Chaos Fractals and Dynamical System
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Lecture No. # 07
The Chua's Circuit

So on the last class there is one question coming from him. The question is that in order to calculate the Lyapunov exponent I said that you have to start from an initial condition and you have to evolve the trajectory and you take a part of position somewhere here say and let it evolve but you cannot allow it to evolve for a long time because in a chaotic system you expect the distances between two nearby points would increase but that cannot go on increasing indefinitely.

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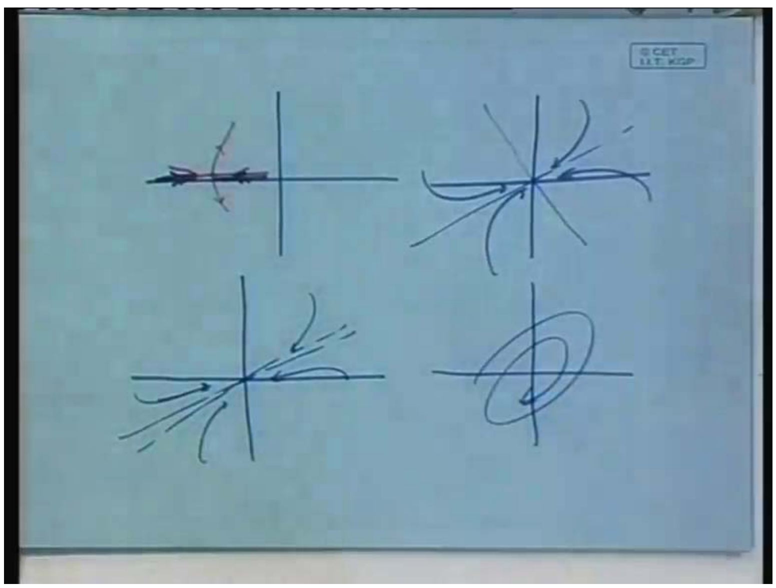

So in order to keep the whole thing bounded, after sometimes this will fold. Folding means that if you are taking the distance between these two points, after sometime it does increase say this two points but after sometime it will fold like this. If it folds like this the distance between the two points will be seen to be decreasing which is an erroneous result because actually it has increased by this moment. So in order to avoid that I said that you have to again renormalize it that means stop the process somewhere here and then come back in the direction of the deviation from here to here but to the same extent of distance and then from here again you start it.

Again at this point you stop it and you again come back to a distance. His question is after how long do we do this? Obvious question yes. In studying any system you really do not know after how long it starts to fold and this is normally done somewhat blindly that means you write a code and you allow it to work on this system equations. So how do you decide?

Essentially what we do is that we allow it to evolve for some time depending on the characteristic time of the system. Any system has some kind of a characteristic time frame. Some oscillate over a period of say milliseconds, some oscillate over period of seconds, some oscillate over a period of; even though the oscillation is a periodic you can more or less identify the characteristic time and this time should be somewhat of that order. So what we normally do is we set around that time and then we run it again by having this time. So you make it half and then run it again. If you get the same result more or less they know that your estimate of the time is more or less okay, if not then you have to take the shorter time and do this procedure all over again that means you have to again make the times smaller and try to estimate the Lyapunov exponent. If you find that it is now more or less close to the last value, you know that you have arrived at the right one. That way it is done by somewhat like a trial and error method because for the full system you really do not know how much should be the characteristic time.

In fact a great deal has been written on this particular problem. there have been papers on this specific problem how long should this be and in the readymade program that are available they home on to some kind of an algorithm to do that. For example in a Runge-Kutta type solution, if you want to vary this stippling how should you vary that has been a matter of great discussion for a long time and now in Matlab you will find there is a code like od 2, 3 od 4, 5. What is it? It is first calculated by od 4 that means Runge-Kutta fourth order then it calculates in the Runge-Kutta fifth order. If he finds that there is a difference between the two results then it knows that the stippling has to be shortened.

If they are more or less the same then it allows the stippling to be longer because if there is a sharp turn, it knows that a higher order Runge-Kutta method will give a more or less accurate result. So if the lower order and a higher order heals a difference it means that it is taking a sharp turn, so in that case it has to take a sharp. So therefore different methods accounts for this kinds of problems where you have to choose some kind of a step length. In general it is done somewhat in case of the Lyapunov exponent, done by trial and error. It slowly reduces this time and find at what level it more or less converges to a number. Now you had a question. His question was in the Lorenz system we find that at a particular parameter value probably it was two. Is it? It was different system.

In general his question is that for some parameter value the two eigenvalues may become equal. So if they become equal then what? Now imagine it from a linear systems point of view meaning that you have suppose a two dimensional linear system so there I can draw. A two dimensional linear system where you are changing a parameter and there are two real and distinct eigenvalues and eigenvectors. As you change the parameter they are coming to be the same and at some point of time they become equal and then after that what happens they become complex conjugate. Something like if you plot the eigenvalues then it would traverse a path something like this. This is another coming.

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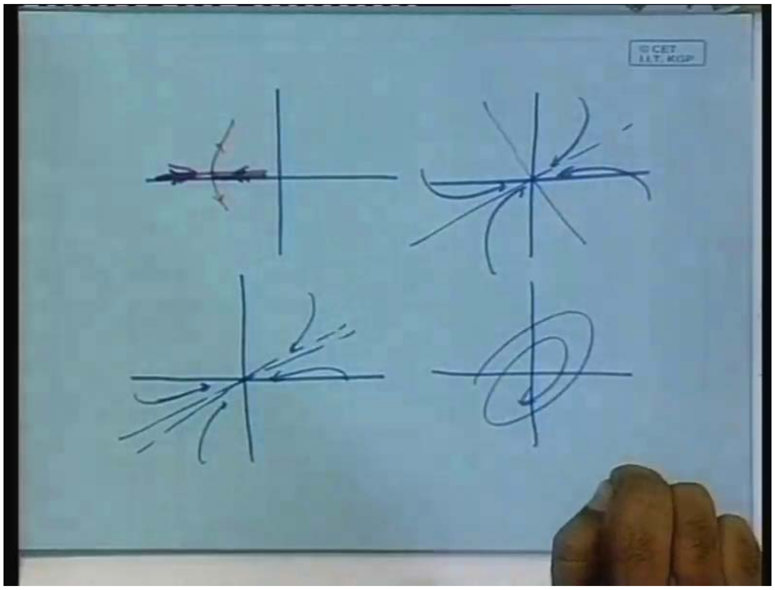

So as you change the parameter there is one coming this way, another coming this way and at some point they collide and then they go out like this. That's what normally happens. So when they are distinct and on the negative side real and distinct then you know what the behavior is like. What is it like? There would be say two eigenvectors and you would find a behavior something like this, if they are real and distinct. Now when they come closer and closer what will happen is that these two eigendirections will come closer and closer and at some point they will merge and after it goes to this side, it develop a spiraling behavior. When it becomes complex conjugate it will develop a spiraling behavior.

So just imagine the situation, just before that you will have one eigenvector like this. another eigenvector very close to it, they will come close to each other after some time they will merge and the behavior will be more or less will be like this so on and so forth but soon after it will turn into a rotating behavior. So there has to be some way to visualize how this changes to this by very small change of the parameter. a very small change of the parameter from here to here or from here to here, a very small change of the parameter should achieve this and as you know if you change the parameter by small amount, the result in changing the vector field will also change by a very small amount. So nothing drastic is happening really. So what will be the behavior like?

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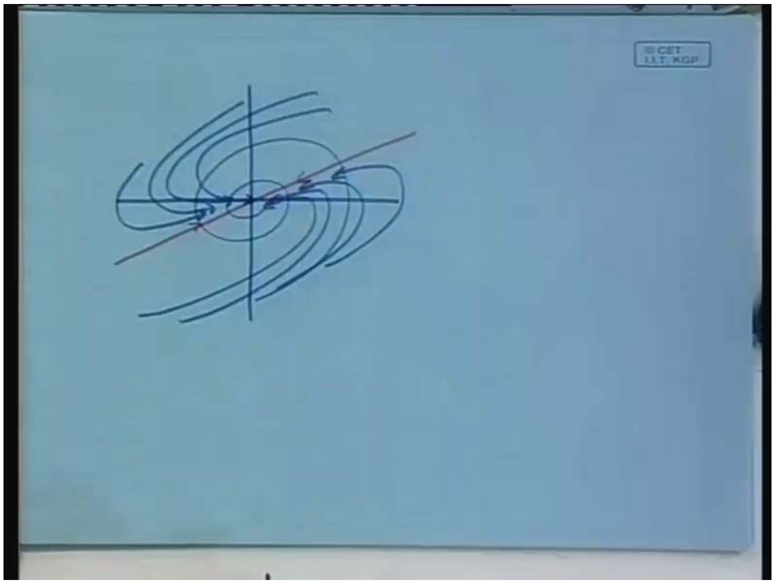

For that particular situation where you have one eigenvector and associated with a one eigenvalue and it is a say converging eigenvector then the behavior will be on this it should have you have to converge if it is this side, so it will converge like this. Note that it will not come in this way where from here it will converge like this. Now do you see it is going into this paralleling behavior, slight change you can visualize, it will go into the paralleling behavior and that is what happens. The moment the two eigenvalues collide and go to the complex side it develops the spiraling behavior that means it starts to go like this.

So that situation is just a critical value between the two kinds of behavior where there is only one eigenvector associated with it only one eigenvalue but the behavior would be like this. Now we have so far seen two different systems mainly. We have seen the Lorenz system, we have seen the Rossler system and we have also seen the force pendulum system and as you can easily imagine that there are very large number of possible systems that go chaotic. Some you may even have seen in the melas. these days you see some toys that have a behavior that never repeats. Have you seen toys something like this with two and in between there is some fellow and he is touching and everybody has seen.

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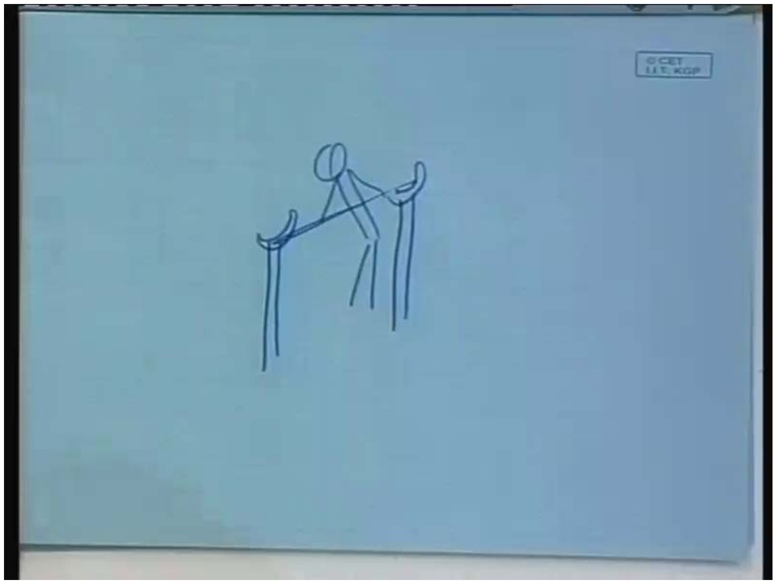

That is again a chaotic system because if you notice it carefully, you will find the same state is never repeating. So these are very common, it is not very uncommon systems and obviously they have to be common in electrical circuits also. So today let me give you a circuit diagram of a very simple electrical circuit that gives this kind of a behavior.

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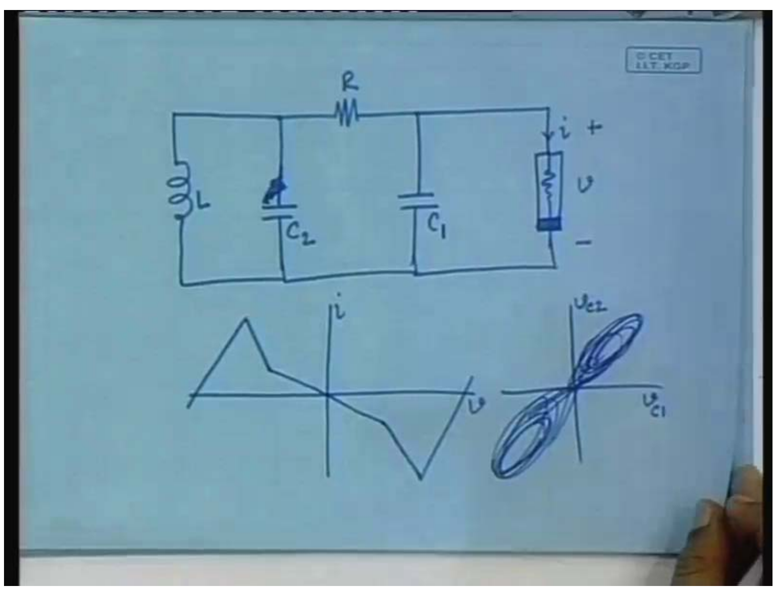

Almost all of them are linear elements, just one nonlinear element, it is an inductor, a capacitor, a resistor here and a capacitor her. All these are linear elements and here there is only one non linear resistance to denote that it is a nonlinear element so I have to give a symbol for it.

So this is a nonlinear resistance say so let's call this C_1 let's call this C_2, let's call this L, R and here there is a non linear resistance so this fellow is nonlinear. Obviously in case it is nonlinear then I have to specify what its characteristic is like. Its characteristic would be like this. You have got the voltage across this and you have got the current through this. So this is my V and this is my i, V is plus minus then its behavior is something like this, a linear segment another linear segment with the larger slope of the same is symmetrical. Here is a symmetrical, here is a positive and negative sides are symmetrical with a larger slope and then you have got another segment which is like so. You can easily notice that this part is a negative resistance part and this part is a positive resistance part. So if an orbit behavior goes to this side it will be dissipative, if it is here it should be expansive. So it should ultimately result in some kind of a periodic behavior or I mean it will normally not behave like just an equilibrium point. So it's a nice oscillator.

Now its behavior would be that because of this LC network there would be an oscillation and the power source for the whole circuit is only this because of the negative resistance part. So this has to have a power source character. So its behavior would be that you can either see an orbit like this or you can see an orbit like this. There are two very symmetrical parts and beyond a certain parameter and finally they will tend to go into that one. So you will see an orbit something like this. The advantage of electrical circuit is that you can see that on the CRO screen and i would request most of you to actually do it on the CRO screen. So this is a v_{C1} verses v_{C2} easily seeable points. You might ask what this circuit consist of this nonlinear resistance. So let me give a diagram of the complete circuit.

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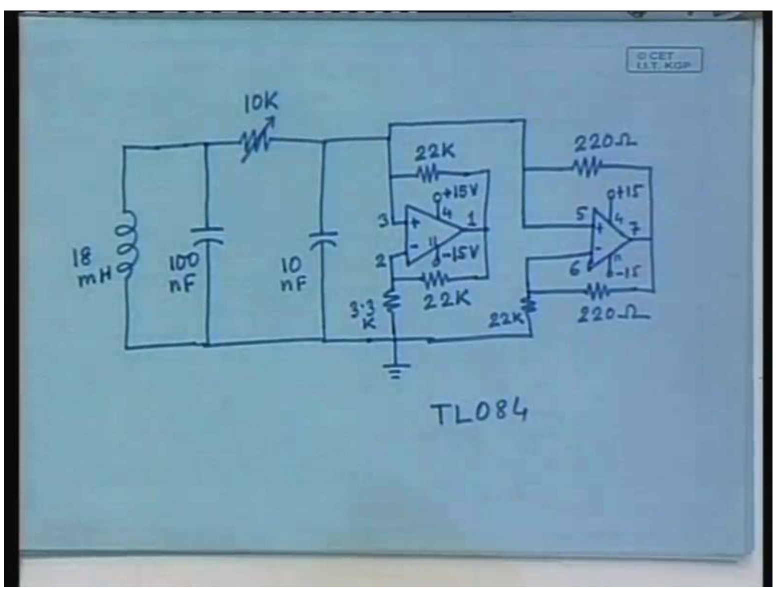

Here you have the inductor, if you want to make it you will made this complete circuit diagram. This is 18 milli Henry. Here there is a capacitor, it is 100 Nano Farad. There is a resistance which is 10 k, easily available component there is no difficult in making it, this fellow is 10 Nano Farad. Now comes the the non linear element that will have this property. It is actually realized by 2 op amps, op amp 1 and op amp 2.

In the op amp 1 I will not spend much time on this circuit because we will talk about it after we have made it and seen the result. This is in 3, in 2, in 1 there will be a source plus 15 volts and there is another source minus 15 volts. This is pin 4, this is pin 11. Now here this fellow also goes to these points here and here (Refer Slide Time: 18:28). From here there is a resistance going, from here there is another resistance going. This again the + 15, - 15, pin 4, pin 11, this is 7 and here is 5 and 6 plus minus. Now this values are this is 22 k. So all this components, these 2 op amps may be TL 084 kind of op amp. Now those of you who are from electrical instrumentation, energy electronics they should all make it on the breadboard. It's very easy to make, there is not more than an hour or so to see the the attractor on the CRO screen. All that we have to do is to put the CRO on the xy mode and to observe the attractor. This is the variable parameter where you connect a pot and as you change the pot, you should see the behavior changing from period one to period two to chaos and all that. It should go through a proper path way to the chaotic behavior.

So the earlier things that I give where meant mainly for simulation studies. this one is meant mainly for experimental study though if someone wants you can also simulate it's equations, you can also derive but it should be like v_{c1}, that's one state variable v_{c2} another state variable and i to be inductor, i_L is another state variable.

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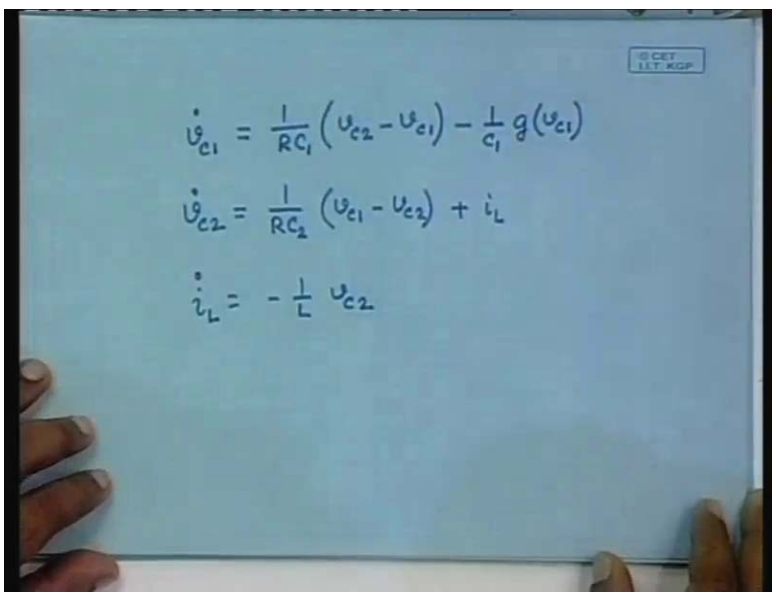

So the equations should be in terms of this equal to; now the first equation would be 1 by RC one ($v_{c2} - v_{c1}$) minus 1 by c_1. This is the functional form of the nonlinear resistance. Here it is 1 by RC_2 ($v_{c1} - v_{c2}$) plus i_L. Here it is -1 by L v_{c2}. So in order to explode the behavior of this circuit, you might take one of the few possible avenues. One which is most preferable, do it experimental. Two use this equation and simulate it. Three use (Not understandable) (00:22:20) or some programs like that to simulate this circuit, may be (simuling) (00:22:25) whatever it is but you may simply put this circuit diagram and do the simulation. So one of the ways you should do it, depending on your back ground if some of you are from electrical electronics, computer, instrumentation energy back ground you should actually do this experimentally.

For those who have do not have the facility do it by simulation, check the behavior and report it. So do it and present it in the next class, tell us the behavior. The purpose of the whole exercise was to give you the feel that there are in fact a large number of systems in this nature where this kind of behaviors are quite natural. Also in engineering there are large numbers of system where this kind of a behavior are quite natural but I cannot really keep on multiplying examples. At some point of time we have to stop and talk about the theory.

So in order to just give me a feel, let me tell you that all switching circuits are nonlinear circuit because there is a switch which is a nonlinear thing. The whole area of power electronics therefore is a heartbeat of this kind of nonlinear phenomenal. Any system that has this kind of nonlinearity will have and as i have already told you that for example any system where there is a heart limit cycle and therefore if there is a limit cycle you know that system is not linear. You cannot have a limit cycle in a linear system.

So in order for the heart to function it is also nonlinear system and of course if you are dealing with a nonlinear system we can expect this kind of linear. In fact that happens, people have really done data acquisition on the human hearts, found what happens when it goes into those irregular oscillatory motions, what happens just before the system collapses. So all that different types of diseases have been analyzed from nonlinear point of view. Remember people have estimated eigenvalues of people's heart. So if your heart is very healthy your eigenvalue should be, that part I have not talked about therefore I am not dealing with that today but you know that stability can be estimated from the eigenvalues.

The solar system itself is unstable do you know, in fact it look very stable, it comes back but of course in a unstable system you still have the characteristic time or if you talk about the Lyapunov exponent we have said that the time after which the prediction will fail depends on the extent of divergence. So extent of non-linearity, extent of instability that will ultimately talk about the the extent of time for which predictions will be valid and we know that for our purpose all our predictions about the motion of the planets have been reasonably successful but still system is unstable.

In fact towards the turn of the century there was a what should I say not a competition but a king of Sweden I think, he announced a award for somebody who can proved that it is stable because he was interested in really proving mathematically that the solar system is stable and many entries came of course and finally the person who own the prize was Henri Poincari. Poincare was a great mathematician theorist. Probably you have heard his name in other contests but before he got the prize he retracted the paper saying that I found an error and then where he found an error I will come to that later but he essentially found an error that proved that should be unstable and in fact that is now accepted knowledge, yes it cannot be stable.

In fact we even use the instability of the system or you can say the sensibly dependence on initial condition but those of you who have done a (Not understandable) (00:27:47) course on say classes mechanics know that any three body system is; heard of these three body system problem? If there are two bodies then that their motion would be around each other and you can easily write down the equation, solve it and you can find out the behavior.

If there are three bodies then you cannot do so, you can still write down the equation but you cannot solve them and it has been the long standing problem. Now it is realize the moment that there are three bodies, the system equation become chaotic behavior and that is why this behavior is such that you cannot really do the prediction. Now often we do use it for fruit full purposes. For example around 1987 or 88 I forgot around that time there was a comet coming. A scientist wanted to have a look at it. Now you are going to really launch a space craft aiming at a particular comet because the comets suddenly come and you do not have the time to make this space craft.

So there was a comet coming and this guy wanted to take a look at it, very carefully look at it. So how to do that? They realized that already they have some artificial satellites circling the earth and they could be used, provided they can hold to such an distance but the difficulty is that none of the satellites have that much of fuel because satellites used fuel only to keep it in track, keep it in the particular orbit so small amount of (Not understandable)) (00:29:26) it takes. So there was a satellite that was almost nearing its life time. It had a remainder amount of fuel available and this scientist decided to use that to observe that comet but the comet was then close to Jupiter.

So you can understand what distance it had to travel and there was no fuel. It so happens that the earth, Jupiter and the satellite is the three body system and therefore it can be easily calculated to be a chaotic system there should be a sensitive dependence on initial conditions. So if you not a system initial condition on slightly there should be large deviation in the final condition. They manage to calculate and then use that small amount of residual fuel to match it to that position to that initial condition so that the gravitational attraction of these three bodies I mean it is basically gravitation problem that itself properly to the vicinity of that comet and that (to be photographed) (00:30:27).

So this way we do use even the instability that is there in the solar system but the point is that there is another very compelling reason why chaos should be very common in nature. The compelling reason is this that if say every system in the world are non-chaotic, what does it mean? It means that if you observe the initial condition with some error ball, you can evolve it and predict the final condition. So the final condition is exactly given by the initial condition. Final state is given by the initial state. If everything has that property that the final state is given by the initial state then one might easily say what we are doing today talking about it the the molecules and atoms in your body they are moving inside your body making you think in a particular way listening to my lecture and all that. That where pre-ordained by things that happened for hundred years back but they are obviously not true so there must be some way for new information to be created and whenever it is not possible to predict even theoretically that’s why the new information is being created.

Since you can see that this pre-ordained thing is evidently false, therefore it immediately comes to the conclusion that much of this system that we see in nature must have that problem then there that must be sensitive dependence on initial condition. So what we are talking in this course is not really the auditing, you might have a feeling that the chaos thing is somewhat odd or I mean you don’t really come across that kind of things but not quite so but it’s quite uncommon. Non linearity is also very common, these things are also very common and if you do not take the nonlinearity into account what happens? I may have a couple of pictures to show you that.

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This is a bridge; we may run it all over again. So this is what a bridge is doing. The bridge that was designed depending on linear system theory. So if you take a nonlinearity into account then that could have been predicted so if that happens that was a Tacoma bridge just before it collapse and it collapsed after some months of operating. While it was operating, it operated like that, whenever there was a strong wind it had both the directions of oscillation. This direction and that direction and both were photographed and on the net if you search for the Tacoma bridge collapse, you will find the stories of people who were actually on bridge on the track when it collapsed.

So you will find those things that is what happens. We do not take non linearity into account than design your system based on linear system theory. So having established the necessity of understanding this, now we set on the task of understanding the tools by which we will try to understand this kind of behaviors before that let me just show in brief what are the different types of behaviors of system that we have heard of.

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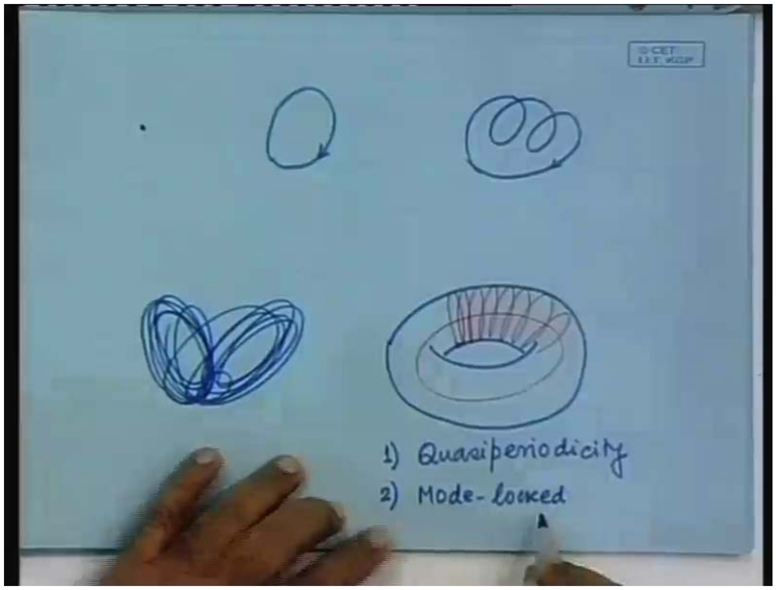

One a simple equilibrium point where every thing collapses in the state space, two a closed loop that is a limit cycle, three a high periodic limit cycle, four a chaotic behavior where the same state never repeats. There is a fifth type which I will mention in passing today and we will take it up in greater details later but since I am putting my cards on the table one after the other, the things that will come in the subsequent lectures, you have to understand what is coming. So there is another type of behavior which is also very common where the dynamic happens in the state space, happens on the surface of the torus. So the Torus the doughnut, imagine the dynamics happening on the surface of the torus it will be so on and so forth, it will wind around and these things are also reasonably common. It is not difficult to see that there is one frequency along this direction, there is another frequency along this direction.

So it is actually a combination of two frequencies such a behavior, if you subject it to a Fourier analysis you will find a two piece. There are two distinct frequency components. Now you may say that is that a periodic wave form. May or may not be. If the same state after having come all through if it merges with this one then you would say that it is a periodic orbit but if it does not then you would not say that. Now can you see under what condition it would and under what condition it would not. There is a frequency associated with this one, there is a frequency associated with this one. There is a time associated with this one, there is a time associated with this one. If these are incommensurate that means there exist no number that can be obtained by multiplying this one by some number, that one by some number; number means natural number then this are incommensurate.

If they are incommensurate the same state never comes back to each other. It will go on winding around but it will not fall on the same position exactly. Therefore the orbit will be a periodic, still a periodic but it will not have the sensitive dependence on the initial condition. So there is also possible another type of orbit, this type of orbit where there is no sensitive dependence on initial condition but the orbit is a periodic.

You would start from two nearby initial conditions, they will always keep the same distance. If they always keep the same distance what is the Lyapunov exponent? Lyapunov exponent was on an average e to the power lambda t times an initial distance. What should be lambda so that the initial distance keeps itself? So in such systems the Lyapunov exponent is zero in fact whether or not the dynamics is actually happening on the surface of a torus that can be simply estimated by estimating the Lyapunov exponent, if you find it to be zero it happens only in torus. Such behaviors that means dynamics on torus can be of two types. One if the two frequencies are incommensurate then the same states never repeats and this behavior is called quasi periodicity.

However if they are commensurate then what happens? Then the same state comes and repeat itself so that it will become actually periodic orbit. It is actually periodic orbit where these two frequencies are sort of locked and you will find that this kind of behavior is also not really uncommon though. So that is called a mode locked periodic orbit. The two frequencies are locked and in such systems even if you part away parameter by slight amount, the locking still continue that's why it is locked. It's not that if there are two frequencies if you try to change one frequency it will no longer remain incommensurate. No it's not so. They get locked and they remain lock for some time and one of the very clear examples is the motion of the moon around the earth.

How does the moon move around the earth? There is one motion of the moon around the earth itself and there is also the motion of the moon around its own axis. There are two things, two frequencies and this two frequency components are commensurate. The two frequency components commensurate with the ratio of one. That's why what happens; you see only one face of the moon. So the speed at which the moon is going around the earth, at the same speed it is going around its own axis and you might ask how did this happen. Is it just a stroke of luck that it happens? No it's not. There is a very well understood, non linear phenomenon going behind it which makes it lock and you will find that out of the hundreds of satellites that they were there in the solar system most are locked but not at that 1:1 frequency. There may be other frequency but most are mode locked.

So this is another component. In order to understand the type of behavior again you can imagine the type of behavior i mean that two frequencies as the motion on the moon around the sun. Now we are not considering the axial rotation but motion on the moon around the sun, the earth is rotating around the sun and the moon is rotating around the earth. Now if I ask you what is the rotation of the moon around the sun, it is a motion on a torus. So this way you can easily figure out that this is not very uncommon, these are reasonably common things but here we are talking about the torus in the state space. So in the state space torus is occurring and you see the state motion in the state space as a motion on torus. That's quasi periodicity.

So here is the complete picture. This is all that can happen as stable behavior of a dynamical system; unstable behavior, transient behavior there can be many other things but stable behavior there can be only these types. Therefore there are two questions that would automatically come to one's mind. one seeing a systems behavior or say an experiment is running, a system is running where we have implanted some kind of a measuring instrument, some instrumentation is done we are getting the data. By analyzing the data, how can we infer which type is it?

Quite natural question in an experimental situation. If say system changes from one type to the other, as you change a parameter how can we predict that and also how can we understand that? There must be a theory behind it, theory behind such changes, will deal with that in this course. So there are would be naturally similar such questions whenever we talk about. For example how does this kind of a behavior coming into being, how does it happen? So if I simply say that happens that's not a good answer. It has to be some theoretically grounded that it changes from one type to this by some very well understood mathematical mechanism but ultimately much of control theory centers around our understanding of stability. Those of you who have done some question control theory, in order to half the control theory course is essentially treatment of stability.

We are trying to understand the stability of systems and in control theory what kind of systems are those whose stability are we enquiring? Essentially we are enquiring the stability of equilibrium points. In nonlinear system we have first time encounted the possibility of a linear cycle and we have also understood that this limit cycles are very common in engineering as well as very common in nature also and therefore we have to be worried about the stability of the limit cycles also. Oscillator for example design an oscillator you are in interested in its stability. So how do you infer its stability? Obviously its stability is not the same as this fellow stability. In case of a simple equilibrium point this stability, you knew can be accessed simply by obtaining Jacobian around it, obtaining the eigenvalues and if the eigenvalues are in the left half plane, it is stable simple stuff.

Obviously the stability of a lever cycle cannot be inferred that way because it goes through various points. It is not just one point, it goes through a wider range of this state space and through that wider range the behavior is not the same. It is nonlinear system so at this point if you locally linearise, at this point if you locally linearise, at this point if you local linearise they will not yield the same thing. So if you locally linearise at every point you would see that the matrix elements are changing as it go through. Obviously it is a not trivial problem. How to assess the stability of this?

Similarly a high periodic orbit, its stability can also be a question. Stability of chaotic orbits, what do you mean by the stability of chaotic orbit? Chaotic orbit itself is the result of instability. At every point on the chaotic orbit, the thing is unstable but it is not going to infinity. The chaotic orbit stability would be understood as when states run to infinity. That means its boundedness is no longer satisfied then it becomes unstable so we would also need to understand under what condition a chaotic orbit can become unstable. We will also need to understand under what condition can they torus orbit also become unstable. Now before we end this class, i will give you glimpse of the method and we will elaborate upon it in the next class. Let us start with this problem.

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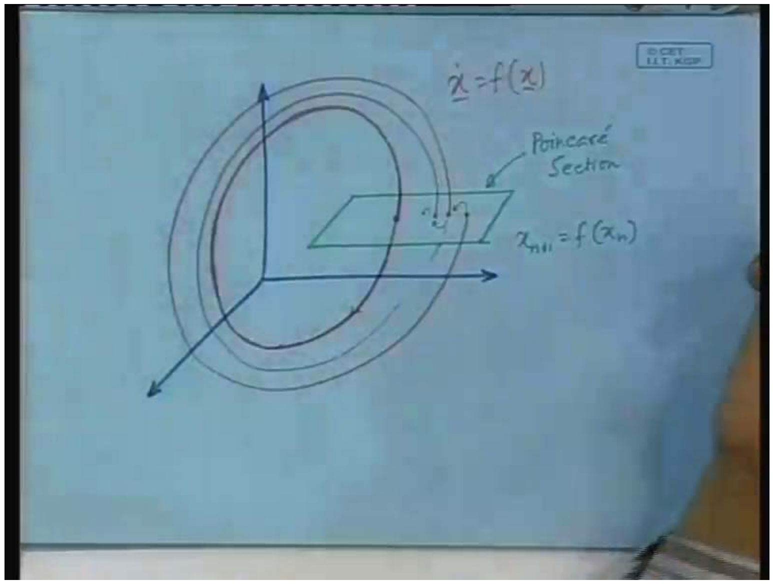

In the three dimensional space we have got an orbit which is a closed loop and I want to find out its stability. The essential method was developed by same man Henri Poincare. His method was something like this. He say that suppose an imaginary plane a section that intersects the orbit, at imaginary section in the state space that intersects the orbit which means that the orbit will peers this section and will go to the other side. So the orbit will say, it will go like this and at this point it will clear a section. Now if it is stable orbit, if you part of it what will happen? The part of version will die down. as a result of which if you start from a different point on the, suppose the starting point is on this plane; suppose you started from here that means you can also imagine it as this orbit having been parted to this point.

What will happen? It will come but after all since it is coming closer and closer to this thing it will come somewhere here closer and then it will again go and it will come to against some point that's closer. So on this plane, you would see not a straight line. You see a succession of points that ultimately convert onto this point. Therefore the issue of this continuous time orbit has now been brought down to the level of what is known as the map. That means this point is mapping to this point, this point is mapping to this point, that point will again map to this map, (Refer Slide Time: 49:29) forget about the whole evolution. Just look at what is happening on this plane. A point is mapping to a point, is mapping to a point, is mapping to a point then so on and so forth. You get a sequence of points and if that sequence is a convergence sequence, you know that the limit cycle stable. This section is called the Poincare section.

So Poincare invented this method of reducing a continuous time evolution into a map. Here is a discreet and dynamical system. See on this Poincare plane it is a discreet and dynamical system, discreetly it jumps from a Point to another Point. It doesn't move continuously, it jumps from point to point and you get a sequence of point and that should be convergent. Now you might say that on this Poincare plane, though the original thing was xyz there was three coordinates; on this plane there are only two coordinates. So you have brought down the system complexity by one degree. The system dimensional has been brought down by one degree that's one advantage.

The other advantage is that a system of differential equations that means original things was expressed in this from, x dot is equal to f (x) these are all vectors but now here it has now become on this plane it will be x_{n+1} n+1 instant the position is function of x_n. Is a discreet time dynamical system and discreet and dynamical systems are far easier to handle than continuous time differential equation. That is why these are actually triggered that means this method is almost universally used in order to estimate the stability of any periodic orbit or this kind of orbit and that is not exactly equilibrium point. So this method essentially achieve this, that this set of differential equations has been changed to this but this a one to one correspondence because here there is a particular difference equation or discreet and dynamical system or a map; is also called a map. As I told you that this is also called a flow, the set of differential equation is also called a flow because its sets into an action a vector field that looks like a flow. So this is a flow to a map, so Poincare section achieves the transformation from flow to a map.

Now if you know the map, you can infer the orbits and then there is a one to one correspondence between the behavior of the map and the behavior of the dynamical system actual continuous dynamical system. So there is a one to one relationship which means that a periodic orbit as seen as a stable periodical orbit in a map, will be a stable periodical orbit on the continuous dynamic system also. What is a stable periodic orbit in a map? In a map a stable point means in that case it means a point which maps to same point. In other words x_n +1 is equal to x_n. So this was actually f x_n, if it that happens to become equal to x_n that is the equilibrium point and that will be related to the limit cycle in the continuous time and you can now reduce the problem that was looking enormously complicated because on the complete cycle at every point there was a different type of non linearity and it becomes hellish to study the stability but now you can reduce the problem to another linear problem. I can look at the local linear neighborhood of this map and study the stability of this fixed point in the same way as we study the stability of an equilibrium point. So the whole problem again becomes essentially the study of stability in the linear stability consideration. We will develop on this issue in the next class, that's all for today.