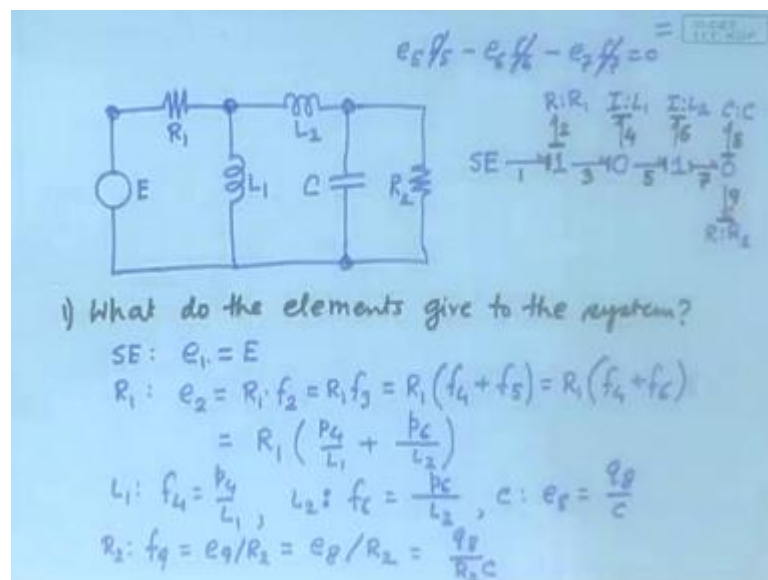


Dynamics of Physical Systems
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Lecture - 16
The Bond Graph Approach – IV

Well, in the last class, we were beginning to learn how to obtain the differential equations from the bond graph. And we learnt that we essentially asked two questions. One we asked about all the elements what do they give to the system, and we asked to this integrally caused storage elements what do, they received from the system. So, just by answering these two questions and trying to express the answers in terms of, the identified state variables, we get the equations. And we had done it with respect to one system, in the last class let us recapitulate by working out another example.

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Let us take a circuit example today, so in this example we have a source of effort E then connected to a resistance, connect to an inductance here, is another inductance, here is a capacitance and finally, another resistance. Suppose, this is my circuit there will be R 1 R 2 here L 1 L 2 here and C, so when we begin to do the bond of this we start from one of the ends say the S E element. So, we start from the S E element here it shares the same flow with R 1, so it will be a 1 junction connected to an R which is R 1.

Then this combine shares, the same effort with the L 1, so it will be connected to a 0 junction, with L with I which is L 1, you see inertial element which is designated as I. Notice that I am not directly writing L 1 here, there is a generic representation of a resistive element as R inertial element as I, so we are writing that way. Now, after this stage it is in series whatever this combination is that shares the same flow with L 2, so it will be a 1 junction connected to I L 2.

And finally, here these two share the same effort, they have the same voltage, so it will be a 0, there will be two things connected 1 I will connect this side it is R, which is R 2 and here it is C, which is just only one C. It is visible, so that completes the story. Now, let us put the power directions and the numbers. Now, when we want to put power directions, we start from the S E element power directed outwards, these ones will have the power directions into the elements and here.

Since, the power is flowing in this direction there is only one power source therefore, it is logical to put the power direction here like this, so you may choose to do it otherwise because these are bond to junction to junction bonds. So, let us do it this way now let us give the causalities, how do you assign the causalities where would you start.

Student: ((Refer Time: 05:04))

Let us start from the source of effort, the effort is causal in what way this way, because it receives the flow information and gives out the effort information. Then this element I normally what does it do.

Student: ((Refer Time: 05:25))

In this one is it completely causal, no because, it has not yet is a 1 junction it has not yet received the flow information, so only when it is does. So, we will say that it is completely causal, so here we will first put this which means that the I element is receiving the effort information and giving out the flow information and it has come to the 0 it is not yet caused, because 0 will be completely causal, if a strong bond brings in what information, effort information.

So, let us move to the next one, here let us put the, the moment you do that the flow information has come into the 1 junction. And therefore, the flow information must go out of the 1 junction, so these two are immediately caused fine. Come to the this point, now see it has taken out the effort information, it has taken out the effort information and

therefore, this bond must be on the effort information, so it must be like this. So, here is a 1 junction, which has received the flow information and therefore, the flow information must go out.

Now, you come to the last one 0, which has received the flow information, but you need a effort information normally, the C element would create the effort information and bring it here. And therefore, the moment you do that, the other ones are immediately caused, because this has brought in the a effort information and effort information will go out through these two which is going out.

So, all these bonds are now caused is that clear, now we let us give numbers, simply 1, 2, 3, 4, 5, 6, 7, 8 and 9 done, so we are through as far as the as far as, drawing the bond of the concern we are through now the next step is to obtain the equations. Now, let us ask the first question, so that is the first question let us ask this question to all the elements let us start.

First S E what does it given to the system.

Student: Effort information.

S E gives the effort information, which is e_1 , so e_1 which is we know E, next come to the R 1, so R 1 what does it give to the system R 1 notice here, you have to refer not to this, but to this in order to find out what it gives, it gives effort.

Student: Effort.

It is not by elements, see apiary unless you draw the bond graph, because a R element could give both the information's. Now, here in this specific case it is giving out the effort information, so you start with e_2 , e_2 will be related to its own character which is,

Student: R 1 times.

Yes R 1 times f_2 , then we have to find out where did this f_2 information come from, f_2 information has come from this as the strong bond. So, this is equal to R 1 f_3 , now f_3 information where did it come from 0. Therefore, f_3 is a combination of this and that and notice the power directions the f_3 , because the voltages are equal efforts are equal, so f_3 is equal to f_4 plus f_5 is that right, because of the power directions remember that.

Had the power directions been opposite, then it would have been it would be some minuses, so here it is $R_1 f_4$ plus f_5 done. Now, f_4 is known why because, it is a state variable, so we are through. f_5 what is it, f_5 should be coming from somewhere and f_5 is coming from this bond f_6 . So, is $R_1 f_4$ plus f_6 actually, now we can write the things properly, R_1 into f_4 is, f_4 is what?

Student: ((Refer Time: 11:18))

Actually, while writing this we should have done it for the storage elements first, because then it would be directly available, but we can see the f_4 is P_4 by L_1 . So, it is P_4 by L_1 plus f_6 is P_6 by L_2 done, have you notice how we went about, we basically try to figure out where does the information get created and we reached that particular thing, this will always be some storage element. And therefore, ultimately we get the right hand side in terms of the state variables fine.

Next we move on to these three, these three would be trivial, for L_1 what does it give to the system.

Student: f_4 .

Yeah, f_4 which is P_4 by L_1 , then L_2 what does it give to the system f_6 f_6 P_6 by L_2 , C what does it give to the system, e_8 can you all see that it gives to the system e_8 , e_8 which is...

Student: ((Refer Time: 12:50))

Yes, q_8 by C , so this is done one thing left here let us let us do that, see here is a space I will push it a little up, so that you can see we can write here, R_2 what does it give to the system f_9 , so $R_2 f_9 f_9$ is...

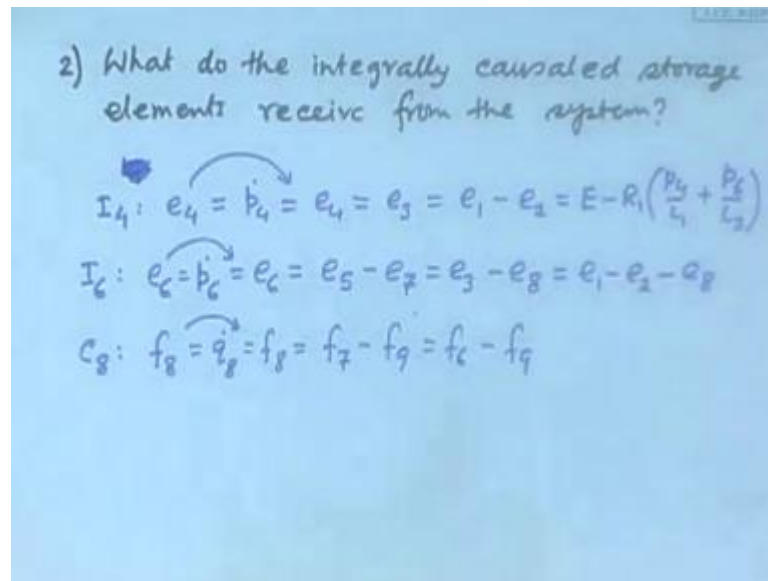
Student: E_9 .

Yeah, e_9 by R_2 fine, so we have related to the character of that particular system element. And then e_9 , we have to scout the rest of the ((Refer Time: 13:54)) to find out, where this fellow has got created. E_9 was created at,

Student: E_8 .

E_8 right, so e_8 by R_2 and e_8 we know, q_8 by 6 is equal to q_8 by $R_2 C$ fine. So, we have obtained everything, we have obtained the answers of this question related to all the elements, so the first part is done, second part.

(Refer Slide Time: 14:37)



2) What do the integrally caused storage elements receive from the system?

$I_4: e_4 = \dot{p}_4 = e_4 = e_3 = e_1 - e_2 = E - R_1 \left(\frac{p_4}{L_1} + \frac{p_6}{L_2} \right)$

$I_6: e_6 = \dot{p}_6 = e_6 = e_5 - e_7 = e_3 - e_8 = e_1 - e_2 - e_8$

$C_8: f_8 = \dot{q}_8 = f_8 = f_7 - f_9 = f_6 - f_9$

The second part, the second question is what do the integrally caused storage elements receive from the system, yes this is important though in this particular problem all the storage elements are integrally caused can you see that. ((Refer Time: 15:21)) for the I element an integral causality is, where the causal stroke is here, because then it is receiving the effort information giving out the flow information and they are integrally related, for this it is correct, for this it is correct and therefore, all these storage elements are integrally caused.

So, our state variables are f_4 f_6 e_8 , so we ask this question with respect to these elements let us start. First we ask this question with respect to L 1 or basically it is better to write that, I am asking the question with respect to I element connected to bond 4 I element connected to bond 4. What does it receive from the system e_4 , e_4 , so we will say it is e_4 and e_4 is equal to \dot{p}_4 dot.

Student: ((Refer Time: 16:33))

No \dot{p}_4 dot.

Student: ((Refer Time: 16:36))

That is the character of any integral, or any inertial element that the effort is the \dot{p}_4 dot that is it, so we bring this thing to this side is equal to e_4 . So, the dotted term is in the left hand side and the un dotted term in the right hand side, and then we try to figure out where this information comes from. E_4 ((Refer Time: 17:03)) e_4 where did it come

from, it is connected to a 0 junction and therefore, there should be a strong bond bringing that information which is e_3 , so that is e_3 , now e_3 yes, this is a 1 junction.

And therefore, that is a combination of no, not e_1 plus e_2 notice, here the power direction has to be considered, e_1 is equal to note the power direction is equal to e_2 plus e_3 and therefore, e_3 is e_1 minus e_2 minus e_2 this is e_1 minus e_2 . Now, both these we already know, e_1 will know e_2 we know from the answers to the last question, just substitute.

So, we get this as E minus $R_1 P_4$ by L_1 plus P_6 by L_2 fine, so one differential equation obtained. Second we ask the question related to the ((Refer Time: 18:26)) I element connected to bond 6, so I_6 what does it receive from the system, ((Refer Time: 18:37)) it is e_6 . So, e_6 is equal to P_6 dot and then we bring it to this side e_6 , now e_6 is,

Student: ((Refer Time: 18:48))

Yeah, this is a 1 junction therefore, notice e_5 the power direction is that way is equal to e_6 plus e_7 , so e_6 is nothing but e_5 minus e_7 , e_5 minus e_7 good. ((Refer Time: 19:11)) Where does the information e_5 come from.

Student: E_3 .

E_3 yes, so e_3 here and where does the information e_7 come from,

Student: E_8 .

E_8 , so minus e_8 fine, now e_3 ((Refer Time: 19:35)) where does e_3 come from,

Student: E_1 minus e_2 .

E_1 minus e_2 yes, so e_1 minus e_2 minus e_8 , ((Refer Time: 20:03)) you see e_8 e_3 minus e_8 , e_3 has come from e_1 minus e_2 and e_8 is fine, e_7 is equal to e_8 . Now, all these are known in the right hand side just substitute, because e_1 is known from here, e_2 is known from here and e_8 is known from here, just substitute you will get it happy, let us go to the C element, that is connected to C connected to 8 bond number 8.

Student: ((Refer Time: 21:08))

How will you write it.

Student: ((Refer Time: 21:10))

F 8, f_8 ((Refer Time: 21:16)) is what it receives from the system and that is equal to q_8 dot is equal to q_8 dot and then bring it to this side is equal to f_8 and then find where the f_8 comes from, ((Refer Time: 21:32)) f_8 this is a 0 junction and therefore, it has to be relate to f_7 and f_9 .

Student: F_7 minus f_9 .

Yeah, f_7 minus f_9 is equal to f_8 , because of the power directions fine f_7 minus f_9 , f_7 comes from f_6 , that is the strong bond bringing in the information to the 1 junction when it becomes distributed to the two sides. So, f_6 , f_6 which is known, ((Refer Time: 22:09)) so we will not bother about that f_9 , f_9 is also known minus f_9 which is also known.

Just substitute f_6 is known ((Refer Time: 22:19)), because of this f_9 is known because of this substitute you will get it, yeah.

Student: ((Refer Time: 22:28))

Where...

Student: ((Refer Time: 22:34))

Wait, wait, let us, let us see, let us see, let us check e this is e_6 , e_6 is...

Student: ((Refer Time: 22:46))

E_6 is e_5 minus e_7 e_5 minus e_7 and e_5 is e_5 is ((Refer Time: 22:56)) coming from here.

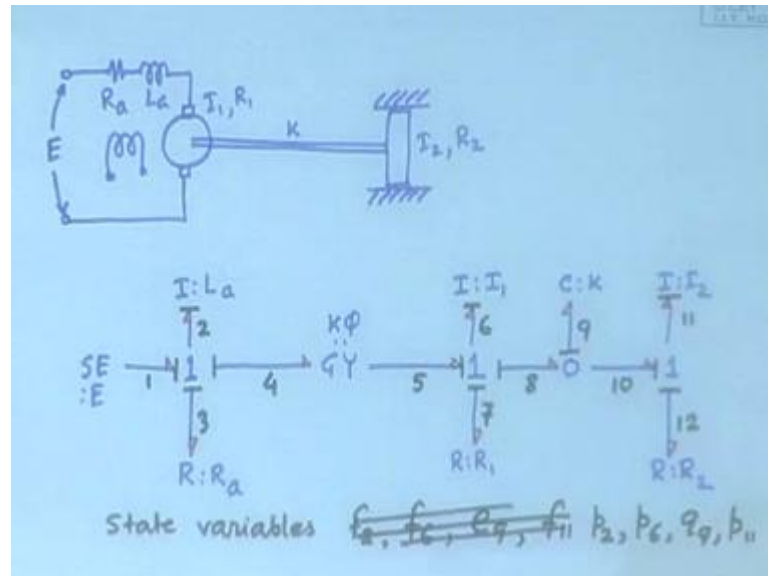
Student: ((Refer Time: 23:02))

Power directions, your question is why is this true this part e_6 is equal to e_5 minus e_7 , let us, ((Refer Time: 23:16)) let us illustrate that, in this particular 1 junction you have the power direction like this power direction like that power direction like this, which means that e_5 f_5 is going in minus, this going out e_6 f_6 this also going out, minus e_7 f_7 should be equal to zero because there is no power here.

Now, it is a 1 junction which means the effort flow equalizing junction, which means the flows are all equal they cancel off, which means we were interested in f_6 e_6 , so e_6 goes this side which is e_5 minus e_7 is that right, yes.

So, e_5 minus e_7 was correct fine have you understood the procedure, then all these are known from the from this, so you just substitute you get the equations fine. Let us do another problem related to we have already done the motor problem.

(Refer Slide Time: 24:45)



In the motor problem your system description was, you had a DC motor and it was connected to a source, and there would be the internal resistance inductance and this was constant field was constant. This was connected by means of a shaft to some rotational mass, which is allowed to move against a frictional element that was the system description. Now, when we obtained the bond graph how did you go about it, we started from the applied voltage here.

So, it would be S E which is E that is connected in series with the inductance and the resistance, so connected with 1 junction to the I element, which is armature inductance L_a to a R element, which is R_a armature resistance armature inductance. And that shares yes that shares the same effort with whatever, is here now what is here, it sees the electrical circuit sees it as a something that gives a back EMF. And that we have already seen that that would be ((Refer Time: 26:27)) by gyrator, so it is a gyrator here with ratio $k\phi$ and the gyrator will be connected to this side.

So, this is the electrical side, this is the mechanical side, the electrical sides effort is a back EMF and that is related to the speed of rotation, in this side that is a flow. The electrical sides flow is the current that is related to the torque in the mechanical side that

is why it is a gyrator fine. So, we have come to this point at this point here is a rotational element, which will have its own mass as well as it is bearing friction.

So, they whatever the mass is whatever the bearing friction is shares the same flow share the same flow with this. So, it will be a 1 junction it will be a I, which was I one and here, it was a R, R 1, so here it is I 1 R 1 here it is I 2. Now, here there was a springiness there was some bit of freedom some degree of freedom, so that they can move somewhat independent of the, that this point and that point share the same effort.

Student: Effort.

Yes, they share the same effort, as a result of which this C element will be connected by means of a 0 junction to the C which is K. Then in this side, you have these two element sharing the same flow, so it is 1 I, I 2, R, R 2, so that is the bond graph, let us give numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Now, let us give the power directions, here these are very standard, so let us not spend time on explaining this.

Now, since there is only one source, it is logical to have the positive power flow assumed to be in this direction though as I told you, you can as well put then on the opposite direction, but let us do it this way. Now, let us assign the causalities causalities, first S E element will be receiving the flow information and giving out the effort information. At this point this 1 junction is not causal, so let us put the causality of the I element integral causality, which is like this.

The moment it brings in the flow information, it has to take out the flow information and therefore, this junction becomes completely caused, well the flow information coming this way into the gyrator. And therefore, the effort information must go out, which means the stroke must be here, so here the flow information is coming in gyrator therefore, it is multiplied the flow is multiplied with the K phi to give the effort information. So, the information flow would be effort in this direction, so effort goes out.

Now, this one is not yet completely caused, because this is not a strong bond, so we have to put the causality integral causality in this storage element, which is like this. The moment, you do this it has already obtained the flow information and must take out the flow information in the other bonds which is right. Next you come here this is not yet caused, so let us put it here, this is the integral causality for the C element and the

moment this receive the effort information, it must give out the effort information in the other two bonds like this.

1 junction, if you put the integral causality like this it has received the flow information and therefore, it must take out the flow information, it is already taking at the flow information here and it must take out the flow information like this. So, this is how you causal it, so it has been power directed numbered caused done, now let us start the business of obtaining the bond graphs, what are the state variables here.

Student: ((Refer Time: 31:38))

Yes, yeah f 2, f 6, e 9, f 11 right here, sorry, sorry, no, no, no, f's are not the...

Student: ((Refer Time: 32:04))

Wrong.

Student: ((Refer Time: 32:06))

Wrong, P 2, P 6, q 9 and P 11 these are the state variables fine, So, let us start ask the question what was the first question. What do the elements give to the system let us ask this question to the first.

(Refer Slide Time: 32:36)

What do the elements give to the system?

$$e_1 = E, f_2 = p_2/L_a, f_6 = \frac{p_6}{I_1}, e_9 = \tau \cdot \kappa, f_{11} = \frac{p_{11}}{I_2}$$

$$e_3 = R_a f_3 = R_a f_2 = R_a p_2/L_a$$

$$e_7 = R_1 f_7 = R_1 f_6 = R_1 \frac{p_6}{I_1}$$

$$e_{12} = R_2 f_{12} = R_2 f_{11} = R_2 \frac{p_{11}}{I_2}$$

$$e_4 = \kappa \phi f_5 = \kappa \phi f_6 = \kappa \phi \frac{p_6}{I_1}$$

$$e_5 = \kappa \phi f_4 = \kappa \phi f_2 = \kappa \phi p_2/L_a$$

So, let us start with first one, e 1, so what does it give, ((Refer Time: 33:05)) it gives e 1, e 1 is E, so the first question answered. Next come to come here it gives f 2, f2 is P 2 by L a, I am just writing with commerce, so that we conserve space. ((Refer Time: 33:33))

This one let us finish the storage element it will be easier, this is f_6 , f_6 is equal to P_6 by I_1 , ((Refer Time: 33:52)) this gives e_9 , e_9 is q_9 times K . And your f_{11} is, P_{11} by I_2 , so we have done it for all the storage elements and the source these were simple, but now let us do it for the three resistances.

First one, ((Refer Time: 34:37)) this fellow what does it give to the system e_3 yes, so e_3 is equal to $R_a f_3$ is equal to now, we start to find out where is f_3 information comes from, f_3 information obviously, comes from f_2 , so $R_a f_2$, which is known $R_a P_2$ by L_a , so completed. ((Refer Time: 35:16)) Second one, what does it give to the system, e_7 , e_7 that is, $R_1 f_7$ and f_7 information comes from clearly from here, ((Refer Time: 35:29)) that is the strong bond bringing in that information, so $R_1 f_6$ which is again simple $R_1 P_6$ by I_1 .

Similarly, the last one, ((Refer Time: 35:51)) this one it gives e_{12} , e_{12} is equal to $R_2 f_{12}$ and f_{12} information comes from simply here. So, $R_2 f_{11}$ which is also known $R_2 P_{11}$ by I_2 , so all these answers are obtained we are happy have, we have we obtained everything. ((Refer Time: 36:23)) No not really, because the gyrator element also receives gives something to the two sides let us text of that, what does the gyrator element give to this side and what does the gyrator element give to this side, well to this side it gives.

Student: ((Refer Time: 36:41))

Yes, effort f sorry, e_4 , so to this side it is e_4 , e_4 , so when we start with e_4 that is $K \phi$ times f_5 , $K \phi$, f_5 yes f_5 , f_5 comes from f_6 .

Student: F_6 .

F_6 is P_6 by I_1 [FL], ((Refer Time: 37:24)) next what does it give to this side it is what does it give to this side

Student: ((Refer Time: 37:31))

No, no what does, what does it give to this side.

Student: ((Refer Time: 37:38))

No, see yes, yes yeah e_5 , because it gives e to this side, it give e to this side can you see that, because the causalities were the causal structure is there, it gives e to this side e to this side. Therefore, e_5 , e_5 is $K \phi e_4$, f_4 sorry f_4 , now f_4 information where does it

come from f_2 simple, so $K \phi f_2$ is equal to $K \phi f_2$ is P_2 by L_a . Had it been a transformer ((Refer Time: 38:35)), yes then it would be different, but since it is a gyrator it gives if either it gives flow to both sides or it gives effort to both sides, so we have done it, now let us answer the second question.

(Refer Slide Time: 38:49)

What do the storage elements receive from the system?

$$I_2: e_2 = \dot{p}_2 = e_2 = e_1 - e_3 - e_4 = E - \frac{R_a \dot{p}_2}{L_a} - \frac{K \phi \dot{p}_6}{I_1}$$

$$I_6: e_6 = \dot{p}_6 = e_6 = e_5 - e_7 - e_8 = e_5 - e_7 - e_9$$

$$= K \phi \frac{\dot{p}_2}{L_a} - R_1 \dot{p}_6 / I_1 - K \phi \dot{q}_9$$

$$C_9: f_9 = \dot{q}_9 = f_9 = f_8 - f_{10} = f_6 - f_{11} = \frac{\dot{p}_6}{I_1} - \frac{\dot{p}_{11}}{I_2}$$

$$I_{11}: e_{11} = \dot{p}_{11} = e_{11} = e_{10} - e_{12} = e_9 - e_{12}$$

$$= K \phi \dot{q}_9 - R_2 \dot{p}_{11} / I_2$$

First, let us ask this question to, ((Refer Time: 39:18)) I_2 what does it receive from the system, I_2 it receives effort 2, so e_2 is equal to \dot{P}_2 . So, bring it to this side is equal to e_2 and then e_2 , where do you obtain e_2 from.

Student: ((Refer Time: 39:44))

Yes, it is a 1 junction therefore, e_1 minus e_3 minus e_4 is equal to e_2 , because of the power directions are you now comfortable with how to set the pluses and minuses. From the power directions, e_1 minus e_3 minus e_4 is equal to e_2 , e_1 minus e_3 minus e_4 , e_1 is known is equal to E minus e_3 , e_3 is known ((Refer Time: 40:23)) e_4 , e_4 is known right all these are known substitute. Can you see that e_1 is known e_3 is known e_4 is known put it, so you get $R_a \dot{P}_2$ by L_a minus $K \phi \dot{P}_6$ by I_1 , so that is one differential equation with a dotted term in the left hand side and un dotted terms in the right hand side. Now, let us ask this question two this one, I_6 here it receives e_6 is equal to \dot{P}_6 dot is equal to e_6 we will start from here. \dot{P}_6 , e_6 , e_6 where did it come from ((Refer Time: 41:33)).

Student: ((Refer Time: 41:35))

Yes, because it is a 1 junction it will be effort summing junction therefore, $e_5 - e_7 - e_8$, $e_5 - e_7 - e_8$, ((Refer Time: 41:55)) e_8 is equal to where did this information come from,

Student: E 9.

E 9, because that is the strong bond, so we can write $e_5 - e_7 - e_9$. Now, you notice that all these are available e_5 here ((Refer Time: 42:12)) e_7 here, e_9 here, just substitute. So, if you substitute you obtain $K_{\phi} P_2$ by $L_a - R_1 P_6$ by $I_1 - K_q \dot{q}_9$ sorry not dot, $e_q \dot{q}_9$ K q 9 sorry, so this is the second.

Now, we ask this question in relationship to this one, which is C in the bond 9, ((Refer Time: 43:16)) C in the bond 9, it receives flow f_9 , f_9 is equal to \dot{q}_9 is equal to f_9 . And then we scout the bond graph to find out where does f_9 come from f_9 , is e_8 sorry $f_8 - f_{10}$ in this bond $f_8 - f_{10}$. f_{10} is actually comes from f_{11} and f_8 is actually come from f_6 , so $f_6 - f_{11}$ ((Refer Time: 44:04)) and both these are available f_6 is here f_{11} is here put it in, is equal to P_6 by $I_1 - P_{11}$ by I_2 done.

Finally, let us ask this question in relationship to this one ((Refer Time: 44:27)) I connected to bond 11, I_{11} it receives effort. So, e_{11} is equal to P_{11} dot is equal to e_{11} we bring it to this side and then find out, ((Refer Time: 44:45)) where does e_{11} come from it is e_9 , no $e_{10} - e_{12}$ e_{11} is $e_{10} - e_{12}$, because of the power directions $e_{10} - e_{12}$. ((Refer Time: 45:07)) E_{10} is coming from e_9 , so $e_9 - e_{12}$ should be directly available, so because, is related to this e_{12} is available and e_9 ((Refer Time: 45:23)) is here put it in, so you have e_9 is $K_q \dot{q}_9 - e_{12}$ is $R_2 P_{11}$ by I_2 , so these are the 4 equations done.

Whenever we ask such questions in exams and when the result whatever you have, obtained are linear you should always give the final result in terms of a matrix equation. So, can we express this as a matrix equation, do that fast do not give the final answer in exams like this, if there if the resulting equations are linear equations always express that as a matrix equation.

(Refer Slide Time: 46:21)

$$\begin{bmatrix} \dot{p}_2 \\ \dot{p}_6 \\ \dot{q}_9 \\ \dot{p}_{11} \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K\phi}{I_1} & 0 & 0 \\ \frac{K\phi}{L_a} & -\frac{R_1}{I_1} & -K & 0 \end{bmatrix} \begin{bmatrix} p_2 \\ p_6 \\ q_9 \\ p_{11} \end{bmatrix} + \begin{bmatrix} E \\ 0 \end{bmatrix}$$

$$\dot{X} = AX + BU$$

So, it will be in this case a, this is \dot{p}_2 , \dot{p}_6 , \dot{q}_9 and \dot{p}_{11} is equal to something times, p_2 , p_6 , q_9 and p_{11} . And whatever this matrix is, for example ((Refer Time: 46:56)) in the first equation it is plus this seems, that are the external inputs, so this will be E , this is related to minus R_a by L_a minus, R_a by L_a the first one, next one is minus $K\phi$ by I_1 times, p_6 and this is these two are 0 0, fine. Notice it is in the right hand side, you only have p_2 and p_6 and not the other two variables, so these two terms would be 0.

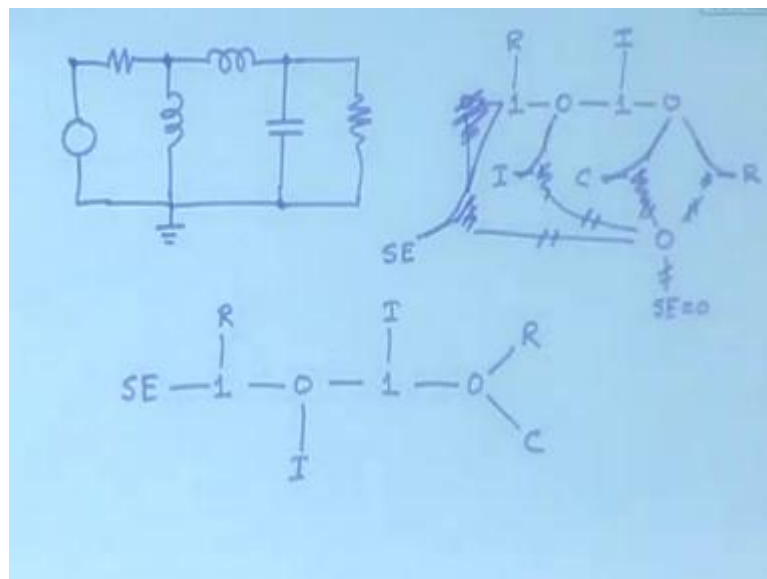
Next one \dot{p}_6 contains this terms it contains p_2 , p_6 and q_9 , so p_2 , p_6 and q_9 , but not p_{11} , so these three will be there. And then you can write, $K\phi$ by L_a , $K\phi$ by L_a for p_2 term, p_6 term the second one minus R_1 by I_1 minus, R_1 by I_1 , third one minus K 0 and so on, and so forth clear. Make it a habit whenever, the equations are obtained in linear form always give the final thing in matrix, this will be the vector of state variables, which is say \dot{X} is equal to this will be called the A matrix A , this is the X vector X plus $B U$, U is the externally applied force, so you will always express these equations in this form fine.

Now, please complete the rest that will be trivial and I am not showing each and every term for it. So, far whenever we were trying to obtain the bond graph, we were we are following some kind of a logical procedure, in case of electrical circuits what did we do we, started from say the left side and went on into the circuit trying to argue, which one share the same flow and which one share the same effort.

For simple circuits, these are these are fine I mean you can argue things, but for more complicated circuits you know something that looks like a jumble of things, it will be rather difficult to do it that way. So, there you need some kind of a blind procedure which will work, the blind procedure is something like this say, ((Refer Time: 50:16)) if we if we take this circuit we had obtained this bond graph by simply working by logic. Now, in this case what we will do the procedure, which is somewhat algorithmic will be that identify the points, that can be at different voltages what are the called the nodes.

So, what are the nodes here, fine these are the nodes, so in this case all these nodes if they are in different voltages, so things that are connected between them should share the same voltage. And it, so point is that in between these two nodes, whatever are connected they should share the same voltage. And in between two nodes, if there is a element that shares the two sides of it share the same flow, so in between you can connect by a 1 junction lets illustrate.

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Let me rewrite, it and then do it, otherwise it will be difficult for you to see, you need to see both the things together in order to appreciate. Here, you have this point, this point, this point and this point at different voltages fine. So, you will say that these are simply blindly put at as the 0 junction, so a 0 here, a 0 here, a 0 here and a zero here, here is a 0 junction, here is a 0 junction, here is a 0 junction and here is a zero junction, so there are 4 0's.

Between these two there must be a 1 junction and a resistance connected to it, between these two, there must be a 1 junction with an inductance connected to it. Between these two there should be a 1 junction connecting a C and there will be another 1 junction connecting a R fine clear. Between these and that you have 1 junction connecting a S E have you connected everything, yes.

Student: ((Refer Time: 53:22))

One more inductance, yes, yes, yes, between these two, between this point and this point fine, so 1 junction connected with a inductance. We have blindly done it, we have just said it that these 4 points represent different voltages in between whatever, are connected or connected by one junctions done. But now we have landed up with a bond graph that will turn out to be containing more number of bonds, that you then you really require.

For example this is a 0 junction, which is connected to only two things therefore, the effort whatever comes here, must go here whatever flow here must go here, the powers are equal, and the effort and flow are also equal. Therefore, this 0 junctions have no purpose you can short it, that is one way of shortening the bond graph. The other way of shortening the bond graph is to say, that we something that we always do we say that let this be at the ground potential, so if that is at the ground potential how is it reflected here, this is this bond we will say that here, you have connected a S E element which is equal to 0.

The moment you say that, this is equal to zero this bond carries no power, therefore it can be ignored not only that it is a 0 junction, where the effort is 0 therefore, all these bonds carry no power therefore, they can be ignored. Have you understood the line of logic, that this we always say that one of the voltage points or nodes would be at 0 potential, there is no harm in doing that that is a datum.

And then the moment you do that in the bond graph, that is reflected as S E becoming 0 and S E becoming 0 connected to a 0 junction means, all these bonds will carry 0 effort. Therefore, 0 power, because the power is effort multiplied by the flow, so all these are unnecessary, now we can we see that these again becomes a through junction is unnecessary, this also unnecessary fine, this is also unnecessary, we can connect it this is also unnecessary, we can connect it is there anything else this is 0.

Therefore, this will become unnecessary this can be get connected, now at this stage it looks ugly, so let us just redraw it in a more handsome way. What will you say S E connected to 1, S E connected to one, connected to R, connected to zero, zero is connected to I, I is connected to one, the zero is connected to one to I to a 0 to R and C that is it.

And notice that this exactly the same as what you had earlier, just check once ((Refer Time: 56:53)), by argument we had obtained this, by blind procedure we have obtained this they are exactly the same. So, the act of drawing bond graph can also be somewhat algorithmically done. I will illustrate that further in the next class.

Thank you.