Dynamics of Physical Systems
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Lecture - 11
The Graph Theory Approach for Electrical Circuits (Part-1)

Where in the last class we have seen that if you apply the Kirchhoff's laws in a raw away, which means simply application of the mesh current and the node voltage methods. Then things have to be attacked ad hoc, the other problem is that if there is some redundant capacitance or redundant inductance, redundant means where the state variables associated with those capacitance or inductances are not really free.

They are, because of this specific connection dependent on the voltage sources the other capacitances, so the voltages become related and so they are not free to choose any value. In that case they do not become state variables and in case of such circuits, where you have this kind of a typical problems, the application of that measure, become somewhat difficult. The third problem is that the mesh current method after all is applicable only to planar surface, if there is a circuit something like this, you have one.

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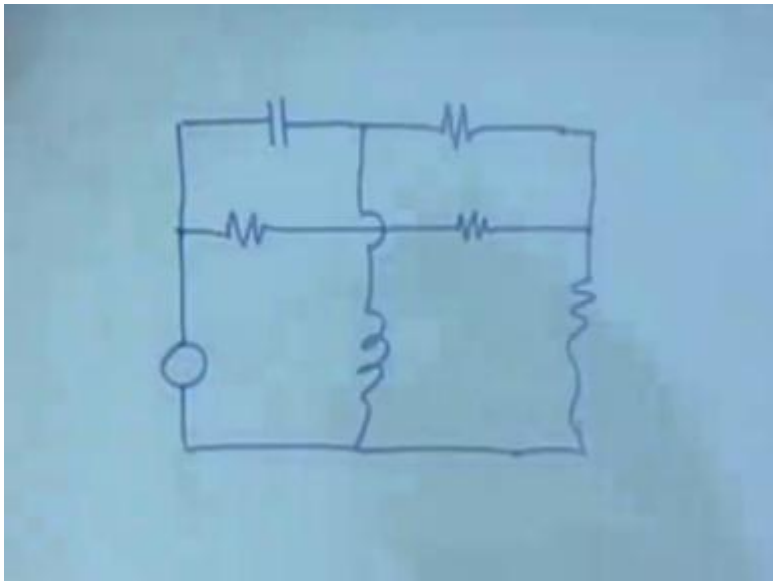

Then you have problem, it will not be easy for you to identify what the measures are, it is not a planar circuit. So, in those cases, we cannot apply the method and that is why, for electrical circuits it is necessary to have some kind of a general easy to use method, with

which we can derive the differential equations for even very complicated circuits and that is provided by the graph theory.

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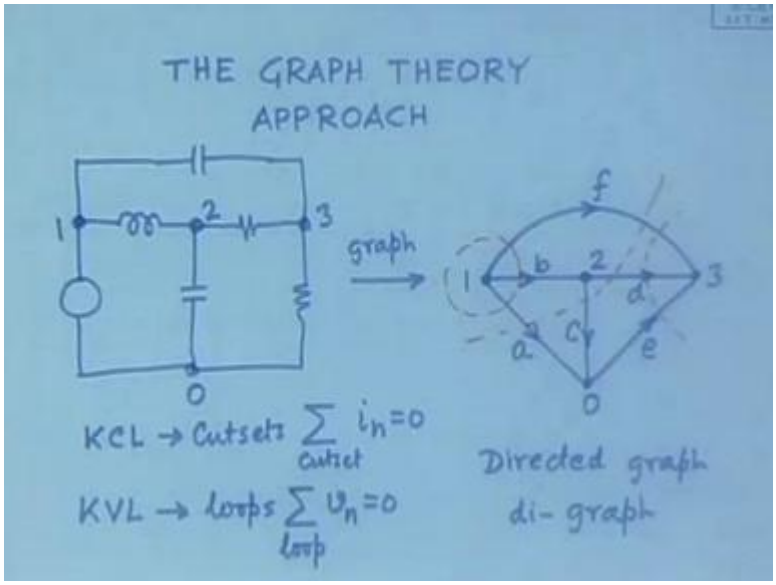

So, today will in the main deal with this graph theory approach of deriving the differential equations. Now, in this approach, whenever you have a circuit consisting of inductances, capacitances, resistances, all these are represented simply by branches, so this will represent a branch, this will represent a branch, this will represent a branch as on as so far. So, there will be, all these are represented branches and the branches are connected through nodes.

So, if you have, the circuit that we did yesterday, if you have a circuit like this, here it was a inductance, here it was a capacitance, a resistor here, a resistor here and a capacitor here. Then this circuit, will be represented by the graph, the graph will be, how many nodes are there, 1, 2, 3, 4, which we denoted as 0, 1, 2, 3. Here also, it will be 1, 2, 3, 0, but in between we will connect simply by lines and this is another branch this, that is it.

So, when we draw the graph, corresponding to a circuit, we initially forget about what are the contents of the it is branch, each branch is just a line, so that is the graph corresponding to this circuit, similarly you can easily imagine, that you can draw graphs for any given circuit. Now, in this circuit, in this graph, I can see there are 4 nodes and there are 1, 2, 3, 4, 5, 6 branches. In the next step, we need to identify them, so the nodes need to be identified.

And we will do similar way, by means of numbers 0, 1, 2, 3 and branches will have to be identified, we will do so by, means of letters say a, b, c, d, e and f. So, when I say, now I am talking about the branch c, you immediately know that this is the branch I am talking about, so it will be easy for us to work with these branches. Now, obviously the branches carry current and there is, there would be voltages between the nodes and this graph, as it does not tell us what are the directions of the currents or the positivity and negativity of the voltages.

So, we will need to assign some kind of a direction and that direction again, would mean that, say if I give the direction like this, what is it mean, it only means, that this is the direction of current which I am considering as positive. So, if the current actually flows in the opposite direction, no problem, it will only appear as a negative value, so my positive direction is like this, so this way we will derive all the graphs. So, suppose I direct it like this, like that, these are all arbitrary, I just gave.

So, whenever I have given the directions, it is called directed graph or in some books you will find also find di-graph. You might ask, I have given the direction of the current, what about the voltages, notice that the moment you give the direction of the currents, the positivity and the negativity of the voltages immediately become define. For example, in this branch, the current is given this direction as positive which would mean, which would imply, in this the node 1 is at higher potential than node 3.

So, the assignment of the direction in the branch, essentially immediately also assigns some direction of the voltages. So, the first step, is to define this kind of graph, the next step is, notice the concept, you can now cut this graph, suppose is you taken a pair of scissors and you simply cut, in how many possible ways can you cut it, in many ways. For example, you can cut it like this, you can cut it like this, you can cut it like this, cut it in such a way, that the graph becomes divided into two disjointed parts.

So, you cut it, so that the graph becomes divided into two disjointed parts, these are called cut sets. So, how many different ways can you think of, just take this graph and try to think, how many different ways you will find, you can cut it in very, very different ways, they are all possible cut sets, that is one concept, the cut set. The other concept is loop, which is the same as what you have already learnt, regarding the loops of this circuit, so you can easily identify there are loops like this.

Now, on the basis of this concept, we will redefine the Kirchhoff's laws, what does the Kirchhoff's current law say, current law earlier, when we wrote the Kirchhoff's current law, then we say that an each node, all the currents entering or exiting, sum to 0. Now, we will restate it by saying, that at any cut set, all the currents going from one side to the other, one sub graph to the other sub graph, sum to 0. So, it is a higher level of definition of the Kirchhoff's current law, is understood.

For example here, this particular thing, it is not really connecting related to any particular node this one, but yet we can say that the I f plus I d plus I c plus I a is equal to 0. So, but this is relative to one particular node, because it is isolating only one node, here again we can write in the old way I a plus I b plus I f equal to 0. So, this is something that we knew already, but this is something that we have learnt, now that which is also related to cut sets, not only nodes.

So, the Kirchhoff's current law, KCL and the KVL would state like this, the KCL is related to cut sets and KVL related to loops. So, in all the loops, the v a plus v c plus v b equal to 0, but notice here, while you do it on the loops, you will have to traverse either the anticlockwise direction or the clockwise direction. For example, if I want to write the KVL equation in 0, 1, 2 this loop, I start from here I v a minus v c minus v b equal to 0, if I say do it on this loop, is also loop v a minus v c plus v d minus v f equal to 0.

So, that is how we will do it on the loops and while we do on the cut sets, we will say, that for every cut sets, for example, for this one, it is suppose the currents entering, this cut set, this particular sub graph is positive. Then, I e plus I d plus I f equal to 0, so this says, that this sum over loop, the v n equal to 0 and this says, sum over cut set I n equal to 0, so where one step higher than where we were earlier, but now, we need to define something more.

So, there are you can easily see, that there are various ways of defining the cut sets and there are, a few a some number of loops. Now, we had started by saying, that the basic step in the obtaining of the differential equations relate to the Kirchhoff's laws and the Kirchhoff's laws are applicable to cut sets and loops and you can see, there are many, many loops here and also there are many, many cut sets. So, out of that, which one would be most convenient for us to use, that is the key issue. The in doing, so let us define one more concept that is called the tree.

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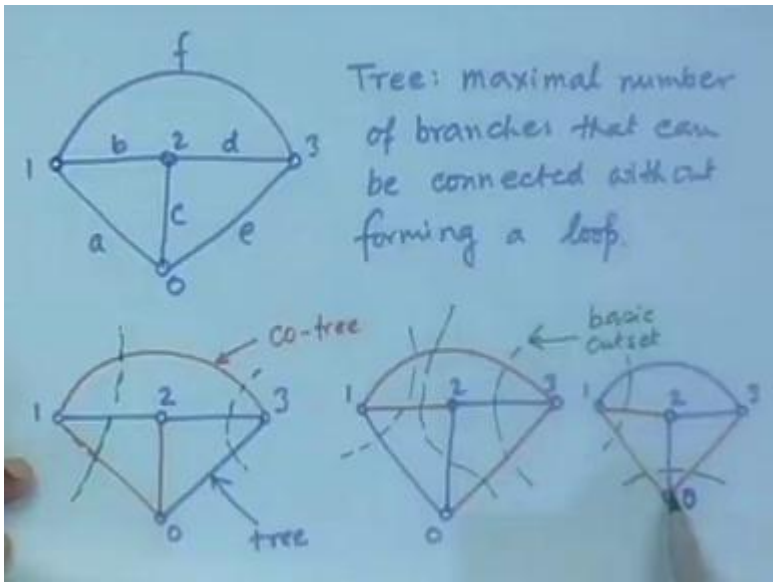

So, let us redraw this graph, it was one node here, one node here, one node here, connected to another node here and a third one. So, that is how, my shape was it was a, b, c, d, e, f, 1, 2, 3, 0, the tree is the maximal number of branches, that you can connect without forming a loop, the tree is the, so from this how many different, trees can you identify, let see. In order to do that, start from any branch any particular node and go along a branch.

So, we can start from here a 1, go along this, go along this, if you go along that, we complete a loop. We cannot do that, then we can come back along this, but if you do this it will be completed, if you do that it will be completed, so that completes a graph, so 1, 2, 3, 0, that is a one possible graph, one possible tree, yes. Now, let us start from another direction, let us start this way, so first, second, third, can you take any more, this does not allow, this does not allow, so 1, 2, 3, 0, another tree.

Let us start from that direction, then I can come this way or you can come this way, let us come this way, then I cannot go there, I have to come here, I cannot do anymore. So, these are all the possible trees, you can easily see, they can be more, what remains if you have connected a tree, the rest of the things remain. For example, if I drawing a different color, this is what was remaining, this is what was remaining and this is what was remaining, so the blue ones connect the tree, and the red ones are the co-tree.

So, the things that remain, after you have identified the tree, they are the branches of the co-tree, so these are the co-tree and these are a branches of the tree. So, here what would be co-tree, this, this and that, here this, this and that, so these are the tree branches and these are the co-tree branches, you might notice one interesting thing, even though we tried to connect the tree in various possible ways, always we ended up in three branches, as the number of branches in the tree, notice.

And you try to connect in any possible way, this particular thing it will always land up in 3, that is one interesting thing. For planar graphs, that always happens, it will always the three number of branches in a tree would be the same, yes. So, we have identified the tree and the co-tree and we have learn to identify the tree and the co-tree in any graph, but still we have the problem, that we can connect in very possible in many possible ways, many possible ways we can connect the tree.

And out of that, sum must be giving as the advantage, but let us see, now the question is, what should we choose as our state variables. We know that the voltage across the capacitor is a logical choice of a state variable, the current through an inductor is a logical choice for a state variable, but then we have also seen that not always, they can be volt some voltage across the capacitor that are not independent, some currents across the inductor that are not independent, so we need to choose properly.

Now, we have identified the trees and the co-trees, so let us go to the next step, how to choose the state variables. Now, you can now apply cut sets on this, let us do it in a different color, we can apply a cut set like this, we can apply a cut set like that and so on so forth. Now, there is a specific definition, if I can identify a cut set, that cuts only one tree branch, then it is a basic cut set, so here we have cut it, in such a way that it cuts only one tree branch, so it is a basic cut set while, is this not.

So, this is a basic cut set, in this case can now identify, this cutting will not be a basic cut set, this cutting will be, this cutting also will be. In this case can you identify, even this cut set will be, it is a basic cut set, so these are the basic cut set, similarly in this case, this will be a basic cut set, that will be a basic cut set, this not, this not, is there any other, no. So, these are the basic cut sets, so basic cut sets are one that contains only one tree branch.

Similarly, the basic loop is one; that contains only one co-tree branch, so let us see, let us try with this one.

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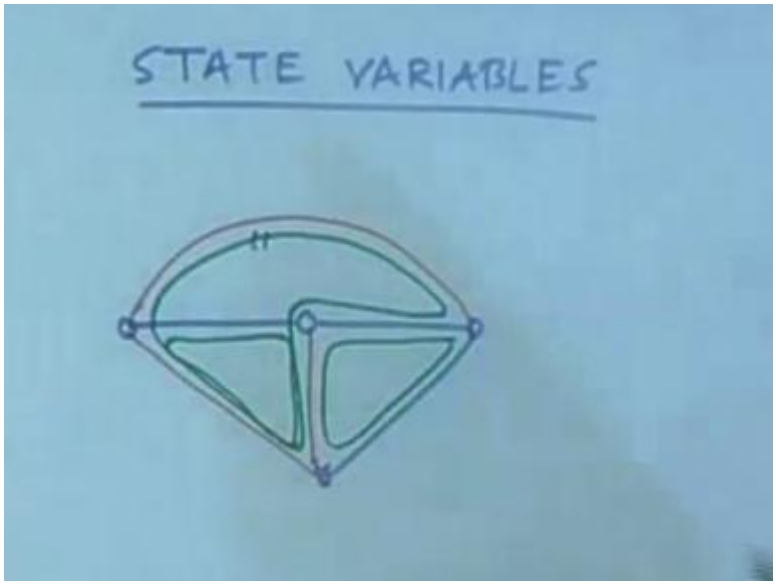

We have done it like this, can you identify the loops, the basic loop, the basic loop would be one which contains only one co-tree branch, all these are then basic loops. So, this also basic loop, start from here, it will no, no this no, this is…

Student: ((Refer Time: 23:47))

Sorry, I made a mistake.

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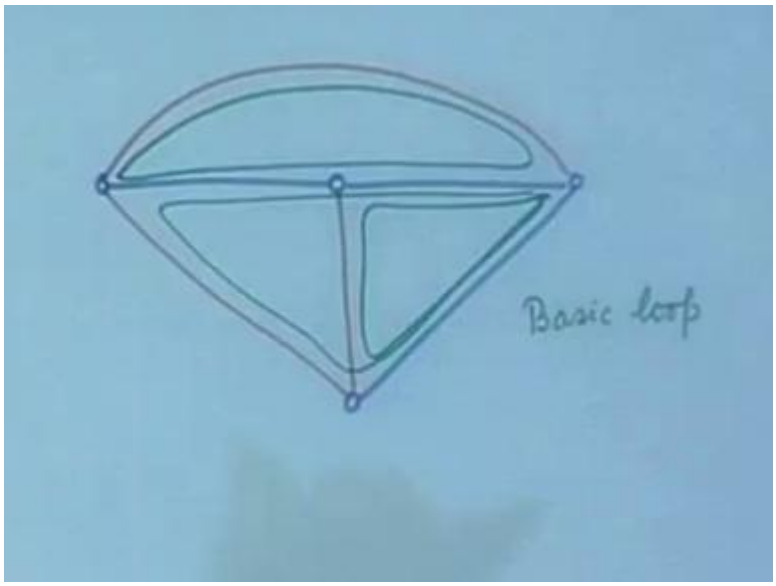

Let us do it correctly, this, this and this and this are the, so only one co-tree branch, so red one is a co-tree branch, so this is one, is there any other, not this?

Student: ((Refer Time: 24:17))

Yes, any other?

Student: ((Refer Time: 24:26))

This yes, so these are the basic loops, the loops that contain only one co-tree branch, so this is the basic loop. So, we have introduce the concept of the tree, the co-tree, the loop, the basic cut set and the basic loop, there can be very many cut sets, but out of that some will be the basic cut sets, there can be very many loops, out of that some will be the basic loops and we will concentrate only on the basic loops, there is a reason that for that. Now, the state variables, we will choose as I told you, the capacitor voltages are state variables.

We will choose only those capacitor voltages, as state variables, that form the branches of a basic cut set and in case of inductors, we will choose inductively we make the current will be a state variable, those inductor currents that form the part of the basic loop, we will let us see, how to proceed that ,with that. Now, let us proceed with this, now this way also we are not very happy, because what we said we will choose those branches, ((Refer Time: 26:20)) that are forming the basic cut sets.

But, you can see there are various ways of forming the basic cut set, out of that which one do you choose. Now, that is a specific choice to that, we will choose that particular thing in some order of priority, we will see out of the various possible trees, we will choose that particular tree, which has a following properties.

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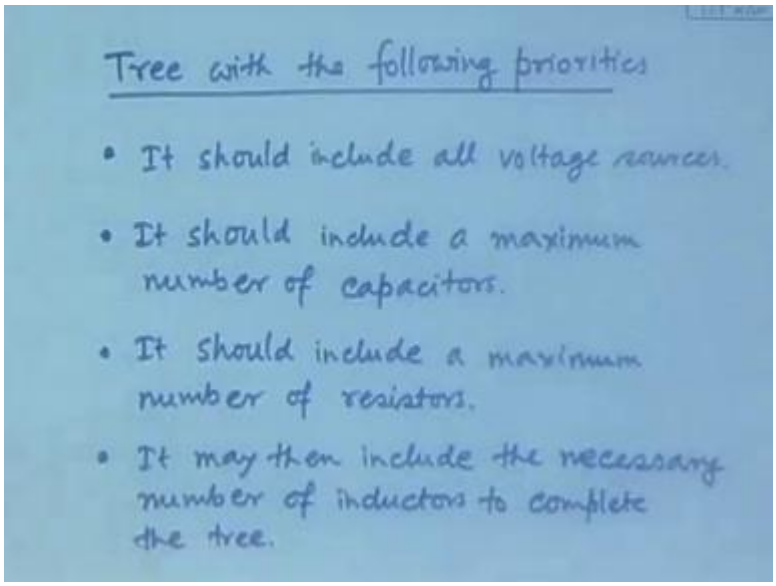

Tree with the following properties, following priorities, first priority is, it should include all the voltage sources, so out of various possible trees ((Refer Time: 27:24)) we will choose only that tree, that includes the voltage sources. Second, it should include a maximum of capacitors three, it should include a maximum number of resistors, finally when we still cannot compute a tree, then we will say that it may then include to complete the tree.

Let us see the point is that, in any given circuit I can draw a tree in various possible ways and for every tree, I can draw many ways to draw the cut sets, I can have many ways to draw the basic loops. So, this give these are giving as a too many options, we do not like that, so we need to restrict our options, so that we can have a very algorithmic procedure to write down the differential equations. So, out of all possible trees, we will choose the one that satisfy these requirements.

Let us see, what was our original circuit, it was like this, ((Refer Time: 29:47)) we said that there are various ways of drawing the trees, do one that we want should be first including all the sources, so we should one include this. So, out of the possible trees, you

can see only this one includes that, can now see, it is this branch you should includes, only this one includes them, let us see. The next point is, that it should include a maximum number of capacitors.

Let us see, there was a capacitor here and there was a capacitor there, so which one will, that do, this one does not do that, because this does not contain that capacitor, which one no, that.

Student: ((Refer Time: 30:41))

This does not include this capacitor, so we need to do it some other way this one does not because this, was the branch with the source it does not included. So, this is out, this is out, this also out, so these are not the right choices of the trees, so let us start doing it again from here, look at the circuit, I will draw the circuit and then start off.

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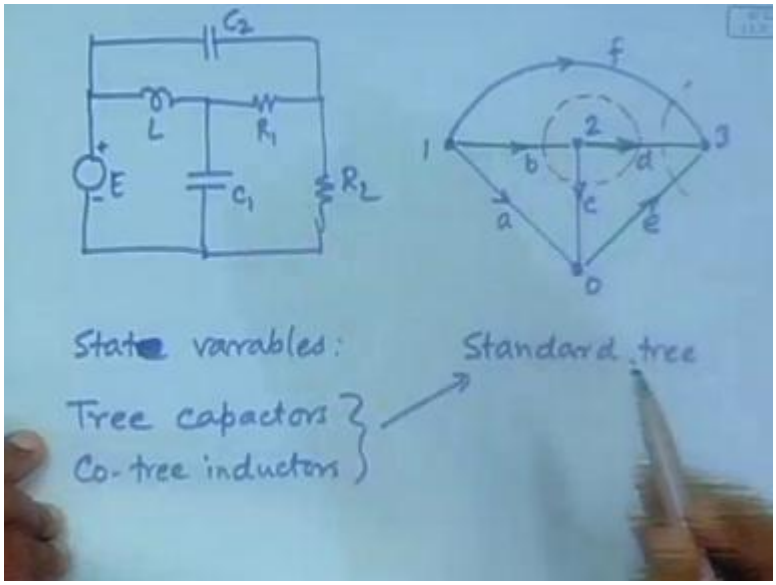

Here was I will draw a little fast, so that we can quickly do it, here there was a resistor, here there was a resistor and here there was a capacitor. Now, it is, we have to include this, so include this, we have to include the capacitor, we have to include this capacitor, that is it. The moment we try to include, the next point was, it should include the maximum number of resistors, the moment you try to do that the loop is closed and therefore, it does not remain a tree.

Here also the same problem and we do not need any inductor to complete the tree, so that, out of the all possible trees, is a very special tree, which we will use, it is called a standard tree. So, what are the co-tree branches, this is the co-tree branches, is it visible as two different colors, then the tree and the co-tree we have identified. Now, we will say, that we will take those, as our variables that are related to the tree and the co-tree, so we will write that.

If, since the capacitors are included in the tree, therefore the state variables will be, the capacitor voltages that are included in the tree. The way we have drawn it, the inductors will be in the co-tree, so the state variables will be those inductor currents, that are included in the co-tree. So, we have identified the state variables, what are they, state variables are tree capacitors and co-tree inductors, these are all related to the standard tree, not just in a tree.

So, we have made a choice of what we call our state variables, notice that in those cases, where there is some dependent capacitance or dependent inductance, immediately that will go out of the number of capacitance that we choose, because they will not form branches of the standard tree, we will see, we will do that by examples. Now, when we try to obtain this the equations, differential equations what will do, first we will say that, now I am trying to obtain the state variable equation for this particular capacitor branch. I am now trying to obtain for this particular inductor branch, good.

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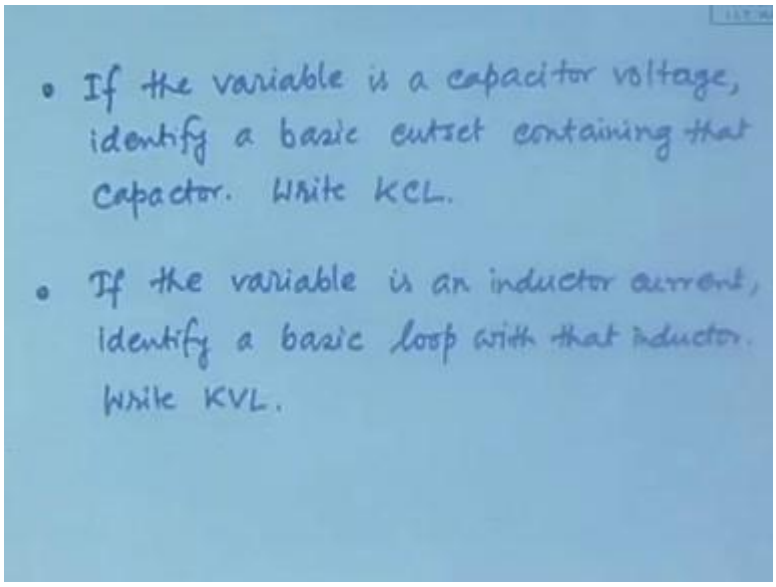

If the variable is a capacitor voltage, obviously that capacitor will be part of this standard tree, so identify a basic cut set containing that capacitor, then write KCL, that is it. Second point, if the variable is an inductor current, identify with that inductor, then write. So, we are not going on writing the KCL, KVL equations for all possible loops, trees, branches, no, we are just doing it for this specific cases, what is it, if you trying to write down the equation for the capacitor voltage, we will identify the basic cut set.

And then we will write the KCL for that cut set, that is it, so let us illustrate, we had this circuit ((Refer Time: 27:33)) and we had this as the standard tree. Let us give the names, 1, 2, 3, 0 and then we said a, b, c, d, e, f, we had also given directions like this. So, that is our starting point, we had identified this as the standard tree and the other one as a co-tree, so now we have to identify the state variables, what are the state variables?

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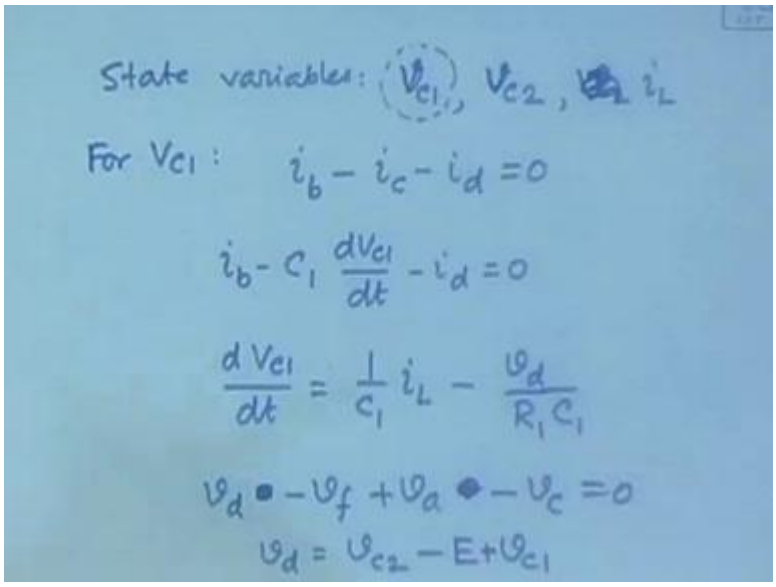

State variables would be, ((Refer Time: 38:33)) the capacitor voltages that are parts of this standard tree, which are they, this was our C 1, this was our C 2, L, R 1, R 2, E. Obviously, C 1 voltage V C 1 is one state variable, because it is a part of the standard tree, this is also a part of this standard tree, so V C 2 another state variable, V C 1 and V C 2 and the inductor current, that is part of the co-tree. So, inductor that is part of co-tree, it is I L.

So, now let me first your job is to identify, suppose you are obtaining the equation for V C 1 here, identify what, a cut set a basic cut set, that contains this branch, can you identify?

Student: ((Refer Time: 39:51))

Here, that is the basic cut set, immediately we can write down the KCL equation as, let me display it here and write it. It is notice the arrows V b or I b minus I c minus I d equal to 0, notice plus, minus, minus. So, for V C 1 we are writing it, i b minus i c minus i d is equal to 0, so that is the starting point, we can easily identify that i c is C d V d t, so immediately that gives. So, we will write i b minus C 1 d V C 1 d t minus i d equal to 0. What?

Student: ((Refer Time: 41:05))

Yes, i b is a state variable, ((Refer Time: 41:09)) here is i L, which is state variable, so we are happy, we will write it back write it down here, but first let us take this one to the left hand side and the rest in the right hand side, the way we write differential equations. So, d V C 1 d t is equal to, it is 1 by C 1, now I can write it as i L, these should all be small letters because these are variables, capital letters imply fixed values i L minus i d, what is i d, i d current to d, here is a resistance sitting.

So, it is a there is a resistance sitting here, so what will how will I write it?

Student: ((Refer Time: 42:11))

Yes, so we can write it as, or V 2 minus V 3 or voltage in d, V d, we can also write that, V d by R 1. Now, we do not yet know V d.

Student: ((Refer Time: 42:33))

We do not yet know V d, so V d is not a state variable, so we need to do something about it, V d where do we it getting from V d,V that should be obtained from a loop, cut sets yield the currents and then this will be obtained from the basic loop that contains this. So, what is that basic loop that contains this branch?

Student: ((Refer Time: 43:11))

2, 3, 1, 0 yes this, so write down the KVL equation in this loop, you get V d is equal to, this equal to minus V f. Now, I will not write equal to, then it will be problem, start from here minus V f plus V a minus V c, we are going around, start from here, V d minus V f plus V a minus V c equal to 0, that gives V d is equal to V f, yes that is V C 2, V a which is E, this come to this side and V C plus. So, you know all these, the right hand side is are the other state variables.

So, you simply substitute it here, you get the equation.

Student: ((Refer Time: 44:48))

Here is a question, V a is E or minus E good question, you have normally this as the polarity of this, which means, that this is at a higher polarity than this, which means this is, that means, in that case the current will flow through like this. So, accordingly you have taken, so substitute this we have got.

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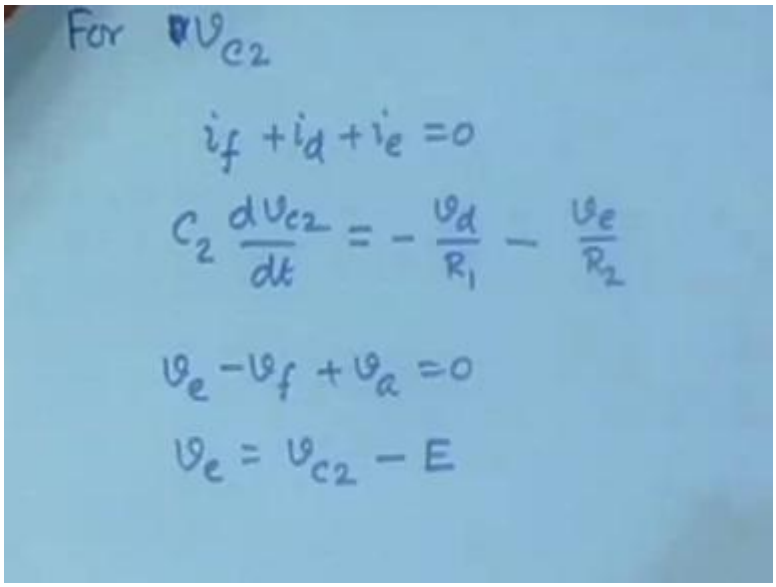

Now, for V C 2, which cut set should we take ((Refer Time: 45:46)) C 2 is here, we have to write that basic cut set that contains only this, what is that, this. Write down the equation, it is i f plus i d plus i e equal to 0, i f plus i d plus i e equal to 0, out of this which one do I need to know, this is C d V d t. So, C 2 d v C 2 d t is this is equal to minus i d, i d is same thing, we have already done it, so we can simply substitute, so i d

is V d by R 1 and V d we can substitute, we know what it is, this one i e, i e is this one, i e is V e by R 2.

We already know this, but we need to know this, for this again we need to know this voltage and for this again we need to identify this particular voltage, we need to know and this voltage will be obtainable from the basic loop that contains this branch, where is the basic loop, can you identify?

Student: ((Refer Time: 47:32))

No, 0, 3, 1, 0 yes here it is, so you write down the equation for that, start from here, V e minus V f plus V a. So, V e I want to find out, V e is equal to V f is V f is V C 2 plus, it will come to this side, minus V a is E, so you just substitute it here, you get it.

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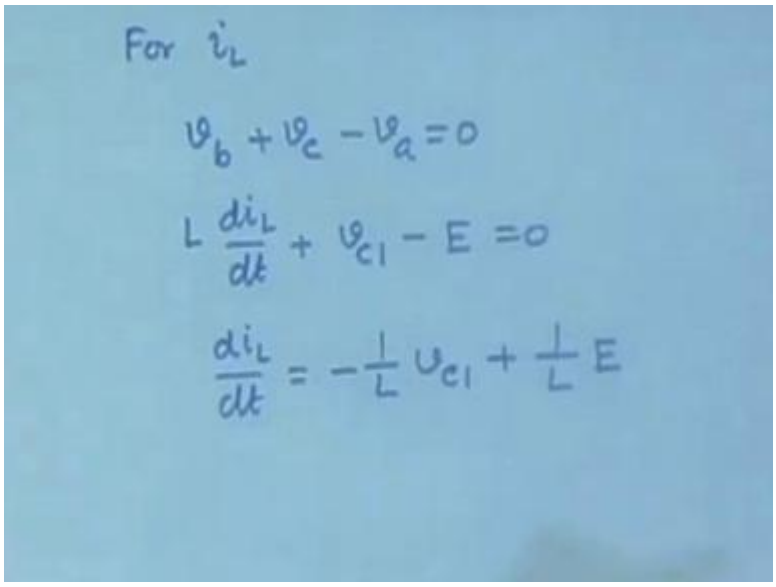

Finally the state variable is, for i L, this is obtainable from, what did I say ((Refer Time: 48:44)) the basic loop equation, that contains this branch, where is the basic loop that contains this i L.

Student: ((Refer Time: 48:50))

Yes this, so write down the KVL equation in this, so it will be, start from here V b plus V c minus V a is equal to 0. Notice V b plus V c minus V a, because the arrow is opposite equal to 0, now we have V b is L d i L d t, so we have got one thing, plus V c is already

known V C 1, V C is V C 1 minus V a is E equal to 0. So, d i L d t is equal to minus 1 by L, V C 1 plus 1 by L E, that is the third equation, so once you identify the standard tree the rest of the things become pretty simple.

Let us quickly do one problem.

Student: ((Refer Time: 50:24))

It will also be useful for circuits that that are not planar, but here we have used the circuit that is planar.

Student: ((Refer Time: 50:34))

Yes, it will be non planar, so what?

Student: ((Refer Time: 50:41))

No, here we are not identifying the measures, here we are identifying any closed loop, which even if it is non planar we can identify, here the idea is not to identify mesh, mesh is a planar, but the loops that satisfy certain property, because still identify that.

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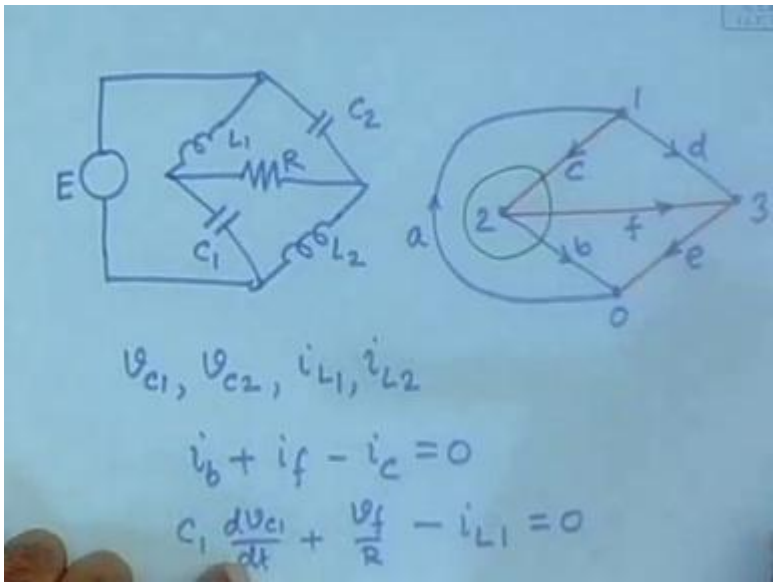

Let us do this work problem, we have here you start doing, because in this class we do not have much time left. Can you do it, by other methods it will be an extremely difficult, by any other method, by this method it will be rather simple, so it is L 1, L 2, C

1, C 2, R do it. First, quickly draw the graph, the graph will be, so the graph will contain these as the branches, now let us try to directly identify the basic tree, the standard tree, standard tree should contain the this thing.

So, it will contain this branch, standard tree should contain this, yes, now you notice that if you close any of the other things it will complete the loop and therefore, this is the standard tree. Once you have identified quickly, give the numbers 0, 1, 2, 3, so the co-tree will be, good what are the state variables, the capacitor voltages associated with tree capacitors. So, this is a tree capacitor, this is a tree capacitor, the state variable will be V C 1, V C 2 and the co-tree inductors, so i L 1 and i L 2 all of them.

No problem, let me just illustrate one equation, just one, suppose we want to find out the this capacitor voltage for V C 1, we have to identify the cut set that contains the C 1 branch, it is simple. So, immediately you write down the equation, we have to give the a, b, c, d yes, so let us say a, b, c, d, e and f, so now we can directly write, directions yes, give directions. Obviously, they are arbitrary, so let me give directions and this, so i b plus i f minus i c is equal to 0, first equation. Out of that, i b is C 1 d V C 1 d t plus i f is what, i f is V f by R that is enough and I C is also state variable minus i L 1, good.

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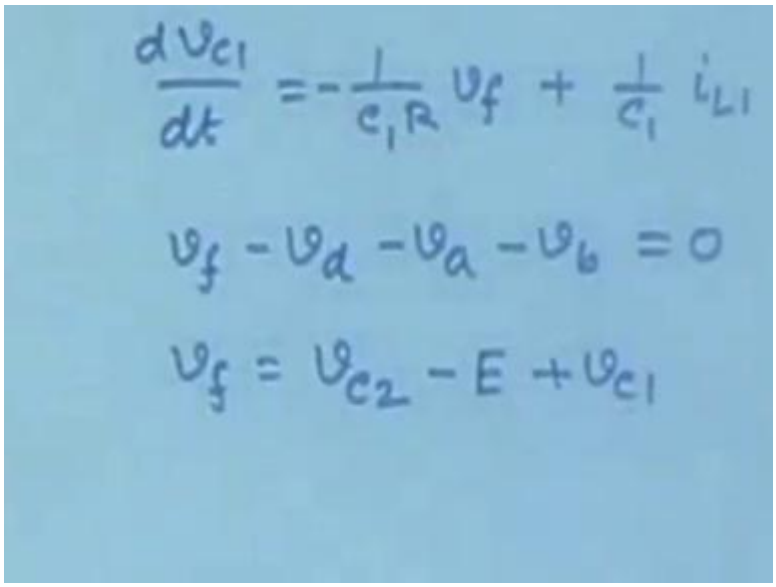

So, we can take this one to the left hand side and write d V C 1 d t is equal to 1 by C 1 minus R, now we have to write V f, we will find what V f is. Firstly, you write it now,

plus 1 by C 1 i L 1, V f is needed, ((Refer Time: 56:12)) V f is a voltage which will have to be obtained from a basic loop, so what is the basic loop containing V f?

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0, 3.

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0, 2, 3, 1, 0 yes, so this is the loop that contains V f, so write down the equation for that, let us start from here, so it is V f minus V d minus V a minus V b equal to 0. So, V f is equal to V b yes V C 2, V C 2 plus V a is yes opposite, so minus E and V b is, substitute you have got it, so this is how the equations are to a.