Dynamics of Physical Systems

Prof. S. Banerjee
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 13
The Bond Graph Approach – I

We have seen that the dynamical equations for systems could be obtained by the Lagrangian method by the Hamiltonian method for electrical circuits by the graph theory method. Through all these we were able to obtain the differential equations. And then the next step obviously is to solve the differential equations in the tiny minority of the cases where equations are exactly linear, it can be solved analytically. But, if the equations happen to be non-linear obviously, we do not have a analytical solution.

So, in most of the cases, you will have to obtain a numerical solution of the equations, so where are we are having to write down the differential equations ourselves, humans have to write it and the computer is given the job of solving the differential equation. Now, there exists one method by which even the job of deriving the differential equations can be handled by computers, so it is very strange that that seems to be a very human affair, but that can also be done by computers.

But in order to do that you will have to somehow convey the essential characters of the system to the computer. So, it has to be conveyed, it has to be conveyed in an unambiguous term, so that when everything is laid out or conveyed to the computer, it can follow some kind of a algorithmic procedure to derive the differential equations. And that is what this the subject of lecture, today the Bond Graph Approach is all about.

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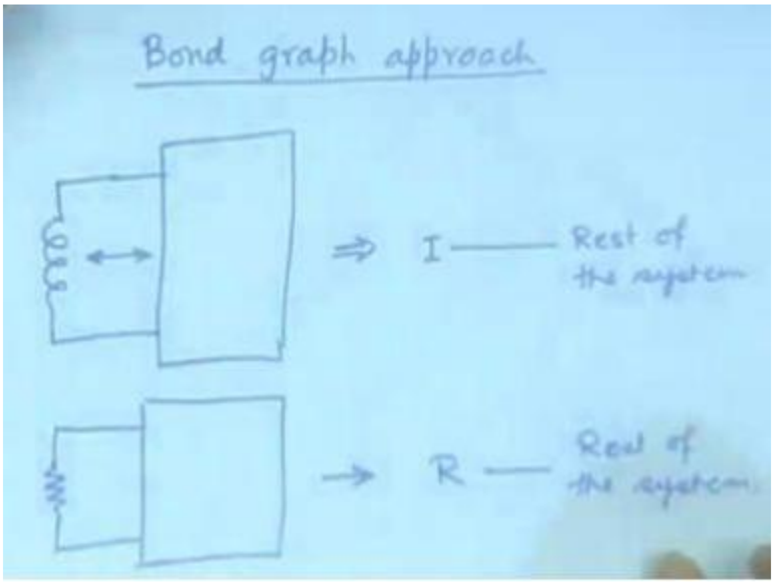

So, the essential idea is to formulate some kind of a language, in which the character of the system can be conveyed to the computer, so that it can then follow a very algorithmic procedure, which is already programmed into it, in order to derive the differential equations. And then, if the computer derives the differential equations; obviously, it can solve it and it can perform a whole lot of tasks. So, that is the basic idea, the idea was germinated very long back in the 1960, suppose the first paper came out by Professor Henry Painter of MIT.

But after that it has gone through stages of development and now every year international conferences on this bond graph technology takes place, so what is the essential idea. Essential idea is that in any system there are elements, we have seen that any system is composed of elements like the inductor, capacitor, resistor, voltage sources, current sources or sources of effort force and all that ultimately make up the whole system.

When they make up the whole system in what way can we define the interaction between the elements how do they interact, they interact essentially by exchanging energy. So, if an inductor is in a circuit, then it essentially absorbs energy for sometime gives out energy for some other time, if it is ideal inductor it does not dissipate. If, there is a resistor then it absorbs energy and dissipates into the environment, if there is a voltage source it gives out energy that is a source of energy, if it is a current source it is a source of energy.

So, it is essentially energy that becomes exchanged between elements, so an element is as if looking at the whole system by means of some kind of a energy transfer, so between that system that particular element and the rest of the system there is a energy transfer. And that, energy transfer completely defines the dynamics, no it is not exactly correct, not only the energy transfer, but there is another thing that is transferred between elements that is to think carefully and deeply that is the information.

What information, you see the energy is composed of two things energy or power, energy per unit time is power, power is exchanged. Power is composed of two things, it is a product of a effort variable and a flow variable, in case of electrical system the effort variable is the EMF and the flow variable is the current, in case of a mechanical system, translational motion if the effort variable is the force and the flow variable is velocity, if it is a rotational motion it is a torque and the rotational velocity and so on, and so forth.

One can even do that in magnetic domain, magneto motive force and the rate of flux, the thermal domain in all domains it can be defragmented, so essentially in every domain wherever dynamics takes place, there is always a effort variable and there is always a flow variable. And these are also exchanged between the elements that means, the effort and the flow product is the power, which is exchanged between the elements.

But, the information about the flow and effort they should also be exchanged between the elements, these are information variable, so flow and effort are the information variables, their product is the power variable. And that is, all that is exchanged between the elements to define the dynamics of the whole system, so professor painter argued that if we can somehow, picturize how the information variables and the power are being exchanged between the elements.

And if we can convey that to the computer, it should be the sufficient information for a algorithmic procedure to be in place to derive the differential equation, so that is what we do. Essentially we sort of this graph is nothing but a pictorial representation of how the information and the power is exchanged between the elements, so how do you go about doing that.

Imagine that there is an inductor connected to some kind of a circuit, this rest of the circuit could contain anything some sources, some in resistances, capacitances more inductances all that put together is the rest of the circuit, but here there is an inductor. Now, what does the inductor do, it sees as if the rest of the circuit and it exchanges the effort flow and the power with the rest of the circuit, which we represent as the inductor is an inertial element, we have already learned that.

The general representation of an inductor or a mass or a moment of inertia that is I, meaning not current, it is inertial element in that sense, in this representation, this will be connected to a bond with the rest, so, I connected with the bond, now this bond will then carry power, flow information, effort information. Actually it will carry the effort information and the flow information and the power is nothing, but the product of them but, so one bond is necessary in order to carry the effort and the flow information.

Similarly, if you have a resistor connected to some kind of rest of the circuit then it will be represented as R rest of the circuit. Now, notice even though the inductor has two extremities, we are not drawing with two extremities because essentially we are abstracting it, what does it do, it sort of exchanges effort and flow, so it is sufficient to represent that by one bond, so never do it like this one side of I another side of I, no it is not like that.

So, we are not showing the actual connection of the circuit rather, we are abstracting it to say that this I element is exchanging flow and effort information through this bond with the rest of the system. Similarly, for the capacitor, even the source of effort and flow, so let us try to understand the character of each, first when we talk about the inductive element.

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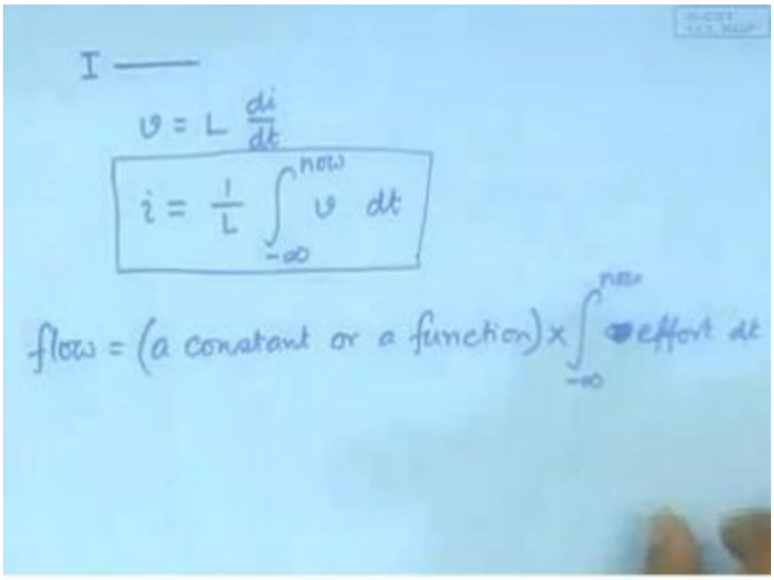

So, I connected to something this bond, what is the character of this bond then character of the bond is represented by the fact that v is equal to L d i d t we know that it tells that i is one by L integral of minus infinity to now hum minus infinity to now this is v d t. This is a specific type of relationship between the effort variable and the flow variable, effort integrated gives the flow, so effort integrated gives the flow that is the special type of relationship due to the characteristics of the I element.

The I element is connected with the bond with the rest of the circuit and the moment we draw the bond, we know that it has this character, so this is the essential character of this, there is a reason why I am not using this as the essential character, but this, you will understand slowly when I come to that, so in general, for this is the character of the inductor or the linear inductor. The inductor could also be a non-linear element or something like that; that means, there could be saturation or whatever, so let us generalize this.

The character of the inductive element or character of the inertial element is that the flow is equal to a constant or a function times this fellow integral minus infinity to now effort d t, so this is the character, effort integrated and there is a constant or a function. If the inductance is constant then it is a constant, but if it could also be a function of the current flowing through it, as it happens when there is a saturation, so it could also be a function there is no problem about it, this could be some L a function of I, there is no problem about it. Whenever, these relationship holds we call that an inertial element and represent it by this. Now, what is the character of the compliant element, we have already learned that in the in the first class that is the compliant element.

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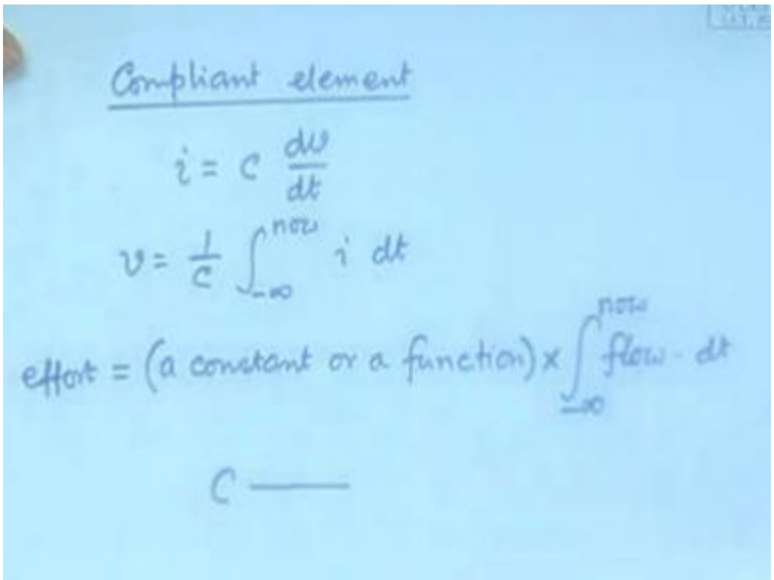

The compliant element is where i is equal to C d v d t or v is equal to 1 by C integral minus infinity to now i d t and if we write it in a general term, then we would write the effort is equal to a constant or a function times integral minus infinity to now flow d t. Now, this particular statement is the definition of a compliant element, so when we learnt earlier it was a bit restrictive definition.

But, now you can easily understand there would be compliant element in the thermal domain, there would be compliant element in a magnetic domain, which is not clear, which was not clear when we discussed about the compliant element earlier when we discussed only about the spring and the capacitor.

But, the moment you write it this way, you can easily understand in any domain this would be there and whenever we have that such a thing we represent as C and with a bond, C is representative of the compliant element with this property. So, the bond will have the information about the flow and the effort, satisfying this condition and that will be connected to the rest of the circuit.

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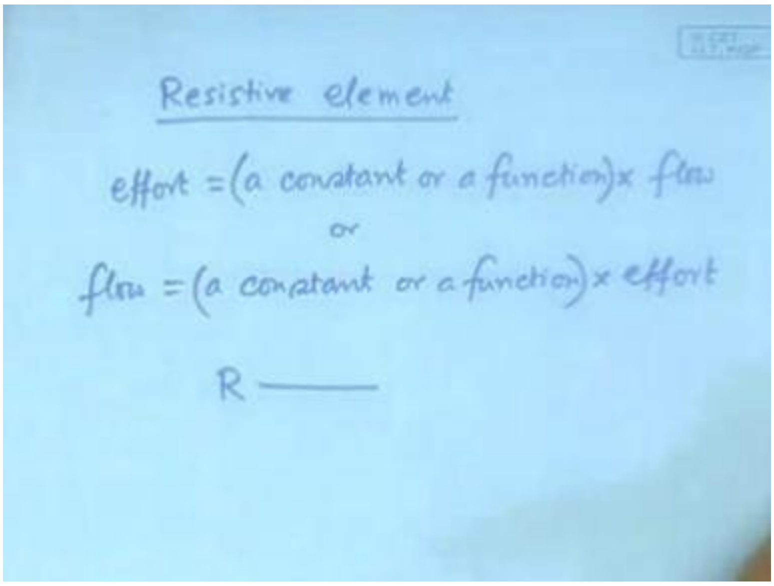

Then the resistive element, what is the character of the resistive element, it is that the effort is equal to a constant or a function times flow or flow is equal to a constant or a function into effort, so this is the character of the resistive element always. So, whenever we have a non-linear resistor, the way we encountered in case of the stick slip kind of oscillation or dry friction then this would be a function that is all, but this character will hold, so it will always be represented by R connected to a bond. And that bond will carry the information about the effort and the flow related by these relationships.

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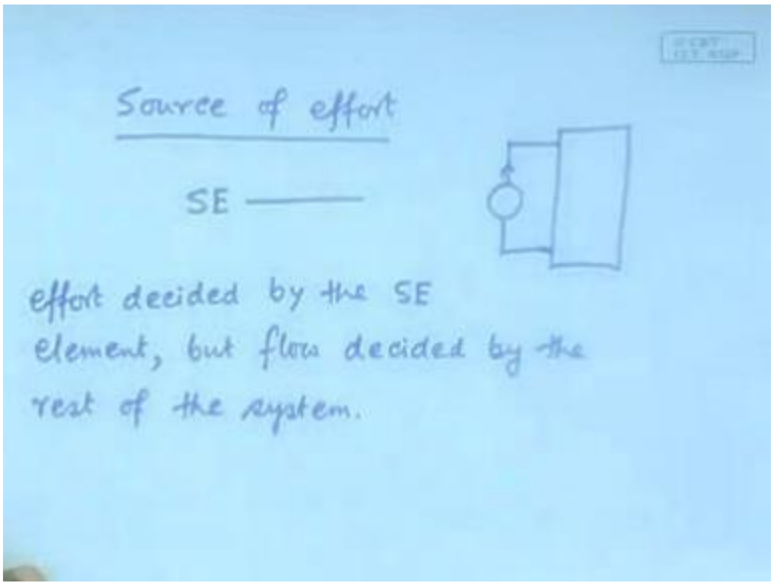

Similarly we have two other elements that can be represented in a similar way that is the source of effort, source of effort will be represented as just note the symbol S E connected to a bond, what is the property of this S E element, source of effort this S E element's property is again to recapitulate that it can independently decide the effort variable on that bond, but it cannot decide the flow variable. Just imagine that a voltage source is connected to some kind of a circuit then it decides what will be the voltage here, but it cannot decide what will be the current here.

The current here is decided by the rest of the circuit, so the character of the S E element is that the effort decided by the S E element, but flow decided by the rest of the system, so something being source does not mean that it can decide everything, it can decide only the effort information, remember effort information. How much power will ultimately flow, also cannot be decided entirely by this fellow because that has to take into account the flow information and the flow information is decided by the rest, not the SE element.

So, I am slowly coming into the special properties by which the flow and the effort are related in every bond, so here the flow and the effort will be related by that, similarly what will be the character of the mechanical S E element, what is that mechanical S E element, mechanical source of effort is a force. So, mechanical S E element is a force, so whenever I am pushing something I am applying the force, but with what velocity it will move that depends not on me, it depends on the rest of the system. A huge system being pushed might not move it very much a small system being pushed will move much, so it depends on the rest of the system. The other source element is the source of flow.

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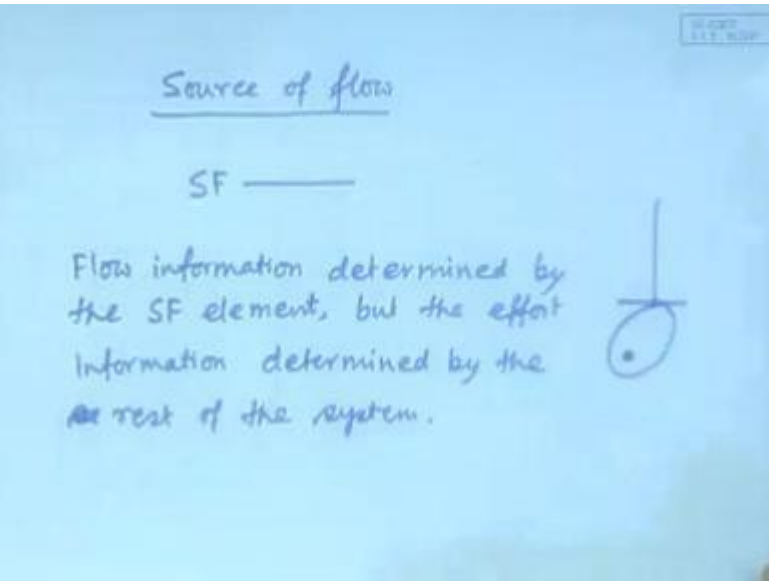

It is represented by S F source of flow, so notice that here we are not talking about the voltage source or current source in general we are writing it as source of flow, so that there will be no distinction between the electrical system, mechanical system, thermal system, hydraulic system no difference, in all systems these elements will be there. What is the character of the source of flow element, that it can independently determine the flow variable, but the effort variable comes from the rest of the system,

So, the flow information, so that is the property of the SF element source of flow, what is the electrical source of flow the current source, what is the mechanical source of flow the camp. So, it is an arrangement where you have an eccentrically placed shaft, so that when it rotates this fellow goes up and down as a result on this point, a velocity is imposed, a velocity is imposed, so that is why this is a mechanical source of flow, so with this, we can start building up with this these many things in hand we can start building up simple systems.

For relatively more complicated system we will need more elements, but let us start at least. Now, all these elements are ultimately connected, ultimately in some way they interchange, exchange information about effort and flow and that somehow must be represented by the bond graph, let us start with a very simple example where you have.

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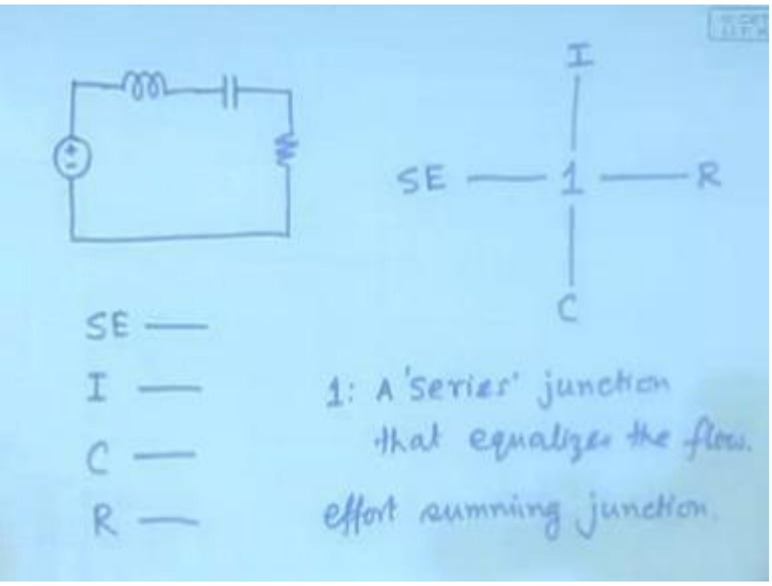

A source of effort connected to an inductor, a capacitor and a resistor, so we know that SE element will be a just like this SE, the inductor will be I, the capacitor will be C and the resistor will be R, but how are they interconnected. Now, that the moment you look at it, you would notice that they are interconnected or they exchange information in a very, very specific way, what is it, that they share the same flow, it is a series connection which means that they share the same flow.

So, in that case that particular specific way of exchange of information will be represented by a junction, a junction in this case it will be called a 1 junction. So, it will be represented this will be represented by S E connected to a bond C connected to a bond, R connected to a bond, I connected to a bond, in between they are exchanging information by a specific way that is represented by a 1 junction, this 1 represents the property that it equalizes the flow in all the bonds connected to it.

So, this 1 has the property it is 1 A series junction that equalizes the flow in some material you will also find S, which means a series junction, but international convention is to write it as 1, 1 and 0. So, this 1 is the international convention of this particular type of junction that has the property that all the bonds will carry the same flow information effort will be different, the efforts will then be decided by all this fellows, but the flow will be the same.

Now, all these will also carry power, so there is a power going from here to here or whatever the direction is, so there is a power exchange between this, power exchange between this, power exchange between this, but the flows are equal and the power is a product of the flow and the effort, what does it mean, the efforts are summed up to 0, so it is a effort summing junction. So, the flows are equalized and effort are summed up, efforts are summed up is essentially the in this case the Kirchhoff's voltage law. But we are writing in a general sense because in mechanical system or thermal system or hydraulic system there is no Kirchhoff's law, so there we have to understand it this way. Let us come to a similar circuit.

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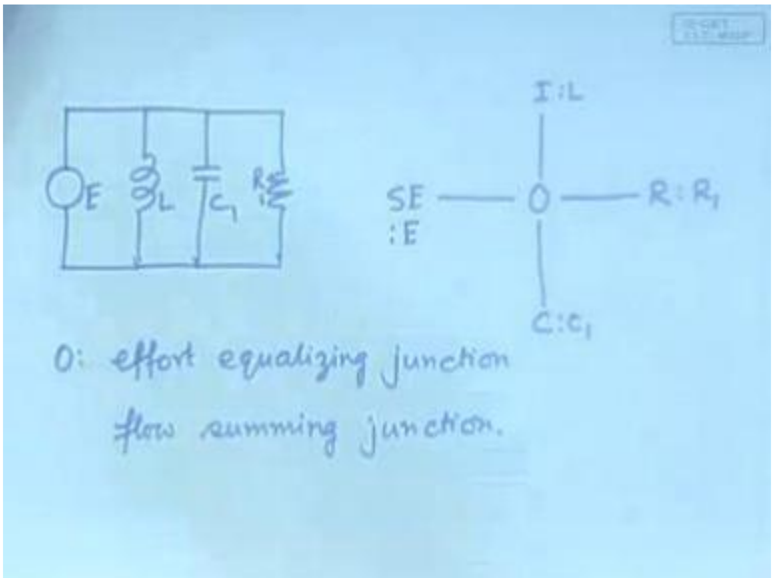

So, this is the E L C R, so in this case also these elements are interconnected in a specific way and we will do it in a similar way S E connected to I connected to R connected to C connected to, but then what is in between what is the property of the junction, how do they in exchange the information, the voltages are the same. So, we have to represent something that is a effort equalizing junction, so it is called 0, so 0 is the effort, obviously since the power coming into the 0 junction is the effort and the flow here, effort and the flow here, effort and the flow here, effort and the flow here and all that ultimately should be 0.

And since the efforts are equal therefore, we in effect get the Kirchhoff's current law, so which is the, this is also a flow summing junction, remember that even though this fellow is actually E, this fellow is actually L or in a circuit there could be more number of inductances, in which case we denote them as L 1 L 2 L 3 C 1 C 2 C 3 and all that, here all these are generic symbols. Since these are generic symbols, there has to be some way of denoting that is SE element is nothing but the E, which is which is given as a colon and then you write the actual thing.

So, this L and, so on and so forth, so this is C and this is R, but it could be R 1 and C 1 in that case you would write C 1 R 1 in stuff, so that is the convention that we write the generic symbol first and then what it is that we write with the colon, so that if you draw a large bond graph there each element is properly specified. Now, let us go about trying to draw bond graphs for a few very simple systems, suppose there is a circuit like this.

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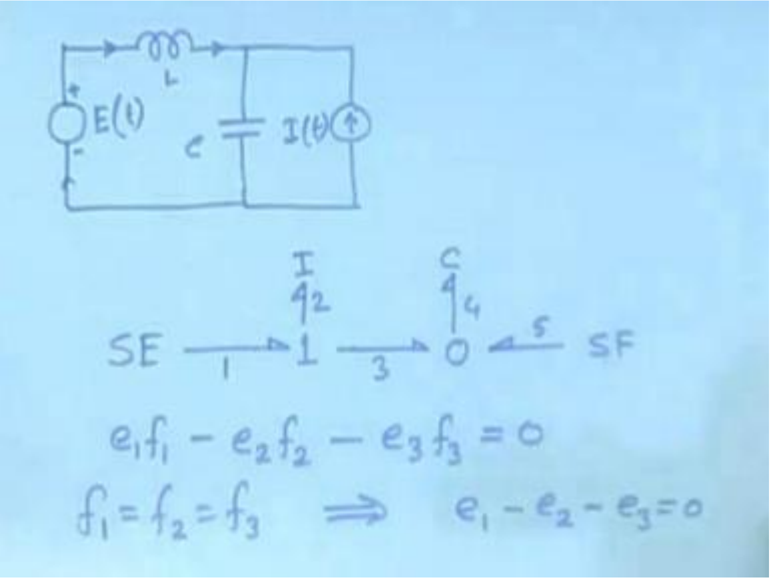

Initially let us start with circuits because they are easier to handle, first it is inductor then we have a capacitor and then we have say a source of flow, this is a E of t, L C this is I of t this is a source of flow. So, it is slightly different, but let us start from one element and then go ahead to draw the bond graph, first let us start with here, so it is a SE element, SE connected to a bond what is it connected to, notice that it shares the same flow with this L element.

Therefore it should be connected to a 1 junction, 1 junction to which the I is connected and the rest of the circuit, this whole thing also shares the same flow, the flow here and the flow here and the flow here are the same. So, this whole block shares the same flow with the inductor and the source of effort and therefore, this is the representation all these bonds carry the same flow, but after this what is it is the capacitor and the source of flow in parallel, which are sharing the same effort, so they will be connected by a 0 junction and what will be connected to it C and S F.

So, do you notice the argument, here the argument is that we start from one end and go by observing what remains, equal between specific parts of the circuit. Here these two shares the same flow with the rest of the system, so this is a structure, these two have the same effort and that is shared with the rest of the system, these two have the same effort that is shared with the rest of the system, so that is the bond structure of this, another example to get us familiar with it.

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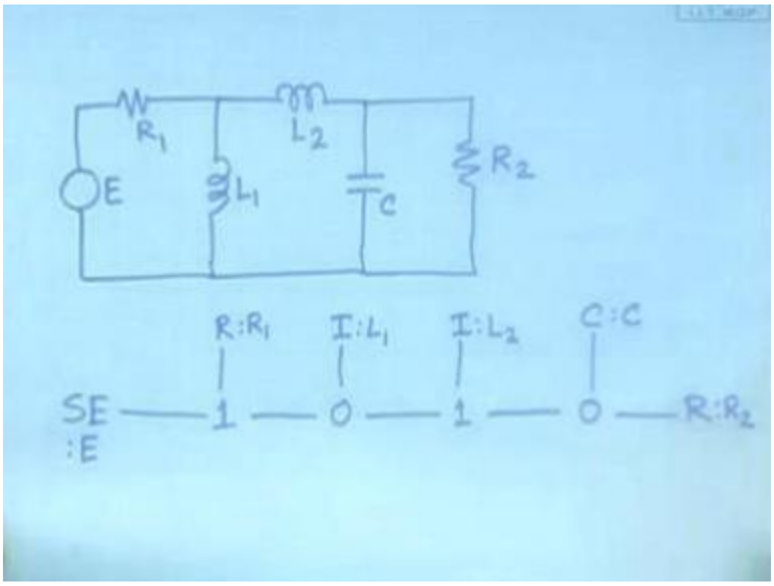

Suppose there is a source of effort connected to a resistor, connected to an inductor, connected to another inductor, connected to a capacitor, connected to a resistor, how we will go about arguing this case. We will start from the S E element, S E here then we will say that this S E element and this R element share the same flow, so they are connected by a 1 junction R and with a rest of the system, so then we argued this way that notice this you have got a S E element R and the rest of the system they share the same flow, so here is the rest of the system.

What is the rest of the system, now move a bit, we have got a inductor which shares the same effort with this combination and with the rest of the system, so it will be a 0 junction with the inductor I element in it. So, we have come to this point come here, this whole thing shares the same flow with this inductor and the rest of the system, so it is a 1 junction with I with the rest of the system. Finally come here, this capacitor and the resistor share the same effort with the rest of the system.

So, here comes the bond representing the rest of the system, here there will be a 0 junction, effort equalizing junction connected to a capacitor and a resistor and how we will write it. We will write S E colon E because this E this is R 1, this is R 2, this is L 1, this is L 2 and this is C, so we will check this colon R 1, here it is R 2, here it is L 1, L 2 and here it is C , let us go ahead. We started with electrical systems, but after all sometime or rather we need to attack mechanical systems, so let us go ahead with mechanical systems. In a mechanical system, the S E element is the agent that is applying the force, how do we identify the character of a 1 junction and a 0 junction, what was the character of the 1 junction it equalizes the flow.

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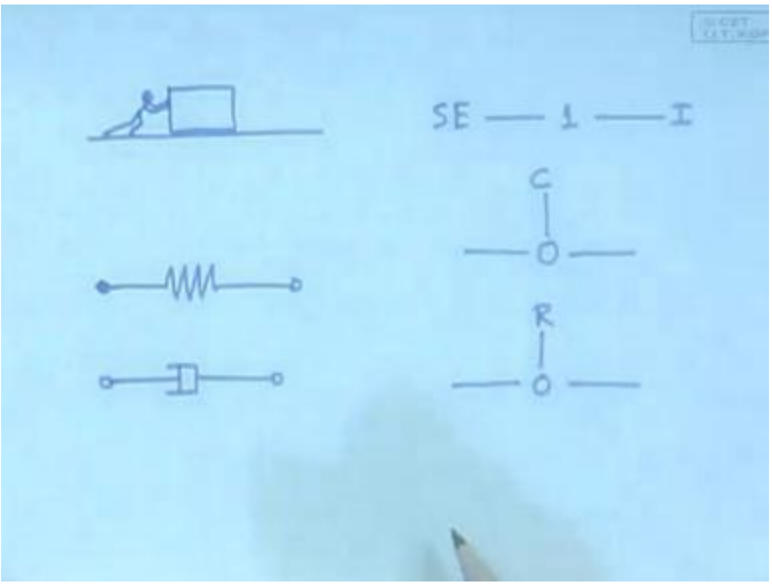

Now, notice there is a mass and say some fellow is say pushing it like this, so this is the SE element and this mass is inertial element I, so this system how will you represent that. Notice that these two fellows the S E element and the I element, they have to share the same flow other if they do not then it will it will go off his hand, so the contact is lost S E element no longer remains S E element, so in order to S E element to remain as S E element he has to, this poor fellow has to run with the same mass, has to share the same flow.

So, its representation will be S E connected to a 1 junction connected to I, so whenever there is a force applied on a mass you by looking at this the system's structure, you have to identify which are the elements that share the same flow and connect them by the 1 junction. Now, what is the property of the 0 junction they equalize the effort, what could be the mechanical representation of something that equalizes the effort just imagine, if suppose there is a spring with two ends, and suppose you are holding the two ends and pulling them, then this end and that end feel the same force.

If they do not then there will be a resultant force and the ideal mass less spring will fly off the infinite velocity; obviously, that is not the case, so these two ends will feel the same force, same is true for a resistive element just move them the two ends must feel the same force. So, if these two sides feel the same force then whatever are at the same the two sides they share the same effort, which means they are connected by a 0 junction, so in that case we will represent this as a bond 0 junction to a C element to another point.

So, whatever is this side, whatever is in that side and the C element see the same effort, for this one it is whatever is in this side, whatever is in that side and the R element see the same effort, so that is how these things are drawn, on the basis of that knowledge, let us start building up the bond graph of some simple systems, for example…

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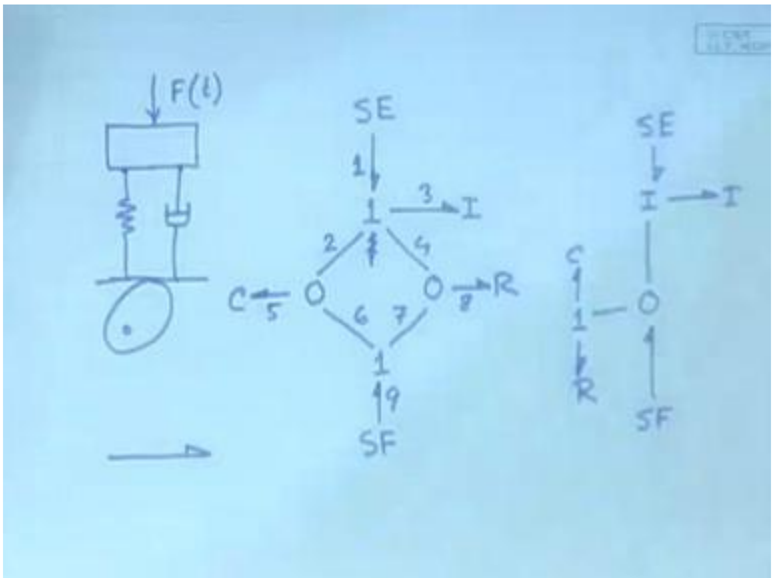

Suppose there is a mass, being act acted on by a force here there is a spring damper, support and suppose here we have a cam, so from below there is a source of flow, from above there is a source of effort and the thing moves with some dynamics. How would we represent the bond graph of it, let us start from the top, we will say that here we have a SE element, now it shares a SE element Ft shares the same flow with the mass therefore, it should be a 1 junction connected to the I element to the rest of the system.

Now, what is the rest of the system, I can see that this end and that end of the spring and the damper they share the same flow, so they are also connected, so I will not do like it, I will do it like this. So, here is the thing that is connected to the spring, here is the thing that is connected to the damper, but both of these bonds share the same flow because these two ends have to move together, but then what is shared between this end and that end same effort, so we will say that here is a 0 junction, here is the 0 junction connected to the R element and the C element and here it has come.

So, this side and that side, this side and that side share the same effort, but ultimately this point, this point and the cam have to move together, they share the same flow, so it is a 1 SF. So, do you notice the structure of the argument, the structure of the argument is that we are starting from one element and just by observing the system, we are finding out what is common between the elements, in this case this point, these, these all these share the same flow and therefore, they are connected by 1 element 1 junction.

Here, the SF and this point and that point they are connected by a 1 junction because they share the same flow and but this point and that point share the same effort, so they are connected by a 0 to this C. So, this bond and that bond whatever the effort is seen by this bond and that bond, it is the same effort that is seen by the C element, here also whatever is the effort seen by these two bonds is the same effort seen by the R element that is why this structure is ((Refer Time: 44:35)).

So, this is the bond graph representation of the system, now you might say that no, no I do not like that argument I can completely different formulate in a different way, for example anybody can could argue this way that suppose, here is a S E which is connected to a I element there is no problem about it I, but what is after that after that one could argue that let us think that these two are combined to an element. So, the top and the bottom share what, same effort, no same effort, this point and that point share the same effort.

So, initially forget about what is inside, this point and that point share the same effort, so just connect them 0 to the S F, now we have to consider what is sharing the same effort in between, you will find that these two the capacitor and the resistor or the spring and the damper they have to move, together these two fellows must share the same flow. So, connect them by means of a 1 junction C and R, so by two different lines of argument we have written one bond graph and we have written another bond graph.

In fact, both are right, there they are in effect, same similar they would ultimately lead to the same differential equations. Now, notice one the conceptual advancement here we have got a specific system, system drawn in a specific way the mass, the spring, the friction element, the cam, but ultimately that is abstracted into a graphical structure, which can now be conveyed to the computer and we all need to do, we need to ensure that this graphical structure contains the complete information about the exchange of the flow and effort as it does not, so we need to do something more to it.

But nevertheless, the concept is that any system will be converted into a graph like this, bonds will tell how the effort and flow information's are shared between different elements. In fact, as you go along you will realize that the information has a specific direction of flowing through the system, from somewhere the effort information is generated and that flows through bonds to reach other elements, somewhere the flow information is generated and that flows through the bonds to the other elements and that is what decides the structure of the differential equations.

We will we will learn that, so let us go step by step in this conceptual advancement, now in the next stage, we need to do something, we need to say when we talk about a specific bond then we need to say, we want to talk about the power or the voltage or the current in this case the effort and the flow through this bond. We will not be able to say that unless we specify this bond and that is specification comes by numbering, so we simply put numbers

So, we number the bonds, the moment we numbered then we can say the effort in this bond will be E 3, flow through this bond will be F 4, so we can specify, so that was necessary in order to specify things, so we number the bonds we give specific numbers to the bonds. Then the next stage is when we were dealing with the graph theoretic approach of electrical circuits, you remember that when we do similar graphs; that means, the lines between the two nodes we denote it by arrows.

The arrows is represented by what, arrows represented not the actual direction of flow of the current, but it was representing our assumed direction of positivity, here also we will do the same thing. Here also for every bond we will need to give an arrow, meaning that that is the particular direction that we consider as positive, but in this case positive direction of power flow, not individual effort of the flow, positive direction of power flow that is why these are called the power directions and these are denoted by half arrow.

For example if there is a bond, then it will be like this the power direction is given like this why, why not a full arrow that is the full arrow is reserved for something else, we will come to that later it is half arrow. So, normally the SE element the sources will give out power, so the power direction will be like this, so the power direction here will be like that, here will be like this and here will be like that.

The resistive element will, of course absorbs power, so it will be like this, it will be like that, now the inductive and capacitive elements do not really absorb power or generate power, for some time they absorb power, for the other parts of the time they give out power. So, in that case we still have to assign one of the directions as positive and it is convention that we denote as positive, the power that is absorbed by that element, so our power direction will be like this.

Let us take the circuits for example here, how will you do this is the power direction, this will be the power direction this and that then you would realize that there are elements that are not directly there are bonds that are not directly connected to the element, they are they are between junctions. So, they are between junctions, what do the elements between the junctions mean what will be the power direction they could have any direction, so you assign say like this.

There is no harm, if you assign it opposite, there is no harm and all that will mean is I will show you what that that would mean, it will mean that what we call as positive or negative that will ultimately change, but that will have meaning when we have already obtained the differential equations, we have solved it and we have found that the say the capacitor voltage varies is like this.

And then when it is positive what is the meaning of that positive, what is the meaning of the positive, which plate of the capacitor is at higher potential, which plate are at the lower potential that when we want to interpret, that we will need to refer to this. But otherwise even the assignment of the power direction can be algorithmized a computer can do that, now once this is done; that means, suppose here we have a numbered 1 2 3 4 5.

Notice that in this junction, we have in this junction, we have the power balance, so e 1 f 1 going in minus e 2 f 2 going out minus e 3 f 4 going out should be equal to 0, now here this is a 1 junction and therefore, all the flows are equal, so f 1 is equal to f 2 is equal to f 3 this immediately implies e 1 minus e 2 minus e 3 is equal to 0. Now, notice the meaning of the power direction by the power direction these minuses and pluses were determined, these minuses and pluses were determined by the way you have given the power directions.

If you had given the power directions this way, then the e 3 will become of a different sign, but that as I told you is something that we need to worry about only at the time of interpreting the results. So, presently we can assign it in any direction and go ahead fine, in the next class we will start by understanding the meaning of this direction and that direction, how it will actually be have to be interpreted that is enough today.

Thank you.