Dynamics of Physical Systems

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Lecture - 17
The Bond Graph Approach – V

In the last class, we were formulating some strategies of obtaining bond graphs for circuits in a sort of generalized way. And we said that one of the ways of doing it would be, if you can identify the nodes in the circuit. And those nodes, since anything connected to the nodes will share the same effort. So, those nodes will be designated the junction zero and then anything connected between the nodes will have to share the same current.

And so they will be connected by a one junction, which means that this is a different way of looking at it. So, far we were doing it by logic we were looking at the what are connected in series and what are connected in parallel and accordingly we are laying out the junction strategy. But, now it is a sort of blind procedure that we are talking about, why is it necessary. In the example that we showed in the last class it is not very evident. But, let me tell you that there are situations where it becomes somewhat difficult to figure out the series and parallelness, for example take this circuit.

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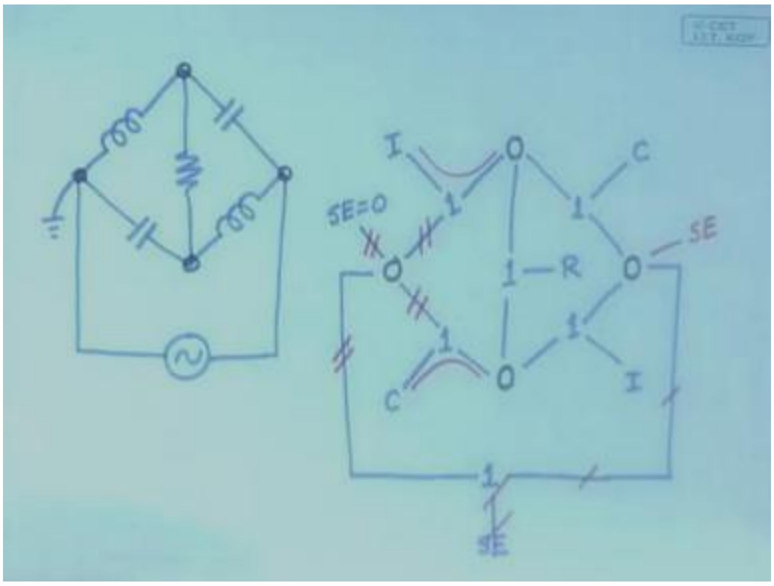

So, this is a source, source of effort. Now, in this case can you see what is connected in series and what is connected in parallel, it is not immediately evident. And in such cases and if such a thing is part of a larger network of elements, it becomes difficult to figure out how to connect this series or parallels. So, in that case it will be convenient to go by that procedure. So, we can identify in this circuit at least a few nodes, this will be a node this will be a node, this will be a node and this will be a node.

So, we will say that these are the four nodes, these are the four nodes will designate by 0 in between this is connected. So, we will connect them by one and what is connected to it an I element, in between these two there is a capacitor. So, that is connected in series between these two that will be our line of logic and that is connected by a I element. Similarly, here it will be a one junction connected here and here it will be a c element, here it will be another one junction.

And you have I element here. Apart from this there is this thing, so we will have to connect it like this, this is what it will again have to be connected by a one junction and what is connected here a S E. So, that is the representation of the circuit now. But, then I said that one of these junctions will have to be in order to.

Student: ((Refer Time: 04:48))

What have I missed, there is the resistor. So, here it is a one junction and connect it a resistor, so that is the complete circuit. Now, we need to reduce this and the way of reducing would be as I told you, that out of these different nodes one would be assumed to be at this zero potential grounded. So, let us assume say this is grounded, the moment you do that you will effectively say that here is an S E element connected which is equal to 0.

The moment you do that, this bond carries 0 power, because the effort is 0. And since, this is a 0 junction therefore, the same effort is distributed equalized. So, all these bonds will carry 0 power and therefore, they would be unnecessary, so this whole thing is unnecessary. Now, the moment you do that you find that this becomes a through junction. So, whatever are the efforts and flows in this bond will be the same as the efforts and flows in this bond and therefore, this one junction is unnecessary.

So, we can then connect it through, similarly here through, similarly here this is not there. So, if this can be a direct connection, so all these will go and we can say here is S E element connected, can there be any further reduction, no. So, we have this situation, but it looks ugly. So, let us lay it out in a more handsome fashion, can you see the whole thing. Let us try to lay it out in a more handsome fashion, you have the this 0 is out, so you have 0 1 0 1 0 1. So, we can do it this way.

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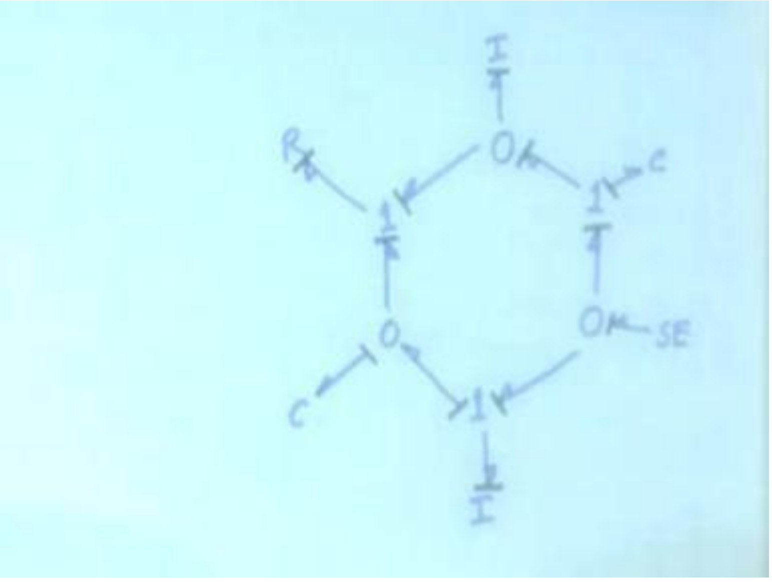

Let us start from here 0 1 0 1 0 1 0 can you see that it forms such a loop with this 0 what is connected the I, with this one what is connected C, with this zero what is connected S E, with this one what is connected I, with this zero what is connected C. So, I will remove it, because then I will not able to draw C and with this one what is connected R. a pretty handsome arrangement. This definitely could not be seen from the original circuit, we could arrive at that only, because we followed this procedure. Now, let us do the usual things in order to practice, one we will have to put the power directions, which will be like this, this, this. Now, you would notice that this bond to bond once, you could put any direction of power.

So, we will arbitrarily put it, it is natural that if the power comes here it will go in this two direction. But, after that it is not clear nevertheless we will put it like that. Remember, these are arbitrary you could as well give the opposite direction it will only mean that the positive and negative would be in the different direction. Let us put the causalities start with S E, it is causality is like this, it gives the effort information and receives the flow information.

This fellow I element it is natural causality is like this, the moment it gives the flow information into the one junction, the flow information must be taken out. So, now this 0 junction has received the effort information and that is why it must distribute the effort information like this. So, it gives the effort information in this direction and receives the flow information in this direction, as yet these are not causaled. So, let me put the causality here C it is integral causality is like this.

So, it is giving the effort information into the 0 junction, the moment it gives it has to be distributed like this R will leave out to do later C, what kind of causality does it have, it is natural causality is like this. So, as yet these two fellows are taking out the flow information and therefore, this bond must bring in the flow information like this. So, in this 0 junction as yet the casual structure is not complete. So, we will put it here, this is the natural causality of the I element.

So, it has taken out the effort information, this one has brought in the effort information 0 junction. So, this has to take out the effort information, one junction both these are taking out the flow information. And therefore, this fellow must be giving the flow information, then it is done. So, that is the causal structure and then you put the numbers. So, this way we can easily obtain by this sort of algorithmic procedure, we can easily obtain the bond graph for any given electrical circuit.

What will the equivalent be in mechanical systems, suppose you argue that for electrical systems we understand, because we can identify nodes, in mechanical systems what you will do. See, we started by identifying the nodes because in a electrical circuit it is easier to identify the different voltage points, in a mechanical system it will be easier to identify the different velocity points. Because, the masses move with velocities and you can identify.

So, in mechanical system we will take the opposite view, we will first identify the points that have different velocities. And the moment you have something at different velocities, something at a particular velocity something must be sharing that velocity connect that with the one junction. Now, wherever there are springs or friction elements the two sides share the same effort. So, those things you connect by a 0 junction, simple logic, just the opposite of what we did in the electrical circuits, let us see an example.

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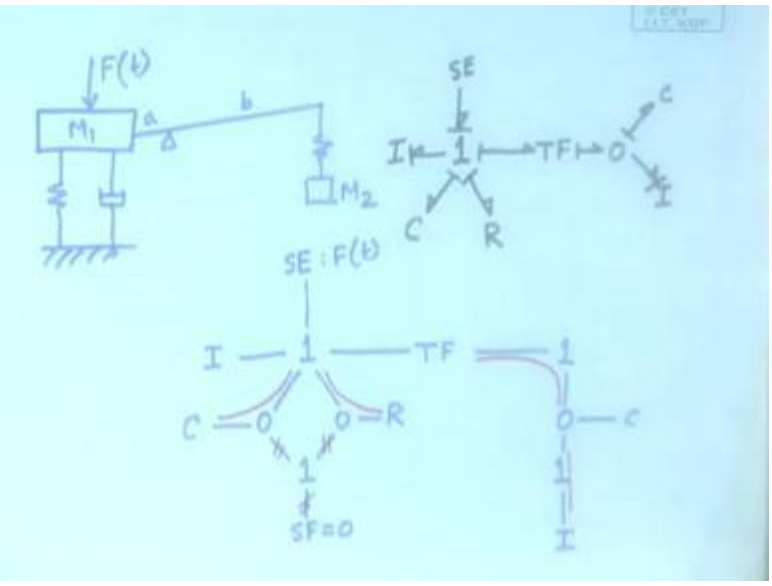

Let us take start with an example that we have already done, the way we did for electrical circuits. For example, this is something that we already did mechanical system with this, this connected to the ground with some force acting on it, this is what we did earlier. A system with a mass spring damper arrangement connected to the ground with a force acting on it and this is a lever with a b ratio and here is another mass M 2, so this is M 1 this M 2.

Earlier we went by logic, but now let us go by this procedure. What are the different velocity points, one I can easily see. So, we will put a 1 junction here two, so we will put another 1 junction here, three we will put another 1 junction here, four that is also another velocity point another 1 junction here done. Now, what is connected to this one junction, the mass definitely.

So, we will connect the mass to it I element, this force also shares the same velocity. So, that will also be connected by means of the same one junction, this is the F of t. But, now these two fellows are there in between these two 1 junctions. And they have to be connected by 0 junction, because their sides share the same effort. So, we will connect a 0 junction like this and connect the this is the C element, we will connect another 0 junction and connect the R element.

Now, here we have between this point and that point we have the lever. So, between this one junction and this one junction we will have.

Student: ((Refer Time15:30))

No transformer.

Student: ((Refer Time15:32))

It is a transformer T F connecting. Now, between one this point and that point we have a 0 to C and this one junction is connected to this I, there is something more. Here, there is a ground something that does not move, how do we represent that.

Student: ((Refer Time: 16:10))

No, it is S F.

Student: ((Refer Time: 16:13))

It is a source of flow, which is forcibly equal to 0, so that is connected to S F. Now, this S F being equal to 0 will immediately mean that this bond carries no power. That will means, since this is a one junction we will mean that these and that these two bonds carry no power. So, we will eliminate these, these, these as a result these becomes a through bond, we can connect directly, this becomes a through bond we can connect directly.

Can you reduce anywhere here, here we can connect directly it is not necessary, here we can connect directly it is not necessary. So, we have a far shorter bond graph now. Let us start with this 1 connected to S E, I, C, R transformer and then it is not necessary to a 0 junction connected to a C and I, that becomes a bond graph. So, that is the simple logical procedure by which you can write the bond graphs.

Now, just as a matter or practice put the power directions and the causalities, here there is a S E looks like this, this, this, this, this done. Power directions were simple causalities how do you put the causalities. First let us start from here S E will take flow information, I will give flow information, the moment you give flow information into the one junction all the others must take it away.

So, the C element is properly causaled, T F will take it the other side 0. So, let us start with the C element, C element will have this casualty it will bring in the effort information. Naturally, it has to give the effort information to the other things, so it is done good. Let us handle now a relatively more complicated problem, in this problem we will consider.

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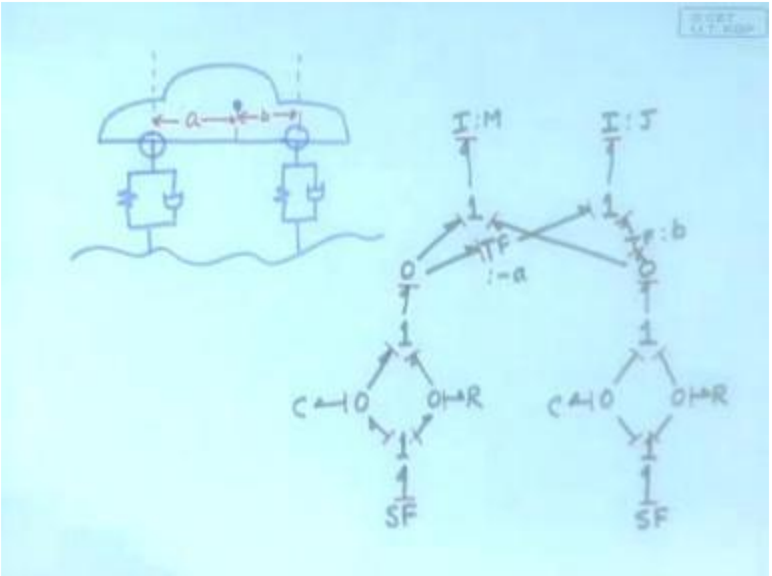

Imagine there is a car and as this car I will not draw the wheels, because the wheels are actually suspended. The car is suspended on the wheels by means of a suspension which is nothing but, a spring damper arrangement. So, it will be good to see this as a connection, where you have a spring damper to something, here also it is we will just draw it this way in order to make the physical property salient.

Now, what is there in at the bottom, bumpy ground a road which is normally bumpy. That means, as this fellow goes ahead and if the road is bumpy, what is seen at these both two points.

Student: ((Refer Time: 20:24))

Source of flow, yes source of flow which is sort of imposed on this two points by the character of the ground. So, at these points you have source of flow, so imagine that you have a bumpy ground here. We are not considering the forward motion, we are considering the up and down motion. But, notice that there will not only be one up and down motion, the fellow will also rock.

That means, some time the front will go up sometime the back will go up. So, there will be some rotational motion also, so how to do this. The points of suspension, if you put this and there would be some point at which the c g d sides say here. So, the c g is here, then the c g is at some distance a from here and b from here visible. So, depending on that here is a c g and here are the forces acting, because of this it will be going up and down.

And not only that this c g itself will go up and down. So, one will be the rocking motion, the other will be the total up and down motion. And for the rocking motion you will have to consider the moment of inertia for the up and down motion you will have to consider the mass. Now, how to make the bond graph of this.

Student: ((Refer Time: 22:09))

First identify the velocity points, where are the velocity points one two direct. So, let us start with this one two, velocity points will be one junction. So, there will be a 1 1 junction here corresponding to this another 1 junction here corresponding to that, there will be 1 1 junction here another 1 junction here, these two are the also the velocity points. So, 1 here, 1 here.

Now, this motion the cars motion will have two components, one the rotational component, another the translational component and this two are also velocities different velocities. So, for that we will put two one junctions like this, we will see what we can do with it. But, we can identify there will be one junctions. Now, what are connected to these two, we have already understood that these are connected to S F source of flow, these are connected to S F.

Now, in between this and that, this point and this point you have the spring damper arrangement whose representation we already known it will be... So, this will be like this no problem. Now, how to connect this point and this point and this point and this point, this point it is this one junction whatever bond is connected to it will have the velocity here, whatever bond is connected to this one will have the velocity here up and down velocity.

Now, if this fellow is going up with some velocity, this fellow is going up with some velocity, how is the total c g determined, how will it be given.

Student: ((Refer Time: 24:30))

How will that be given and if this is moving with some velocity, if this is moving with some velocity, how will the rocking motion be given try to understand these two points. Notice that, from here, here it is giving not only it is giving a force to the car, here is a force being applied and this force is being is causing this motions. So, you are giving a force to the mass that is causing a motion. So, in order to have the force effort what we will do is we will connect it by a 0 junction, a 0 junction.

So, that at this point what we have it, what are these this will have the force that is acting on this. Now, if a force is acting here another force is acting here what will determine this up and down motion, the addition of the two forces will ultimately cause this motion, which kind of junction causes a additional two forces one junction. One junction is a flow equalizing junction, but the effort summing junction. So, if you connect it like this, this point and this point, here the effort that will be seen is a summation of these two efforts.

And what will be connected to it, it will be the I the mass done. Now, this is related to the rocking motion. The rocking motion, suppose you have applied a force here, you have applied another force here what creates the rocking motion.

Student: ((Refer Time: 26:44))

No not the difference yes is a moment.

Student: ((Refer Time: 26:49))

Yes, so this one if there is a force applied times a will be the moment, the force applied times b will be the moment. And so the torque fell by this c g will be...

Student: ((Refer Time: 27:02))

Will be yes, but in that case when we are talking about at this point the torque being applied. The torque is or the moment is how will you see that, this one and that one. Suppose, the motion in the counter clockwise direction is assumed positive, then this fellow will try to move it that direction and this fellow will try to move it in the opposite direction. So, they will act in opposite ways, so it will be see in that case what it means is that, whatever is the effort here is the effort means force. That will be applied to this junction through...

Student: ((Refer Time: 27:53))

A transformer, because the force will be converted into a moment and there by it will be applied. So, it will be a transformer connected to this, here also there will be a transformer connected to this. And the transformer ratios will be…

Student: ((Refer Time: 28:19))

One into…

Student: ((Refer Time: 28:25))

B by minus a, so this side will be b and this side will be minus a. And what you have you got here, the total in that case after summing these two torques what you have here is the total torque applied and that will be connected that will be causing, the motion in the that direction. This will be a I element, which is be the moment of inertia is it right is this be b or 1 by b.

Student: ((Refer Time: 29:18))

It will be multiplied by b to give the ((Refer Time: 29:30)) yes right done. So, you have this can we reduce it, no there is no 0 velocity point. So, we cannot reduce it, so this is the bond graph just causal it, put the power directions first and then the inter junction bonds we can any way do it.

Causalities S F will give this, S F will give flow one junction it will equalize, one junction it will distribute the flow, C natural causality is this, the moment you have it, it has brought in the effort information, it must give out the effort in information. Similarly, here what does the R do, as yet you do not know. So, let us start from the other side, here it is a I element, here it is a I element, so integral causalities 1 junction. So, it will distribute, transformer it will take out the information.

So, this 0 junction has received both the flow information's and distributed the effort information. So, the effort information must be coming from this bond like this, here also it will be like this, here is a 1 junction where the flow information has come. And therefore, it should distribute the flow information has come should distribute, this junction is now bringing in the flow information and taking out the effort, taking out the effort, therefore the R must produce a effort information like this done.

So, even this kind of relatively complicated things which have two different types of motion in the same body we can represent like this. And in fact, if you obtain the differential equations from this, if you simulate this, you can easily obtain how a specific car is going to bounce. And this is actually done, this things are actually done in industries in order to find out how should these things be tuned. So, that there is a least amount of bumpiness.

So, we have understood how to obtain the bond graphs of relatively complicated circuits, notice that there are things that repeatedly appear. For example, this combination that repeatedly appears, you will see later that these combinations the whole thing can be abbreviated. This spring damper arrangement as a sort of cushion is a unit that can be abbreviated with one bond going up and one bond going down.

So, that later all these thing can be just put as one element. But, that comes later, first you have to understand this, but then you will see that there are many things that come as an unit. And that unit has a specific characteristic, that I given by two estimated bonds. Now, there are some situations where we run into some difficulties when writing the differential equations why or giving the causaled strokes. See, what we did I was all the time giving you practice in putting the causalities, why because that is where people often make mistake. But, then all these were sort of straight forward, there is no difficulty. But, let us illustrate a situation where you do have difficulty. Suppose, there is a circuit something like this.

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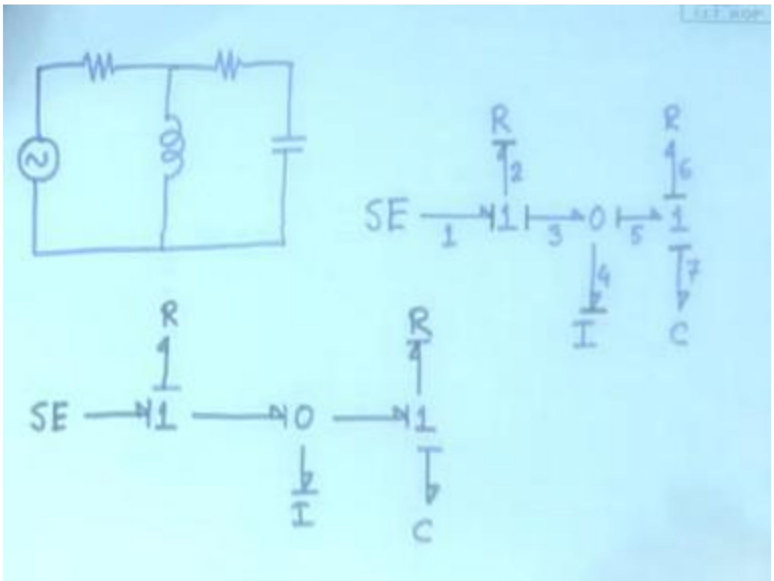

A simple circuit that does not show any sign of being difficult, no difficulty with the circuit, you can easily connect it. Now, let us draw the bond graph of it, S E connected to I connected to R in parallel with I, here it is a series arrangement of R and C. So, give the power directions which would be somewhat simple, now start giving the causalities S E element will give the flow effort information to the rest of the system. So, the causal stroke should be here.

As yet this fellow is not determined. So, let us go straight to the I element, I elements natural causality would be like this, it has brought in the flow information into the 0 junction. But, as yet the other bonds will not be determined, so let us do it here C, the C element has brought in the flow effort information and taken out the flow information all right, but this a one junction.

So, as yet these things are not determined, how will you complete it, in order to complete it you might argue that let us what is the problem. The problem is that this one junction does not yet know where the effort flow information is coming from, let it come from here. Suppose, what does it mean, it will immediately make this one complete, because this has to take out the flow information.

The moment this one has taken in the flow information and taken out the effort, taken out the effort this bond must bring in the effort. So, it has to be like this, so this is a one junction in which the flow information has come and it must be like this done. But, notice that you had to somewhat out of your hat put this causality another fellow can say then no, no I do not want to put the causality here, let us start from here.

Let us see let us do it do it here again S E to an I to R to 0 to 1 to R, I, C. Now, the one that we know already let us put. Then, now that fellow said that let us start from here. Now, this one junction needs the flow information in order to be complete, so let us do it like this. So, this fellow brings in the flow information, the moment it has got in it has got it, so it has to distribute like this.

So, this 0 junction it has taken out the effort information, it has taken out the effort information, therefore this bond must bring in the effort information like this. So, the flow information has gone one junction and must distribute again complete it. But, you see these and that both are true, but they are differently causaled, what does it physically means, if you try to understand that. Let us obtain the differential equations starting from this premise.

We either have to choose this or that in order to obtain the differential equations. Let us choose this, this one, so let us number them first, it will be an 1, 2, 3, 4, 5, 6, 7. So, based on this we are about to write the differential equation, so let me place it here, so that you can see, can you see visible we have some work space here then. So, we first start it by asking the first question which is what do the elements give to the system.

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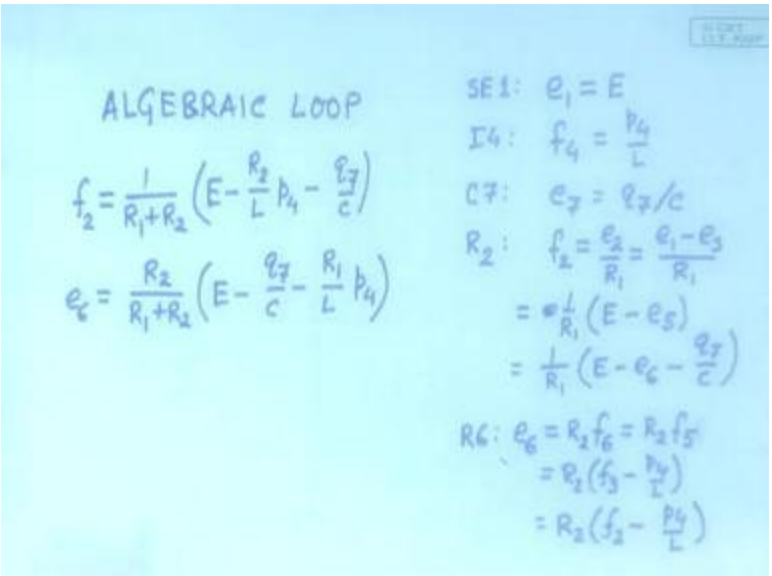

So, the answer to that question with respect to the S E element would be S E 1 would be what does it give to the system e. So, E 1 is equal to the E 1 is this e R 1 R 2 L C. Now, next is R 2, so R 2 gives what...

Student: ((Refer Time: 40:22))

Let us do it first with the storage elements. So, I 4 what does it give to the system F 4 is equal to p 4 by L. Next question is asked to this element C 7, what does it give to the system e 7 which is q 7 by C. The next question is then we start asking the question to the resistances, so R 2 what does it give to the system F 2 F 2 is e 2 by R 1. Now, where does the e 8 information come from e 2 would be this and that. So, it will be e 1 minus e 3 is equal to e 1 minus e 3 by R 1, e 1 is known e 3, e 3 where does it come from.

Student: ((Refer Time: 41:59))

Yes, that is a strong bond. So, is equal to let me put R 1 by R 1 this is known e 1 is known minus e 5. So, e 5 is the one that has brought in the information E 5, E 5 is obtained from where.

Student: ((Refer Time: 42:22))

Yes, so is equal to 1 by R 1 E minus e six plus ((Refer Time: 42:42)) so this will minus. Now, e 7 is known q 7 by C, but e 6 we do not know it yet keep it like this. So, presently keep it like this, now let us ask this question to R 6, so R 6 what does it give to the system e 6. So, e 6 is equal to R 2 f 6 f 6 comes from f 5, R 2 f 5, f 5 come from.

Student: ((Refer Time: 43:27))

F 3 minus f 4 yes is equal to R 2 f 3 minus f 4 is known P 4 by L, f 3 comes from.

Student: F 2.

F 2 is equal to f 2, so R 2 f 2 minus P 4 by L. So, you see you have landed up with one equation, where the left hand side is f 2 and you have e 6 in the right hand side. And another equation in which e 6 in the right and f 2 in the right, which means that in order to obtain f 2 you need e 6 and in order to obtain e 6 you need f 2. This problem is called a algebraic loop, can you see that it has run into a loop, logical loop in order to know this you need to know this, in order to know this you need to know this. So, this is called an algebraic loop.

And whenever there is a indeterminist of the causal structure. So, that you are having to I mean out of your hat put a causality in one of the R elements, it immediately indicates that you will have this kind of problem, while obtaining the differential equation. But, you would notice that there is not a big deal, why because you have two equations and two unknowns, the moment you have two equations, two unknowns you can always solve it and obtain these two individually.

So, they are actually determinable no problem, but you have this causaled loop. So, how would you obtain this, can you just do this bit of algebra to obtain this individually, it will require a small bit of algebra I think it will be f 2 is equal to 2 by 2 plus R 3 E minus and e 6 is equal to check that. So, all if you have done is using these two, here is a quantity, here is a quantity and here is a quantity here is a quantity we have simply solved the two equation two an unknown problem in order to obtain this individually obtainable.

So, then the rest of the problem can be done exactly the way we have learnt. So, after we have obtain the answers to the first question, then we will ask the second question namely what do the two integrally causaled storage elements receive from the system and we will go exactly the same way I will leave these exercise to you. So, far we were considering the R element, C element, I element as a one port element like this.

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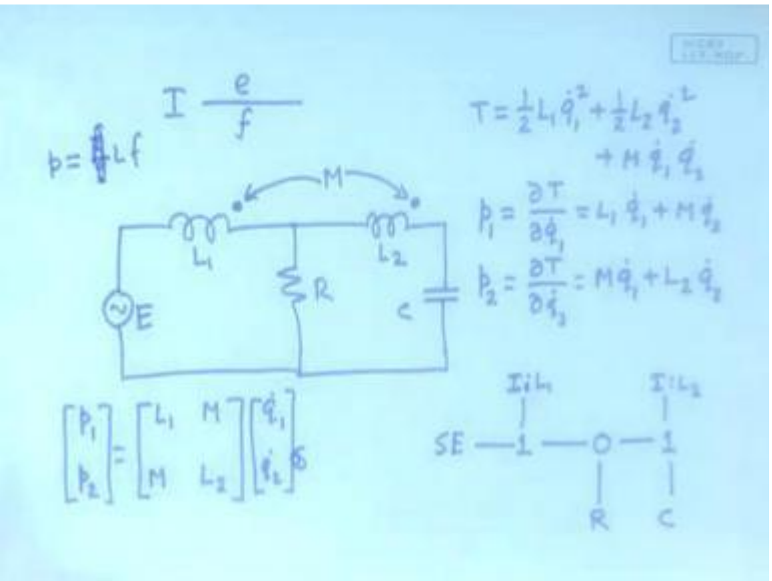

I connected to rest of the system, what does it mean, means that here is a say this is bond number 1 it is a e flowing and there is a f flowing. Then, this e and f completely determine the characteristics of the I element, which means it is effort is completely given by the history of it is flow. So, that is the character of the I element that the voltage is equal to L d i d t or I flow is integral of the effort.

So, the flow now is completely given by the history of the effort in this bond itself. But, there are situations where the flow could be given by the effort not only in this bond, but in another bond also, where imagine. Imagine the situation of a mutual inductance, mutual take a circuit, say you have got a source and you have got a inductor and you have got a resistor here and there is another inductor here something here and imagine that there is a coupling between this, so this L 1, this L 2 R, C, E. Now, how will you do if you try to obtain the bond graph directly, you will say that if I ignore this M part what we will do. We will do it simply like this no i will do it here S E in series with an I colon L 1 in parallel with R in series with 1 it is a I L 2 and C. If this inductor this in this mutual coupling where not present this would be the diagram, but the moment the mutual coupling is there what happens. Then, the voltage induced here will be dependent not only on the current here, but also on the current here.

So, the current here is available at this one junction, the current is available here in this one junction. So, some way these two will have to be coupled, coupled how, this how question how are the momentums and flows are coupled in order to probe that, just let us just get back to the question of the kinetic energy of this whole system. Kinetic energy T was how will you write it half L 1 q 1 dot square plus half L 2 q 2 dot square plus M q 1 dot q 2 dot.

So, your momentum P 1 was the derivative of T or derivative Lagrangian is the same as in this case T and derivative with respect to q 1 dot. This will come to be...

Student: ((Refer Time: 51:42))

L 1 q 1 dot plus M q 2 dot. Similarly, P 2 will be it is M q 1 dot plus L 2 q 2 dot, so how are the momenta and the flows, these are the flows right and the flows related you would notice that you would be able to write a matrix equation P 1 P 2 is equal to L 1 M M L 2 q 1 dot q 2 dot. If this were not there, then how would you write the equation, in that case if you only consider one inductance how would you write, you would write P is equal to flow by L that is all no L i, so L into flow.

So, in this case the momentum is just this number times the flow in the same, that is why it can be represented by one bond. But, here you see they become coupled in this matrix way, now this is the flow in this particular junction, this is the flow in this particular junction. And therefore, these two will have to be coupled by means this of this two matrix in order to obtain the character of this. So, the way to do that is I will start from here and do it.

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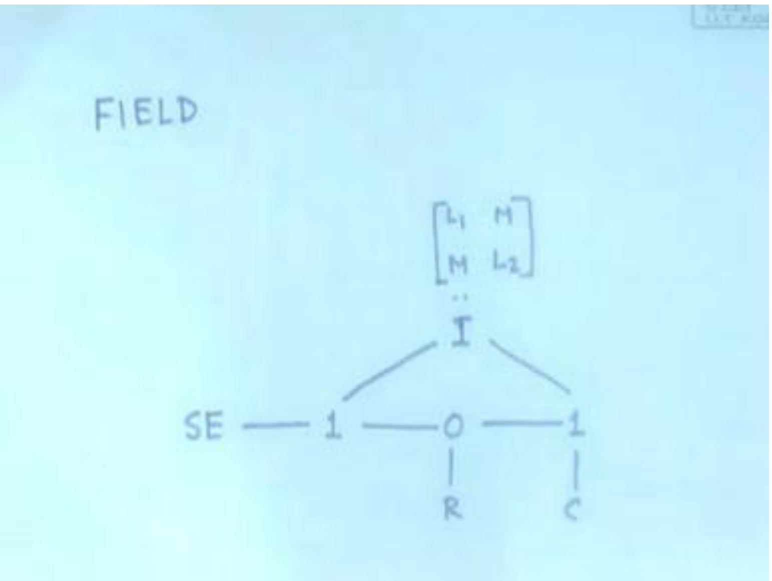

S E to 1 to 0 to 1 R C. Now, here can you see that, can you see the whole of this. This particular thing is available here, this is f 1 f here flow, this is available here the flow. So, these two will have to be coupled to this is also having the character of the inductance, this whole thing also having a character of a momentum. Why, because in a momentum this is how there it is related, this is a momentum like thing. So, it will also be an I element, it will be also be an I element.

But, it is character will be represented by a matrix L 1 M M L 2 that completes the story, these are two port I element. Under such situations, you can you will have an I element with two ports is the concept understood, why you need a two port I I element sometimes, these are also called fields. So, this is a I field, I field means a two port I element. That is a natural necessity when you want to represent a mutual inductance, there will be situations where you will need similar representations for C for R things like that. And in the next class I will illustrate under what condition you would need that fine, that's all for today.