Dynamics of Physical Systems
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Lecture - 14
The Bond Graph Approach – II

In the last class, we were learning how to express any given system in the language of Bond Graphs and we learned that the essential interactions of the power as well as the information variables are represented by means of bonds between elements.

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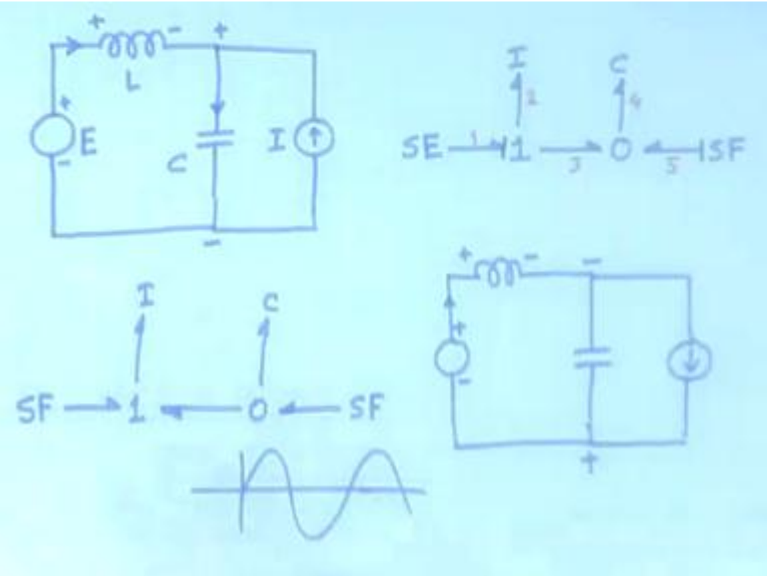

So, that if you have a simple circuit something like this, you have a say voltage source, you have a inductor, a capacitor and say in addition to that there is suppose a current source here, so this is the voltage source E, the current source I C and L. Now, in that case, we would write the bond graph by first observing for one end say we start from the element E which is a source of effort, so we will write SE and then this is in series with the inductor; which means, they share the same flow with the rest of the circuit.

So, we will say it is the 1 junction connected to an I element, connected to the rest of the circuit, now what is the rest of the circuit they share the same effort with this, so it is a 0 junction connected to this C and to the S source of flow. So, that will be the bond graph of this and then we say that each of this bonds have to be numbered, so we will put some kind of a number 1 2 3 4 5 then what is the step, you have to give power direction, now the power direction from we assume that power goes out of the sources and these are denoted by half arrows.

So, we are first giving this power directions then we assume that even though these things are not ultimate consumers of power, they take power for some time and give out power for some time. But we assume the incoming power direction as positive, so when power goes into this storage elements we assume that is positive, so far, so good, but we notice that there is a bond that connects two junctions and there is no rule for assigning the power direction, so what will be the meaning of that.

So, you can easily see that we can give the power direction this way, as well as this way, let us figure what will be the implication of that, if the implication will have to be sort of this way that here we have got the SE element which is giving out power. Here is the plus terminal, here is the minus terminal of that and so when it gives out power what is the direction of the current, It has to be that, so this will be the positive direction of the current.

So, this power direction immediately say that this would be the positive deviation of the current, now here is an inductor, this inductor is the power is positive when it goes into the inductor, so power is positive when it goes into the inductor means what, what will be the polarity of the voltage across the inductor, it will be plus and minus. Then it will consume power, so current going this way and this is plus and minus means that it is consuming power.

Now, come to this particular position, here bond three which is between this, now this says that the power goes into the capacitance source of flow combined here, In order for the power to go into the capacitance source of flow combined, the current is flowing this way what will be the polarity of the voltage, positive voltage, yes plus minus correct, so in that case this power direction implies that when this terminal is positive and this terminal is negative that will be the positive direction of the voltage.

And accordingly the capacitor voltages are defined no problem, the capacitor consumes power because the power is positive when it goes into the capacitor and so the direction of current, positive current will be like that, now this fellow is the source of flow, a current source and it is generating power can you see that. The power direction is positive when it goes out of the source of flow, which means that what is the positive direction of this current in that case.

As shown as here, so if you if the current actually flows in that direction and the polarity of the voltage is like this then it pushes power into the rest of the system, so this configuration means that this will be the positive direction of the current through the capacitor. This will be the positive direction on the polarity and this will the positive direction of the current through this thing.

Let us now draw the same one with a opposite, here we will put the opposite direction and this 0, C now what does it mean, again let me draw the circuit, circuit was like this now here again let us start with assuming that this is positive this is negative so let us start by assuming that this sense of voltage is called the positive voltage reaction. In that case this remains the same, it giving out power, so the current direction positive is this, this one consuming power

Therefore the voltage positive voltage direction across the inductor is this, but now this whole combined is consuming power, consuming power means it is not actually consuming power again understand the essential meaning. It will be called positive when the power goes in this direction; that means, from this combined to this combined. Which means that now what will be the polarity of this current is going in this direction and this whole thing has to generate power and push into this then it will be minus and plus.

Then only if it is minus and this is plus, then only whatever is this combined will be generating power and giving to this, now if this is true then here is a source of flow, a current source what will be the positive direction of that, that current. Notice that is generates, it generates power the power is positive when it is coming out of the source of flow, so when will it be, when it is right, so what was the implication, implication of this bond was that we always started from by assuming something as positive we started by assuming the E with the positive polarity here and negative polarity here.

The rest because of this specific power directions automatically their positivity and negativity become defined, so here this becomes positive, this becomes negative, this is positive, this is negative, as a consequence this direction of the current becomes the positive direction of the current. While if you put it like this then this direction of current becomes the positive direction of the current, fine physically what physically you will see that all these elements say the voltage across the capacitor.

After all you have you will write down the differential equations, you solved the differential equations, after that you will see that there is some fluctuation in the voltage across the capacitor say the fluctuations something like this, if you had done it like this then you would say that this is my positive, if you had done it like this then will say that this is my positive, that is all, so it implies that while you are doing this, let us note bother too much about this small directions.

Because in the junction to junction power bonds they could be any way and this alterably give it, it would not really matter, but when you want to interpret what does it mean ultimately I am give getting this waveform what does it mean, which one is positive, which one is negative then you have to worry about that. So, the worrying about power direction actually comes because power direction essentially talks about not the actual direction of flow.

But it talks about the assume direction of positivity, therefore it comes, its implication comes only when you want to interpret the waveforms, so at the time of deriving the waveforms I am deriving the differential equation they will not really matter and that is why even in computers these are automated, there are programs in which you simply ask the computers to give power directions it will give. I will illustrate that fine, so you have understood how to give the power direction, by the way notice one thing.

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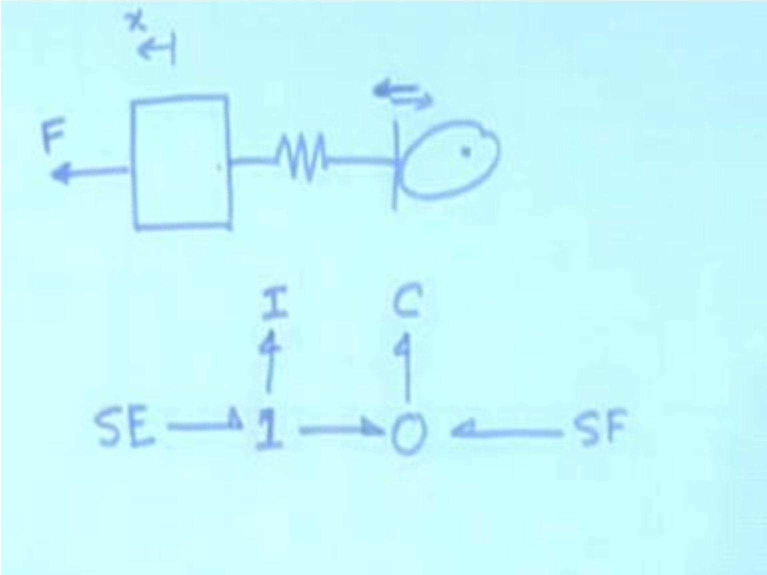

I can draw a system something like this, a mass being applied on by a force say there is a cap spring and say at this point there is a camp, can you draw the bond graph draw, here there is a mass I element which is sharing the same flow with the force. So, it is SF 1 junction to an I to the rest of the system and this part and that part are sharing the same effort, there is they are sharing the same effort, efforts are the same at the two ends forces which will it be 0 junction connected to a capacitor C element spring and to this is a camp, camp is source of flow. Notice that you have arrow that exactly the same bond graph; which means, that circuit and this mechanical system are physically identical.

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So, here also you will give the power directions like this and then this one could have this power direction of that power direction what does it mean, it will mean physically exactly in the same sense that the two directions of the current, this was positive in case of this direction and this was positive in case of this direction. We saw that here it is implication equivalent will be that you will find that this motion in one case will be positive and this direction of motion in another case will be positive that is it.

You can just work it out, how it comes because if you put it like this, if you put it like this it means that the power essentially goes from here to here and accordingly if this say this direction is your positive direction of say x, then F causes the positive power flow. So, in that case it goes into this particular direction will become the direction of positive flow, if the power direction like that so that is all there is no further difference now, so far we have been handling the one port elements.

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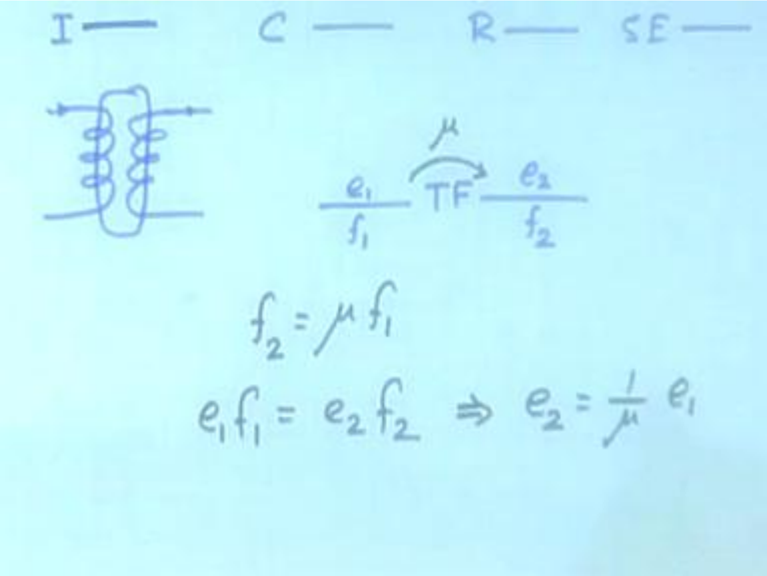

We have been handling elements like I with one port, C with one port, R with one port, SE with one port and, so and so forth, why do they have one port because only with one bond you can define the effort on the flow variable. One bond carries its effort and the flow, so if the effort implication flowing this way, the flow information's going that way, so one bond suppresses, but there are some elements in which one bond does not suppress, imagine that there is a transformer with some kind of a magnetic link between them.

What will happen, you apply a voltage you see another voltage appearing here, with some transformation ratio turns ratio if you apply a current here, a current will flow here again with the turns ratio; which means, there will be an effort and flow in this side and there will be a effort and a flow in this side correspondingly. And they are related by they are actually drawn in terms of this been whole thing being represented by just one element T a transformer TF, with two bonds in the two sides because without that you cannot completely representing, two bonds in the two sides.

So, here is a e 1 f 1 and here is a e 2 f 2 and they are related by what, the conversion see the voltage here is a turns ratio times of voltage here, a flow here is a turns ratio times the flow here, but then that transformation ratio is represented by this a mu this is represented with arrow like this why, the conversion is that you could express the effort here as a function of effort here, you could also express the flow here, as a function of the effort the flow here. But the conversion is to go by the flows, so it says that f 2 is mu f 1, that is it.

F 2 is equal to mu f 1, now the power that comes this side should be equal to the power that goes in this side, so e 1 f 1 is equal to e 2 f 2, which immediately implies that e 2 is equal to 1 by mu e 1, so this is the relationship at that is you know depicted by this symbol. The symbol is note this that it carries a arrow because if you give the arrow in the opposite direction then it would be f 1 is equal to mu f 2, in that sense it has to have an arrow, so which one is in the left, which one in the right does not matter what really matters is the direction of the arrow. And mu is a transformation ratio what can be a mechanical transformer, gear box, yes gear is a mechanical transformer, not only gear, levers are a mechanical transformers.

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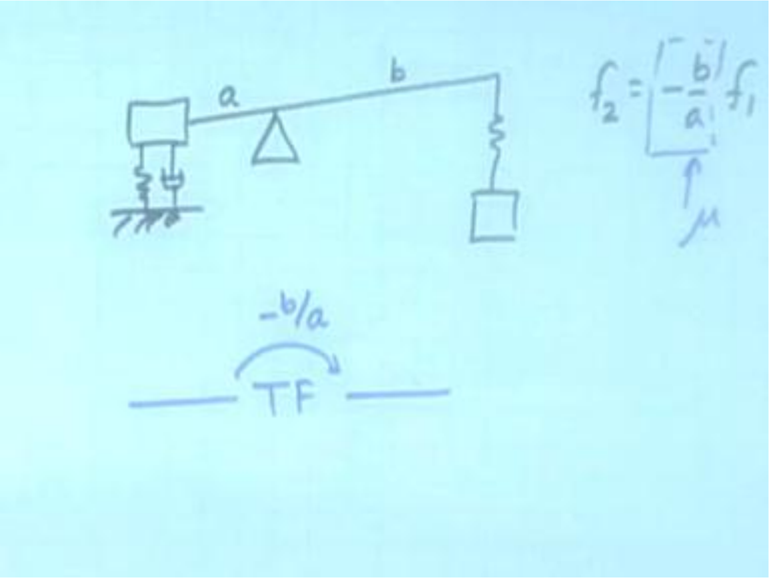

Imagine that you have got something here connected to a lever to something here, so that is also, let it be connected by means of a spring damper arrangement to it is ground, these with a ratio a by b, that also a lever why, because the flow here is related to the flow here by means of what relation, no minus b by a because it goes up it goes down. So, this motion here, so f 2 is minus b by a times f 1, so this minus b by a becomes the mu.

So, that is also to be represented by means of a transformer, in that case you would write it something like this transformer with a transformation ratio minus b and of course, gear box is where you can directly see the flow here is giving rise to a different flow in the other side and accordingly the efforts are also divided. So, these are the transformer elements, so again what is the transformer element doing, it is converting a flow into a flow and effort into an effort, is it possible for a effort to be converted into a flow and flow into an effort, yes it is possible.

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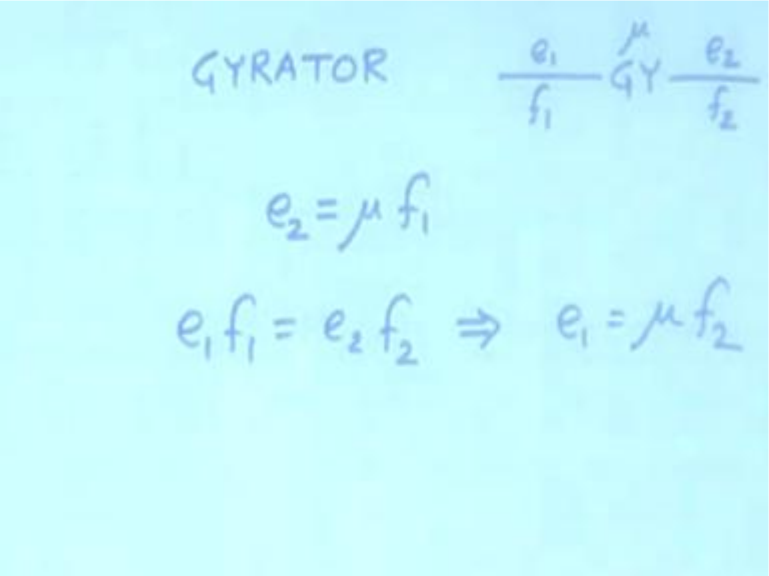

And that element is called the gyrator, which is represented as with some mu as the transformation ratio, but in this case I will show that there is no need to put an arrow suppose it is e 1 f 1 and e 2 f 2 . In that case the relationship would be e 2 is equal to mu f 1 not e 1, so the flow here multiplied with the multiplying factor mu gives with the effort here and how is f 2 given by e 1 let us see, since this is also not a power producing element, so you can write e 1 f 1 is equal to e 2 f 2, so this and that combined implies what.

e 2 is equal to mu f 1 and e 1 is equal to mu f 2, which means that there is no meaning in putting an arrow, both sides the same thing effort here is mu times f 2, effort here is mu times f 1, likewise flow here is 1 by mu times e 1, flow here is 1 by mu times e 2. The question, next question, natural question where is such thing to be found, let me give an example, suppose you have a DC machine, in a DC machine you have applied something and something results and what is the relationship between them.

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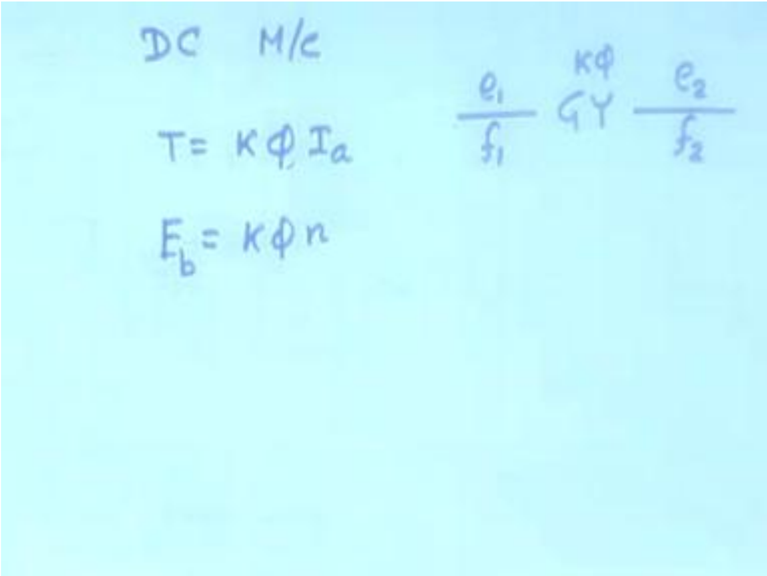

Relationship between them is a DC machine, in a DC machine what is the relationship between the torque produced, torque is K phi armature current I a, torque is the effort in the mechanical domain and I a is the flow in the electrical domain. So, this is the multiplying factor and this whole thing is converting a flow into an effort, so you would have to write this representing as a gyrator with multiplying factor K phi, so here it is e 1 f 1 and here it is e 2 f 2.

This bond is number one, this bond is number two in that case, notice that here is a mechanical side say and here is the electrical side say, we have already shown that e 2 which is T, is K phi times f 1 which is Ia. For what about this relation, let us see e 1 what is the effort in the electrical side back emf, so E b is K phi into the number of, a speed of rotation n in rps, so you have n as what, the flow in the mechanical side, so exactly this relationship is holding.

e 1 which is the E b is K phi into the speed of rotation, which is the flow the mechanical side, so the decimation as easily the ideal decimation is nothing, but a gyrator. In fact, all machines are you can easily see that in all machines the current signal is converted into a torque signal, in all electrical machines or actuators, current signal is simply, if you if you assume that there is a magnet is calling another pulling a piece of iron then the magnetism, magnetic force is depended on the current and the force is being produced in the mechanical domain, so that is a effort.

So, wherever you have such things it is essentially gyrator action, so we understand this things, now comes a very interesting concept. We have said that in the bond graph, we are representing the structure of the system completely, so for we have been worrying about the direction, the elements representation of the elements and how power flows, but then power is composed of two things the effort and the flow and we have already made the statement that no single element can decide both.

So, these are information variable, the effort and the flow both are flowing, but no single element can control both, so we draw the complete structure in that complete structure the information must be flowing in a very specific way. How is it flowing, who is deciding that let us that what we will concentrate on, now and that issue is called the causality.

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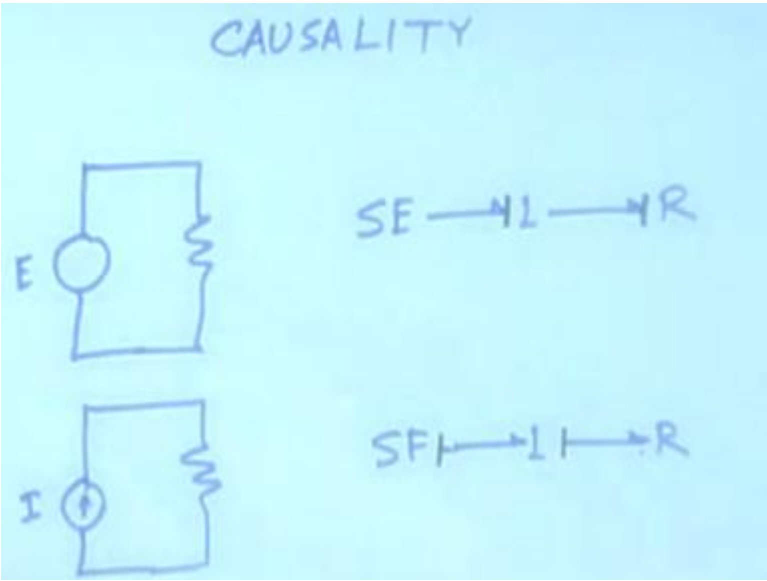

To illustrate say you got a voltage source connected with the resistance, who dictates what the voltage source the SE element, here the bond graph will be simply SE connected to one inductor connected to the R and the power direction will be like this of course, but then who decides what, notice the SE element decides the effort, but it cannot decide the flow. So, the effort information flows this way and the flow information, the resistance the dictates the flow that flow information goes this way.

So, there is a direction of flow of information through the whole structure of the system, everybody is deciding something and an interesting thing is to keep track of who is deciding what, for example in this simple thing, the SE element is telling the rest of the system, in my bond the effort is this, but he is not being able to decide the flow. The rest of the system will decide the flow, while the resistance in this particular case is deciding the flow, now that till take the opposite case.

Suppose this fellow is the current source, then it is SF connected to 1 junction connected to R again power direction like this, who is deciding what, the source of flow the SF element the current that is deciding the flow, but it cannot decide the effort. Who is decide the effort the resistance, so that the resistance is deciding the effort, so that flow direction flowing information going this way and the effort information is going that way, now these are marked by something called causality. But before we start marking it, let us try to understand why is it causality, let us try to understand who decides what is what again, but in the in the relationship to the capacitor inductive elements.

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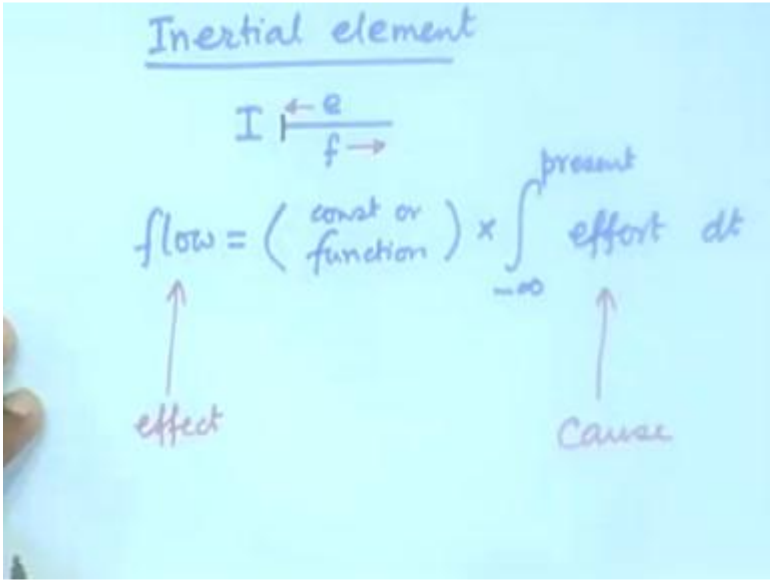

Say, take an inertial element in the inertial element I, we have got a bond in that bond suppose it is e and this is f., there is specific relationship between the two, what is the relationship between the two, the flow is equal to something a constant or a function times integral of minus infinity to present effort. So, here it is constant or function, now notice the relationship between these two, in order for the flow information to be generated, it requires the whole history of the effort information.

So, what causes what, if Ii ask you what causes what; obviously, effort causes the flow because the effort information is prior, it has already happened and that is why the flow happened, can you see the flow now is determined by the history of the effort and any causality system, causality means that something causes something the what causes should be preceding what is the effect. So, cause precedes the effect and that is why we say that there is a causality means I have done something due to that something else is happening.

So, that is a history and that effect is what happens, so out of the flow and the effort what is cause, what is effect, effort is the cause and flow is the effort. So, that is a very important, so this is the cause and this is the effect, so whenever there is a inertial element that is the very important concept that in order for a flow to happen there has to be history of effort and in order to calculate the flow you will need the history of effort, so it is not only concept is also need needed for your calculation.

We will ultimately write a differential equation that has to take this effect into account, so if you say that here is a I element along with it a bond then what is the direction of flow of this informations, this says that the effort information must be going this way and then the flow information comes this way. In this cases we could say that the SE element gives out the effort information receive the flow information, this resistance gives receive the effort information and gives out the flow information, here similarly this fellow gives out the flow receive the effort.

This fellow gives out the effort receive the flow fine, but in this case in this bond, it is very hard way understood that this fellow must be receiving the information about the effort in order to give the information about the flow. So, the effort information must flowing this way, so I will put an arrow direction, this is the effort information direction and this is the flow information direction.

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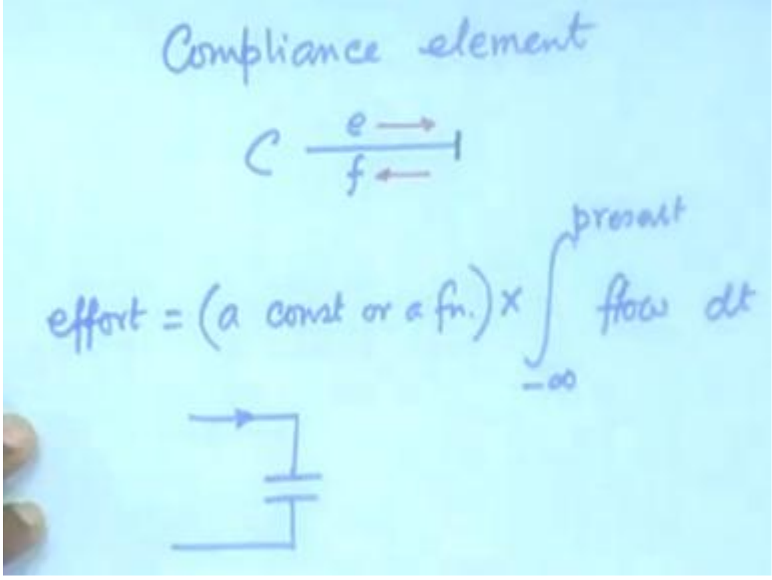

Similarly, in case of the compliance element in C connected with the bond with a effort and the flow, what is the relationship between them, the relationship is effort is equal to a constant or a function times integral minus infinity to present flow d t. So, the capacitive element must receive the information about the flow, in order to generate the information about the effort, see we do not notice this all the times say capacitive element like this, it must be receiving a flow in order for it to generate a voltage difference.

So, the voltage difference is the effect, the cause is the flow cause is the current, so in this case this particular element has that casual structure in it without it without that it cannot function, so here the directions would be effort information is given out by this fellow, but the flow information has to be received by this fellow. Now, therefore, we understand that whenever we draw a bond graph, in the whole bond graph each bond has a specific direction of the flow of the informations.

If effort goes this way, flow goes that way and each of the elements has specific preference in choosing one of them, compliance element will naturally receive the flow information give out the effort information, an inertial element will receive the effort information give out the flow information, so on and so forth. This is normally denoted by a specific convention who determines what; who causes what, that is normally given by means of a specific convention.

The convention is imagine, you are holding a syringe in your hand like this, can you see yes a syringe in your hand, so if you press it will go that way, so imagine that is the flow then you are holding the base of the syringe and you pushing it. The convention is to give a stroke at that end, so here flow is that way, so I will put a stroke like this means that we are imagine that we are holding a syringe and pushing it will flow, will go in that direction.

The moment you have you see a stroke here, it called the causal stroke you would mean you would know that the effort is coming this way and the flow is going that way, so that is the convention of the inertial element, similarly for the compliant element what will be the point at which you put the stroke here. Because the flow is going that way how do you put the stroke in case of the SE element, SE element determines the effort, but has to receive the flow.

So, flow goes this way, like this SF element here what about the R element, when the R element is something that can take both and depending on the way it is connected in a circuit, it either determines the flow information or determines the effort information. It cannot determine both, but in this particular case the way it is connected it is doing what, no it is receiving the effort information giving out the flow information, the flow information is going that way so it is like this.

In case of this, it is receiving the flow information and flow information is going this way and gives out the effort information, so flow information is going in this direction, so it will be like this, so far so good. Let us now talk in terms of the transformer element.

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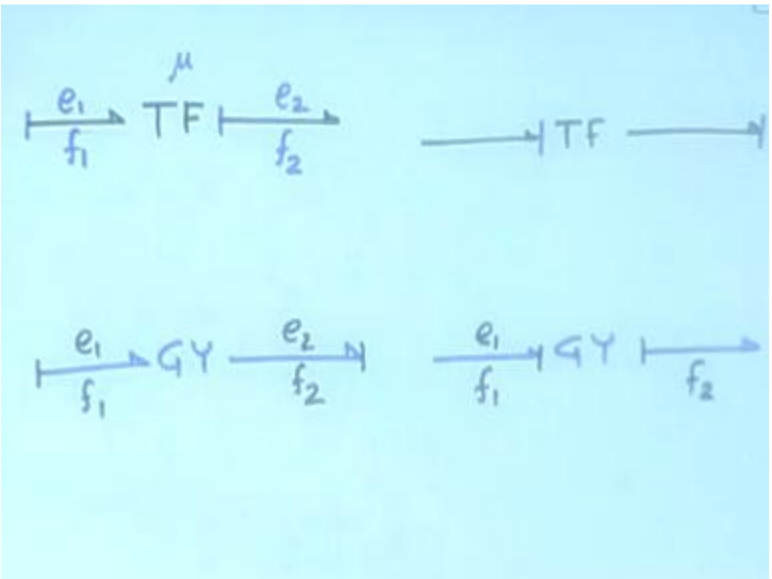

And this gyrator element, transformer here is a e 1 and here is a f 1 and here is a e 2 f 2 how will you first give the power direction, if power direction is positive this way, it has to be positive it that way, so power direction is draw the simple, now in this case if the flow information is coming this direction. Since f 2 is equal to mu times f 1, the flow information is going in that direction of course, the opposite two is also the true, if the flow information come this way it will be in the opposite direction, so it will be either this stroke or that stroke.

Both are possible either it goes this way into the other side or the flow information coming this way and goes through the transformer and goes through the transformer, what about the gyrator, here also the power directions would be in the same direction because power goes through that two possibilities again, so let me draw two, now here say here is e 1 here is f 1 here is e 2 here is f 2. What is the relation, the relation is we have already written that relation e 2 is equal to mu f 1, so if this bond f 1 is bringing in the flow information is goes out as the effort information.

Is that clear, so if this bond brings in the flow information as this, if you put the stroke line here means the flow information is going this way in that case e 2 is mu times f 1 and so what goes out is the effort information. If this fellow go takes out the effort information it must bring in the flow information, so the stroke has to be here in a similar way in case of the situation where this bond is bringing in or say this bond is bringing in the effort information not the flow.

So, here is e 1 f 1 and here the e 1 is bringing in the information, in that case flow information is going that way and then the flow here f 2 is then determined by the gyrator element times the e 1 and so the effort information is coming, the flow information must go out and so the stroke has to be here. Now, once we have understood this the complete information necessary to draw the bond graph is ready and in the next class itself we start deriving the differential equations, only this much was necessary.

The main advantage is that once a physical description of the system is converted into its equivalent bond graph structure, the complete information necessary to write the bond graph is there and then not only that the derivation of the differential equation will be very algorithmic. You, have to blindly follow a certain procedure in order to obtain the differential equations, it is so algorithmic that passing an exam is very simple, it becomes very simple because you have to almost blindly do it.

Some people may like it, some people may not like it, but the major likable aspect is that you do not even have to derive the differential equation, it can be done by any damp fellow and the dampest fellow is the computer, so we generally give the job to a computer to do it. So, normally yes for our practice, we will practice obtain the differential equation because you have to have the feel for it, but in actual practice, when you have the task of writing the differential equation for a very big system.

Obviously, if want to do it by hand you will take ages, so all way do is to just the figure out the bond graph, give it to the computer it may derive something like 55 dimensions of differential equation no problem, but if the same thing has to be done by you, you know that it is going to be a night mare. So, that is the main advantage of the bond graph methodology, so let us now figure out a few simple bond graph structures and how to give the mark.

Let us start with this simple system, we have already done the rest how will you give the casual directions, let us do that and they will make a very strong point that is say the AC element, AC element gives of the effort information and this is the flow information, so that the casual stroke will be here. The SF element, it gives out the flow information and receives the effort information, so the casual stroke will be here, fine. Now notice an important thing I will come back to that.

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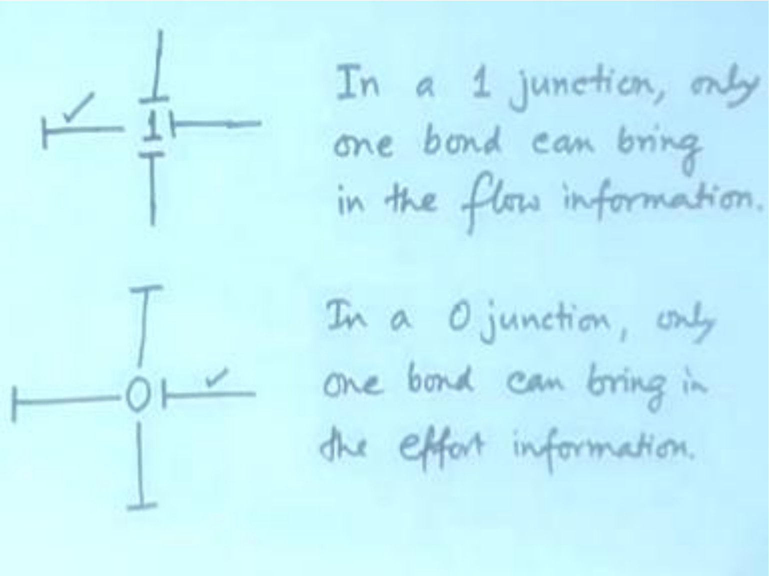

Suppose there is a 1 junction connected to four bonds and suppose one is bringing in the flow information, 1 junction is the flow equalizing junction, it is a series kind of electrical circuit it equalizes the flow and immediately you would notice that it will equalize the flow; that means, the flow information if it comes to one bond it must go out through the other three. It cannot be that a 1 junction is bringing is receiving the flow information from two directions, it cannot happen forbidden.

Always if there is a 1 junction, only one bond must bring in the flow information, so let me write it, similarly if there is a 0 junction connected to four bonds then the flow information can be brought in by any number. But the effort information if 0 junction receives the effort information then it must distribute that information in the rest of bonds it cannot help it, because it is a effort equalizing junction, so if one bond brings in the effort information, it is flow this way effort this way.

Immediately the causal structure of the rest of the bonds should be determined, so it will be this means that the effort information is coming in by means of this bond and getting distributed by means of the other bonds. So, these are, so in a 0 junction only one bond, fine good on that basis let us attack this problem, here we have got a SF element here we have got a SE element and here there is a I element, SE element has brought in the effort information.

The 1 junction can still accommodate more, so there has to be a source from with the flow information comes and it is easy to see that here is a I element inertial element which gives out the flow information and this is the effort information. The moment is 1 junction has received the flow information, it must give out the flow information in the this two directions, immediately this direction become determined. Now, the 0 junction has received flow, flow there has to be something that determines this effort.

Obviously, this C is that element, which receive the flow information and gives out the effort information, so the moment to give it is fine, notice the logic that we followed here if a 1 junction receives the flow information then it is its casual structure is completely determined you cannot help it. Is 0 junction, receiving the effort information is casual structure is completely determined you cannot help it, that is why this one and this one will be called the strong bonds and the others are weak bonds, in the sense that this fellow in a strong way brings in the flow information. This fellow in a strong way brings in the effort information, here in this case, can you identify the strong bonds is this the strong bond no, is this a strong bond no, but this and that 2 and 4 are the strong bonds. Let us do another circuit.

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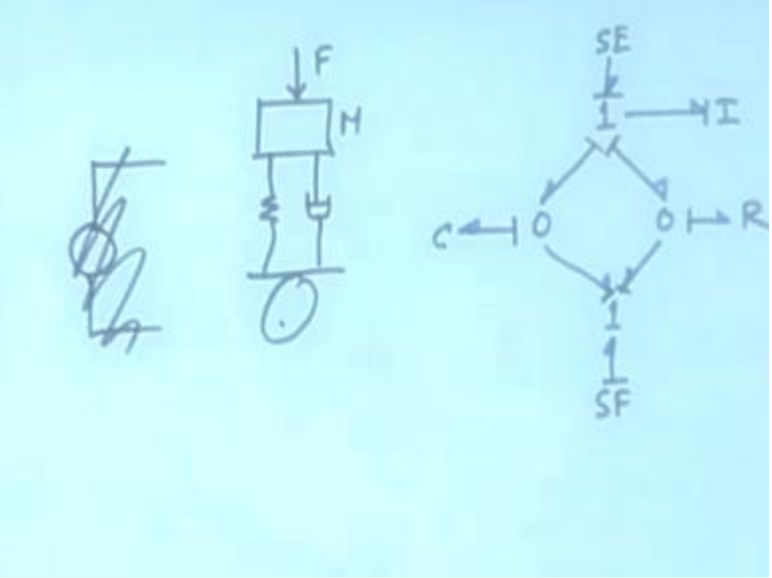

We have done a problem in the last class, where we had know let us not do the circuit, circuit we have done enough, we have applied a force on a mass, there is a spring damper arrangement connected to say a camp, how did we make the bond graph in this, case we said that the 1 junction is connected to the SE element here, connected to I element here and that is connected in the two directions of two 0 junctions. Here, it was a C element here it was a R element, we have already done that that is why I am not repeating the whole logical structure.

In that case, in order to complete this bond graph what we will have to do, we will have to first put the power directions and power directions can be put started from the sources like this. Then, the individual elements like this, now these can be power directed anyway, so let us just put that any power direction no problem, this only say what will be the positive direction of the power that is why, now give us the causalties, the causalty when we start assigning it, we start against from the sources.

Here is a source of flow it gives the flow, so it will be causal this way, here is the source of effort which give the effort and receive the flow, so it will be causal this way, so this one has become a strong bond can you see that. Naturally it immediately tells that these two must take having the flow information, it gets determined, but then nothing else is now determined because here the flow information has come, but this is zero junction, this here a effort information has come back it is a 1 junction, so let us go to the storage elements next.

Here is the I element that naturally receive the effort information and gives out the flow information, so it will be causal this way, so the flow information goes this way at it goes into a 1 junction which is a flow equalizing junction. Immediately the flows in the other junctions will become determined, so here is a 0 junction in which the flow informations have come, but as it I do not see an effort information coming in, so that the effort information must be generated by the C junction, which is it is natural property, it is property is that it receives the flow information gives out the effort information.

So, it is causality will be here, so the flows added up goes into the C and the effort is given out by the C, which is equalized by this 0 junction done, now let us come to this R element which can as we have seen take both the causalities. But in that case what will be the causality, will it generate the effort information or will it generate the flow information, effort yes because of the same reason that it has already received the flow informations and the flow information going into that and the 0 junction must receives the effort information. So, it has to have the causality then this bond graph is complete fine, so with that, that much understanding will embed the next day on the auto difference obtaining the differential equation, fine.

Thank you very much.