Dynamics of Physical System
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Lecture - 1
Introduction to System Elements

One of the important elements of the engineering practice is to be able to predict the behavior of the system that we design. For example, if we design a circuit, we should be able to predict, how it will perform, how it will behave under certain circumstances. If we are designing a mechanical system for example, then also we should be able to do that. And that is generally done by forming a mathematical model of the system, applying different times of inputs and seeing it is performance.

So, one elementary thing that most engineers should know is to be able to form mathematical models of systems. And these systems are normally very complicated in nature, the actual physical system practical; systems that you will have to deal with in your engineering life are normally more complicated in nature. But, also the advantage is that, in general that composed of simpler elements.

So, these simpler elements interconnected in specific ways, ultimately form the whole system. And that, is what we are trying to model and what is the purpose of the modeling, we are the purpose of modeling is to predict the dynamical behavior, under certain circumstances. What do you mean by the dynamical behavior, if I give a certain input, what will be the output, like how it will say, if it is a mechanical system, how it will oscillate, how it will different parts will move, these are the things that you are trying to predict.

Now, these as you know are done with the help of differential equations that you have probably learnt in your first year mathematics that all systems are changing in nature, moving in nature, dynamical in nature. So, whenever we are trying to find out how things change that is represented by differential equations. Why because in a differential equation in the left hand side, you have got d x d t type term, where x is some variable, so what does d x d t say, it says, how x varies with time.

And in the right hand side in the differential equation, you will find certain function of the variable. So, if the variable has a particular value, if you say 1.2, you can find out what d x d t is; that means, you immediately know how it is changing at that instant of time and then this way the whole thing will ((Refer Time: 03:49)). It is means that for a more complicated system, we will have to write down differential equations.

The one major element of this course of dynamics of physical system would be to learn how to write down the set of differential equations for any given system. Mechanical, electrical, electromechanical, so however a complicated it may be you should acquire the ability of writing down the differential equations of the system. In the second part of the course, that means mainly after ((Refer Time: 04:35)) examination.

We will learn how to understand the differential equations, how to handle them, how to solve them, but at the same time, here the treatment will be slightly different from the standard differential equation treatment in the mathematics classes. In the sense, that our aptitude would be mostly geometrical, we try to understand, we try to our focus will not be just to be able to algebraically solve the differential equation.

Our focus will be to understand the dynamics of various same types of system, so with that objective we will learn how to solve differential equations and how to understand the dynamics, so that is what the whole plane of the course is. Now, as I told you a physical system that we often have to model, these are in most cases very complicated. But, that complication is on the surface; in reality they are not very complicated, why because they are composed of elements.

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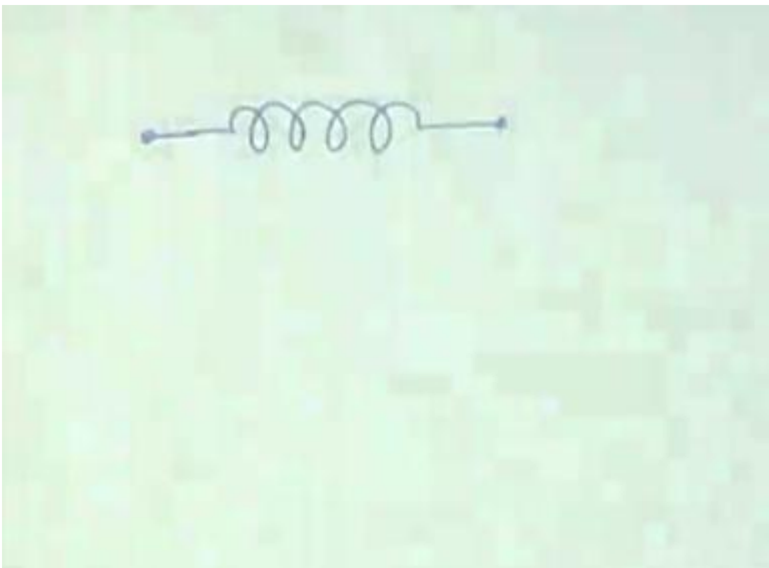

Now, here we will have to make one simplification for example, suppose you have a spring something like this. Then, if we want to understand the character of this spring, then as you compress it or elongate it, various parts will develop stresses and how this stresses developed in individual points and how they move. If you try to consider that that will become very complicated, it is spring itself become very complicated.

But, then notice that ultimately whatever the spring is, whatever happens inside it, ultimately it interacts with the external volt, through the two extremities. So, other things are connected through the two extremities and whatever happens inside. We will not concern ourselves with that integrity of whatever happens inside, we will talk in terms of whatever is seen at the two ends.

This is called lumped modeling; that means, this whole thing is treated as a lump, as one single entity, which in a particular way interacts with the rest of the volt, so that will be one of the simplifications that we will add up. And secondly, even though many of these individual elements like this spring, can be very complicated even the terminal characteristics can be very complicated. We will approximate by the simple characteristic, which you will for information is not very perform the actual reality.

So how, first let us come to the elements one by one, even though you may understand the character of these elements, it will be illustrative to just pin point, what we are interested in this course and then go ahead. The first element that everybody knows is the mass, mass is a technical term, but it is general term is the inertial element, something that has inertia.

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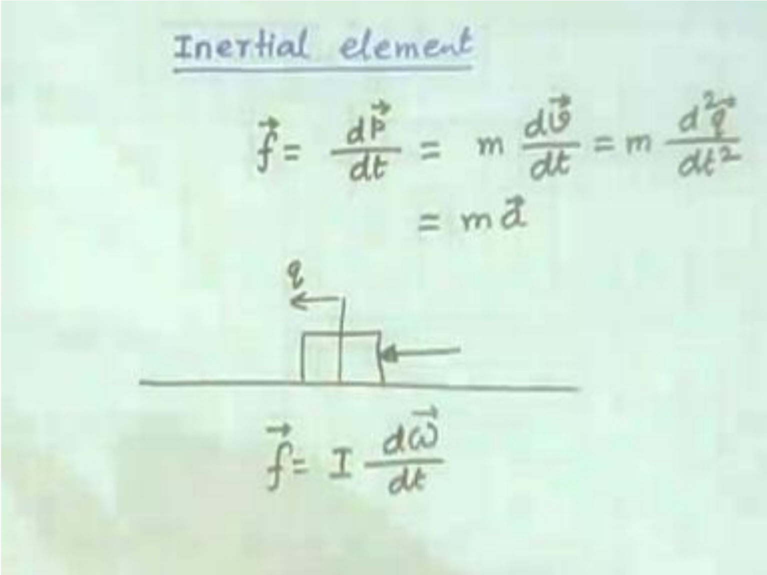

So, inertial element is anything, now we are generalizing it, you know that mass is an inertial element, but anything could be an inertial element. If the governing equation of that particular element is f is equal to d p d t, where p is the moment, for the mass you know, but you will see that later we will generalize this concept. So, this is the essential idea that you apply a force on the mass, there is a mass we apply a force as a result of that the moment changes and the rate of change in moment are exactly equal to it is force.

Now, the moment let us now consider the mass itself, the moment is the mass times the velocity. So, it becomes m d v d t or if the position of the mass is a measure for a particular point position is q. So, if you write this, it is mass into acceleration this part, that you would notice, I have written very simply, but the force is a vector. Moment is the vector, v is a vector, q is a vector and acceleration is vector.

So, in general these are all vectors and the moment you write it this way, the definition of mass, do you know what the definition of mass is; how do you define mass, I mean for you it becomes a individual concept, but there has to be a definition of mass. The definition of mass is proportionality constant between these two quantities. So, the moment you say between these two quantities, you will see there are complications, because this is a vector this is a vector and you can have a proportionality constant between two scalars.

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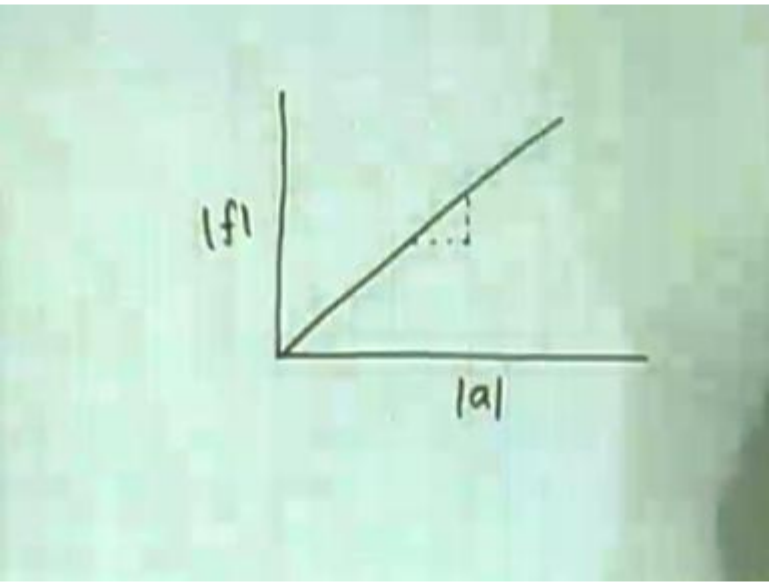

So, what we actually do is to plot the magnitude of a f, then you have a slope and that slope is a mass clear, then you cannot define mass and for you information there is nothing at this, if something addressed, you can define mass. Mass is defined by the property, that it actively tries to resist any motion, any change in motion, that is always defined. So, only when there is change in motion, acceleration, change in velocity, you can defined mass, so that is the definition of mass.

Mass could also be change changing in the equation that happens only in the context of the general relativity. Else, we do not consider mass as changing, unless you are putting something with that mass that is the different issue all together. In case of a rocket, his question is that if the mass is changing in case of a rocket, then these slopes will be changing all the time that is it nothing more. This will not be a straight line, it is a straight line, so along as the mass is constant.

In case of rotational motion, it is the same thing, in that case force is torque applied, in that case the position is the angular position, in that case the velocity is angular velocity, angular acceleration and exactly the same equation holds. Only in that case, we often write it as f is equal to I d omega d t, omega is the angular velocity and I is the moment of inertia by the way these were vectors.

So, in this case is that a vector, if I say it is a vector then what is the direction, you are rotating it after all. In that case, it is still a vector and the vector direction is by the corkscrew rule, so that is the direction of the vector. So, f is a vector, omega is a vector, all these are vector by the corkscrew rule, so that is the definition of the mass or the moment of inertia.

Now, let us come to the second type of this was remember, this is in general called the inertial element. We will not use the mass element; it is an inertial element, an element that has the property of inertia. Again, the property of inertia is that it tries to maintain it is velocity status, it does not want it to change and if it you are applying a force to make it change, then it start of resists, this is the property of inertia.

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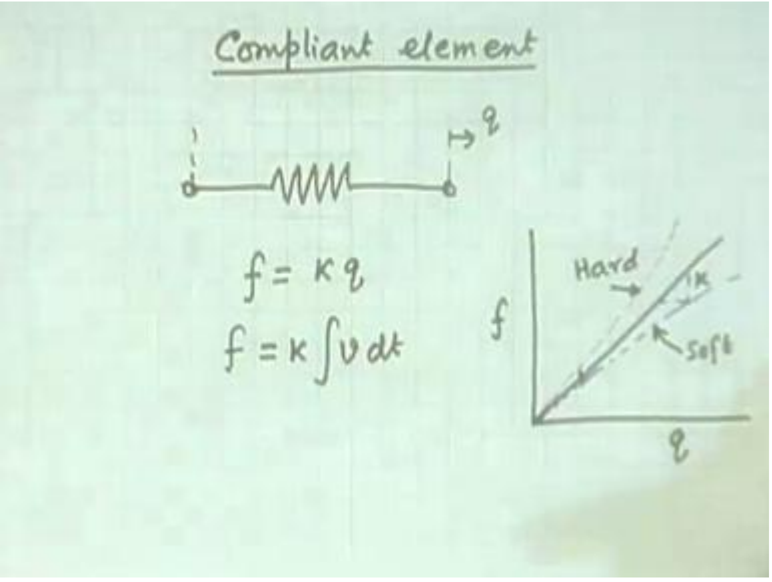

The second element that we will be considering is the compliant element; the compliant element is essentially the spring. So, these two extremities have certain positions in the unstaged condition and if you pull it, thereby change the position. Suppose, these two position, suppose this is fixed and this is defined by the position variable q and you are pulling it, what happens, what is the terminal characteristic, I am not concerned about, what happens inside the spring.

What is the terminal characteristic, what is seen by the terminals, a force is developed that is equal to k times q, q is the position, so this q has zero value. When, it is unplaced, we have assumed, we have defined the coordinate axis, that way and as the q changes, the force is developed which is proportional to the q and proportionality constant is k that is spring constant.

Earlier, mass for the proportionality constant, here the spring constant is the proportionality constant. So, you can say that, there will be a similar graph drawn, where q is here and f is here and you would normally get a straight line. If you do get, then only this equation is valid else not. But, in actuality what happens is that, you either have characteristics something like this, slightly off or you have characteristics something like that slightly off and is visible.

Now, if the characteristics are the black 1, then it is a linear spring, what is the characteristics of the blue 1 that as you elongate it, the force is somehow going down, so it is softening, as you are elongating it further, so that is the soft spring. On the other hand, this is a hard spring, even though these possibilities are there and you have to be alive to the idea that the actual spring in it is characteristics, may not be exactly linear.

But, for our purpose we will approximate it by the linear 1, which will again be within a certain amount of elongation not very far from the reality. If you stretch it too far, then only all these softness and hardness appears, else you see the difference between these three lines up to this point is very small. So, we will go by the linear approximation up to a certain point in this course.

And then we will release it, but up to a certain point, we left to go with that linear approximation, because we want to make a life simply at least to start with it. At least, when you are appearing in the examinations, you want to do things with simpler equations. So, this is the equation of the and you can write this also as k and q is the change in the position, if you want to write it in terms of the velocity, then how do you write it, integral of velocity d t.

So, that is the character of the, so velocity and the force are related by this integral relation v d t. This is a relative symbol, this will be you later see, this will be for more meaningful in our ((Refer Time: 19:35)), so f is equal to k integral v d t. So, this slope is in this case k, the third element that, we have to deal with is it a frictional element or a resistive element or dissipative element, whatever you call it frictional element.

What is the character, in general you may have ((Refer Time: 20:25)), that there are two types of frictions, either it is a viscous friction or a static friction. Viscous friction happens, when two bodies are moving against each other, but there is a film of liquid in between them. It is something with viscosity in between them, that is a viscous friction and if to dry bodies are sliding against each other, then it is a dry friction or static friction, the characteristics are entirely different.

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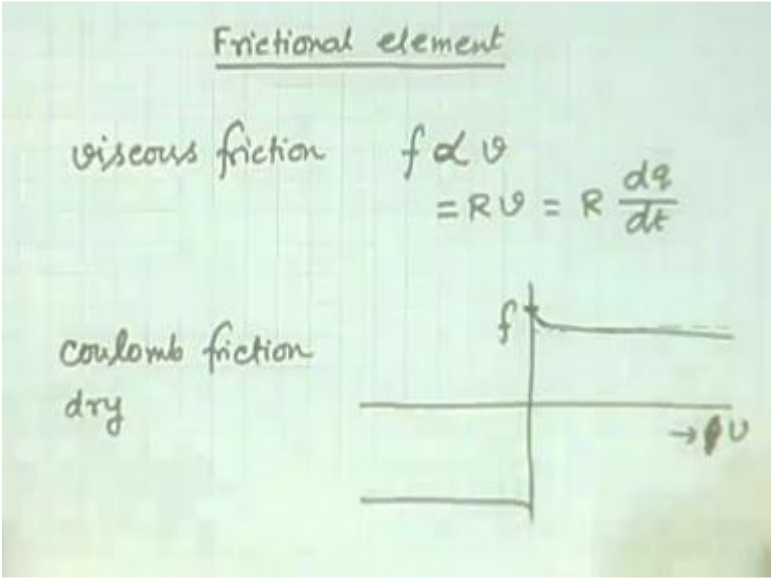

Let us first deal with the viscous friction, for viscous friction the force is equal to or force is proportional, if you can write in this way, force is proportional to the relative velocity between the two bodies. And we will in general, represent that proportionality constant as R, because it is a resistive kind of element, we will in general denote it as R. So, this equal to R, so the relative change in the position in terms of that, if you write it is d q d t.

Now, in case of the coulomb frictions or dry friction the character is that, if you have such kind of a body and if you are applying the force, for sometime it does not move, it does not move the force builds up and still it does not move. Till a certain point, when it starts to move, so there is a critical value of the force below, which it does not moves, above which it moves.

So, if you plot the characteristic, then it will be said here is the force, no let me here force and v here, for zero velocity the force builds up and till a certain value, still it does not move and then it starts to move. When, it starts to move immediately the forces falls and it drops like this but not much, force is still there and thereafter as the velocity is increased, it short of limit is more or less constant. But, actually it slightly drops not much, if you draw the constant value something like this, but it slightly drops.

It can be approximated by a constant value, that is why in most modulates; you will find it is approximately a constant value. If it is moving in the opposite direction, it is like this, so it is a very clearly non-linear behavior, a coulomb friction a very clear non-linear behavior, where as the velocity increases, it is here is more or less straight line, here is more or less straight line. But, in between there is jam, so that is the coulomb friction or dry friction.

Now, that is the characteristic, see there is stick slip kind of motion, so elongated as it sticks a certain force builds up and then slips, when it slips, then that much of force is not available between the two bodies and so it just stops. Now, out of these 2, again for our purpose, till a certain point in this course, we will mainly deal with this viscous friction. Why, because that is the most prevalent type of thing that we encounter in industry.

Because, all the motors, all the elements that you design, you always put some oil lubricant, why because you do not want this. This results in sound, chatter, when somebody enters there was a cracking sound in the door that is because of the static friction and if you put some oil in this the sound goes that it becomes a viscous friction.

So, we always want to use viscous friction and since most engineering systems are designed to use viscous friction for all practical purposes, we will use this equation. But later, when we want to model more accurately, we will also include, what can be done with it coulomb or the dry friction.

Now, at this stage let us see the other end of the story, the electrical systems, in the electrical systems, our approach would be, why you are discussing the electrical system. Because, when we try to model actual system, they could be electrical, they could be mechanical; they could be electromechanical, mixture of the both. So, we will also need to understand electrical systems, but the main approach would be, in what way they are similar, you notice that they are in many way similar, that is why you may have heard the term electrical equivalent most mechanical systems can be made into electrical equivalent, why.

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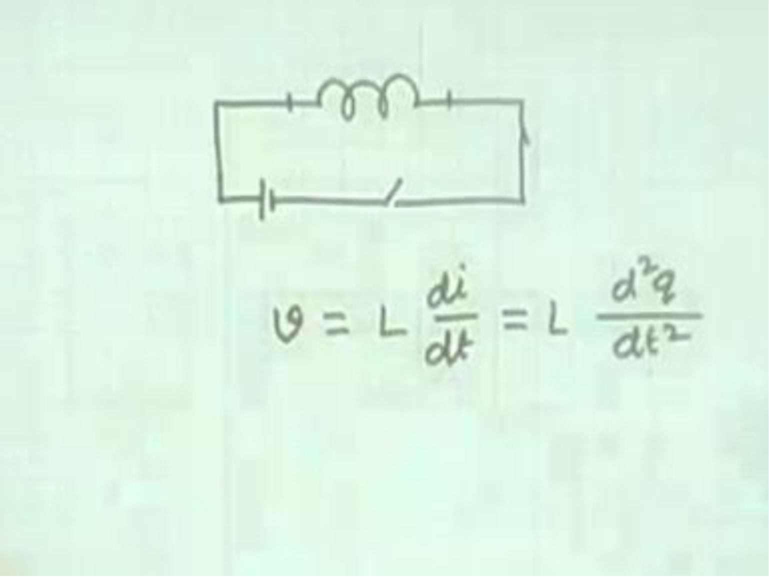

Notice, that if you consider, suppose we consider a inductor and you have got some kind of voltage source and switch what is the character of this inductor as seen between the terminals. It is some kind of a relationship between the voltage and the current and what is the current, what is the relation, simply stop. Now, the current is the rate of change of rate of flow of charge.

So, you can write and the voltage is enough electromotive force that is why you can easily introduce that this voltage is similar to the force in the mechanical system. And if we look at the inertial elements slide, you would notice that the force is equal to mass into acceleration, mass into d 2 q d t 2. So, immediately the L appears to be equivalent to the mass, inductance appears to be equivalent to the mass, but that equivalence will be valid, if the charge become equivalent to displacement positions.

So, the equivalence will be the charge is equivalent to the position, current is equivalent to the velocity, rate of change of current is equivalent to the acceleration, inductance is equivalent to the mass. And the voltage is equivalent to the force, electromotive force is equivalent to the voltage, then the whole thing is exactly the same.

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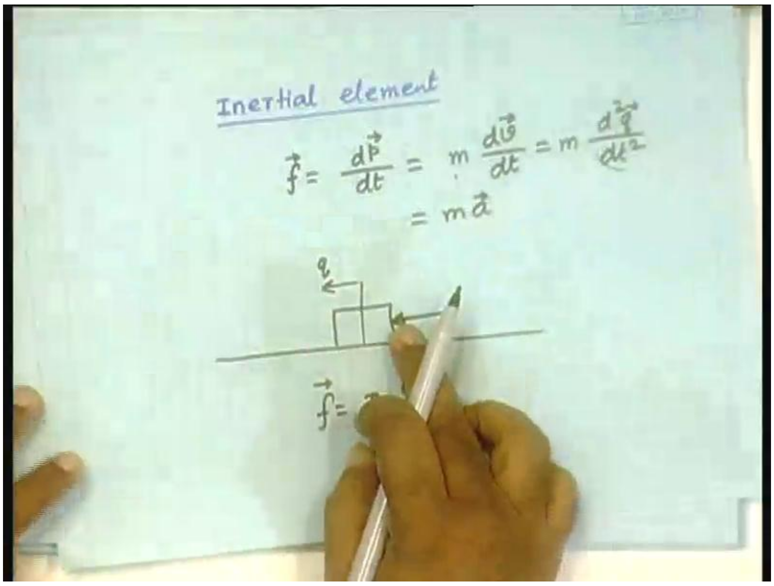

To the extent, that if you now close the switch and along the current to pass through the inductor and if you see a certain way of rise, if you do the same thing here is very well we will change exactly the same fraction. There is no fundamental difference between the electrical system and the mechanical system; they can be exactly equivalent clear. So, that is the character of the and also there is another initiative similarity that you have consider.

What is the character of the inertial element; that if you push it, if you apply a force, it is start of resists and it does not want it is inertial status should be changed. It was already moving with a velocity v, if we apply a force, you are changing the velocity v and that is what it resists. Inductor, it resist the change in current, it resist the change in that there is one of the thing that you learn in first year electrical engineering, that inductor does not allow it is current to be change instantaneously, same thing with the mass.

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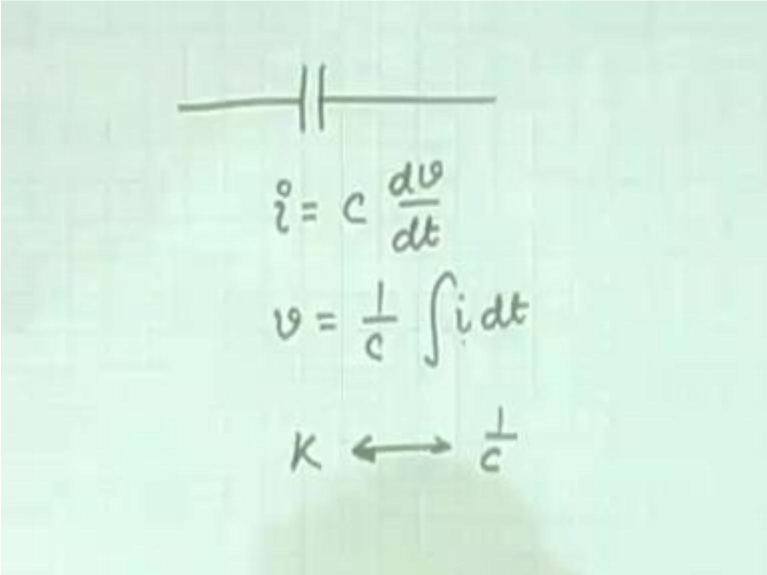

Now, let us talk about the next element, that is the compliant element, compliant element in mechanical was the spring, compliant element in the electrical domain is the capacitance. Here the equation is i is equal to c, that is the equation that we have learnt and where is the slide for the compliant element, spring slide here. Force is k times v d t, here it is the velocity, here the force that is the problem.

Since, the velocity is represented as v in mechanical domain, while the voltage is represented as v in the electrical domain do not make mix them up, it is the equivalent of the force. Force is one by c times velocity d t, either velocity, so you noticed that still the same equivalent force, equation is exactly the same. Only, the k of the spring constant is equivalent to 1 by c not the c, many students make this mistake equivalence is to 1 by c.

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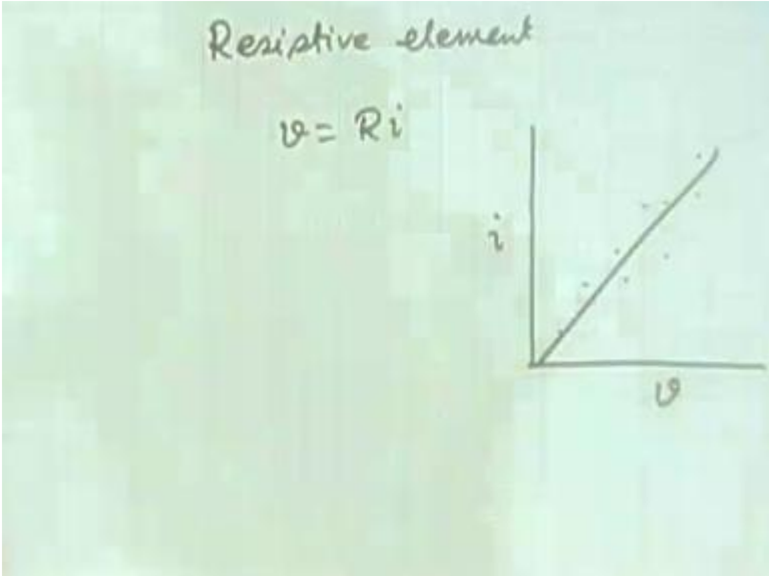

Now, let us come to the resistive element, where that is the electrical resistance property Ohm’s Law, here remember the Ohm’s Law actually defines the term resistance, because that is the proportionality constant between. So, if you are asked to measure the resistance, how do actually do, most of the students if I ask this question, how do measure the resistance, he or she will say that I have the resistance, I apply a voltage battery and then I measure the current that is it, no that is not it, that is not it. You have to obtain this slope of a curve of this line.

So, you have to apply a very well voltage, for each voltage you have to measure the current and thereby you have to draw the line, that will be give the R. Note just one value, because there are always measurement errors, if one value comes here, you will get a completely wrong value there. But, you have many ones then you will get a more or less this elements.

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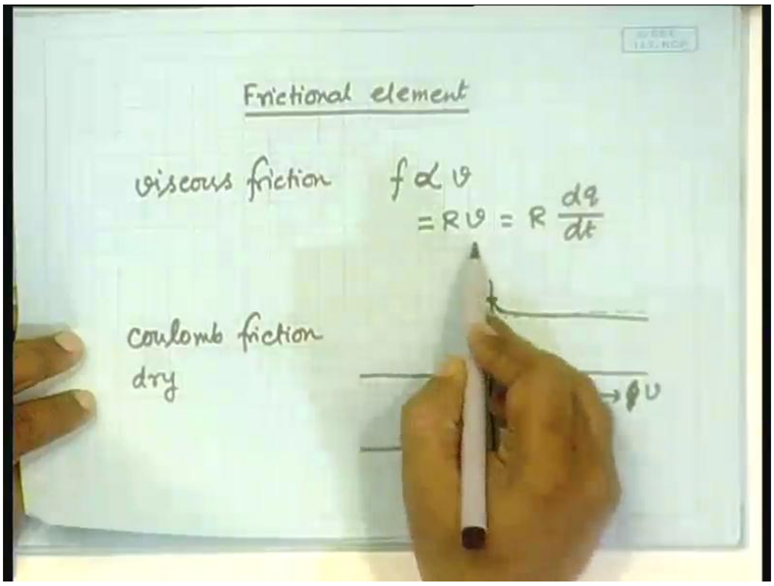

So, that is the property of the resistive element, but you notice that the resistive element in general is exactly the same as the viscous friction element in the mechanical domain. Because, here the force is R times the velocity, here the force is R times capacity clear, so we have established the equivalence of the electrical and mechanical system.

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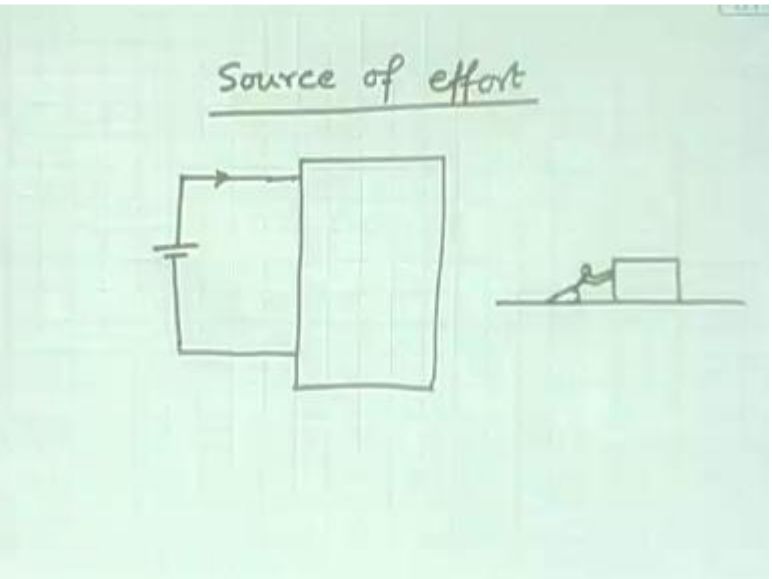

There are two other elements that would require a various tension, one is the voltage source and we will in general name it as source of effort. In electrical domain, that is the voltage source, in a mechanical domain has the external applied force is something is applied on a body, that is the source of effort. Now, what is the character of the source of effort, let us try to understand this, because most people I do not know why, they do not understand this elementary point.

Suppose, you have got a voltage source say a battery and you have got something here, some kind of a little circuit here. What is the character of the voltage source, it gives a definite voltage across this point, but the current that will flow is not detected by this fellow. Current, that will flow is not detected by this fellow, it is detected by the rest of the circuit.

So, it is a voltage source in the sense that the voltage is the independent variable, but things behavior is composed of two things, both the voltage and the current and the current is not independent variable, it is dependent on the rest of the system. So, here it is the source of effort in the sense that it is effort is an independent variable. Notice the mechanical domain, again something that very few will seem to understand, suppose there is a mass and there is a fellow, who is pushing it like this.

He is applying a force, but notices the flight of this poor fellow, if this must moves, he has to move with it and how it will move that cannot be detected by this fellow, that will be detected by the that system. So, the velocity or in electrical domain the current is not dictated by this fellow, but the force will be dictated by this fellow, that is the source of effort.

Yes, it is a strong man every day going to a gym; obviously will apply a more force that is okay, but he is dictating how much the force is, he is dictating how much the force is. But, the rest of the system is dictating, how fast he has to move in order to apply the force, in order to keep applying the force. Like here, the rest of the system, whatever is the circuit; that decides what will be the current flowing for that system exactly to the system.

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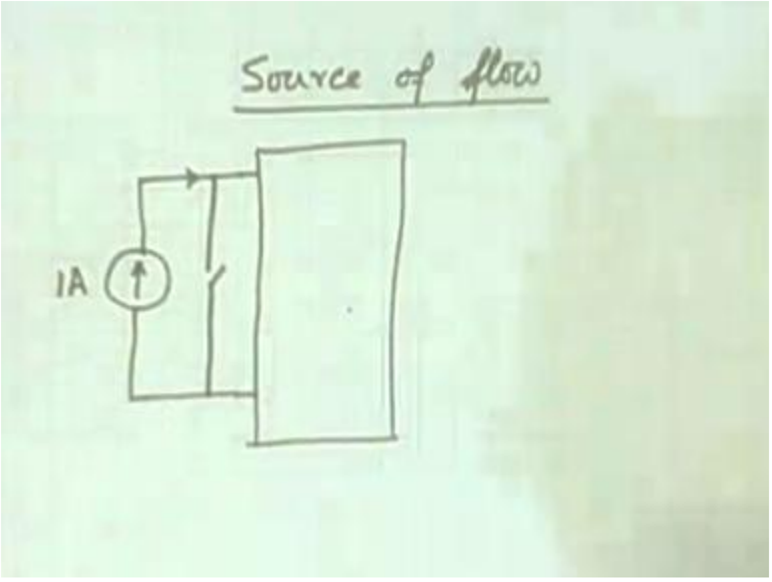

Then, we have to talk about the source of source of flow, in electrical domain; it is the current source, so it is drawn like this, with a certain amount of current, say 1 ampere flowing. There is some kind of system here connected, that immediately says that the current here is dictated by this fellow, a voltage is not. A voltage here, whatever is at work, whatever appears occurs is not dictated by this source elements.

It is dictated by rest of the system and both these variables the voltage and current can be dictated by no element. Nobody can dictate both the things and that is what the elementary thing about the interaction between the elements ease, I will come to that in details later. But, try to understand that here the current is the independent variable, the current is dictated the moment, do you understand that this source of flow cannot be open circuited.

Because, if you open circuit, the current goes to zero, then it ((Refer Time: 38:18)) condition, it cannot be open circuited, so it can only be short circuited, if we open it goes to the rest of the thing and ((Refer Time: 38:29)) it will be very interesting to know, what is the mechanical equivalent of the current source is, do you know mechanical equivalent of a current source. In every machine, it is used actually, every say car motor cycles scooter everywhere is used.

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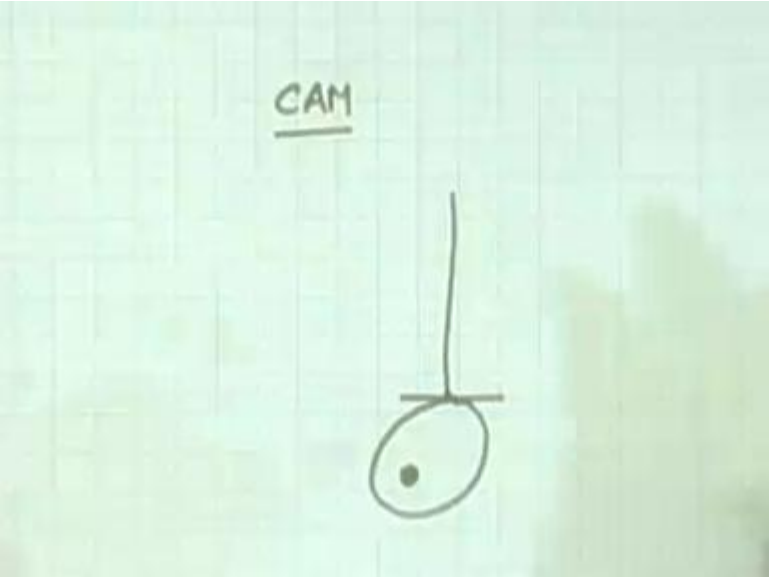

It is something like this, it is called a CAM, where an oval shaped body and there is a plate this is the saft and this is the axis on which it rotates. So, depending on the shape of this, this saft will go up and down, so on this one it is actually forcing a certain flow, you see, it is not giving the force. It is forcing a certain flow, it is a source of flow, it is equivalent to the current source and can you see that, so it is equivalent to current source, it is called a CAM.

So, depending on the kind of movement that you want here, you give a specific equation to this body, it is a closed loop, you give a specific equation, accordingly as it rotates it goes up and down in that specific manner. So, at that point, you have applied a velocity, so it is equal to the current source, so that more or less, completes the things that we have to consider initially in order to build up very, very complicated things. All the things that you see engineering things that you see around you are essentially composed of these things.

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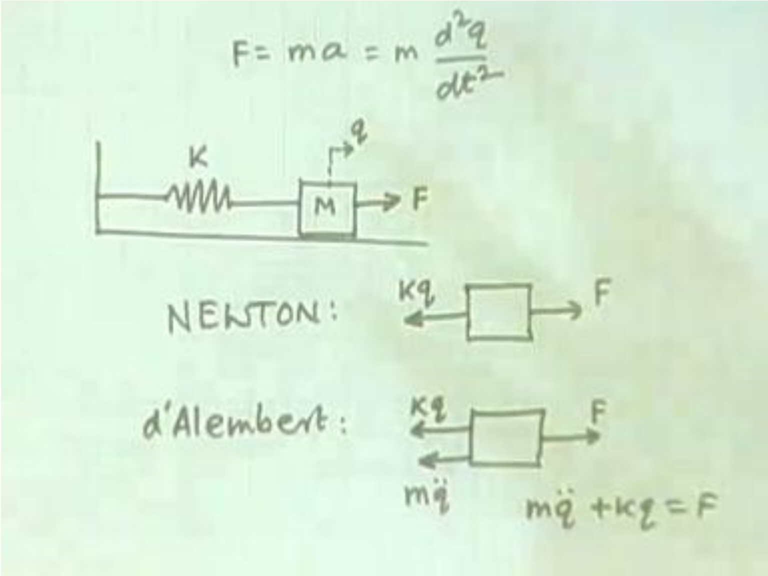

So, let us start what we will do is, first let us go by the mechanical way; that means, let see the mechanical systems and in the mechanical systems, you know the elementary equation is given by Newton. That is force is equal to mass into acceleration or this being a double derivative of the time, immediately you can seen that this is the differential equation, so simple. For anything, if you can identify the force, you can identify the mass; then you can write down the equation.

Saying is the easy, but doing is not, we will see slowly that in case of more complicated system, this becomes somewhat difficult actually to do. But, let us start with a simple system, suppose you have a system like this, a mass M, a spring K and a q variable, the position variable is measured from the point, where the point k is unstated and suppose you pull it with the force F, then how do you look at it.

Newton says that what will happen is that because of this application of the force, the moment will change, so rate of change of moment is what is writing here, so we will write the equation like this. Very close to the time of Newton, D'Alembert a probably you have heard his name, D'Alembert gave a very similar, but slightly different way of looking at it. He says that if there is a application of force that tries to unsettle a mass from it is own inertia status.

Inertia status means, either a state of rest with respect to something or state of uniform erecting in a motion. If you try to disturb it from that, then it short of resistive and how does it resist by applying an equivalent opposite force and the equivalent opposite force has the magnitude of this. So, according to Newton, if you draw the original free body diagram, it is like this F and here it is. If it is moved by an amount q, then the force that the spring applies is k q, so these two are.

In D'Alembert, he says that the free body diagram would be F k q, but at the same time because of this difference the position of status will change and here an opposite force will be applied. So, what we normally do is, if it is a complicated structure that for each mass point, we draw such free body diagrams and simply equate the elements appearing here, simple. So, in this case you can write down the equation as m q double dot plus k q is equal to F that is it, simple stuffing, so that is the equation.

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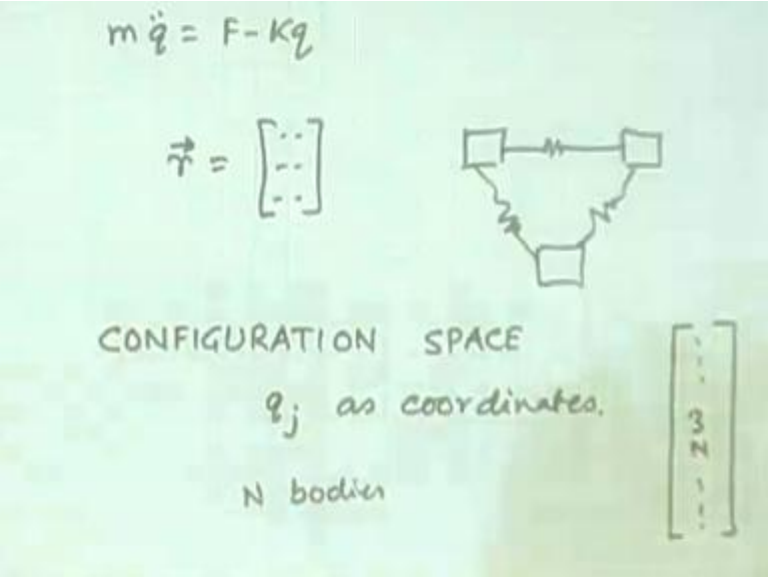

So, you would say that normally the dotted variables are in to be left and the undotted variables to the right, so you will say m q double dot is equal to F minus that is it, this case it was simple. Why was it simple, because this fellow mass has only one degree of freedom, it can only move this way, it cannot move this way, it cannot move this way, that is why, it was a very simple equation.

But, in general it is not difficult to see that the position of the mass is a three dimensional quantity, there will be three variables x y z, so in general the position will have to be written as a vector r, with three numbers in it. For one mass, suppose there is a mass connecting to something with another mass, then for both you will have to write such things. If there is three masses, if you want increasing, just imagine the solar system, they are all interacting with each other, then how many.

Then depending on the number of mass points, we will lump, again these sun into 1 mass, the earth into 1 mass point and then we will say for each for each mass point, I will need three coordinates to define it. So, the position of status of each mass point is defined by 3 numbers, 3 real numbers. How many real numbers do you need, in order to specify the position of status of such a system 9.

Now, in mechanics, we have a nice way of looking at things, if there are 9 numbers required specifying this, there is no fundamental distinction between the x coordinate of this body and the y coordinates of that body, there on the same pedestrian, same kind of variable. So, there are 9 variables equal importance, what we do is, we will say that is okay.

Now, let us imagine a space in which that these coordinates are all this 9, you will not be able to draw in 9 dimensional spaces, but do not draw just imagine. That there is a space with each of these variables as a coordinate, x coordinate of this one, y coordinate this one, z coordinate of this one, xyz of this, so there are nine coordinates.

This space, the advantage of this formulation is that in that case the position of status of this whole system is just one point in that space and if the position changes the point move that is it clear. So, that formulation that is space is called the configuration space, with q j as coordinates, the q j's is the coordinates. Notice the advantage, this concept came very soon after Newton.

That we can represent the whole positional status of a system, by means of an imaginary space in which the position of each body, there is represented by 3 numbers and all these bodies taken together their position coordinates taken together give me a space. It will be a vector of how many dimension, yes in this case 9 or in general, if there are N bodies, there will be thrice N of this numbers, that we would write in that vector.

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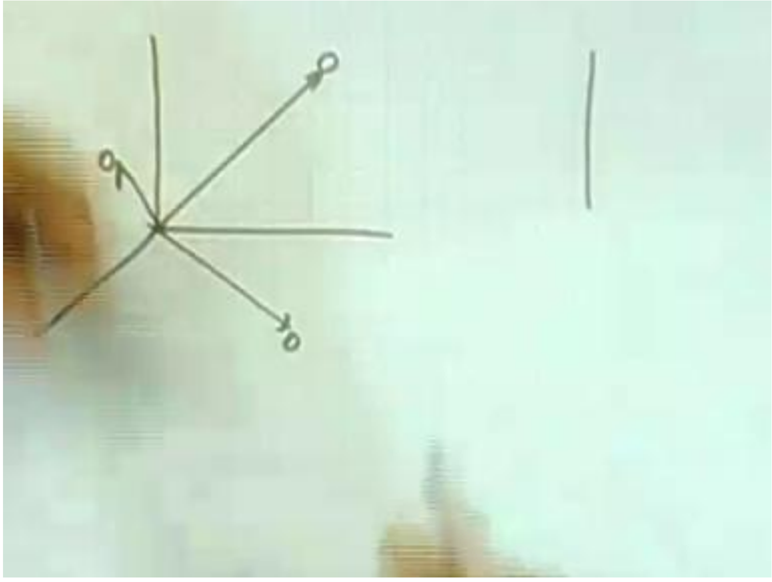

It is just one vector, the point is that so for a rather simplistic way would be that if there is a, if this is the actual space, three dimensional space, there is a body here, there is a body here, there is a body here. Then, you would say here is a vector, here is a vector and here is a vector and so on. But, now in our in this representation that whole thing becomes as one vector.

This whole thing becomes as one vector in just one space, where it now it will not be a three dimensional space, it would be hike in dimensional space. Nevertheless, that many vectors in a single point in the space representing the positional status of that system clear, so that is the concept of configuration space.

Obviously, the configuration space is many dimensional, there is a conceptual advantage in writing it this way, but then it is many dimensional. But, in actuality all that many dimensions are not necessary, why because most of the motions are constraint; most of the motions are actually constant. You cannot have an every freedom in motion, for example, if this fellow is here, I am trying to motion; obviously, I am trying to move it; obviously, it is not free to move it in this direction, it is constraint that is a constraint.

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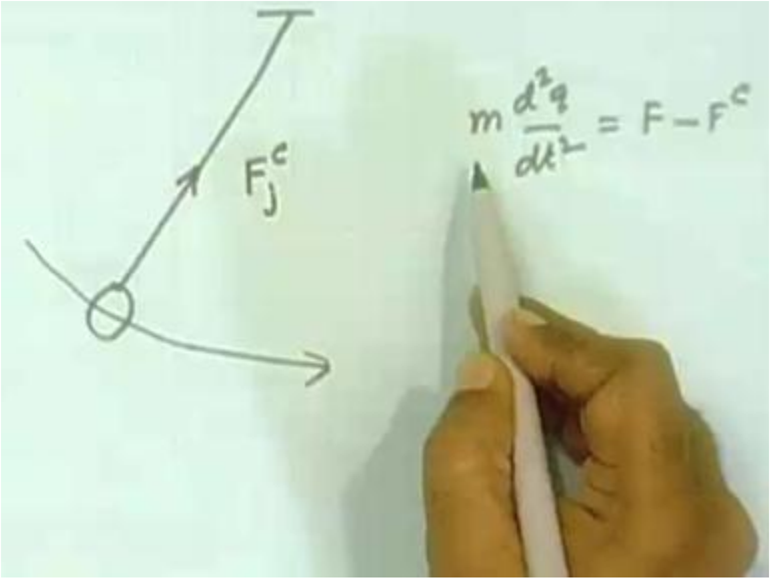

If you have a pendulum, now what is the constraint, now in that case pendulum moves like this it is constraint to move on the surface of a sphere, it is constant to move on the surface of a sphere, this fellow is constant to move on a plane. If you have a planar pendulum; it is constraint to move on a circle. So, these are constraints and have you ever ridden on a roller coaster in the phase, you are made to move in a specific strategies stomach turning thing.

But, nevertheless I was felt that that is also a constraint motion, you are you are giving a specific equations of motion forced on it. So, whenever actually most systems are these kind of constraints. And, I will show that is actually advantage, why because, if the motion of this ball is constraint to move on the surface of sphere, it reduces the dimensional reality of the system. Originally, it was three dimensional, but we do not need 3, you need only 2 in order to position on the surface of the sphere.

If it is a planar pendulum, you need only one, just the angle in order to specify it, so if you have this one on the sheet of paper on the table, then you will need only 2, you do not need 3. So, even though in theory, you need thrice number of coordinates of the configuration space, you do not feel need them that is the advantage. We can simply find the formulation a lot by using that advantage.

But, notice that each in order to force it to remind on that constraint, whatever is constraining it has to apply a force on it. For example, right now this paper weight is being applied on the force by the same force as it is gravitational force by the table, there is a force acting upwards that is what keeping it here.

In case of this pendulum and it is motion, what is the force, now the constraint force is this, that is what is keeping it constant, if you do not have this force, it will move away. And in case of the roller coaster, you can feel that constraint force in a stomach, you can feel that constraint force. The point that I am making is that the bodies motions are constrain, because the whatever is applying or forcing it to that constant motion is applying a force on it, that is called a constraint force.

So, we will call it f c and if there is j number of bodies with j number of constraint forces f j c clear. So, again to repeat the motion of most bodies, not all bodies, for example the motion of art in the solar system is not constraint. It can move any which way, it moves in a particular way, because of the equations of motion. But, the motion of most bodies that we have to deal with is constant and that gives as a major advantage and that advantage we will try to deal.

But, then the moment of consider the constraint force, what becomes the equation of motion Newton in equation of motion for each body and then it becomes m for each body, m times acceleration or equal to the applied force minus the constraint force. You can have these bodies, you are forcing by something, that is the applied force and this is the constraint force.

If this is attached to other bodies by means of springs or whatever, then those things are also the applied forces, applied on this body by some other body. So, there will be categorized under the category of F, in general quality given forces and these are the constraint forces. Everybody, will have this kind of motion, for N number of bodies then what we will you do, you will write this equation, you will immediately see that you are not getting the advantage out of this simplicity.

Because, for each direction you will have to write this, each coordinate you have to write this, if there are N body, you there will still be thrice in equations, so that is one major disadvantage of the Newton equations. Newton gives us a way of writing the equations, but soon after, within a span of some 100 years, some additional thing had to come in order to enable people to write the equations in a more elegant manner. And that is, what we will studying in the next class.

Thank you.