties of magnetic field lines know, the magnetic field lines never intersect, so magnetic field. So, more or less they have the same kind of property, but magnetic field lines do intersect where at the poles. So, will there be something like this here, yes they can intersect at the points where the vector is 0, then it does not lead to any contradiction. So, where would the vector be 0 the equilibrium points.

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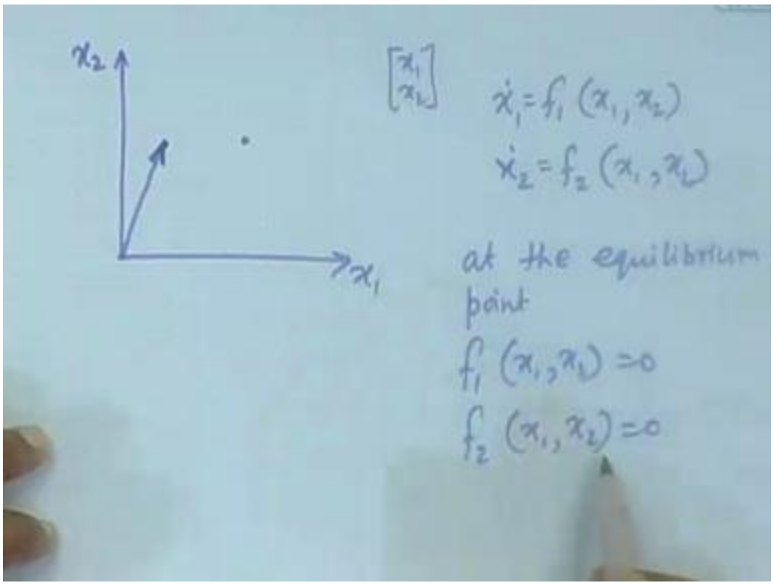


So, the equilibrium point is where suppose I have a space x 1 and x 2, and here is equilibrium point, what is the property of that equilibrium point, it is that if I start it here it will always remain here, it will not move, which means your x dot is equal to some function of x 1, x 2 and x 2 dot is equal to some other function of x 1, x 2, it means that it does not move means x 1 dot is 0, simply x 2 dot is 0, which means at the equilibrium point f 1 x 1, x 2 is equal to 0 and f 2 x 1, x 2 is equal to 0, so that is the property of the equilibrium point. So, let us just do an exercise to find out to practice this how to locate the equilibrium point.

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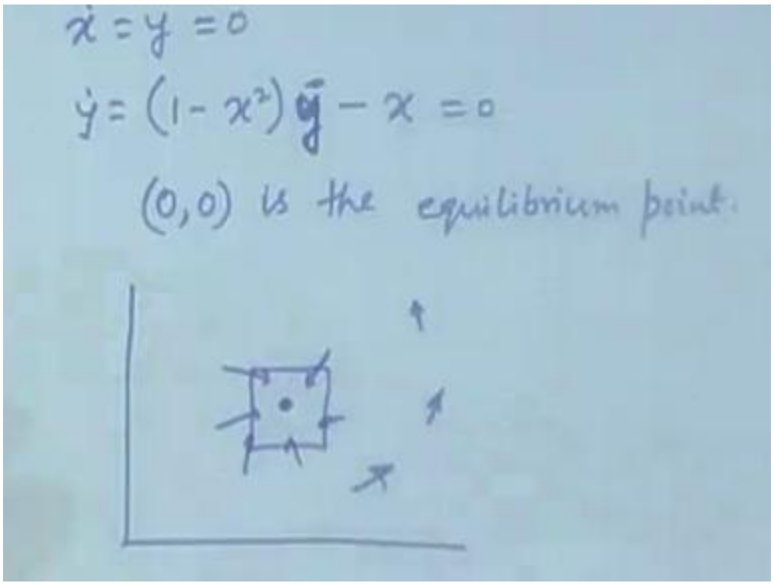

Suppose there is a system x dot is equal to y and y dot is equal to 1 minus x square x dot minus x or this is y minus x, so what is the equilibrium point. Well this term we put equal to 0, we will say y is equal to 0 and if y is equal to 0 x is also 0, this equal to 0 means, so 0 0 is the equilibrium point. So, the point is that, if you have a state space like this normally any point is given by a vector, the state vector. So, when I say x 1, x 2 it is algebraically a vector, geometrically also it is a vector.

So, this similarity between algebraically writing at as vector x 1, x 2 and geometrically seeing it is as a vector with an arrow it should be clear, there is a one to one correspondence between the two. And then this x 1, x 2 point this particular point will be an equilibrium point when this is satisfied, if an equilibrium point then it does not move. So, that should be understood, but it does not move, does not immediately mean that in physical system it will not move for example, if I have this one standing like this.

If it is exactly vertical it will not move, but will that carry much sense engineering wise no, because slight perturbation will make it fall. So, even though that is an equilibrium point it is an unstable equilibrium point. So, we need to understand whether the equilibrium point is stable or unstable. So, in order to do that we need to understand the behavior around the equilibrium points, what do you mean by behavior around the equilibrium points.

See normally you will have a vector here, a vector here something else happens elsewhere, but here supposing there is a behavior something like this. So, that this fellow is going to be stable, but that stability is can be understood by considering only a small neighborhood of that equilibrium point. So, I do not need to consider the things elsewhere. Now, even when an equation is non-linear like this, I can get away with studying only the neighborhood at which it would be approximately linear, or it can be approximated by a linear equation. So, how to do that, how do you linearize a non-linear function, this side is non-linear I can see that and how do you linearize the non-linear function.

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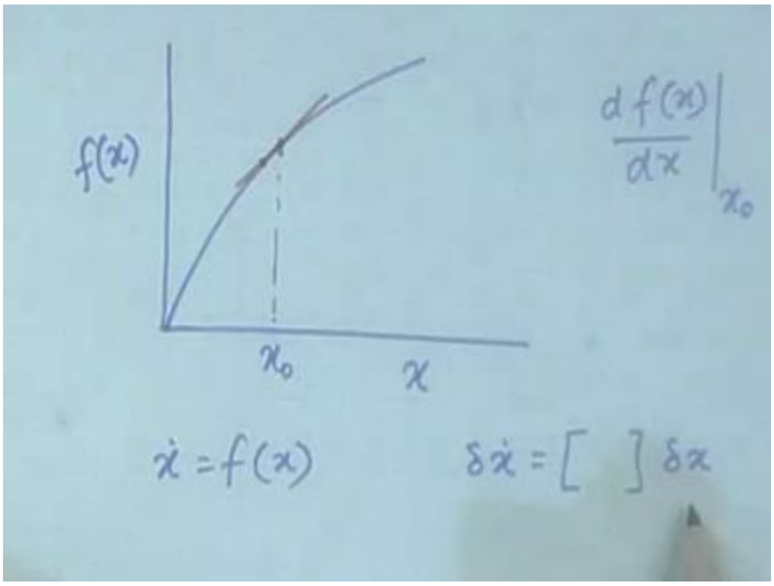


Now, if you have a functions something like this, say x and f x and say something like this, it is a non-linear function. How do you linearize, suppose I want it to linearize at this point, how do you linearize, you simply draw the tangent. So, you draw the tangent and say that this tangent represents my linear approximation around this point, that is what you say. Now, what is this tangent, the tangent is nothing but, calculated at that point, say if this value is say x naught then calculated at x naught that is what you do. When you want to linearize a function you do exactly like that. Now, if you have a function like this in 2 D, then how do you visualize that. It is for example, imagine only this y dot is a function of x and y.

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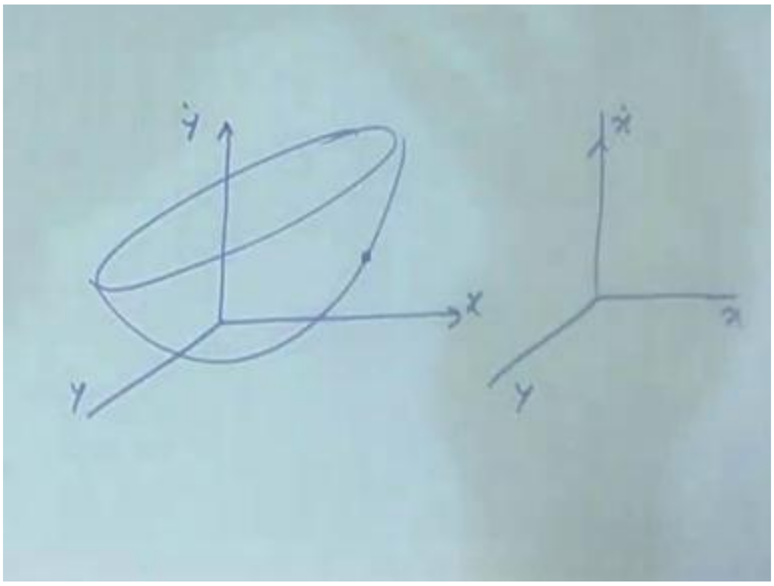

So, you have, something like this you have x y and the x dot is some kind of a surface here, imagine it will be some kind of a surface, some surface. And suppose now you want to locally linearize at this point what will you do, the local linearization of a surface will be a plane the plane will have two inclinations, one this way along the x direction another along the y direction. So, that will suffice in defining a plane at this point that is exactly what we do.

So, in defining the y dot you will have two such values, one how does it vary with x and how does it vary with y. Similarly, for x dot you will have two similar values, so you will have actually the thing given by there is another issue here, that is suppose your equation is x dot is equal to f x, the f x any problem.

Student: ((Refer Time: 18:40))

No, no I did not draw it that way here, I am not representing the equilibrium point.

Student: ((Refer Time: 18:48))

Yes, I am linearize just at any point and there is no reason to believe, that the equilibrium point will always be at the origin, it could be at any point. So, I am just saying that the equilibrium point, wherever the equilibrium point you will have to locally linearize it, schematically I have shown it here. That does not mean it, it cannot be done here it cannot be done here provided the equilibrium point is here.

Now, suppose this was the original function and I want to locally linearize it here. Then what would the local linear equation be, this is the differential equation locally linear differential equation will then be notice that, here from here the deviation, so it will be the delta x dot is equal to some linear function, so something here times delta x. Notice that earlier it was here, starting from here where the 0 was that is a different issue, but now that you are linearizing around that point, now you have to consider deviation from that point.

In other words, your origin shifts to this point, in the new representation, the origin shift to this point that is a vital issue that some people miss, while developing the understanding. While you are doing the local linearization, you have to move the origin to this point around which you are doing the local linearization. And then, the variable becomes the deviation from that point, deviation from that point not the original one and then only this local linearization will be valid. So, when we do the same thing here in such a system, we will also do the same thing.

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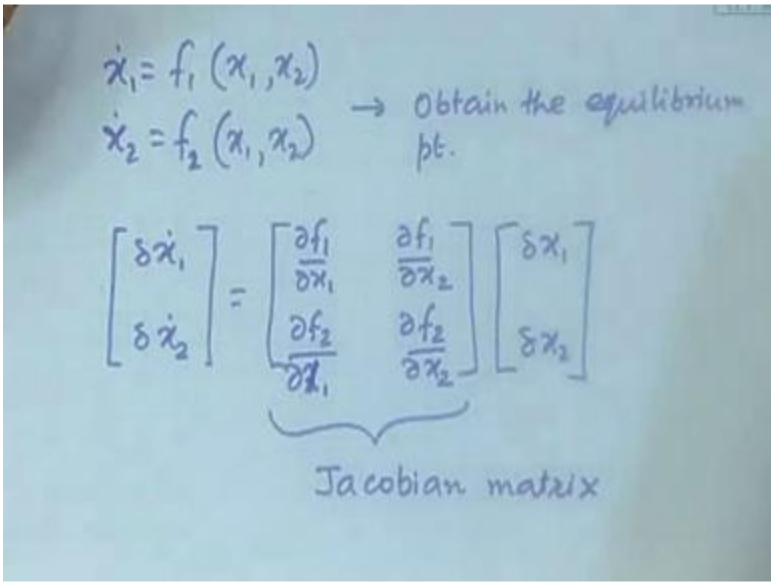


So, ultimately if you have originally the equation given in the form x 1 dot is equal to f 1 x 1, x 2 and x 2 dot is equal to f 2 x 1, x 2. Whatever the right hand side first obtain the equilibrium point, there is no reason to believe that always for whatever value of this f 1, f 2 are it will always at the origin, it could be some other places. Imagine, if here x dot is equal to this y minus 3m then itself it becomes a different position.

So, you have to first obtain the equilibrium point and then, this particular thing, then we will have to write it as delta x 1 dot delta x 2 dot is equal to something times delta x 1 delta x 2. Where this deltas are the deviation from the equilibrium point that you have found. Now, what is this, this has to tell how does f 1 vary with x 1, that is what we did. How does f 1, suppose you are approximating by some kind of a plane here, how does f 1 vary with x at this point, how does f 1 vary with y at this point similarly for the other.

Than this effectively becomes x 1, so starting from here you can locally linearize to obtain this equation and this matrix is called the Jacobian matrix. Now, often in most studies in engineering we only consider this equation why, the reason is that you have when we did the whole gamut of obtaining differential equations, you have found that 80 percent of the cases to obtain non-linear differential equations. Even the simple pendulum is a non-linear differential equation.

But then, we mostly study this kind of equations, because most engineering systems are designed to operate at an equilibrium point. And the deviations from the equilibrium point are generally small, if they are small, then this local linear approximation is more or less valid. But, you will soon see that the moment you are considering this equation, the solution and other things become very simple.

That is why most of the engineering studies you will find that we consider mostly this equation. But, you have to understand that this equation is actually a local linearization around an equilibrium point, but the system was actually non-linear. Why is that understanding important, because you will be able to understand with the help of this, only when the deviations from the equilibrium point are small.

If they large you will not be able to, but once you have locked to this understanding that the system is this, then you never understand what is going to be happen if the deviations are relatively large. That is why it is necessary to understand this as, than approximation from a physical system whose representation is really non-linear. But, we have simplified it by considering the behavior around an equilibrium point, so we have obtain this. Let us just do this for one example, we have taken this examples, let us do it for this.

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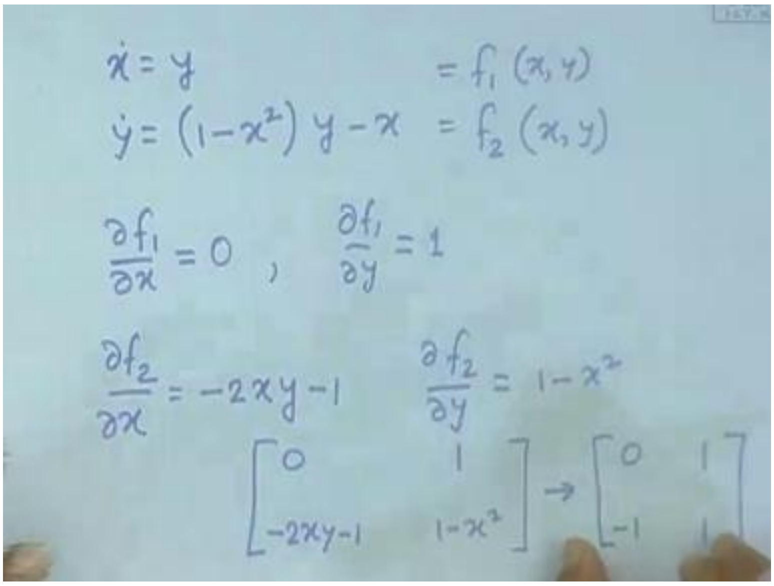


So, you have x dot is equal to y and y dot is equal to 1 minus x square y minus x, so what will be the Jacobian matrix. So, this is our f 1 x, y and this is our f 2 x, y, so when I say doe f 1 dou x is what is it 0, dou f 1 dou y is 1 nice, but here there will be something, dou f 2 doe x is...

Student: ((Refer Time: 26:42))

Minus twice x y minus 1 and dou f 2 dou y is, so your Jacobian matrix essentially becomes 0, 1 minus twice x y minus 1 and 1 minus x square. But then, this will make sense only when evaluated at the equilibrium point and the equilibrium point we have already understood it is 0 0. So, if you put the 0 0 value here you get 0 1 minus 1 1, so that is the local linear representation of the system. No, you have to understand correctly this is the local linear representation, but this is the matrix, but then what is the equation, local linear equation.

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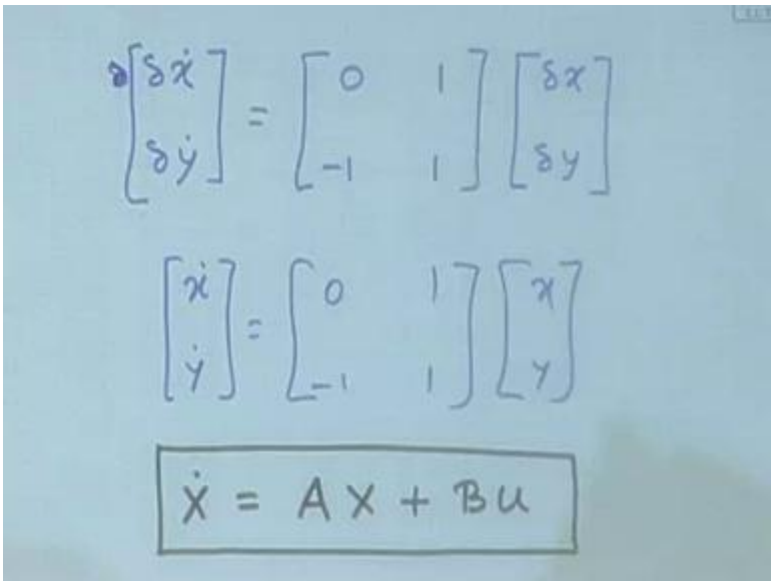


The equation is delta x dot delta y dot is equal to 0 1 minus 1 1 delta x delta y. Now, when we are considering linear systems like this, we have the understanding that we are talking about the deviation from the equilibrium point. Not just from the origin, systems origin, deviation from the equilibrium point. Once you have this in mind, once it has sunk in our mind, then this delta would be pretty redundant, because whenever we see this we always have to remember.

That now we have shifted the origin to the equilibrium point and therefore, we will often write this equation as simply x dot y dot is equal to 0 1 minus 1 1 x y. Do not be confused if we write it like this, this is after local linearization and therefore, after having shifted the origin to the equilibrium point. Now, this equation is normally would be written as, this is the X vector, capital X dot is equal to this matrix would be written as the A matrix times the X vector.

But, so far in this we have not consider the forcing function, if the system is non autonomous, then that forcing function will also appear in the local linearization. Because, the local linearization cannot be independent of that, let me clarify this I will clarify a little later. Point is that, then it will have some addition term representing the time variation, it is customary in engineering practice to write this, in terms of two different things as B u both are matrices.

B is a constant matrix not a time varying matrix, but u is a function of time, so this is the forcing function. So, if the system has a forcing function if an electrical circuit has a sinusoidal input, a square wave input, then this is representing that. And how it is related to the rest of the system and that is represented by vector B. So, this would normally be the local linear representation of a generally non-linear system.

So, now instead of trying to understand the whole thing, let us try to start piecemeal, we will first look at what can happen in the neighborhood of this equilibrium points. And then, try to integrate the whole story, that will be the the nice way of of understanding the whole thing. So, first we will understand the behavior of a system like this, what can such a system do. Notice one thing where is the equilibrium point of this well, so far as it is only this, the origin is the equilibrium point it is origin.

But, the moment you have this no longer, because equilibrium point has to be obtained by making this 0. And immediately you can see that X will be dependent on the time varying term. So, if there is a sinusoidal input for example, what does it physically mean the equilibrium point moves in the space. So, you can imagine this way that the equilibrium point is moving in the space and the actual state is trying to follow, because it is trying to converge on to the equilibrium point.

While it does try to the equilibrium point itself moves, so the equilibrium point goes on moving in the state space and the actual state is trying to follow it and that will give rise dynamics. But, whether or not it will try to follow or will try to go away depends on the stability of that equilibrium point. So, in order to study the stability of the equilibrium point, we need to take this away. We need to keep this wondering of this equilibrium point out of our purview keep it static and then, understand what its behavior will be is that understood.

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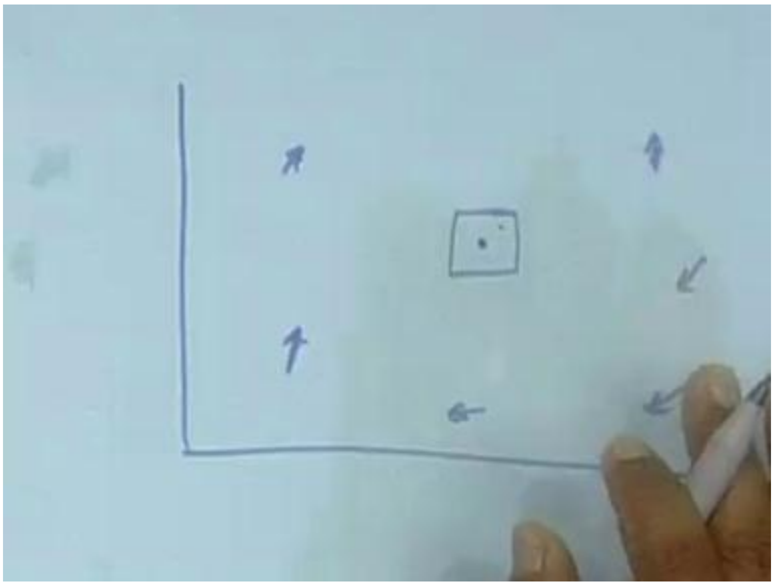

So, we will break up this problem, first we have broken up the problem in to normally there will be the state space with all sorts of behavior and all sorts of places different vectors. But, in the neighborhood of equilibrium points, there would be a specific type of behavior given by this A matrix which we will try to understand first. And then, we will argue that if the deviation from the equilibrium point is small, then it is behavior would be given by this local linear representation we will understand that.

And then, we will try to understand what happens if the excursions are larger. If the deviations are larger, then it goes into the non-linear region and then, we will try to understand that. But, while we try to understand this local behavior, then also we will first keep out of our purview, what happens if this equilibrium point itself wonders. So, we will consider only the local linear representation without that B u term, simple A dot is equal to A X what can it do. So, we are progressing in a specific way, what can it do let us start from the one dimensional case.

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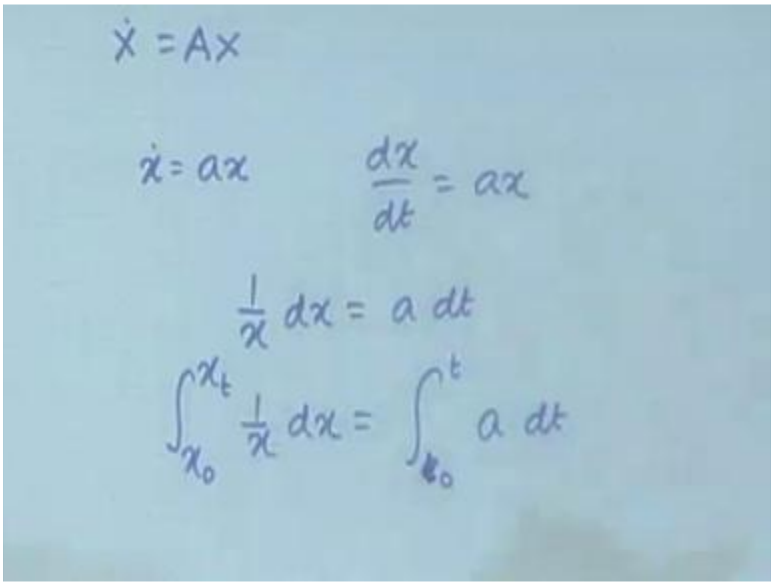


In a one dimensional case x dot is equal to A X will be written as x dot is equal to a x one dimension. What will be the solution of this equation, this equation can be easily variables can be separated, so you can easily integrate it is very in trivial. So, how do you separate the variables, this is d x d t is equal to a x, so it is 1 by x d x is equal to a d t integrate it over. Start it from starting point to some ending point, so from some initial condition to some final condition.

So, starting function say x 0 to x at t 1 by x d x is equal to starting from t 0 to t a d t, not t 0 let it be as 0 to t initial condition can be 0, this integration is rather trivial. So, write it this is log x l n x, so l n x evaluated at x naught to x t is equal to this is a times t t minus 0, so a times t.

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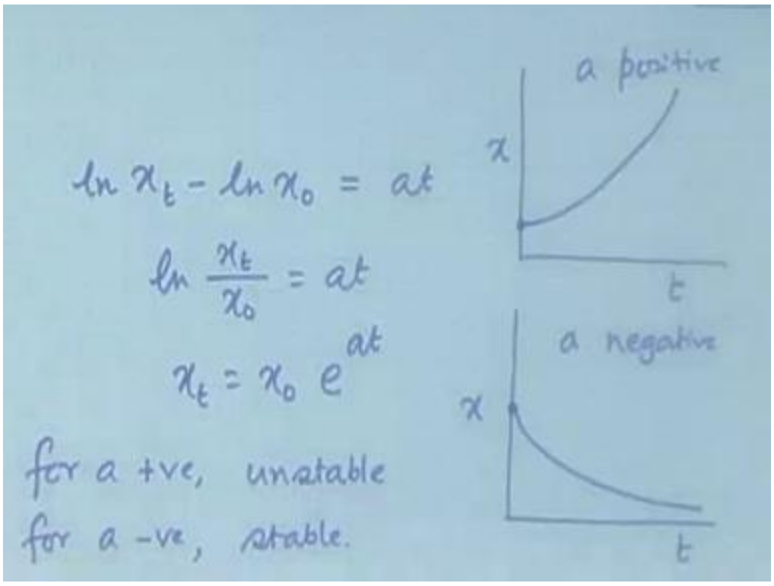


So, this give l n x t minus l n x naught is equal to a t, so you have l n x t by x naught is equal to a t, so now you can extract it x t is equal to x naught e to the power a t, so that is the solution. This essentially means that the result how it will behave depends on the initial condition, but more so depending on a. What will happen a is a number, a could be a positive number or negative number, if it is positive then what happens, it will go on increasing.

So, for a positive it will be time dependent on x start from some x naught and it will go on increasing, t x a negative it will be starting from some initial condition and then, it will go, that is the only thing possible. Now, this goes to infinity unboundedly, means that such a system would be unstable. Starting from some initial condition it will go away, it will go away from where from the equilibrium point, so it is a unstable thing. While here this is the deviation and deviation is exponentially decaying, therefore that is a stable system, so which means that for a positive, that is unstable and for a negative stable, this is as yet rather trivial. But, we need to understand this, because that is what will be extrapolated in high dimensions, how to do it in high dimensions.

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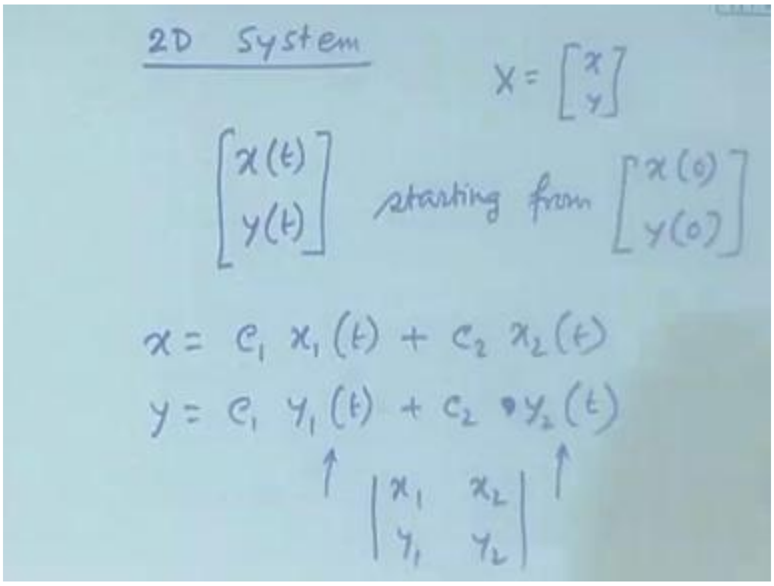


In say 2 D, consider a 2 D system, in a 2 D system your vector would be X is equal to x y. So, actually you will be seeking a solution of the form x t y t starting from that is what you are seeking, the way you are seeking here this as the x as the function of t, here you are seeking this value. Now, probably you have come across the theorem, while you were studying Maths that there exists a theorem that, if you can somehow how we will worry about later, if you can somehow identify two solutions in a 2 D system.

If we can somehow identify two solutions, then any given solution starting from any initial condition will be a linear combination of these two solutions, if it is a theorem discovered pretty long back. So, I am not going in to the proof of the theorem, but let us talk about the application of the theorem. So, if you have find somehow given two solutions in some convenient way, then we have any given solution as x is equal to linear combination means c 1 x 1 t plus c 2 x 2 t, y is equal to c 1 y 1 t plus c 2 y 2.

So, x 1 y 1 t is one solution how we have obtained we will come to that later. So, supposing we have obtained this and this somehow, then any given solution will be dependent on this. Now, what determine the c 1 and c 2 it is an initial condition, so initial condition will determine the c 1 and c 2 and somehow we have to obtain this. So, the cracks of the solution then in 2 D depends on how to obtain this how to obtain this.

Well there is a condition, do you remember the condition the condition is that, these have to be linearly independent solutions x 1 and x 2, not just any solution, linearly independent solutions. Because, you can obtain this only if the determinant x 1 x 2 y 1 y 2 this determinant is non zero. So, you have this condition impose that these two are linearly independent solution, what do you mean by linearly independent, physically what does it mean.

Student: ((Refer Time: 42:11))

You cannot multiply a constant to this to obtain this, if you can obtain, then it is a linearly dependent, if you cannot obtain then it is a linearly independent as simple as that. So, you need to obtain two linearly independent solutions that is crux of the matter. How to obtain this linearly 2 D linearly independent solution that is the essential things in any solution of a differential equation, linear differential equation. So, we need to solve the differential equation.

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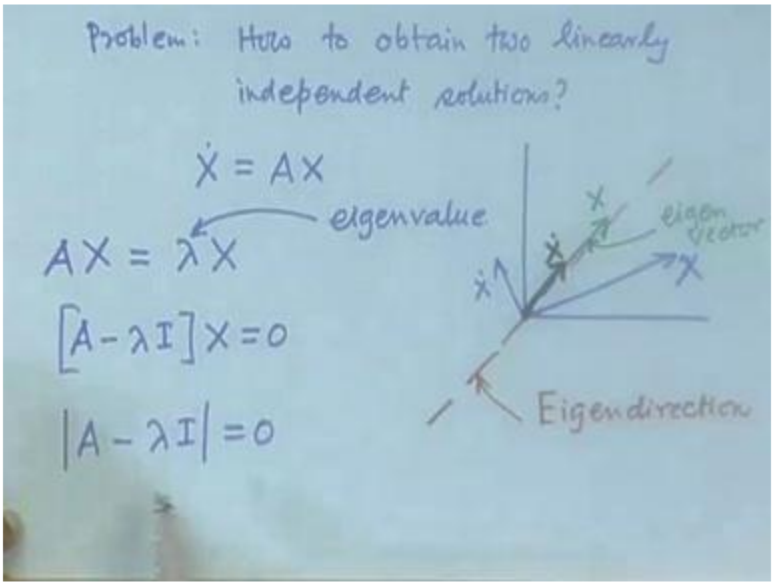


So, let me state the problem, the problem is how to obtain two linearly independent solutions. Now, what is our equation our equation is this ((Refer Time: 43:17)) except this, so X dot is equal to A X, where the X dot is a vector X dot Y dot and the capital X is X Y. Now, notice what is it doing, here is a vector, so here is a vector on which you have operated the matrix X to obtain this, this is also vector.

A vector geometrically is a vector X, on which you have operated by the matrix A to obtain this another vector say like this. I do not know if your mathematics teachers have told you this, that a matrix the main role of a matrix is to map a vector in to another vector that is it. Often we do it and do everything very algebraically, but geometric consumptions are better, that a matrix is primary function is to map a vector in to a vector.

If the matrix is a non-symmetric matrix, then the vector from which it maps in to another vector they have different dimensions. While, if it is symmetric matrix as it will be in this case, square matrix then it will be not a symmetric a square matrix. Then this is the source and this is the result source and the result will have the same dimension, but the point is that it maps a vector in to a vector.

Now, this vector from which it starts operated on gives this, they will in most cases be in different directions, different magnitudes quite natural. There is no reason to believe that they will be same or something like that, they will be different. But, there exist two directions in a 2 D system, two directions such that, say if this is the direction. Such that if you start from a vector in this direction, the result will also be a vector, where is the black 1 in the same direction.

So, if this is your X that will be your X dot, so what statement have we made in a 2 D system there always exists two directions such that, with the property that, if the initial vector X is in that direction, the transformed vector will also remain in that direction. So, the advantage is then that if you have somehow identified this take a initial vector here, the final vector is along that direction. So, the whole things it becomes one dimensional whose solution we already know, that is the clue.

So, we have already obtain the solution of the 1 D system, who which came out to be where is it, ((Refer Time: 46:53)) x t is equal to e to the power a t we know the solution. So, that is exactly what we will do, we will take the initial condition on that direction and the everything will then remain on this direction, because the initial state is this how it moves is also along that. So, in the next instant it will again remain on this line, so everything will be on this line it will become one dimensional problem, good.

So, in that case we somehow to need to identify this direction. Not only that this direction is called a Eigen direction, so these directions are called Eigen directions. And any vector along an Eigen direction is an Eigen vector. Not only that, here you have a vector X vector on which A has been applied, A has operated on this X vector to obtain this. So, this will be some since it is the same direction this will be nothing but, a number multiplied by X.

That means, this number if it is small, then it is squeezed, if it is long it will be enlarged, but whatever it is, if it is the same direction we can say that it is just a number multiplied by X. And that number is called the Eigen value. So, we will be able to write from here, X dot is equal to A X, which means A operated on X will be nothing but, a number lambda times X; this lambda is the Eigen value, the lambda is in the Eigen value.

So, the essential point then falls down to how to find the Eigen vector and how to find the Eigen value. In fact, in order to find the Eigen vector you first have to find the Eigen value, so let us find the Eigen value that will turn out to be trivial. Here this is the equation then, from this equation you can write A minus lambda I, where I is the identity matrix of the same dimension as A times X is equal to 0. Now, this equation is will be true, if this determinant of this will be 0, so the result is A minus lambda I determinant equal to 0.

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$$\left|\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right| = 0$$

$$\begin{vmatrix} A_{11}-\lambda & A_{12} \\ A_{21} & A_{22}-\lambda \end{vmatrix} = 0$$

Now, this will have to be then written as A is A 1 1 A 1 2 A 2 1 A 2 2 minus lambda I, which is lambda 0 0 lambda equal to 0, this determinant equal to 0, which is nothing but, determinant of A 1 1 minus lambda A 1 2 A 2 1 A 2 2 minus lambda equal to 0.

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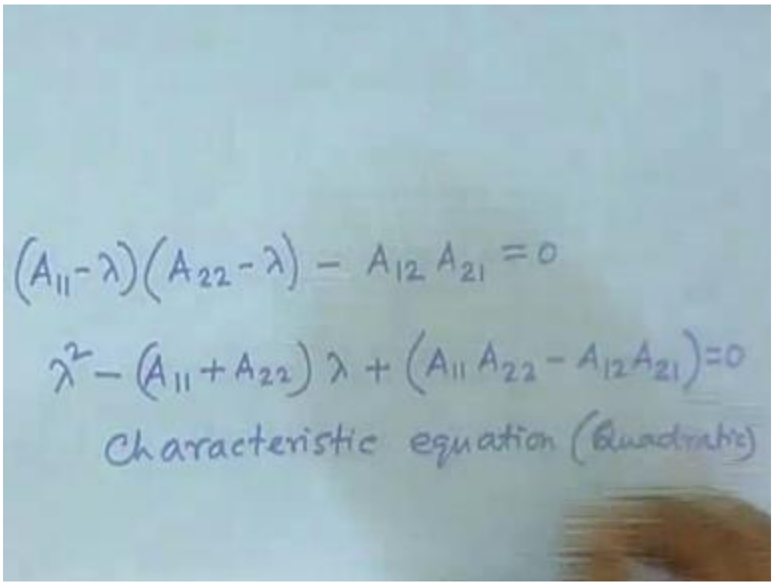


Now, from here then we can write A 1 1 minus lambda times A 2 2 minus lambda time minus A 1 2 A 2 1 is equal to 0. Now, we can multiply this lambda square minus A 1 1 plus A 2 2 lambda plus A 1 1 A 2 2 minus A 1 2 A 2 1 is equal to 0. Notice that it is a quadratic equation this is called characteristic equation, so this is the quadratic for a 2 D system. For a higher dimensional system it will be as many order, good. So, quadratic system we know always that it has two solutions, so there will always be two Eigen values there will always be two Eigen values. So, you can identify, now let us see, let us just do one problem.

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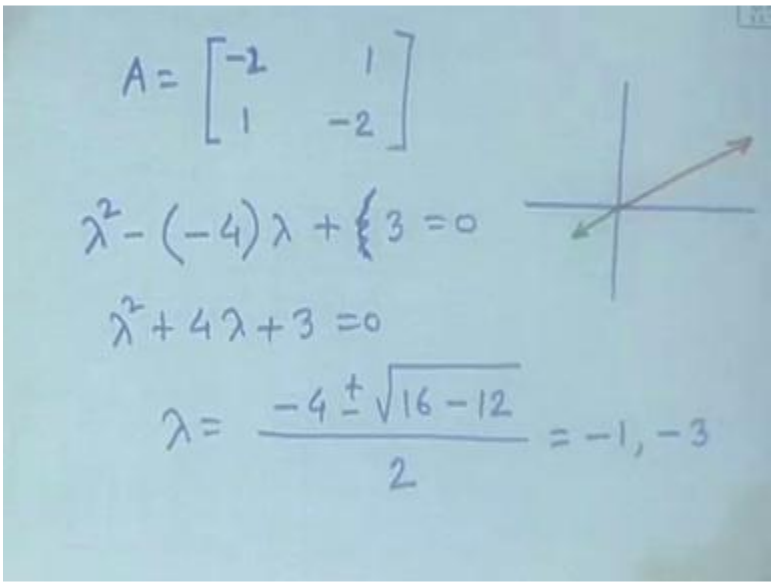

Suppose your A matrix is minus 2 1 1 minus 2, then what are the Eigen values, the characteristic equation will be lambda square minus A 1 1 plus A 2 2, A 1 1 plus A 2 2 is minus 4 and lambda plus this comes to 3 is it, 3 is equal to 0. So, lambda square plus 4 lambda plus 3 equal to 0, so the solutions are minus 4 plus minus root over 16 minus 12 by 2 is equal to this is 4 root over 2, so you have minus 4...

Student: ((Refer Time: 53:52))

Goodness, how can all of you say different things to this problem.

Student: ((Refer Time: 53:59))

See there is by 2

Student: ((Refer Time: 54:08))

Yes, so minus 1 and minus 3, so you have two solutions which means these are the Eigen values. What is the meaning of minus?

Student: ((Refer Time: 54:24))

It means, yes it means that if you have the original vector like this. The transformed vector will be in the opposite direction, but in the same along the same line that is all, so that is the meaning of the term minus. And it is also easy to see that if it minus, then if the initial deviation is this, in the next instant it will move in the opposite direction it will come closer. At this point it will again be along that direction it will come closer, so it will slowly home on to that equilibrium point. We will look at that later, so this is the the way to obtain the Eigen values. And how to obtain the Eigen vectors that is simple.

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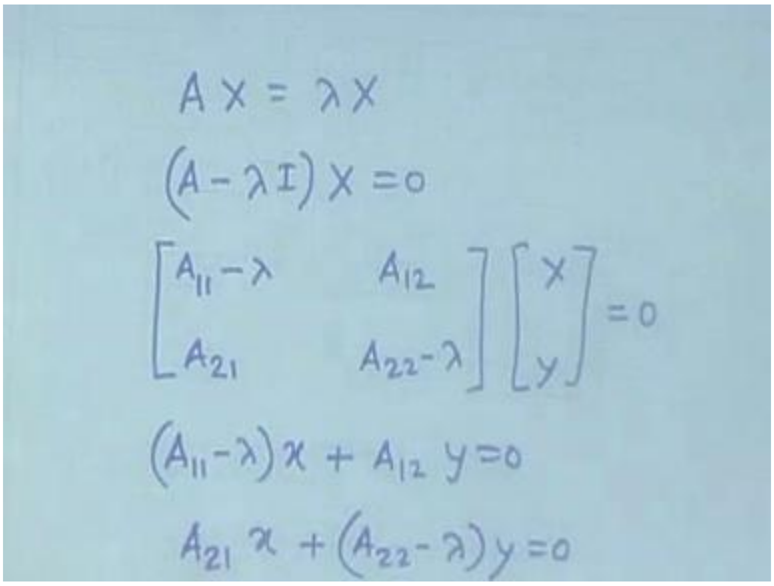


We have A X is equal to lambda x that is what where we started, so you will have to say A minus lambda I x is equal to 0. Now, you can expand it and write it this will be we have already done that A, where is it 1 1 minus lambda A 1 2 A 2 1 A 2 2 minus lambda times X Y is equal to 0. So, this gives two equations, one is A 1 1 minus lambda x plus A 1 2 y equal to 0 and the other 1 is A 2 1 x plus A 2 2 minus lambda y equal to 0, there are two equations, there should be, yes there is a point there is a cracks.

There are two equations and you must say that now I solve it, no you cannot solve it because these two equations to your great surprise, will always turn on to be the identical things. These two equations will always to be identical why, because you are trying to find the Eigen direction not the specific vector. Therefore, X Y should not be determinable, only the direction should be determinable.

So, if you have this equation is same as this equation. Then this and that will independently give just the direction each will give the direction, that is the Eigen direction. So, for this problem you work out the Eigen directions, that is all that for today.