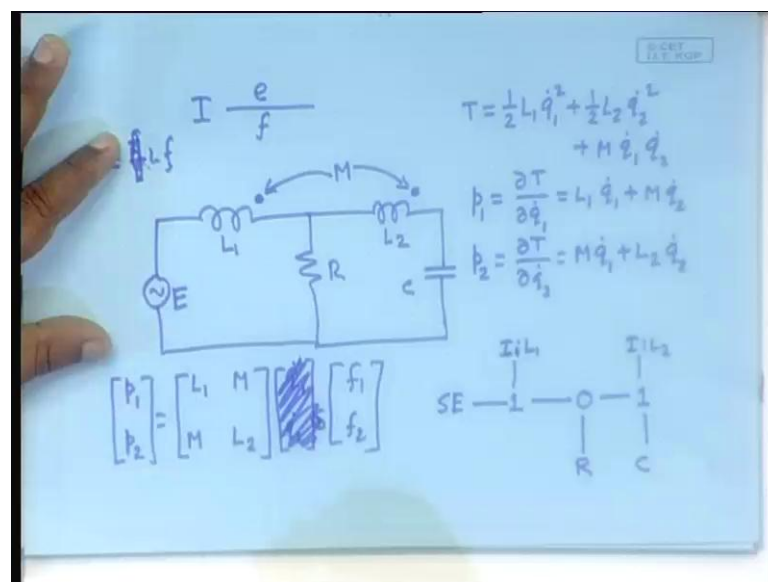


Dynamics of Physical Systems
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Lecture - 18
The Bond Graph Approach – VI

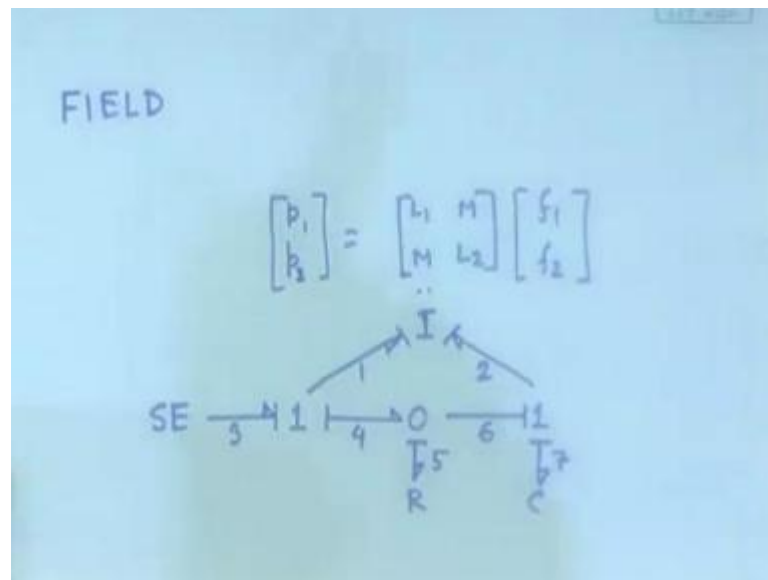
In the last class we saw, that under certain conditions, certain situations the inductive element or inertial element will need to be represented with two bonds, when that will happen when the effort variable that is created from the history of flow. That is dependent on the history of flow in two different places, that we have seen happens in case of mutual inductance and where we considered this circuit.

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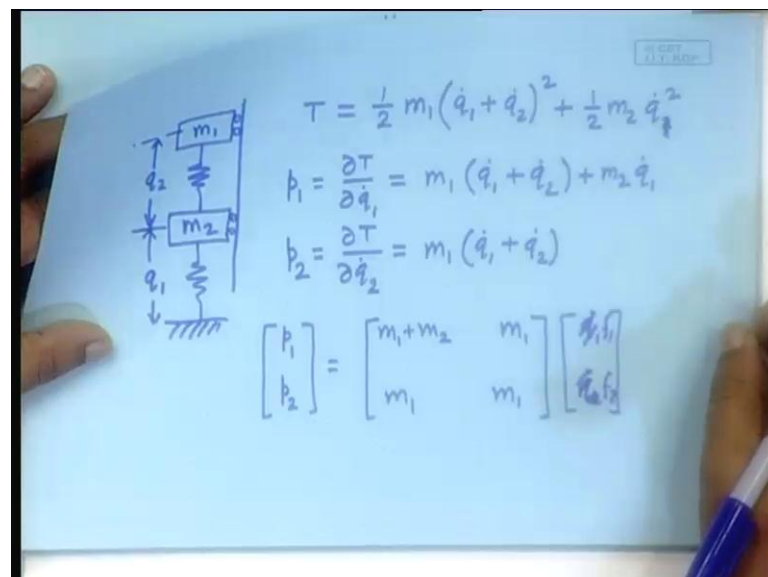
We considered this circuit and we decided that the momentum here and the flows are related by this kind of a matrix equation. So, under that condition it would be necessary to represent it not as this.

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But as this, so that is the point to which we went in the last class. It is easy to visualize this in the electrical circuit. But, can we think of as a mechanical situation that would necessitate the use of two port I element or I field, such a situation actually arises when for example, let me give an example.

(Refer Slide Time: 02:27)



If you have a mass here, that is connected to another a spring and to another mass. Say, that is connected to another spring to ground, do not argue how it will stand. Because, it may be by means of some kind of a guide, that is made to stand like this, we are

considering the up and down motion. But, in that case we have to find out, we have to specify what we call as a datum from which this mass its position is measured. And also the datum from which this is measured.

Now, you might say that let this be the datum from which this and that both are measured or you could say, that I would say this is my q_1 and this is my q_2 , that is also possible. You would notice that, if you do that then it would be necessary to represent it by means of a I field let us see. If you have this, then the way we did it in the last class for this one ((Refer Time: 03:55)) we wrote the kinetic energy equation and from there we obtained the momenta. Let us see, what would be the kinetic energy of this set up, the T would be half m_1 this is m_1 and this is m_2 , m_1 times what square. Now, its motion will be...

Student: ((Refer Time: 04:23))

Will be dependent on both. So, $\dot{q}_1$ plus $\dot{q}_2$, $\dot{q}_1$ plus $\dot{q}_2$ these fellows square plus it is kinetic energy which is half $m_2 \dot{q}_2$ square.

Student: Sir, $\dot{q}_1$ square.

$\dot{q}_1$ square, so in that case your P_1 which is the derivative of the kinetic energy with respect to $\dot{q}_1$ will be do that, what do you have...

Student: ((Refer Time: 05:12))

$m_1 \dot{q}_1$ plus $\dot{q}_2$ plus $m_2 \dot{q}_1$. Similarly, p_2 it will be...

Student: m_1 .

Fine, which means again the P_1 , P_2 as a vector becomes related by some kind of a matrix to the two flows, these are the flows, so what is the matrix.

Student: ((Refer Time: 06:22))

No, P_1 .

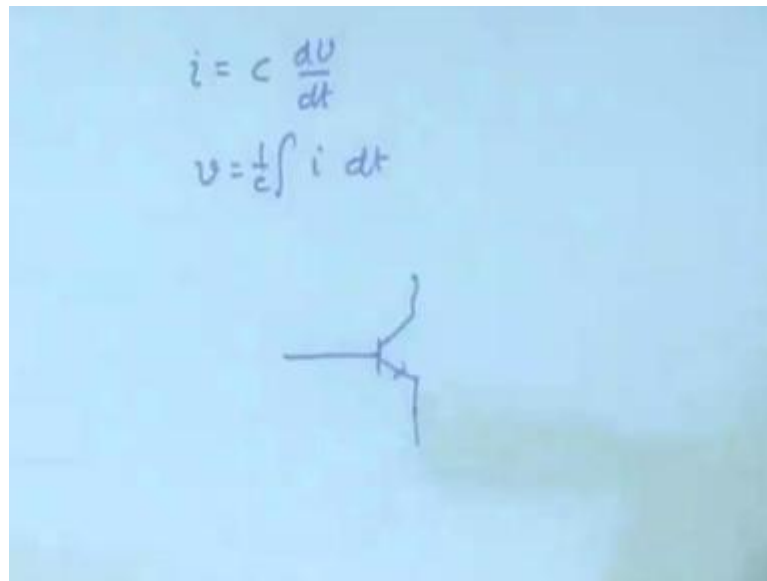
Student: m_1 plus m_2 .

m_1 plus m_2 .

Student: ((Refer Time: 06:32))

M 1 yes fine, so this is after all related by a matrix equation. And therefore, in order to represent this you would similarly need a two port I element an I field correct. When would there be a C field similarly well, capacitances are never coupled. So, we would not directly consider, since we consider the capacitances as isolated elements. So, it is in electrical sense.

(Refer Slide Time: 07:21)



An electrical capacitor will be an electrical capacitors governing equation is i is equal to $C \frac{dv}{dt}$ or v is equal to $\frac{1}{C} \int i dt$. So, the history of the current determines the present voltage. Now, this is normally in a capacitor it is current determines it is voltage, can you figure out a situation where somebody else's current will determine it is voltage. No, not in a normal capacitor, not in a normal capacitor. But, there can be cross coupled capacitances, where there are various lines and in between there are capacitances.

So, these kinds of situations relatively complicated situations would need this representation. But, let us leave that for now, in mechanical system for example, there are beams and whether there is a question of beam bending, that is something like a capacitance. And that can be dependent on the motion in this part, that part they can be related, so the beam bending problems often result in cross coupled capacitances.

But, let us leave those complications for now. But, in those cases remember you would need a C element with a two ports C field, when can you think of having a R field. Wherever, the voltage and the current, what is a relationship between the voltage and the current in a R element V is equal to $I R$ simple. So, the effort is equal to the constant times the flow that is right.

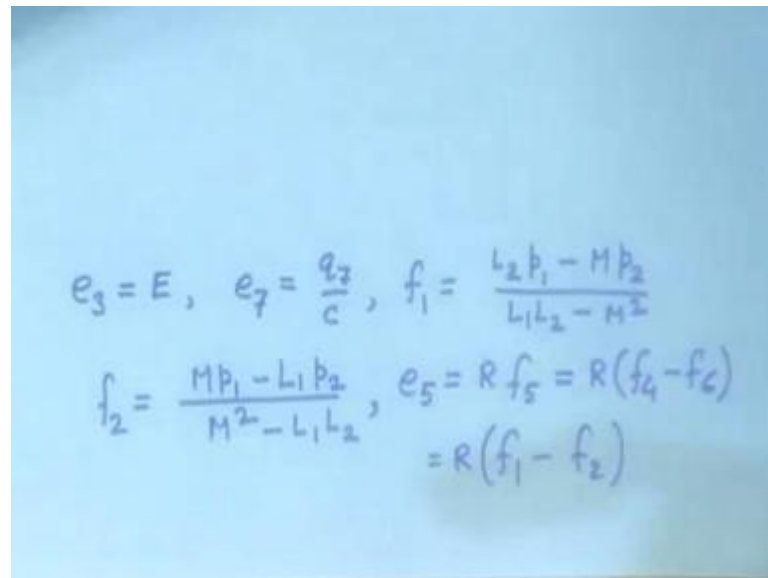
So, if you find a situation where the effort in two bonds are related by some kind of a matrix to the flows in two bonds or the flows in two bonds is related to a some kind of matrix to a effort in two bonds then; obviously, you need an R type of element. But, such kind of situations very often occur; for example, in electronics, if you have say this kind of a situation. If there is how many voltages say, there is a current, here is a current, here is the voltage, here is the voltage how are they related, you would notice immediately that they are related by that kind of a relation.

So, whenever a circuit contains a say a any of these devices, then you would need a R element. But, presently we are considering in this course we are considering mainly electrical circuits, mechanical systems and electromechanical systems. So, let us we understand this would occur, but let us not concern ourselves with this right now, if they occur we will take now. But, we need to understand how to obtain the differential equations for this kind of a system.

So, let us take that up, ((Refer Time: 10:49)) it is bond graph we had concluded would be like that. So, let us power direct it, it would be this, this fine, so the power directions are done, you have the causalities now. S E will be receiving flow and giving effort, C will be receiving flow and giving effort I will be giving flows, the moment the flow comes here, one junction it has to distribute, so this is done. The moment the flow comes to this one junction it will distribute, so this is done.

So, it has received the flow from both the directions it has taken out effort in two directions. And therefore, this bond must bring in the effort information, which is like this done. So, you have caused it and I will now assign the numbers 1, 2, 3, 4, 5 and 6 and 7. Now, let us start asking the question with relationship to all these elements, let us start or shall we displayed like this can you see.

(Refer Slide Time: 12:42)



$$e_3 = E, \quad e_7 = \frac{q_7}{c}, \quad f_1 = \frac{L_2 p_1 - M p_2}{L_1 L_2 - M^2}$$

$$f_2 = \frac{M p_1 - L_1 p_2}{M^2 - L_1 L_2}, \quad e_5 = R f_5 = R(f_4 - f_6)$$

$$= R(f_1 - f_2)$$

So, let us display it like this and then write it here, first we ask the question with respect to the S E, what does it give to the system, it gives e_3 which is the externally applied voltage E . Next, we ask this question to this C what does it give. So, it gives e_7 , e_7 is equal to q_7 by C , then ask this question to this inductive element, which is now two port. In this particular bond what does it give f_1 no f_1 . So, f_1 is equal to what as yet we do not know that. But, we will soon know, but in this bond what does it give f_2 , so we will need to know f_2 also ((Refer Time: 13:40)).

Now, we have seen this is f_1 , this is f_2 and they are related by this relationship to the momenta. So, let me substitute this as f_1 this is f_2 and then can we extract this f_1 , f_2 from here yes we can, essentially two equations two unknowns. Just obtain f_1 and f_2 individually from these two equations do it.

Student: ((Refer Time: 14:18))

What.

Student: ((Refer Time: 14:21))

Oops sorry, sorry, sorry, yeah it was here ((Refer Time: 14:29)). So, these two are I will just these two are actually f_1 f_2 . So, from here can we find out f_1 and f_2 individually, it will turn out to be $L_2 p_1 - M p_2$ by $L_1 L_2 - M^2$. Let us check that f_2 will turn out to be $M p_1 - L_1 p_2$ divided by $M^2 - L_1 L_2$. So, these

two can be obtained from the original equation, let me write down the original equation ((Refer Time: 15:49)).

So, that you can feel comfortable with it, what we said was here the relationship is $P_1 P_2$ would be times $f_1 f_2$. Normally, the momentum is the mass times a flow, but here the momenta in the two bonds would be related to the flows in two bonds by this relation. So, from here we obtain f_1 and f_2 , now in addition to that we will need to know is it visible yes, we will need to know the what does R give to the system what does it give.

Student: ((Refer Time: 16:36))

No, it gives $e f$ comes this direction, e goes that direction what does it give to the system $e e_5$. So, comma e_5 is equal to R times f_5 , so first we relate to the character of the system and then we find where does f_5 come from. So, f_5 where does it come from sorry I have not given the power direction here. So, where does this come from f_4 minus f_6 , so...

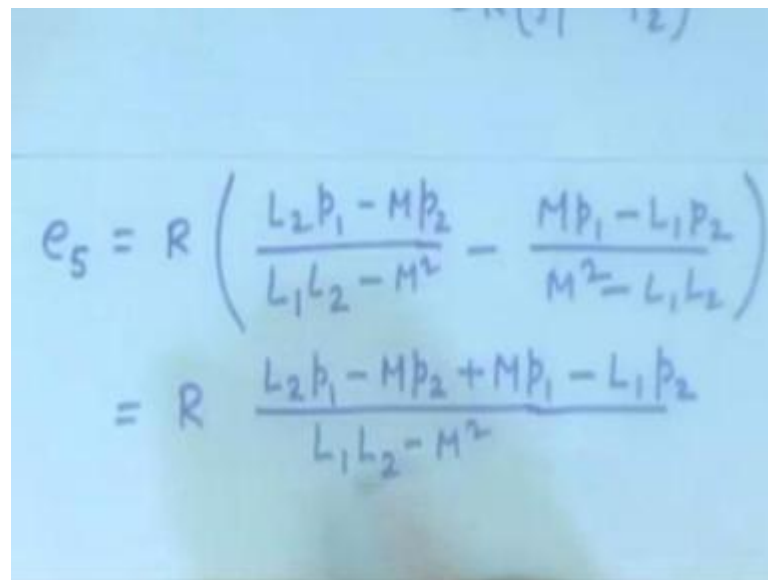
Student: ((Refer Time: 17:19))

No.

Student: ((Refer Time: 17:23))

No, no f_4 minus f_6 is equal to f_5 . So, $R f_4$ minus f_6 , now f_4 is what it is a f_1 because it is a one junction, where the flow has come here. So, this is the strong bond bringing in the flow information f_1 , so this is equal to $R f_1$ minus f_6 is what again $2 f_2$. So, subtract these two you will get it subtract these and that, you will get it I will just write in the next page or may I write I will write in the next page.

(Refer Slide Time: 18:19)



The image shows a handwritten mathematical derivation on a light blue background. The equation is written in two lines. The first line is
$$e_5 = R \left(\frac{L_2 p_1 - M p_2}{L_1 L_2 - M^2} - \frac{M p_1 - L_1 p_2}{M^2 - L_1 L_2} \right)$$
. The second line shows the result of simplifying the expression in parentheses:
$$= R \frac{L_2 p_1 - M p_2 + M p_1 - L_1 p_2}{L_1 L_2 - M^2}$$

We start from here can you see our e_5 then is R times f_1 minus f_2 . So, we will if you substitute these two it will be $L_2 p_1$ minus $M p_2$ divided by $L_1 L_2$ minus M square minus this fellow $M p_1$ minus $L_1 p_2$ M square minus $L_1 L_2$ is equal to just shorten it, you have you can do it $L_1 L_2$ minus M square.

Student: ((Refer Time: 19:02))

This remains $L_2 p_1$ minus $M p_2$ plus $M p_1$.

Student: minus.

Minus.

Student: ((Refer Time: 19:22))

Let us check, because I tend to make careless mistake, so please check. So, you have f_e 5 and then we proceed to obtain the equations for each of these, ((Refer Time: 19:50)) what are the storage elements this, this and this. So, we will have to do it accordingly.

(Refer Slide Time: 19:59)

$$\begin{aligned}
 & q_7, p_1, p_2 \\
 & \overbrace{f_7 = \dot{q}_7} = f_7 = f_2 = \frac{Mp_1 - L_1 p_2}{M^2 - L_1 L_2} \\
 & e_1 = \dot{p}_1 = e_1 = e_3 - e_4 = E - e_5 \\
 & \quad = E - R \frac{L_2 p_1 - Mp_2 + Mp_1 - L_1 p_2}{L_1 L_2 - M^2} \\
 & e_2 = \dot{p}_2 = e_2 = e_6 - e_7 = e_5 - e_7
 \end{aligned}$$

So, what are the state variables q_7 ? So, that is the state variable related to this storage element, but though this fellow is one element it is actually treated as two storage elements, because there are two momenta, so p_1 and p_2 . So, you have p_1 , p_2 we will have to find out the equations for each of them, let us start with this. ((Refer Time: 20:30)). So, there in order to find out this $\dot{q}_7$ we ask this question, what does this element receive from the system.

So, what does it receive from the system. So, f_7 is equal to $\dot{q}_7$ then you bring to this side start with f_7 , and then we write f_7 is what f_7 is f_2 , f_2 which is known we just substitute it from here, where is it f_2 you just substitute it from here done. So, this is $Mp_1 - L_1 p_2$ divided by $M^2 - L_1 L_2$. Then, so we need to ask the question now with respect to p_1 what does this element received from the system through this particular bond.

Student: e_1 .

E_1 , so e_1 is equal to $\dot{p}_1$ is equal to, so we write e_1 in this side and then we start of e_1 where does this information come from $e_3 - e_4$. Now, e_3 is known which is E externally applied voltage e_4 is e_5 yes e_5 and e_5 is known here. So, just substitute it there you have the equation. So, is equal to $E - R$ times this big stuff $L_2 p_1 - Mp_2 + Mp_1 - L_1 p_2$ divided by $L_1 L_2 - M^2$ done.

Third question we ask with respect to this particular bond, this element receives e_2 through this bond. So, we start with e_2 is equal to p_2 dot is equal to e_2 and e_2 is e_6 minus e_7 . Now, what is e_6 .

Student: e_5 .

E_5 , so e_5 minus e_7 and both these are known e_5 is this and e_7 is this substitute. So, see the rest of the procedure is pretty simple, all you need to do conceptually is to draw the proper bond structure that is it. So, if you draw the proper bond structure and properly represent this particular element, then everything falls without any problem. So, you have understood the issue of fields.

There is one more issue I will need to cover before I can end this topic. That is, see always we have considered the bonds, that carry power and that is true for most of the cases. But, there are situations where there will be bonds that does not carry power. Example, suppose there is a circuit and you have placed a ammeter, the ammeter does what it measures the current, but the ammeter itself does not consume any power, because the voltage across it is 0. So, it is an observer in the real sense it the ammeter is an observer.

So, whenever we have a observer in a system then; obviously, the bond to which it is connected will carry no power, you might say that what is that point of connecting the observer at all. We do it for two purposes, one to know what it is, what the current or the voltage is and two for doing some kind of feedback action. Because, whenever we do a feedback action we sense something and on the basis of that we do the feedback.

So, that sensing is of that type ammeter, voltmeter that kind of measurement or we measure the speed, we measure the torque something we measure. So, we measure something on the basis of that we put some action. So, there are two places where we would need such things, one where we have to observe something. Now, well you might say that here as far as the bond graph is concerned you after all have the state variable. So, if you solve the equations you get the state variables and therefore, you are already observing the state variables true.

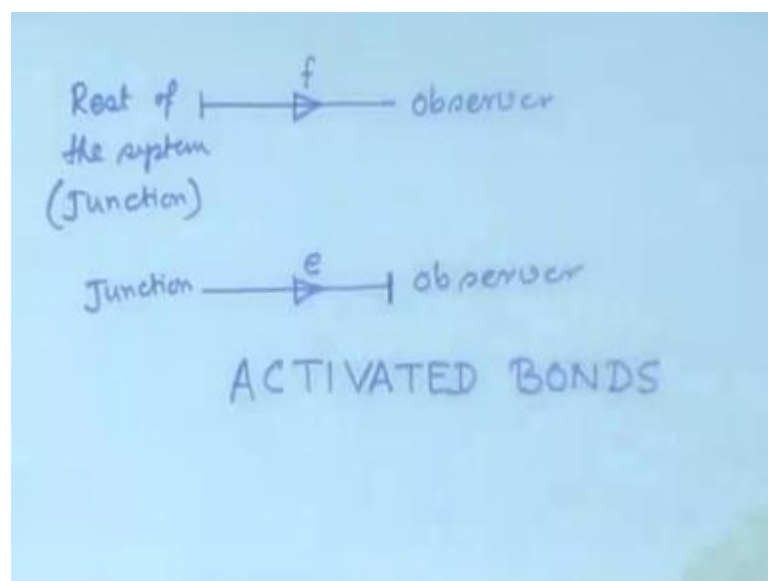
But, there are situations where you want to observe something other than a state variable. For example, what is the state variable related to a mass it is momentum, but suppose

you want to observe it is position will it be available from the from the equations, no. Suppose, you want to observe a mass's position and depending on that you have to control something, very possible situation in a control system.

So, in that case that particular variable the position will not be available, imagine a capacitor what is a normal state variable through it, the charge. So, now if you think that no, no I want to observe the current through that capacitor is it available in the equations, no it is normally not available in the equations. So, if you solve the equation you will get only the charge through the capacitor, so in order to sense the current.

That means, you are laying of your model in such a way, that the current will also be available as some kind of a sensed variable, on the basis of which you can do something. In that case, it will not normally be available in the existing bond graph, you have to do something in addition to it. Let us see how we can do that, but then it is clear that wherever we are observing something it will have to be done by means of a bond.

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See suppose there is a bond like this connected to rest of the system, say the rest of the system. And here this bond is connected to some observer, then either it observes the flow or it observes the effort, but it cannot observe both, it will not observe both in the same place. So, if it observes the flow the effort is 0, then only it carries no power, if it observes the effort the flow is 0, then only it carries no power, which means that for observing element something that observes one of the variable has be must it has to be 0.

Now, whenever we have this kind of a situation where we are sensing it, there we put a full arrow meaning that this bond particular bond graph particular bond will carry no power. Or you can say in place of the rest of the system, you can also say it is a junction, because it is always connected to some kind of a junction.

So, in this part, so from a junction to an observer it has to be activated, this is called an activated bond. Now, it is clear that it could be either carrying the information of the effort, masking the information of the flow or it could be carrying the information of the flow masking the information of the effort, if it carries the information of the flow we will simply write f . It could also be a bond that carries the information the effort we will write e , because without writing this you never know what it is observing.

So, the junction here and here is the observer. Now, if it is observing the flow in a junction then; obviously, the junction has to be what kind of junction, it has to be a 1 junction, because it has to be a 1 junction and the direction of the information flow would be like that, flow is going that direction. While, if it is an observer connected to the junction which is taking away the information the effort.

Effort is going in that direction, flow will be going in that direction logically then the. But, flow actually does not go, but nevertheless the moment you say that effort is going that direction the causal stroke has to be given there. So, these are the activated bonds, remember the word. Let us illustrate a few situations where you need that.

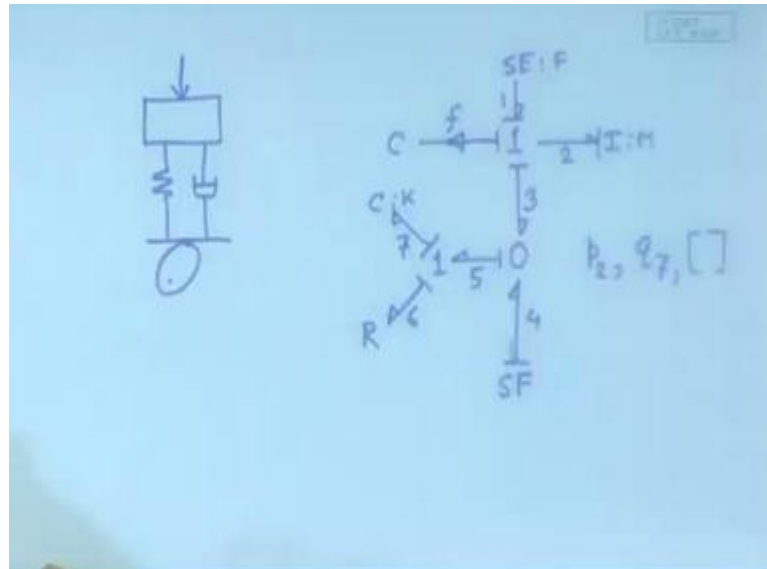
Student: ((Refer Time: 31:10))

Wattmeter actually does not measure the current and the voltage at the same place, it measures the current through a line and voltage between things. So, it has to be taken into account in that way, so whenever you talk about a wattmeter it is not really measuring the voltage as well as the current in a specific bond no it never does. So, just keep that in mind.

So, all that will be needed in order to do the effect of a wattmeter is to have an activated bond to measure the flow another activated bond to measure the effort product them. So, you have understood the concept of the activated bond let us come to some...

Student: ((Refer Time: 31:58))

(Refer Slide Time: 32:17)



This is already done, that is why I am not talking about the whole details once again. Normally, if you put the power directions they will be like this and here you could put it both ways. So, let us put it say like this, now causalities start here done, start here done, then the storage elements like this, like that, the moment you have done that it is totally caused, the moment you have done that it is not yet totally caused, so...

The R these two I have already done. So, these two are bringing in the flow information and taking out the effort information, therefore the effort has to come from here and if the effort has to come from here the stroke is here. So, the flow went already in it has to

be distributed like this good, so it is caused. But, notice at this moment if you solve this the if you write down the differential equations and solve them, what do you have, you have here is the K and here is the M .

So, you have let me give the numbers say 1, 2, 3, 4, 5, 6, 7. So, what are the state variables, state variables will be p_2 and other storage element is this one q_7 . The momentum of this mass, which is proportional to the velocity of this mass therefore, in solving this, you will get the velocity signal. And start from some initial condition, solve the differential equation you will get a waveform that will represent the velocity, but not the position.

So, the position is not yet observed, but suppose you want to observe the position. That means, you want to create another some dummy variable here you want to create another dummy variable. So, that there will be three equations solving which you get this as well as the position, the question then is what do you need to do in order to create that additional thing, obviously position is the natural variable of a capacitor.

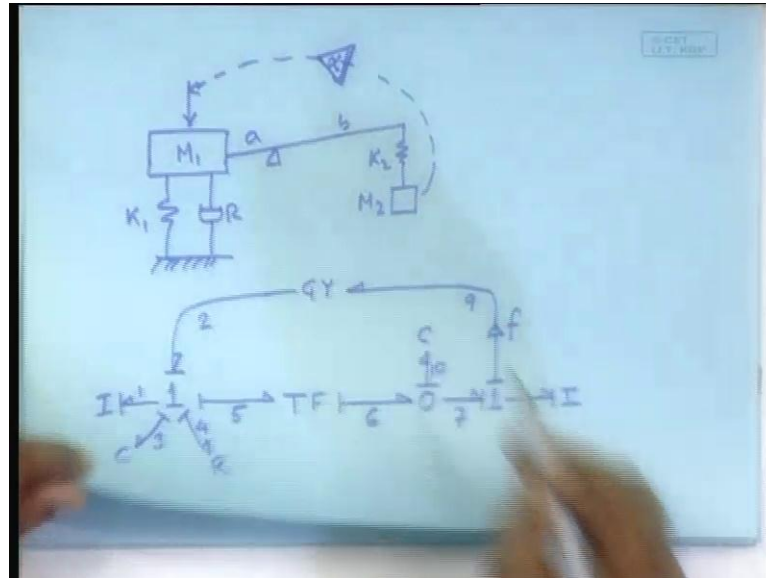
So, you need to connect a capacitor here, so you connect a capacitor C it is a dummy capacitor. But, then this particular capacitor whatever you have connected should not affect the rest of the system, because if it does the dynamics changes it is an observer, it is a measurement instrument. So, this bond has to be activated, it receives the flow and gives out the effort and that is why the effort variable becomes a state variable.

So, we will have it, but what does it receive actually, what actually flows through this bond the flow. It receives the flow information and it generates the effort information, it generates the effort information means it that is why it becomes the state variable q , so you get it. So, this particular addition is the activated bond, so why did we need it, because we wanted to observe the position of this mass we want to observe the position of this mass.

So, in this situations we will need to do that, a whole circuit in which there is a capacitor you want to observe the current through that capacitor what will you do, current is proportional to the momentum. Therefore, you want to measure the current means somehow you have to get the momentum as a state variable. But, momentum is not a state variable of a capacitor, what do you have to do, you have to connect a dummy inductor through an activated bond, the moment you do that it will generate a the

necessary variable, is that question clear now. So, the other situation that can often arise is where this particular thing; that means, the position sense is used to do some control. Let us illustrate one situation.

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Suppose, we consider the whatever we did earlier you have got a mass a spring damper arrangement as we have many times considered this, there is a force and there is another mass. But, now notice suppose that this mass it is position is being observed and not only that depending on the observation of it is position, suppose you are controlling this force. Earlier, we considered that this is an externally applied independent force.

But, now imagine there is a control system you are observing this particular mass's position and then your I will do that by a dash line. You are controlling it hum; obviously, when you are measuring the position and you are controlling the force, here at this point the what is available a flow variable, at this point a flow variable available this flow is being sensed. And therefore, it has one unit, when it is get started into a force it has completely different unit.

So, there has to be some kind of a multiplying factor which I will represent by means of a sort of a gain a some gain block, say the gain is alpha good. Now, how do we represent this, the rest of the thing is already known to you that is why I chose this system. So, that the additional incremental thing that we did, that is what we try to understand the rest of the things is known. So, first let us quickly draw the bond graph, let us draw the bond

graph here, you have got a one junction here 1 connected to we have already some other did this.

We do not need to do this in a very long issue, here we have already seen that this will be simply C this will be simply R. And then this fellow will be your mass will have to be connected. So, I then it will be connected to the transformer, the transformer will be connected to this one, so it will be 0 to the C to the mass connected by a 1 junction to the I element. That is the normal thing and we said that this 1 junction would not be necessary, because you can do it do it through, but we will not do it now that way.

Because, we need this sense here at this point the flow is available. So, we are measuring the flow, so what we will do is we will put a bond starting from here going all the way. But, then here is a flow variable and when it gets input at here, what is it an effort it is force. So, in between you have to connect a gyrator, so there has to be a gyrator in between. So, what are we doing, we are observing the flow here and we are applying a effort here, so it is a feedback control system.

So, that has to be represented by this, but then the moment you observe the flow then it is a observer. And therefore, this has to be masked and what is going through is flow and the other one is not going through, if it went through then it would give a back force on this. So, which we do not want, so it is not giving any it is taking the flow, but it is not giving the effort, so it is done.

So, now let us put the power directions, here it the power comes and naturally we will say the power goes. This is; obviously, not carrying power, but the power direction is still meaningful, because that states the positivity and the negativity here. Now, let us give the causalities, the causality of this one is integral causalities like this, flow has already come into this one junction, it must take out the flow in all the directions, it is a transformer. So, it must transfer the flow it is a 0 junction, so let us put the causality of the C element this.

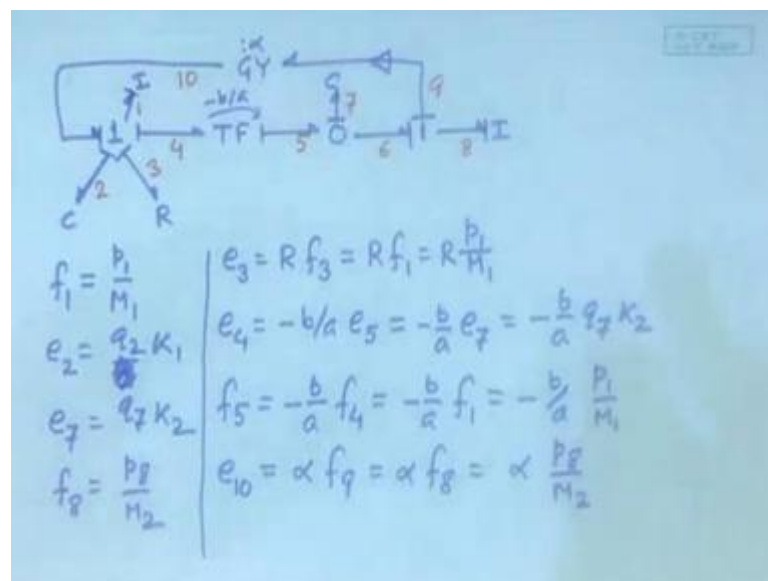
The moment you do that the if information of the effort has come into this 0 junction it must distribute like this at this point it is not yet determined. So, let us put the normal causality of the I element, it gives the flow receives the effort, the moment it is given the flow this one junction is completely determined. So, it must take out the flow information. Notice that the gyrator's proper causality is done.

So, this is where the flow is going in and this is where the effort is going out, one more important thing to consider is, that even though this particular bond is not carrying power, this particular bond is carrying power, because this is applying the force. So, in this case this is not power conserving, that you have to keep in mind. Let us obtain the differential equations of this.

Student: ((Refer Time: 44:09))

Here, yes it is after all taking the flow information yes you can do that. So, his question is cannot we blindly put the causal stroke here, yes you can do that. Because, if you are trying to observe the flow it must be connected to a bond, where the flow information is already available and that happens in a 1 junction, so it is already there. Now, let us number them 1, 2, 3, 4, 5, 6, 7, 8, 9 C 10 there are many things, let me if I have to do this here in this packet page it will be difficult.

(Refer Slide Time: 45:38)



So, let me just in short draw the diagram, otherwise you will not be able to follow. So, it is a 1 connected to a C, connected to a R, connected to transformer, connected to 0, connected to C, connected to 1, connected to I and then from here it gets to a gyrator to this, oh there is an inductor I. So, done I will have this, C will be this, R will be this all the causalities put yes, let me...

Student: ((Refer Time: 46:49))

Activation yes I will give the numbers in another color. So, that otherwise it will be too jumbled 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 all of them done. So, now let us start asking the first question, first question is what do the elements give to the system, so the first this is bond 1, what does it give to the system flow 1. So, let me say f_1 is equal to what?

Student: p_1 .

P_1 by M_1 , ((Refer Time: 47:58)) this is M_1 , this is M_2 , this is K_1 , this is K_2 , this is R , this is a . So, the transformer ratio is b by a and the gyrator ratio is α , so f_1 is done, then C what does it give to the system e_2 , e_2 is q_2 by C K q_2 it is a mechanical thing.

Student: $K_1 q_2$.

$K_1 q_2$, then the R element better let us first complete the easier things, the capacitor the this is spring, that would be e_7 which is $q_7 K_2$ right and this one is f_8 is p_8 by M_2 . So, we have obtained the ones related to the storage elements, now let us start asking the question to this particular element. So, it is giving what is giving effort, so e_3 is $R f_3$ and then $R f_3$ comes from...

Student: ((Refer Time: 49:46))

Yes $R f_1$ and f_1 is known $R p_1$ by M_1 it is done. So, what is remaining this and that, what does the transformer give through this bond to the rest of the system, it gives effort. So, e_4 is equal to b by a times e_5 , e_5 comes from e_7 , 0 junction e_5 is equal to e_7 , so b by a e_7 which is $q_7 K_2$ done. Then, it gives in this bond what does it give f_5 , that will be related to f_4 , so it is b by a f_4 , now where does f_4 come from f_1 . So, b by a f_1 is equal to b by a P_1 by M_1 done, what else remains?

Student: gyrator.

Gyrator, what does it give to the system.

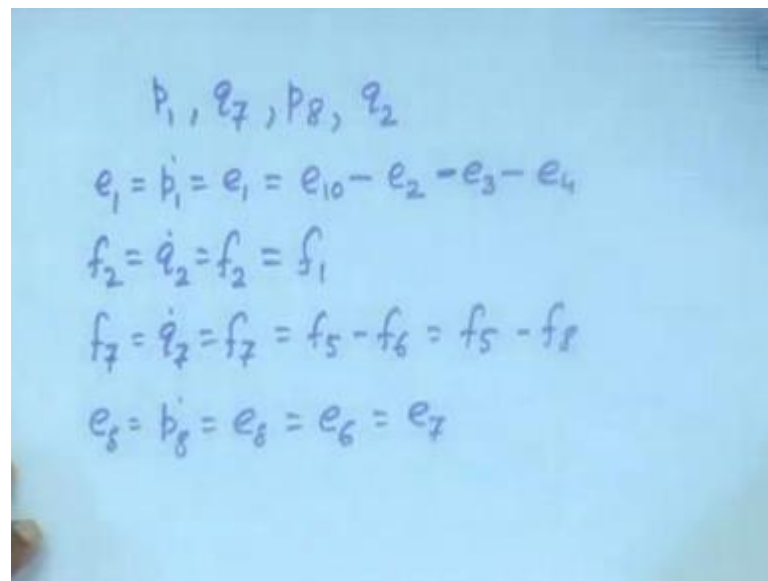
Student: e_{10} .

E_{10} , so e_{10} is equal to it is related to something here αf_9 it comes from here is equal to αf_8 is equal to αP_8 by M_2 done. What does it give to this side,

nothing, because it receives a flow here, but it gives nothing to this side. So, we will not ask this question to this bond, that is the specialty of the activated bond.

So, when we ask the question what do the elements give to the system, we will answer this question related to this side of the gyrator, but not to this side, because this side is activated. So, all these things are done, then come to the next question what do the integrally caused storage elements receive from the system.

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$$\begin{aligned}
 & p_1, q_7, p_8, q_2 \\
 & e_1 = \dot{p}_1 = e_1 = e_{10} - e_2 - e_3 - e_4 \\
 & f_2 = \dot{q}_2 = f_2 = f_1 \\
 & f_7 = \dot{q}_7 = f_7 = f_5 - f_6 = f_5 - f_8 \\
 & e_8 = \dot{p}_8 = e_8 = e_6 = e_7
 \end{aligned}$$

So, that is what are the state variables p_1 , q_7 and p_8 . So, these are the storage elements this, this and this.

Student: q_2 .

Yeah q_2 there are four storage elements. So, let us ask this question to each one p_1 what does it receive from the system e_1 , e_1 is equal to $\dot{p}_1$ is equal to e_1 and this fellow comes from e_1 comes from is a summation of all these. So, we will write it as a summation, notice now we have to do the power directions correctly e_{10} minus e_2 minus e_3 minus e_4 .

So, e_{10} minus e_2 minus e_3 minus e_4 , now all these are probably known just check e_{10} is known. So, e_{10} is known, e_2 is known, e_3 is known, e_4 is known just substitute good, next question let us ask with respect to q_2 . What does it receive from the system f

2, f_2 is equal to q_2 dot is equal to f_2 , then f_2 where does it come from f_1 . So, f_1 which is known p_1 by M_1 , so you substitute it here then we ask the question q_7 .

So, it is f_7 it receives f_7 is equal to q_7 dot is equal to f_7 , where does this information come from f_7 , f_5 minus f_6 it is a 0 junction. So, the flows are summed, so f_5 minus f_6 , now f_5 happens to be available, f_6 happens to be no we have to find out where it comes from f_6 comes from f_8 is equal to f_5 minus f_8 both these are known. So, now the finally, we have to come to 8, so what does it receive from the system e_8 , e_8 is equal to p_8 dot is equal to e_8 , e_8 comes from.

Student: ((Refer Time: 55:37))

Notice, e_6 only because it does not give. So, it is e_6 only these are the things you have to remember, e_6 only not e_9 because it does not give the effort here e_6 only, e_6 is...

Student: e_7 .

E_7 and e_7 is known put it there. So, all that you have to do are remember while dealing with activated bonds is this here, one of the variables is 0, rest of the things are simple. So, we will continue in the next class, let us end here.