**Dynamics of Physical Systems**
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**Lecture - 15**
**The Bond Graph Approach – III**

Let us start by getting some practice in the art of writing the Bond Graph and assigning the power direction and causalities.

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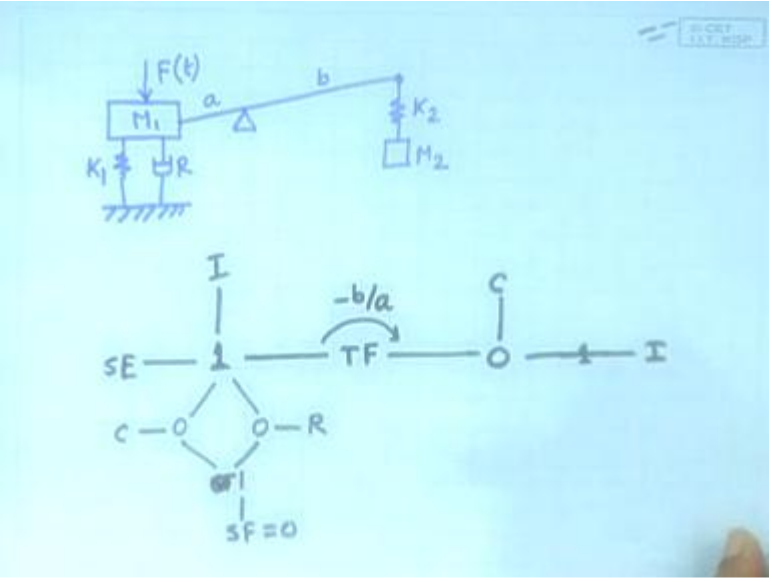


Let us start with this problem, suppose you have got a mass is acted on by a force F. And here we have a spring damper arrangement connected to ground. And suppose at this point, we have a liver and other end we have a spring and then a mass and say the ratio is a by a and b. So, if that is the system description we can say this is K 1, this is K 2, this is R this is M and so on, and so forth and M 1, M 2.

I suppose we have given everything, so how we would write the bond graph for this, then starting to write let us start from this point. You have got a source of effort sharing the same flow with the mass, so we will put a 1 junction and connected to it there would be the S E element, connected to it will be the I element so far so good.

What is there, at this point we have what, a spring damper arrangement which is share in the same effort between the top and the bottom. So, we had earlier done it by 2 means 1 in which, we have a 0 junction connected to a C a 0 junction connected to an R, but that is also connected to something. It shares these points, share the same flow with what is this ground, it is also a source of flow which is 0.

So, the ground which is not moving means that it is imposing a specific flow on this point, which can be represented as S F sorry 1 S F. Now, this is equal to 0, we understand that. Fine, then connected to this we also have this point and this point is sharing the same flow, this point is sharing the same flow with the mass. So, that should also be connected to this 1 junction, so we have this bond, this bond is representing the rest of the system, now what is there in the rest of the system.

We first have a transformer, so this a liver which is the mechanical transformer, so we have to represent that as we have told in the last class by a transformer element. Now, this transformer element, what we will have to assign its transformation ratio, yes it should be minus b by a b is in this picture the other side, so minus b by a, so it has gone to the other side.

We have actually come to this point, so the flow here in this bond is nothing but, the up and down motion of this one. So, this point and this point, share the same effort this point and that point share the same effort, so they should be connected by a 0 junction connected to the C element here. And then, we have this point, this point and the M 2 they also share the same effort, or this point and that you can also think that it is connected by a 1 junction to I.

Now, notice that this 1 junction actually, is a through junction we had argued that this point and the mass share the same flow that is why, but actually it is a through junction nothing there, is nothing other thing connected to it. So, we can simply connect it directly, because the if it is a through junction then effort and flow here must be equal to the effort and flow there, so there is no point in putting a junction at this point.

Notice a few other things here, this source of flow is 0, which means this bond carries no power, if this bond carries no power and here is a 1 junction this is the flow is 0. Here,1 junction will equalize the flow, which means in these two bonds also the flow will be 0, if the flow in these two bonds are 0 then obviously, the power is also 0. So, if power is not flowing there, is no point in putting the bonds, so this part is actually unnecessary.

The moment you have this part unnecessary, these become through bonds this effort and flow should be equal to the effort and flow here. So, there is no point in putting this also, so that significantly simplifies, so let me freshly draw it in another page.

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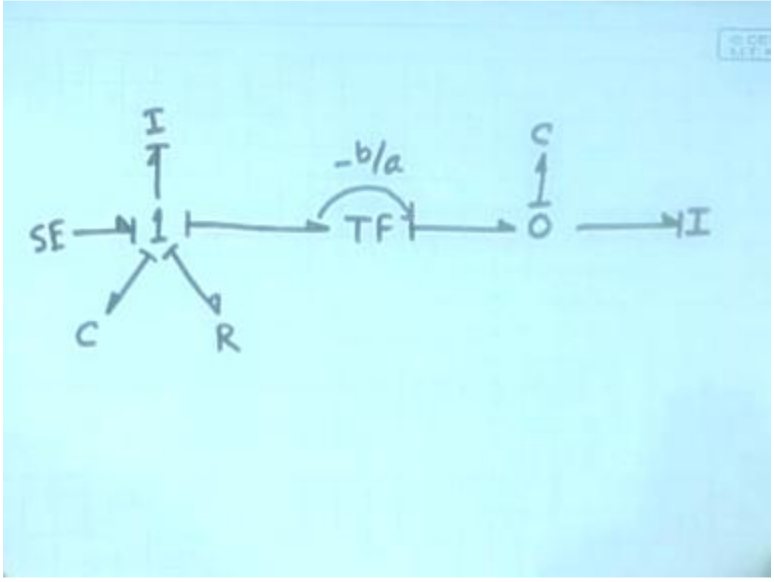


We have got a 1 junction connected to a I element, we have got a S E here there would be a C, here there would be a n R, here there would be a T F connected to minus b by a, you have got a 0 connected to a C connected to I. So, that becomes the bond graph, fine is that well understood, now let us assign the power directions, in assigning the power directions, we first start from the source its power directions should be like that hum. So, its power directions should be like that, there is no other source, so we now choose the individual elements and give the conventional power direction fine.

There are two bonds, which are between these, so again we can any anyway you can do that assign the power directions, but you can see that the power source is only one. So, the power should actually flow in that direction, so it will becomes natural to assign these directions. For the transformer element if the power comes from one side it should be going to the other side, so it should be like that.

Again it is perfectly fine, if you choose to do it opposite this direction and that direction only thing is that the positivity of the specific things would be different, but let us not worry about that at this stage. Yesterday, we talked about the causality and let us worry about that now, how should we go about assigning the causalities, first let us start with the sources. We have got a source of effort here and we know that its causality must be like this that means, it gives out the effort information and this should be receiving the flow information, so it is like this, so the causal stroke should be there.

What about the I element, I element is 1 that naturally received the effort information and gives out the flow information, so it should give out the flow information. The moment the flow information has come into the 1 junction, that 1 junction will distribute the flow information it is a flow equalizing junction, so immediately the other things attain this.

Now, here you have a C whose normal, structure is that it receives the flow information and gives out the effort information which it is doing, so we are happy, R could do both, but in this specific case it is receiving the flow information and giving out the effort information, so ultimately we have come to this bond. In this bond, I can see the flow is going flow is flowing, this way flow information is going that way this way. And naturally the transformer element the flow here is a constant times of flow here, so the flow information should be going that way, so is it is like this.

We have come here and there are two things connected to it, it is a 0 junction lets first causal the C, C is the C elements natural causality is that it receives the flow information and gives out the effort information like this, 0 junction has already received the effort information. Therefore, it should now distribute the effort information, which is already doing in this bond, but this should also be distributing the effort information which is like this.

You can see that I also get properly causaled, I should normally receive the effort information and give out the flow information, which is correct here clear, so that is how we assign the causalities. So, while I was assigning this, I was speaking and elucidating the line of argument, if you tend to forget just write that down otherwise, later it might be a bit difficult to recall. So, the logic was that we started from the sources, then we went to the individually the storage elements I C and all.

The moment one storage element is connected to a strong bond that means, a 1 junction, if one bond is bringing in the flow information it is a strong bond, here is a strong bond similarly here is a strong bond. The moment you have that the others become automatically determined and you do not have to bother about the rest of the ((Refer

Time: 12:47)). Let us do another problem, a again a relatively simple problem, but should be illustrative.

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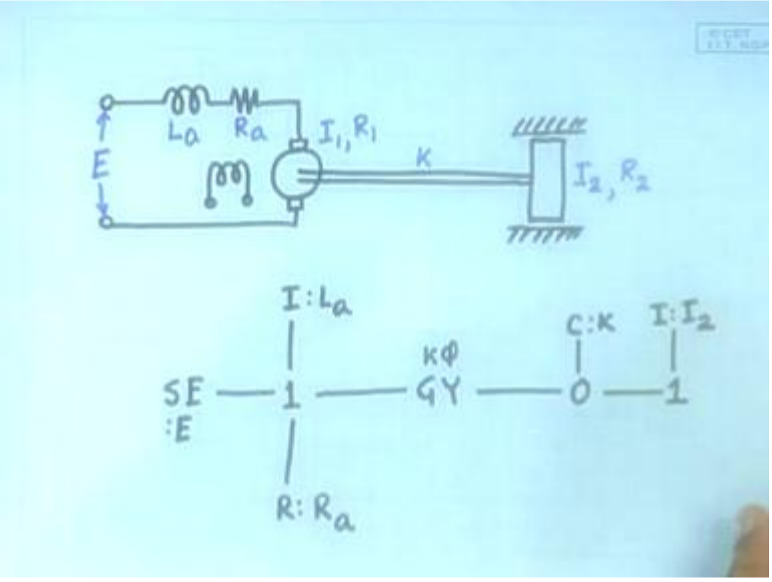


Suppose you have a circuit in which, we have got a, let us, let us do the DC machine problem. So, here you have a DC machine imagine that it as a constant field it is a constant, the machine should have it is armature inductance and armature resistance fine. And the machine is connected to a shaft, and say the shaft is connected to something that rotates a mass against some body.

So, now if we assign, the names to different things this will become the inductance of the armature this is the resistance of the armature, these two things should be there. This rotor should have it is own inertia, let us assign that as I 1, there should be a bearing and the bearing should have some friction, so at this point there should also be a friction. Now, we have the shaft here, now this shaft normally it should have some kind of a freedom, so these two ends are slightly able to move independently of each other, so you have some kind of a springiness, represented by a spring constant K.

This fellow should have, it is own inertia and this point should have its own friction good, at this point we can start if there is, a applied voltage which is say E this is D C applied voltage. We assume the flux to be constant, now let us start we start from the source of effort here, so we have S E connected to a bond. This S E shares the same flow with these two elements, so it is a 1 junction connected to there will be two things one an I element which is this.

This is S E element is the E applied voltage, there will be R fine now, then what this is also connected to something, imagine the electrical circuit what does it see, it sees the back EMF here. So, this back EMF is created by the electro mechanical action and as we have understood that these electro mechanical action is to be represented by a gyrator, so here is a is a bond which is connected to a gyrator. Let us, let us try to understand, this bond its effort should be the back EMF, this side is mechanical side effort should be the torque.

So, here the flow should be converted into the torque here and here, the back EMF should be converted from the speed here. And we know that t is equal to K phi I A, I A is the current flowing here current here, so the transformation ratio is K phi and we do not need a arrow in this case in gyrator, because in both the sides the transformation ratio happens to be the same. So, we have come this side we come to this side, in this side what do you have, this point and that point share the same effort.

So, it is a 0 junction to which, we will connect a C element, which is our K, now that is connected this side is sharing, the same flow with this mass and the frictional element, the frictional element is seeing the same flow as what is being seen by the mass as what is being seeing by this end of the spring. So, we have all that connected by means of a 1 junction, so what do you have here, we have an I which is, I 2 where is I 1, we have missed, sorry this point, we have missed no, no, no then just let me do that once again.

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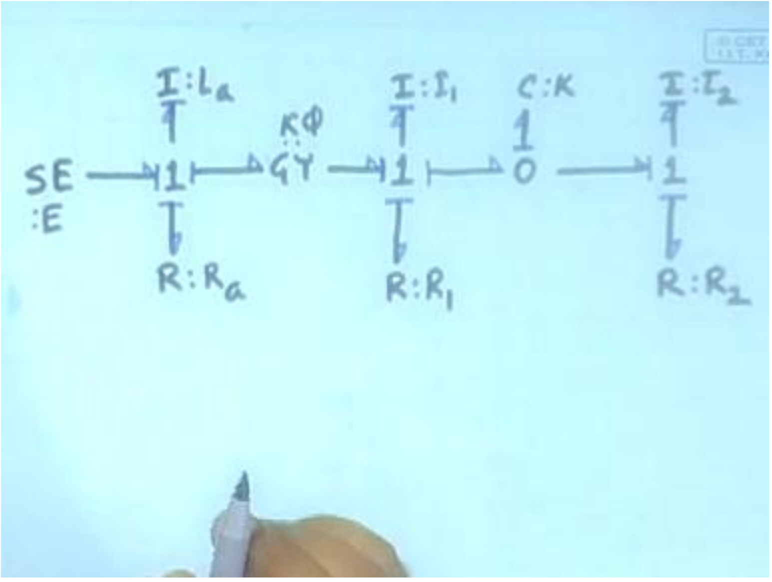


We have this picture here and will do that, S E connected to a 1 junction connected to a I connected to a R connected to gyrator. So far so good its fine, But after that we have this first, after that it is connected to that, after it is connected converted into torque, the torque is applied first on the I one element. So, that has to be a 1 junction connected to, a I and connected to another R, these two, now then that 1 junction here, it shares the same flow with this end, but this end and that end the same logic then continues 0 to a C 1 to i R is that clear. So, let us now assign what is what, this fellow is our E this fellow is our L a, this is R a, this is I 1 this is R 1, this is K, this is I 2, this is and this is fine.

Let us assign, the causalities let us assign the power directions first, we will let it go this way, so the first part done. Now, we need to assign the causalities, let us start with the S E, S E is something that gives out the effort information and should receive the flow information, there is no other source. So, let us come to the next storage element this one, which receives the effort information and gives out the flow information.

So, the flow comes this way it is a 1 junction, so this becomes the strong bond bringing in the flow information the other bonds must distribute. Here, is gyrator element which has here the flow is going. And therefore, the effort should be going, so the stroke should be here, effort should be going this way means flow should come this way and therefore the causal stroke should be there.

We have come to this 1 junction and here is a storage element I, whose natural causalities should be like this, so this again becomes a strong bond, because it is a 1 junction it has already received the flow information it should distribute like this. So, we have a 0 junction in, which a flow information has come that means, that it is not yet complete there has to be a effort information coming into this 0 junction from, where does it come this, C like this, the moment effort information has come it has to be distributed by the 0 junction like this.

And then, I have a one junction in which here is a effort information coming and therefore, it is not a strong bond here, has to be the strong bond, because this fellow is a I junction whose causality like this should be like that. And this brings in the flow information, which then must be distributed is already distributing this side and this fellow should also be distributing, that completes that story fine.

Now, after having understood this one, if you have any question about this please ask me this is the stage to ask, if not lets go ahead, when referring to these elements we have been saying that these are the natural causalities. What do you mean natural causalities, what do you meant is that for the I element the natural causality is one, where it receives the effort information and gives out the flow information receives the effort information in what sense.

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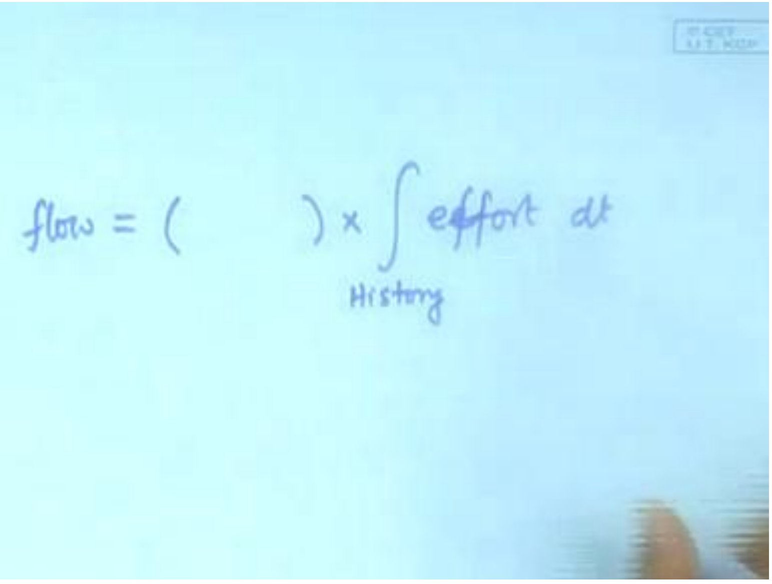

In the sense that we can write the flow is equal to something times the integral of the effort sorry, so this is integral over the history. Therefore, the effort is the cause and the flow is the effect and that is how we said it is a natural causality, now there can be certain situations where you will not be able to assign the natural causalities.

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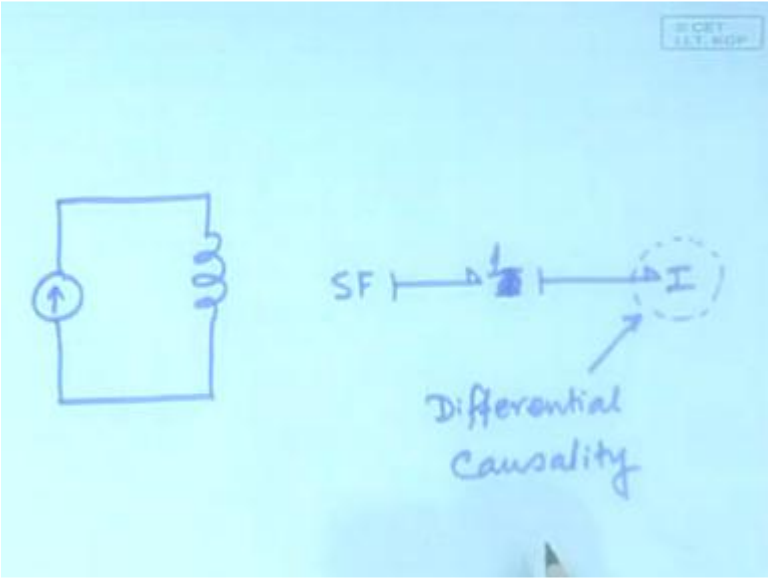


Imagine that you have a current source and an inductor how will you draw the bond graph, it is a S F I to sorry, 1 to I Power direction no problem. Causality S F is the element, that gives out the flow information, therefore the 1 junction should give out the flow information and see the I element, now receives the flow information. Here, we have a problem it is not integrally causal, so here we have a problem and in such cases, we will see, we will say that it is a differential causal it is differential causality here, because the integral relationship that, we wrote here will not hold in this case.

Why did this problem happen can you physically understand this, because the inductor is an element, whose natural behavior is that it receives the effort information gives out the flow information, but here it is connected in series with something that imposes the flow on it. Therefore, it is not able to determine the flow independently that gives rise to the differential causality, let us try to figure out another situation.

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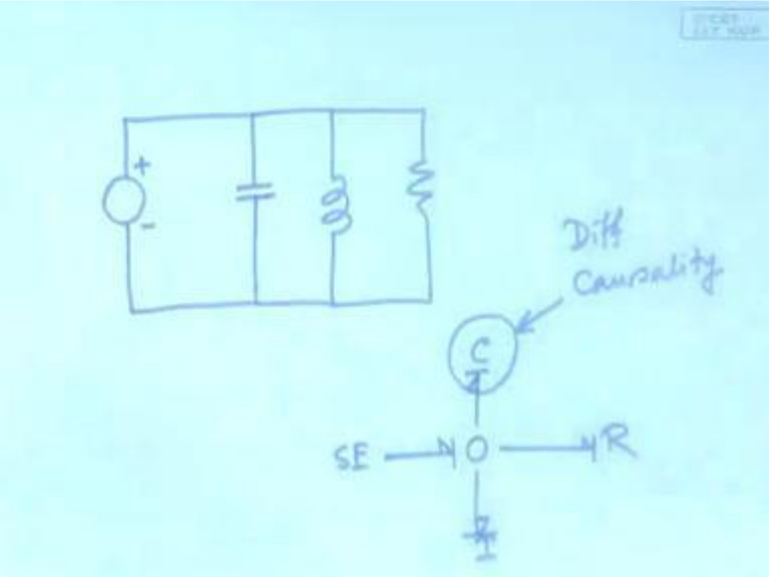


We have got say a voltage source connected in series with a capacitor, it is not difficult to say that it is going to give rise to the same problem. So, instead of doing that let us do something that will make the picture a little more, normally if you get such a circuit you will not see any problem with it. Let us do the bond graph, bond graph would be a parallel junction, so it is a 0 connected to four bonds.

In this side, it is S E here, it is C here, it is I here is R, no problem in assigning the power directions. But here we have, when we have try to assign the causalities S E element is one that gives out the effort information receives the flow information. And the moment the effort information has come to this 0 junction, all the other bonds should be taking out the effort information, so their causality should be like this.

For this one there is no problem, because it is receiving the effort information giving out the flow information for this there is no problem still. But, for this there is a problem, because now the C element is receiving the effort information giving out, the flow information which is not its natural causality, so this is also a differential causality understood.

Now, let us get to the point, the point is we have already said this, when we were dealing with the graph of electrical circuits that whenever, there is a loop containing only voltage sources and capacitances we had a problem. At that time we have dealt with the situation whenever, there is a loop containing only voltage sources and capacitances then the

Kirchhoff's voltage law will say that, there is at least one capacitance which is not independently determinable, so that situation is leading to a differential causality.

Similarly, we have also learned that, if there is a cut set containing only current sources and inductances a cut set anywhere in this whole circuit containing only current sources. And inductances, that is going to give rise to differential causality in this case, so what we have understood there as a particular element, it is not being able to determine its own or it is not free to determine its own state variable. So, that we saw that there these elements did not give rise to a state variable

These elements were, what did we do at that time when we were dealing with graphs, we said that we will draw the standard tree and only the capacitances that appear in the standard tree would be ours, the one's that will create a state variable. So therefore, we saw that some capacitances, may go out of that definition we saw that some capacitances will not produce state variables it is these.

So in bond graph terminology, the equivalent of that kind of a situation is the differential causality, when can differential causality occur in a mechanical system. In a mechanical system also if there are two things, which are apparently different, but they are not independently able to decide their own state variables, then that will lead to a differential causality. Imagine we have done this problem let me draw once again

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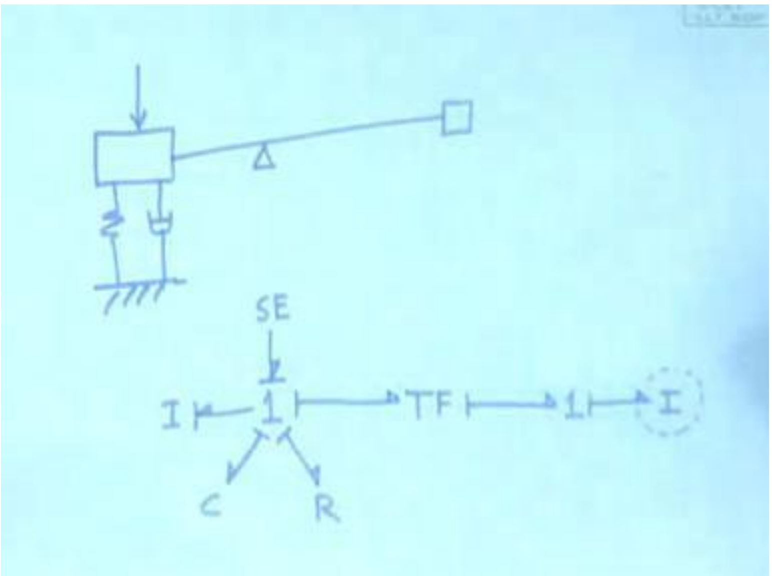

We have a mass, we have this arrangement connected to the ground, and we had this and we had a, at this point suppose the other mass is directly connected to, let us, let us do the bond graph of this. Here, we know that it is a 1 junction that is connected to the S E connected to the I and connected to these two, we have already seen that, so we will not go the long way.

C R and here it is a transformer connected to in this side what, in this point, so the flow here has come to this point and this fellow shares the same flow it is a I. So, let us keep this through junction for now, so the power directions should be like this there is no problem there, let us try to assign causalities. Here there is a S E element which should be causaled like that and I, which should bring in the flow information.

The flow information has come to the junction 1 and therefore, all the other bond should take out the flow information. And the flow information now has to travel through the transformer element, so it should be causaled like this, the 1 junction has already received the flow information and therefore, the other end should take out the flow information. So, the I element is now differentially causaled can you see that, so it is a differentially causal, because the other side of the system that is forcibly determining it is velocity and this fellow is unable to decide it is own velocity.

That is, why it has become this particular element has become differentially causaled fine is that clear, good, so there may be a few ways of getting rid of this problem. One you could say that no, let us make it a bit flexible the moment you make it a bit flexible it becomes, proper integrally causaled. Say in this problem, ((Refer Time: 33:52)) how can you get rid of this differential causality, simple assume a bit of resistance here 0.000001, then also it becomes proper, just check that, then this point has freedom of choosing its own voltage.

Or you can also check that in this problem, ((Refer Time: 34:21)) if this shaft is stiff, which means that there is no freedom of motion in the two sides, then also it give rise to differential causality that is why we had put some stiffness here. Now, we are in a position, once we have done enough practice understood the details, now we are in a position to write down the differential equations. Now, the differential equations writing process as I told you is a somewhat algorithmic process. So, let us talk about that algorithmic process.

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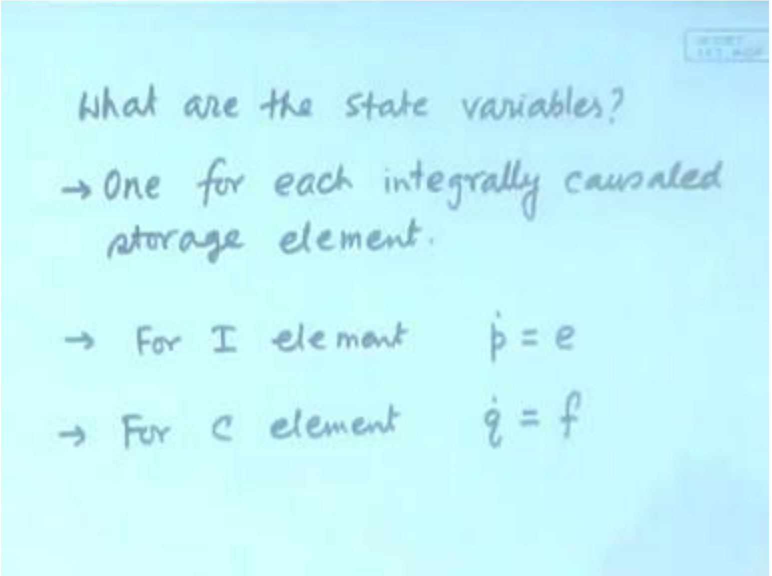


First what are the state variables, the answer is one for each, so for every integrally causaled storage element there will be one state variable and this is important. Because, if there is a differentially causality that particular element will not produce a state variable, that becomes somewhat standard. So, what are the state variables, there will be one all, Which are the state variables, that is for the inductive element what is the constitutive relation, p dot is equal to e p is the momentum right simple equation. Rate of change of momentum is nothing but, the effort that is the constitutive relationship of any inductive element

So, this tells you that the momentum should be the associated state variable, then the equation should be simple. That is what we have also understood, or realized when we talked about the Lagrangian formalism or Hamiltonian formalism. Momentum is the right state variable for the inductor, for C element we would like to express the right hand side in terms of the effort or the flow, these are the variable that we know in bond graph.

So, in the right hand side it should be the flow, in the left hand side it is the simple q dot, so it is the charge in the C element is the natural choice of the capacitor. That is, what we did earlier when we wrote the equations with help of the Lagrangian methods, so the whole thing is consistent it is only a different language in which, we are doing the whole thing fine. So, we have which means that for the I element the momentum is the choice of the state variable, for a C element the charge is the choice of the state variable.

And for every integrally causaled I element, there should be a 1 p for every integrally causaled C element there will be 1 q clear. So, the choice of the state variable should be clear from this, next we asked exactly two questions and the moment we answer the two questions the equations come out, simple.

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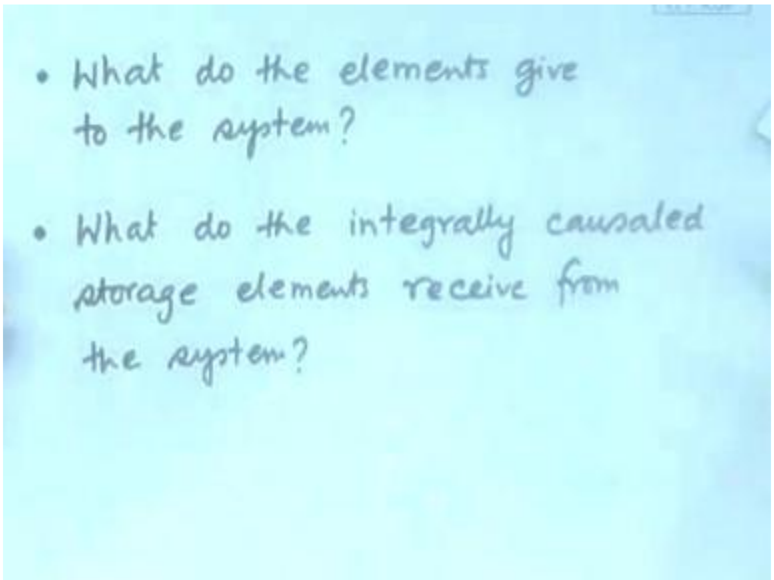


So, what are the two questions, first, so every element actually gives something to the to the system, first we will test of that, what does each element give to the system. And second question is related to only the state variables, these are the two questions, see the integrally causaled storage elements, in this case you can easily see that it the I element receive the effort information, C element receive the flow information. So, the right hand side is what this things, will tell if you can express that in terms of the other state variables, you get this differential equation. But, in order to do that we need the answer to this question, let us illustrate with the help of one example.

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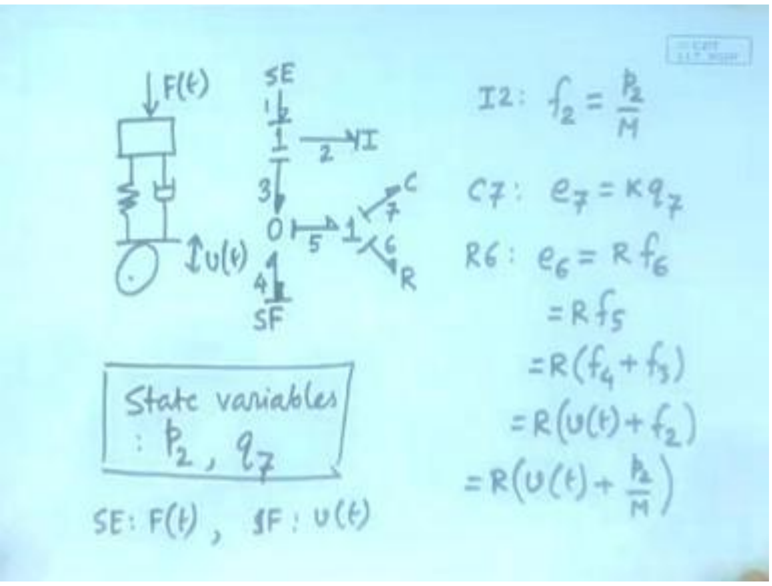


Let us start with the mechanical example, that we have already seen, got this one we have already written the bond graph as this, again I am just repeating, I am not deriving the whole thing, because you already done it. Here, there would be a 1 junction connected to a S E element, connected to a I element and here it is connected to a 0 junction and the 0 junction is connected to a 1 junction, connected to C connected to R.

And this is connected to here, there is a S F element, notice the logic that here, is a 1 junction. And that 1 junction is distributing the same flow in all the sides and that is exactly what is happening, what are sharing the same flow, this S E element the M and these two ends correct. And whatever, are at the this end and the whole end they are sharing the same effort, and that is why we have connected the top and the bottom through, a 0 junction.

And then, we are starting to worry about what is in between and what are in between this friction element and the compliance element, they share the same flow they have to move together, so they are connected by a 1 junction and that was the logic. Now, let us do the power directions and causality, it is like this, like that, like this, that this, the other things can be assigned any direction, so how do you assign it, you can assign like that.

And since, the S E element is stopped there we can of course, you can give both the causality both the directions power directions this way or that way, let us give like this. So, what is the fourth flow, Student: ((Refer Time: 43:07))

Sorry, sorry, sorry, yes yeah it is not that sorry, this not right. It should be giving power into, into the system ((Refer Time: 43:18)).

Now, let us assign the causalities, S F should be giving flow information into, S E should be receiving the flow information, I should be giving the flow information. And this fellow should go into the 1 junction and therefore, it is already receiving the flow information it should give here. So, you have this junction bringing in the flow information, but as yet nobody has brought in the effort information somebody must, so it is like this, this fellow is bringing in the effort information.

So, it has already brought in the effort information that way and flow information has flown this way, 1 junction therefore, it must distribute the flow information. Now, give them numbers simple, 1, 2, 3, 4, 5, 6, 7, now once we have given that now let us start asking the questions, what were the questions ((Refer Time: 44:52)), what do the elements give to the system what do the elements give to the system. We have many elements, so we will ask this question with respect to each element.

One here is an element I 2, what does it give to the system, the flow it gives to it gives the flow, so it is f 2, so f 2 is the flow is in terms of state variable, right pretty simple. So, we have I 2's answer is obtained as, this it gives f 2, which in the right hand side, we have expressed in terms of the state variables, P 2 is the state variable in this case. So, let us write, what are the state variables, one you have these as the integrally causaled thing these as the integrally causaled thing there are two things.

So, there should be, p 2 and p 2 and q 7, these two are the state variables fine, so we have asked this question and got this answer. Now, let us ask this question with reference to the C element, so C 7 what does it gives to the system, effort in C in 7. So, e 7 express it in terms of the state variable, which you know, it is actually spring constant K q 7 done, now ask this question with reference to the R, R 6 what does it give to the system.

It gives the effort, e 6 can you see that it receives flow and gives effort, so e 6, E 6 it could give because it is received flow 6. So, first relate it to that, so this is R f 6, R times f 6. Now, f 6 is then we have to find out where does the f 6 come from, f 6 it is connected to 1 junction therefore, there must be a strong bond bringing in that information where is that.

Here, so this is equal to R f five f five is not a state variable, where does this information come from, it comes from 3 and 4. And notice now, we have to worry about the power direction, because power direction shows that f 5 is e 3 plus e 4, had the power direction been opposite here, which it could then it would be 4 minus 3. So, now we can substitute here R f 4 plus f 3 f 4 is known it is the externally applied current source.

Say, if this current source is say v of t we can write R v t plus f 3, f 3 is not state variable, but this information must be coming from somewhere, where is it. Where does this flow information come from, yes this I element, yeah here is the one junction. So, therefore, there must be a strong bond bringing in that information, which is this f 2, so this is f 2, then R v t plus f 2 we already know done.

So, we have expressed the right hand side, so notice how we did it, we started with the variable that it gives it could give that information, because it is received some information that is related first by this. And then, we scout the whole bond graph to find out, where was this information generated and that is how we have arrived at this has generated that information.

See how as, information is generated in a whole system it is not easily visible, but the moment you draw the bond graph, it becomes visible you can see that the information channel by which its flowing is like this. The flow information that this fellow has received was actually, generated by this and that that is how ultimately you express it. Now, let us go to the other things, what remains are the two flows, two sources, so what does the source of effort give to the system, the effort what does the flow give to the system the flow, so these are trivial.

So, when this question is asked with respect to let us block it, S E then the answer is answer is, suppose this is F t and this is v t this gives F t and for S F it is, so these are trivial. So, let us not bother about that, these are the main information that it has generated. Now, let us ask the second question what was the second question, what do the integrally causaled ((Refer Time: 51:19)) storage elements receive from the system.

So, there are only two integrally causaled storage elements, ((Refer Time: 51:32)) 1 I 2 and C 7. So, let us first ask this question with respect to I 2.

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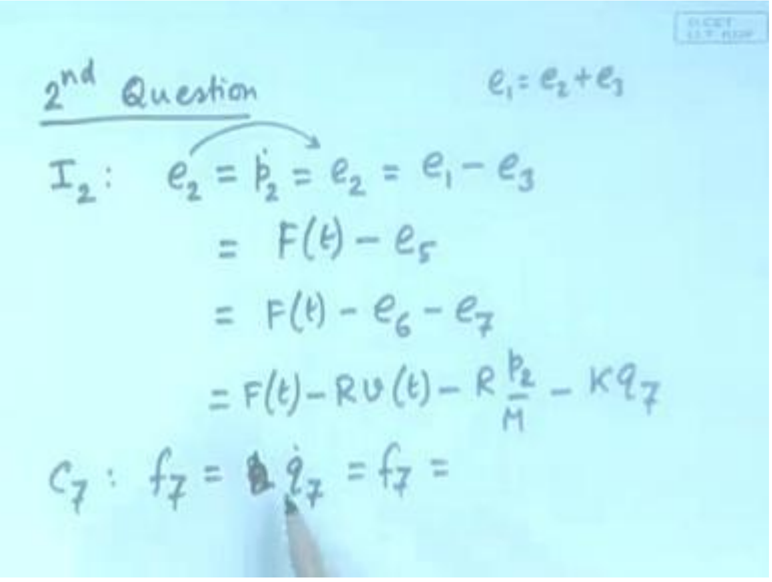


Second question, I 2, I 2 what does is has it received, it has received e 2 it has received e 2, so it has received e 2. E 2 is nothing but, p 2 dot, so we have the dotted term we normally express, that in the left hand side, so we bring it here, so is equal to e 2 we can write it. So, we have got the equation differential equation with, p 2 dot is equal to e 2, but we do not know e 2 and we have to scout the whole bond graph to find out, where this information is generated E 2.

((Refer Time: 52:37)) E 2 is a addition of E 1 minus e 3, because of this power direction. In this particular 1 junction, because of this power direction E 1 is equal to E 2 plus E 3. So, E 2 you can separately write, e 1 is equal to e 2 plus e 3, so we want e 2 is e 1 minus e 3, so we can substitute e 1 minus e 3. ((Refer Time: 53:20)) E 1 is known, e 1 is F t, we substitute is equal to F t minus, now e 3 ((Refer Time: 53:33)), e 3 where, does the information come from e 3.

E 3 is connected to a 0 junction therefore, there must be a strong bond bringing in that information which is this, so e 5 is equal to F t minus e 5, where has information come from. E five ((Refer Time: 53:57)) yes, it is e 5 is e 6 plus e 7 it is a 1 junction, 1 junction means, it is a effort summing junction and because of, the power directions, e 5 is equal to e 6 plus e 7, so we can write e 6 minus e 7. Now, e 6 ((Refer Time: 54:25)) is known e

7 is known, substitute, so it is F t minus e 6 was R v t minus R p 2 by M minus e 7 is K q 7 done equation ready fine.

Now, just do the same thing with respect to C 7, C 7 is the other its state variable is what does it receive from the system. It is f 7, f 7 we express it as p, oh sorry, q 7 dot and then put in this side, f 7. And then, scout the bond graph ((Refer Time: 55:32)) to find out, where this information come from, so now complete that exercise, yourself complete that exercise that here is f 7, we have written as f 7 is what is its receiving from the system.

And in the right hand side, you have got the state variable which we bring to the left hand side. So, f 7 is ((Refer Time: 55:54)) the thing that we need to find out f 7 comes from this 1 junction, there must be someplace, where it is the strong bond it is 5, so f 7, f 5. F 5 should come from somewhere, f 5 is definitely f this f 4 plus f 3 and f 3 should be f 2, so it has come from here and here. Just write it you will get the differential equation clear, we will stop here and we will take up more examples in the next class.

Thank you.