Chaos, Fractals and Dynamical Systems
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Lecture No. # 06
The Rossler Equation and Forced Pendulum

In the last class we have seen a particular way of creation of a chaotic orbit. There in that system you had essentially two planes, two eigenplanes and you had outdoing spiraling orbit in those outdoing planes and they somehow interacted so that the overall outgoing behavior was arrested; from one plane the orbits got thrown into other plane and from there it got thrown back into this plane and so on and so forth. So that is how the chaotic orbit is organized that is one mechanism. Today let us look at some other mechanisms of derivation of chaos just in order to get you comfortable with the idea. Today let us start with what is known as the Rossler Equation.

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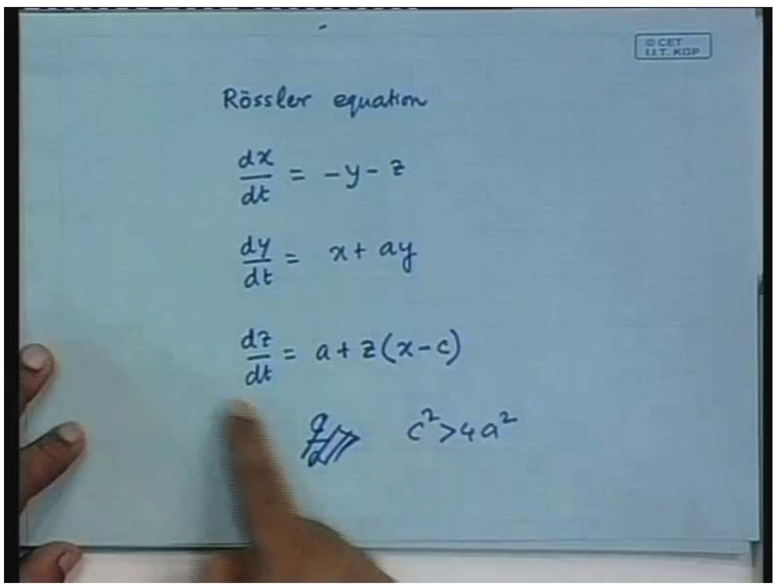

This is the German pronunciation it's pronounced as the Rossler. You will have to make your mouth look like as if you are trying to pronounce o but you will actually pronounce e. So you it will make that o kind of sound this is Rossler. So this man proposed a certain set of equations not exactly in order to represent a particular physical system but it was just a set of equation that has certain property which will be very interesting for us to study. It is simple linear equation. Here is the beat of nonlinearity this is z extra except for that everything is linear. So in a linear system you would expect the kind of behavior that you already know. So if I ask you to analyze this set of equations what will you do, what will be the procedure? You will first locate the equilibrium point. Where is the equilibrium point? It's 0 0 0 in equilibrium point not exactly because of this one. How many equilibrium are there? Try to find, both of them are actually existing.

Now depending on the values so depending on the relative values of c and a. So under what condition would they exist and under what condition they would not? c square is greater than four a square then they exist. If this not true then no equilibrium point. Now here in this system since these two equations are perfectly linear I might argue in the following way, this is a three dimensional thing. I will look from the top as if I am looking at the projection into the x y plane, I can do that. If I do that then I see only these set of linear equations and in these set of linear equations 0 0 is the equilibrium point and by ignoring this I can find out just by looking at this because if I am looking from there, I will not see this, I will see only that and I can more or less in a sort of highly simplified line of reasoning, I can talk about this stability of that 0 0 equilibrium point in the x y plane. Here if you consider only these two equations then it will take this form x dot y dot is equal to 0, -1, 1, a.

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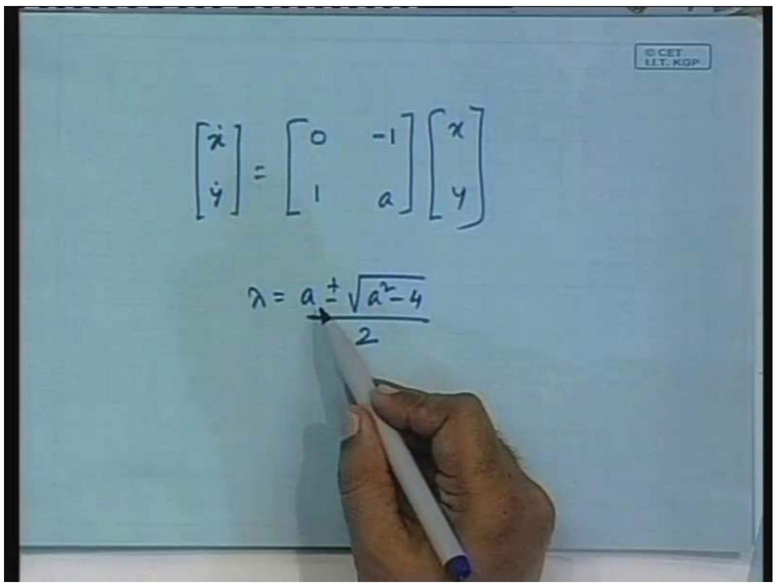

If some condition is satisfied then you would notice that this will become an outgoing spiral behavior. Can you find out that condition? Easy from this, just a two dimensional equation. So what is the condition, what is the expression for the eigenvalue that's it. Lambda is a plus minus a square minus 4 by 2 and the condition for stability is this term (Refer Slide Time: 8:15). This term should be negative then it is stable else it will become unstable. So you can easily see that it will become unstable at a is equal to zero. At a is equal to zero it will become unstable fine. This is negative term, a is equal to zero means it would be a purely imaginary pair of eigenvalues and beyond that it will become outgoing spiral so you would expect outgoing spiral behavior. So this is all you can infer by looking at only the projection, completely forgetting about what is happening in the z axis.

Now notice the character of the thing in the z axis. So long as x is less than c this is a negative term. So depending on z, if z is say you start from the z is equal to zero plane then if there is some amount of z then it will create a negative rate of change in z. It will come down to the z plane. Can you see that?

So if x is less than c then that is happening. Any deviation along the z coordinate will tend to die down of course but I am sort of making hand waving argument to give a flavor. So a value is there but if a value is sort of overwhelmed by this negative thing that will happen but then if x is greater than c then you see plus a positive number. So this will fellow will tend to build up. So things will start to happen in the z plane also. As it happen in the z plane, as it goes in z plane here is a negative term which means it will make x smaller. If x is smaller then again you can see these become linked up but essentially you start to get a feel of why things will not blow to infinity.

If x becomes large then z will tend to become large that will again tend to reduce x so on and so forth. So this is something we can infer that this fellow will tend to create a bounded orbit but then here you can see a and c are the two parameters and these two parameters are in some books we will find the abc. They put the parameter this way but for the sake of simplicity we have taken a as the same and there is an equilibrium point on that plane which has become spiral. Let us look at the behavior there.

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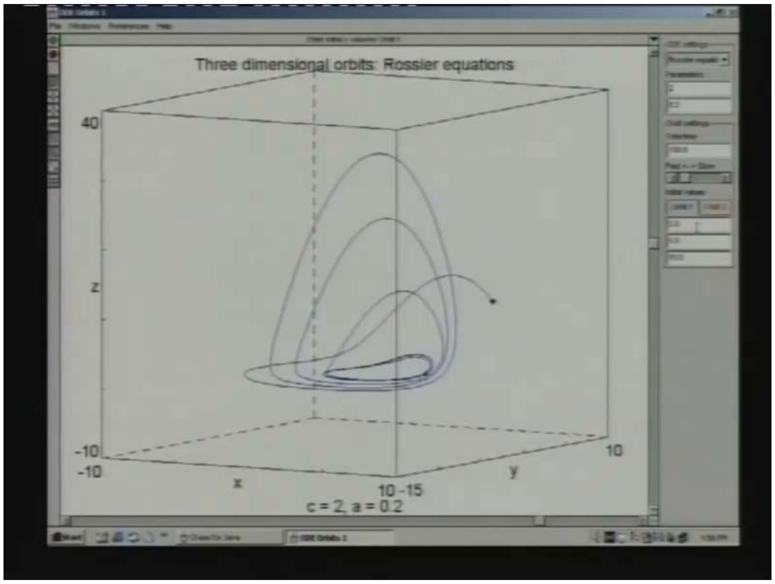

I have started just from anywhere and it goes through some oscillation and finally it converges on to something that is almost on this plane. Almost there is a beat of excursion along the z direction but nevertheless it is a periodic orbit it has created a periodic orbit. Notice that first I argued that there would be a unstable equilibrium point with outgoing spiral kind of behavior somewhere here. So it will tend to go out let’s check. Let’s set this initial condition to be zero and very close to zero, 0.001 and let us restart the whole thing and see how it behaves.

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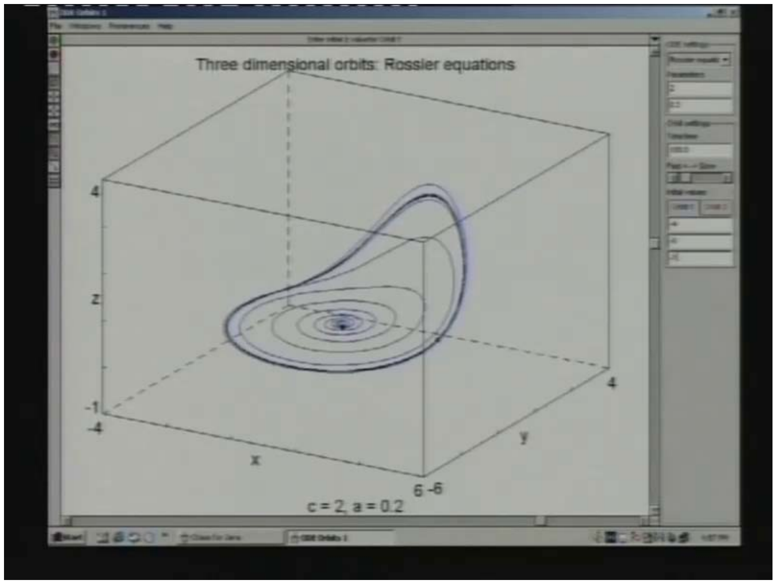

So you can see that here there was a plane that plane I was talking about, if you look from the top you will see this outgoing spiraling orbit that you see right now but it has ultimately gone out. So that the nonlinearity will start having its plane and as a result of it there will be excursion along the z direction which will keep the whole thing bounded again which means that this out going spiraling behavior could not really continue. If you to start for some where here then let us see what happens? Where do you want to start so that you can see the incoming spiral orbit say x is equal to -4, y is equal to -6 and z is equal to -1.

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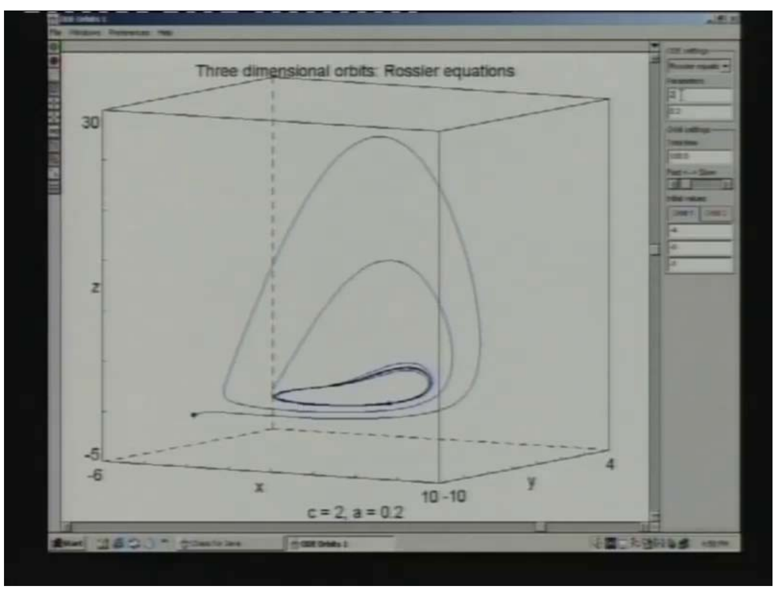

It has converged on to that from the outside which means it is a limit cycle. We have already talked about that situation where it is a outgoing spiral behavior on the inside and incoming spiral behavior on the outside. Ultimately there is a closed loop behavior, a periodic orbit that is created. So it is a periodic orbit that is created, from the outside also it is coming in because of this a feedback term. Now let us change the value, a little more let's see 3 and let us do it.

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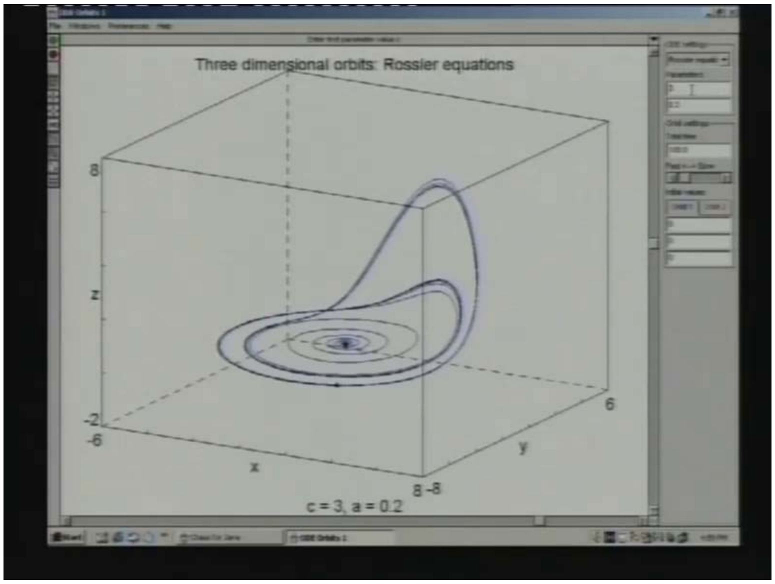

From inside it looks better. Have you noticed what has happened? No, not only the radius has increased something more has happened that too looks now.

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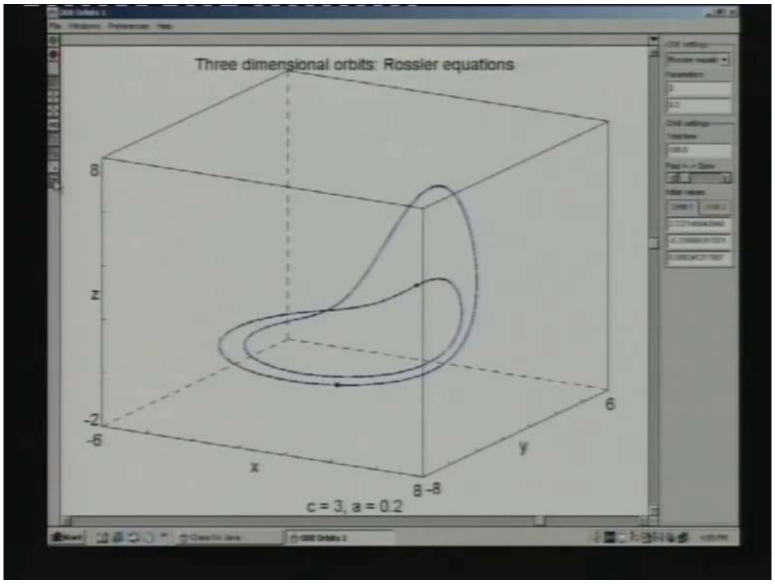

There are two loops now. The one that I talked about as a period two orbit. This is a period two orbit. Initially it was a period one limit cycle. Now it is a period two limit cycle. If you increase the parameter even further say what you actually saw in the first computation. So you can see that now it is still a period two orbit. Let us increase the parameter in steps. I am increasing in very small steps though.

It seems to be a still period two orbit. What did I do? When I am pressing this one what I am telling the computer to do is where it ended the last computation, you start from that as a initial condition so that you delete all the initial transient and then you will be able to see the final steady state behavior. So as you increase it further, here you can see a lot of things happening but is it really there? No, it is still a period two orbit. Now I am making a 3.9.

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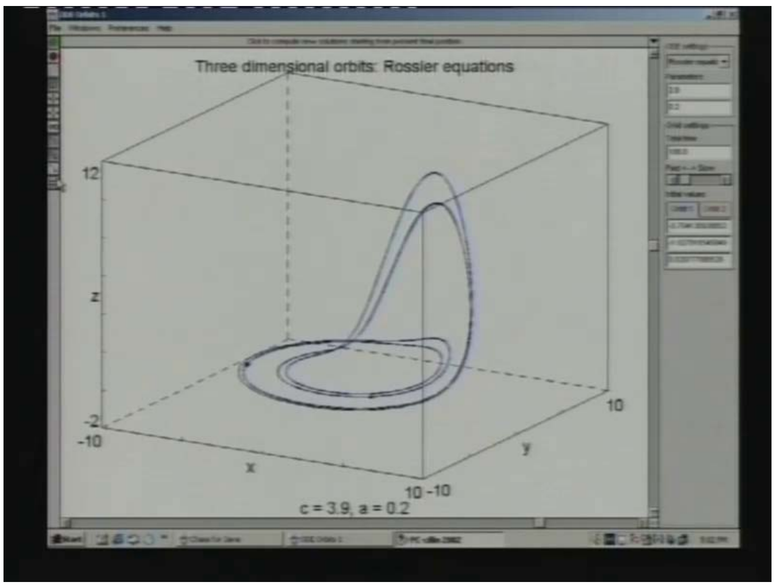

Something has happened. What has happened? It is now period 4 orbit. So you see the period 2, now it has separated and these were earlier together. Now they have separated and as a result of which it has created a period 4 orbit.

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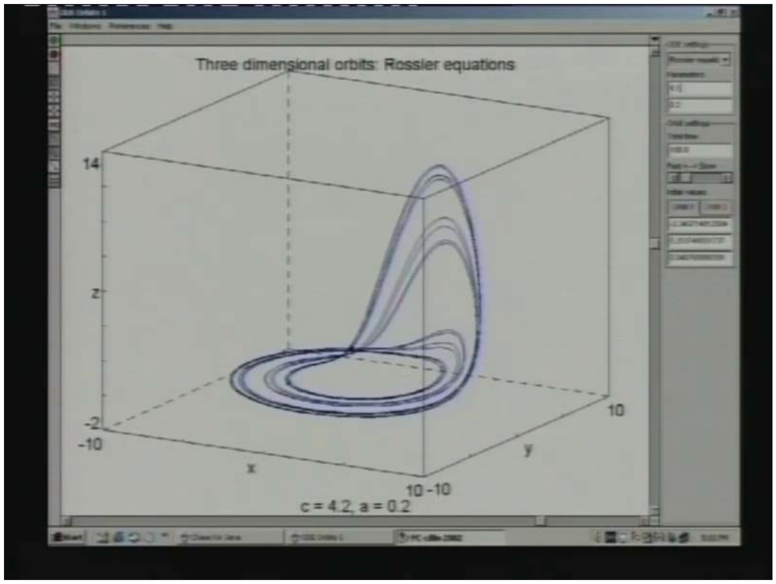

Increase it further. How many? Still 4. How many? Too many but nevertheless a periodic orbit.

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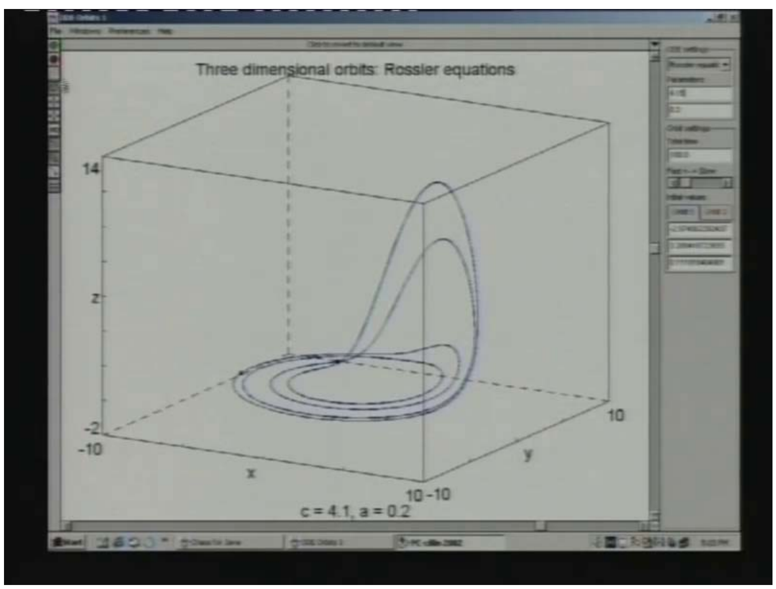

I can see that it was changing too fast so let me see what is in between. This is still a period 4 orbit. At 4.1 it is still a period 4 orbit.

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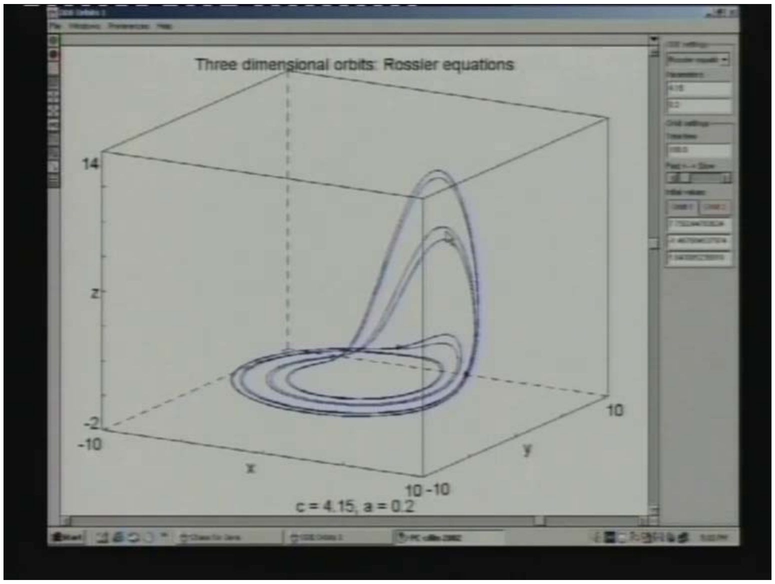

4.15 the period for each of these branches have now broken up. As a result it is a period 8 orbit and then it will enable you to understand what we saw at 4.2.

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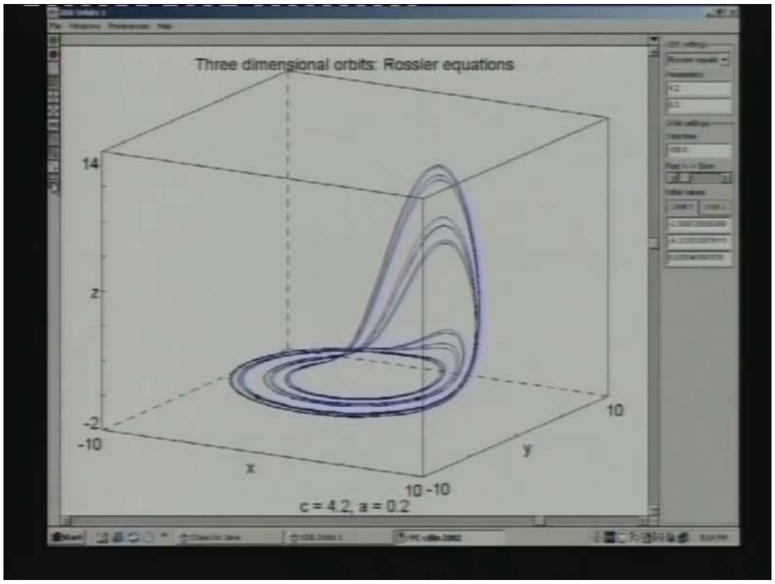

It is period 16 orbit so we see a succession in which it was initially a period one orbit then it change to a period two orbit, change to a period four orbit change to a period eight orbit change to a period sixteen orbit. Do you see it is a progression and where does it end? If we have a series 1, 2, 4, 8, 16, 32, 64 and so on and so forth where does it end? Infinite, so it is not really difficult to predict that going by this sequence after sometime you will end in a period infinity orbit or in other words a chaotic orbit.

You can easily see, in once step I will change it somewhat more.

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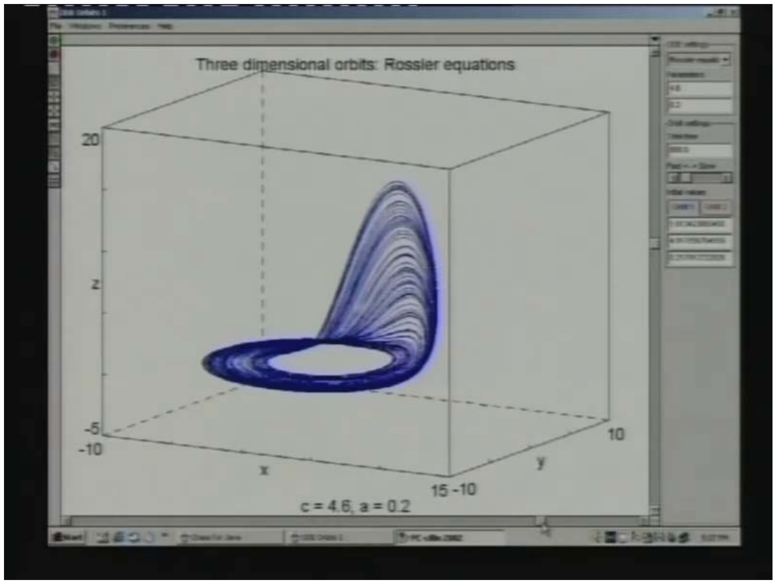

Let me change it a little more so that you can see it clearly. 6 is there. Now see the evolution how it goes? So as it goes outwards it is strong up and it lands inside again it goes outwards and so on and so forth. So you can increase it and see. You can see it has turned into chaos. So what was the sequence in which it was going? In this case you could clearly see that it was not just happening any way, it was going in a particular sequence in order to reach chaos it was what is known as a period doubling cascade. It was following a period doubling cascade. Can you see the Eigen plane even though the system is non linear you can still see that more or less it is following the the eigenplane here in this part and then because of the non-linearity it is going out. So this is character of the Rossler attractor. You can see it from various angles for example I stop it.

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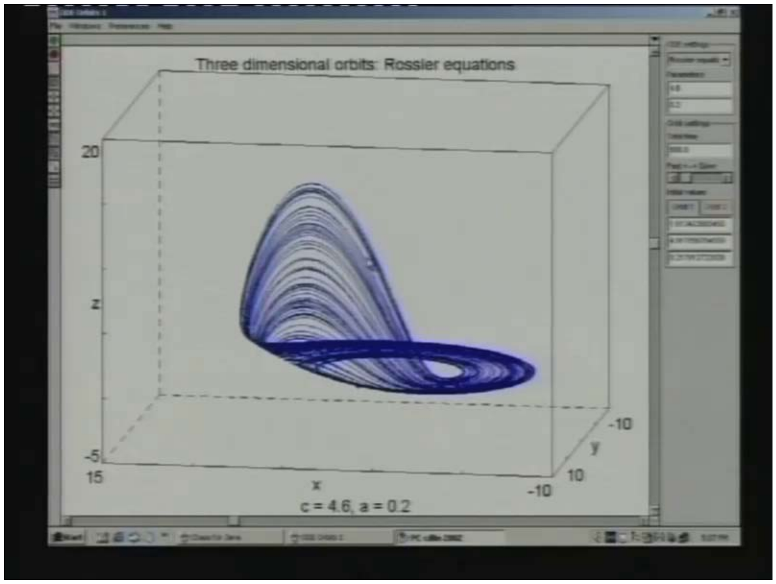

If you see from another angle, you can see what is happening. It is you see the threads going up and coming down, it’s like this. Do you see the structure? You might see it from various angles to make it clearer.

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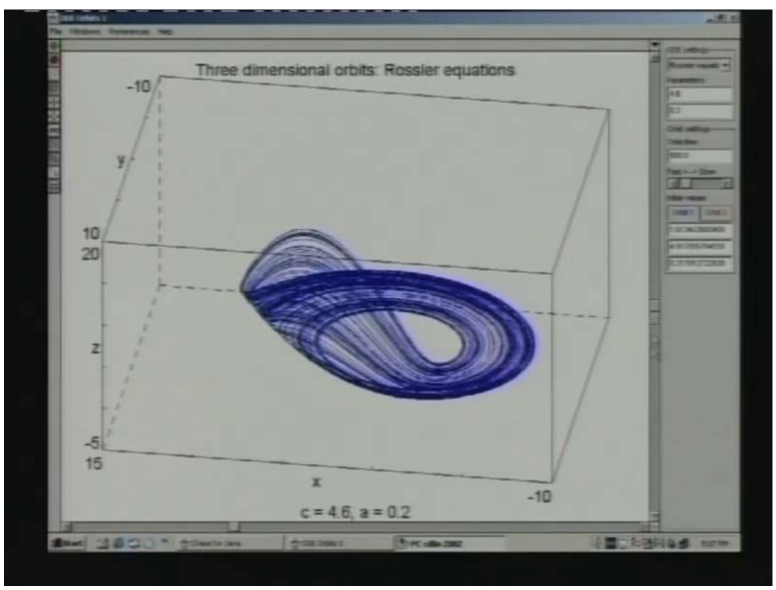

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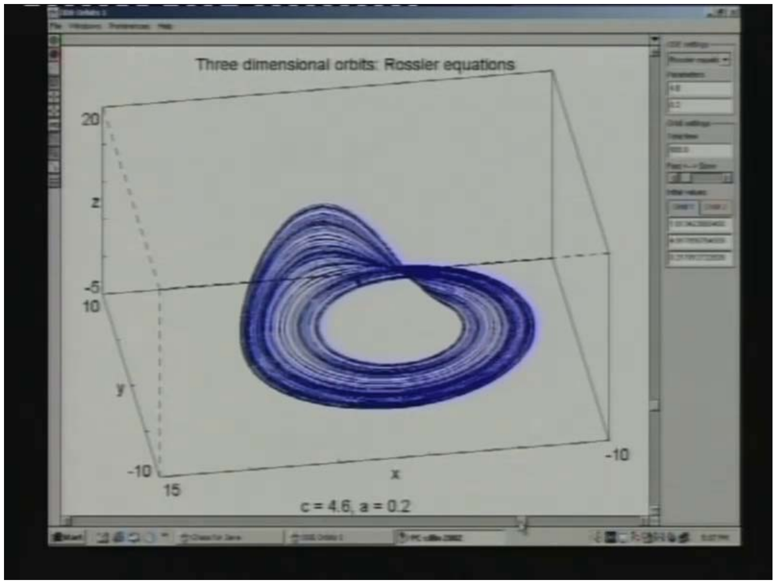

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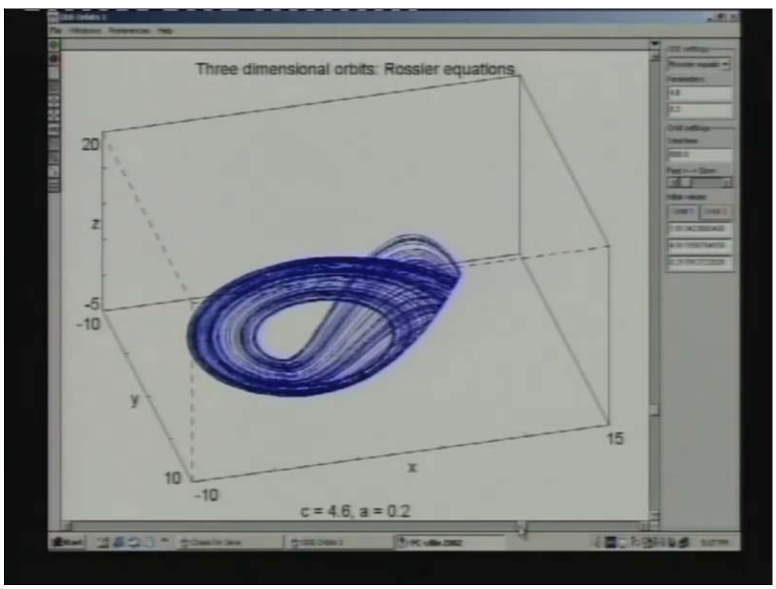

So you have the structure more or else clear now. So that was one another very clear example of how it happens, how it finally builds in to a period infinity or a periodic orbit. We will see why this happens, what is the mechanism in subsequent lectures. Right now I will not explain why it went from period 1 to period 2, period 2 to period 4, period 4 to period 8 and not something else. Today I will not do that but later I will come back to this question. That is obviously a very pertinent question why does it happen that way. Why doesn't it go from period 2 to period 3 for example? So we will come to that.

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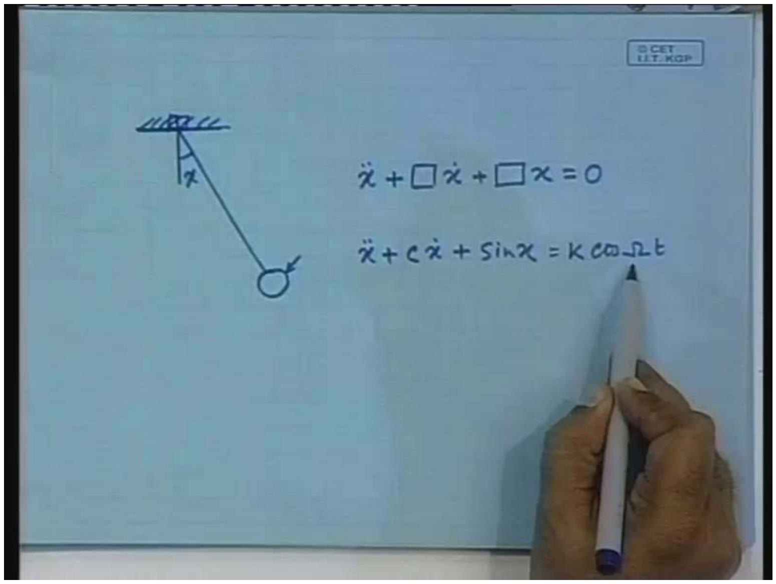

Let us now take up another very interesting and very simple system. Suppose you have a pendulum. If it is simple pendulum its equation would be say if this is your x in normal theta, we now call x because that is the only position variable necessary in order to specify it. So it will be x double dot plus something times x dot this is the dissipation term plus something times x is equal to, this is actually the forcing term. This is actually the sin x term as you have already seen. So let us write it this way that we have already seen g by l sin theta that term comes, x double dot plus cx dot plus sin x.

Now this will be a very simple is equal to zero will be very simple. We have already seen its behavior. Its behavior would be that there would be an infinite number of equilibrium points, there would be a spiral in inverse than the saddle again the spiral, again the saddle so on and so forth we have seen that. Let's make it a little more complicated. Suppose you have a forcing function that applies the force like so in the tangential direction. You might do it like this that you have a point of suspension and suppose this point of suspension will be moved that's one kind of forcing. That means the pendulum is hanging and here is a point of suspension and that fellow is being moved. Those of you who have done the dynamics of physical systems course, we take that derivation of that equations has one of the standard problems very simple but ultimately it leads to somewhat complicated equations.

Let us not today land into such complicated equation because we are not looking at the equations today, we are looking at the behavior. So let's take the very simple system where the forcing is exactly on the tangential direction and the forcing at tangential direction can be a sinusoidal term. So that sinusoidal term let that be some amplitude k cos omega t. by the way here there was a g by l term which we assumed to be in unity where sort of scaled everything as a result of which this sigma which is the frequency of the forcing function that has become a function of the natural frequency oscillation of this fellow. Natural frequency of oscillation of this fellow was root over g by l.

So this is actually expressed in terms of that. In every case ultimately we learned of this equation we are trying to study the behavior of this equation. Very simple system just a pendulum with some kind of damping and you are giving a forcing function. Let's study its behavior. Now for that let's start of all over again. I will go to the driven pendulum. Now here our parameter is, if the value of k here so let us start which say 0.4. See it is homing on to a periodic orbit. Can you see that? It's a nice periodic orbit. So it has become a periodic orbit, in what way that this fellow is making it oscillate and the pendulum is also oscillating just like that.

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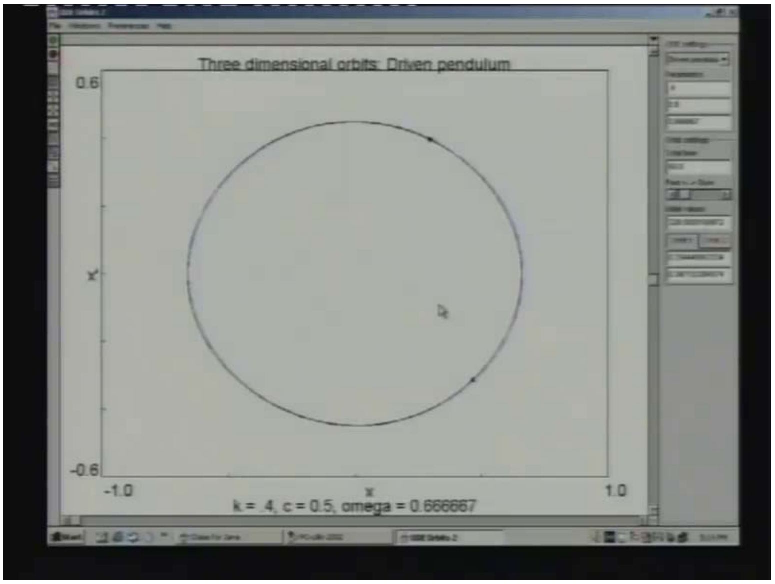

This closed loop here is nothing but an oscillator emotion in the real physical system. This frequency of the oscillation, the forcing frequency really because here there is a natural frequency of oscillation and there is a forcing frequency but here the forcing frequency will dominate its behavior and therefore will make it oscillate at that. Only when the two frequencies interact in a way somewhat moving out of state with each other, you see start to see other type of behaviors. Let's now increase it further, increase the parameter further.

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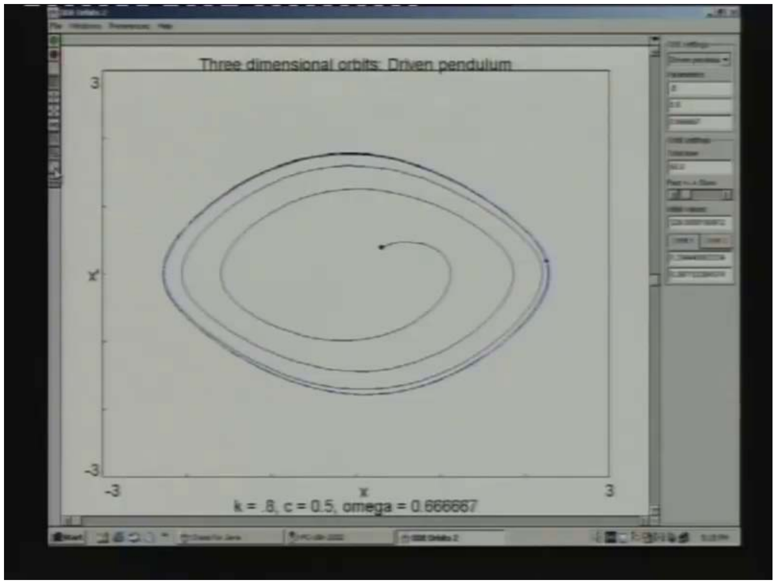

See here it is still going into a nice periodic orbit. Is it completely true in the sense that if I start from another initial condition say -3 and 3 does it land there?

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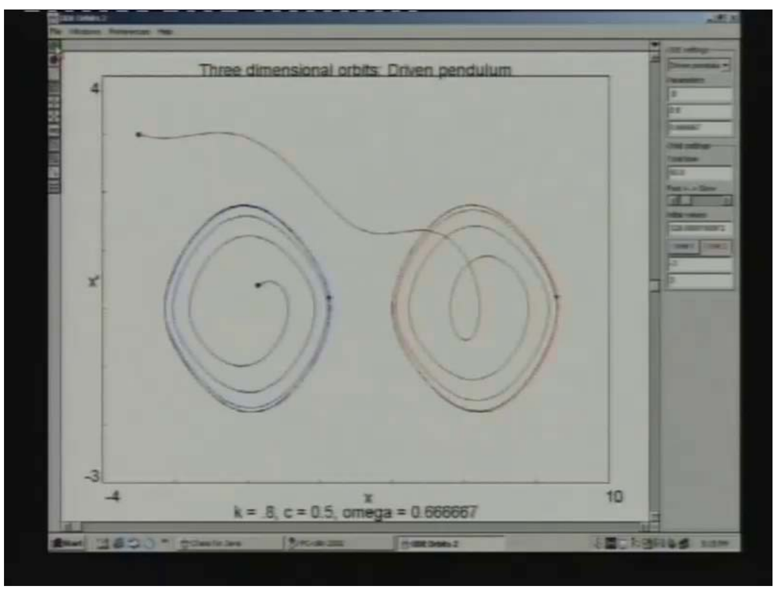

This implies that there are actually two coexisting behaviors. You start from somewhere else it goes here, you start from here it goes here. Can you see that? So it is not really just one oscillator emotion, there are two possible oscillator emotions of the system. These are two coexisting attractors. So for such a simple system you have coexisting stable periodic orbits.

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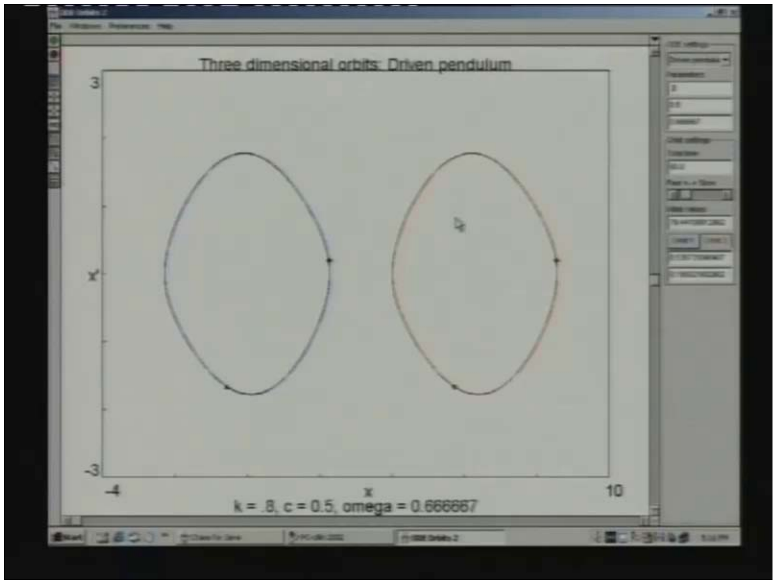

Stable in the sense that if you keep running like this they stabilize on to these two orbits. Now let us increase it further. Still periodic orbit starting from two different initial conditions.

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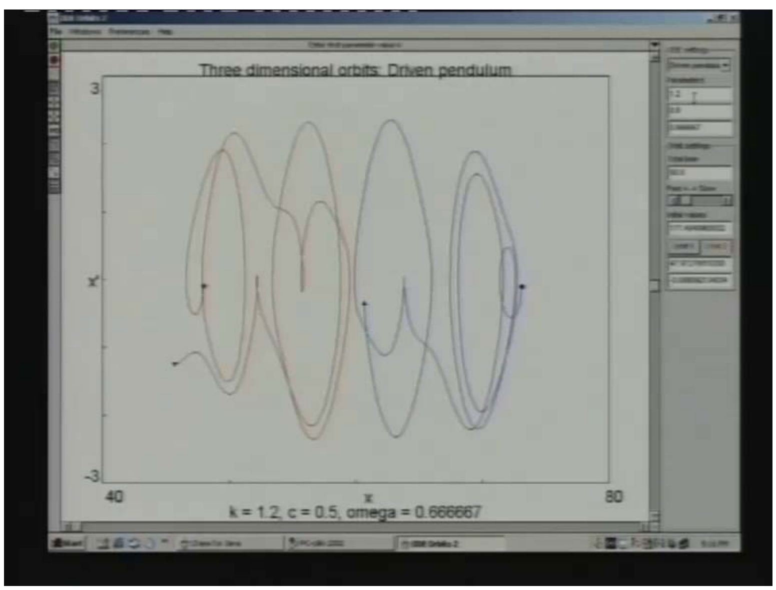

Not coming anywhere right? Let's then I saw that at 1.0 it was nice to coexisting periodic orbits and then this fellow starts doing this wide things. So let us increase slowly, let's see.

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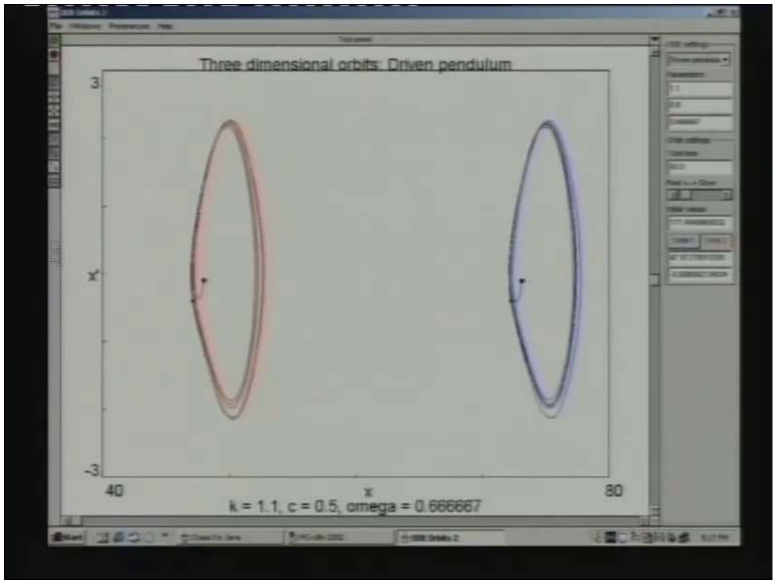

1.1 it is still, no it's not coming to the same behavior. Can you see there? It's not periodic orbit yet. If you are not convinced let's see not periodic orbit really. Let us allow it to run for longer time. See what happens. I want to run it fast and so that don't waste time on this, both individual and not coming back to the same behavior. If you want to start all the time you are seeing this. So you see it has a large periodicity. So in between 1.0 and 1.1 something more happened. So we need to increase in even more smaller steps. I am illustrating this because on various systems you will have to do this procedure. At this state you still a nice periodic. Now look what has happened. See what is it now? It has really gone into a period to orbit both of them. Can you see, there is a difference here?

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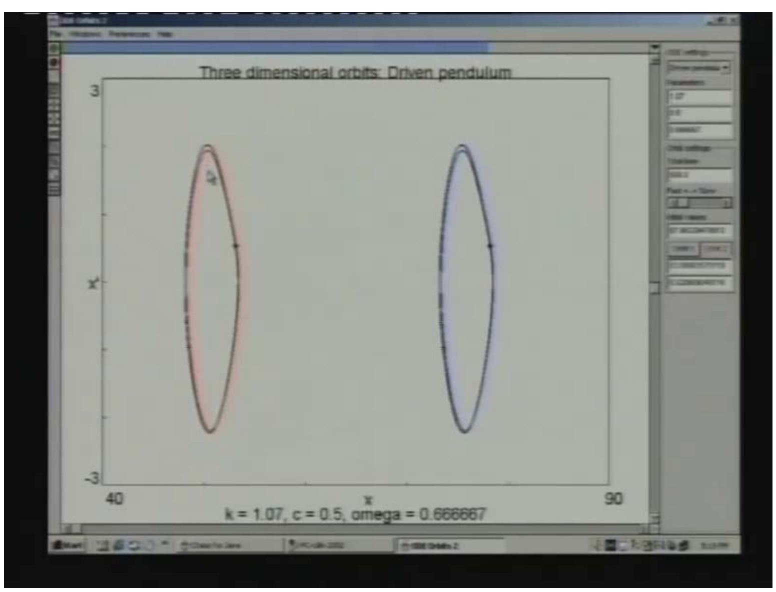

So both of them gone into period 2 orbits and if you increase it slightly more, in this system the changes are relatively faster. You would notice that it is now changing into period four orbit almost. So here also you see a succession of periodicities period 1, period 2, period 4 but here there is a additional complication that there are two very symmetrical attractors but coexisting attractors. Now we are in a position to change it significantly and see the result.

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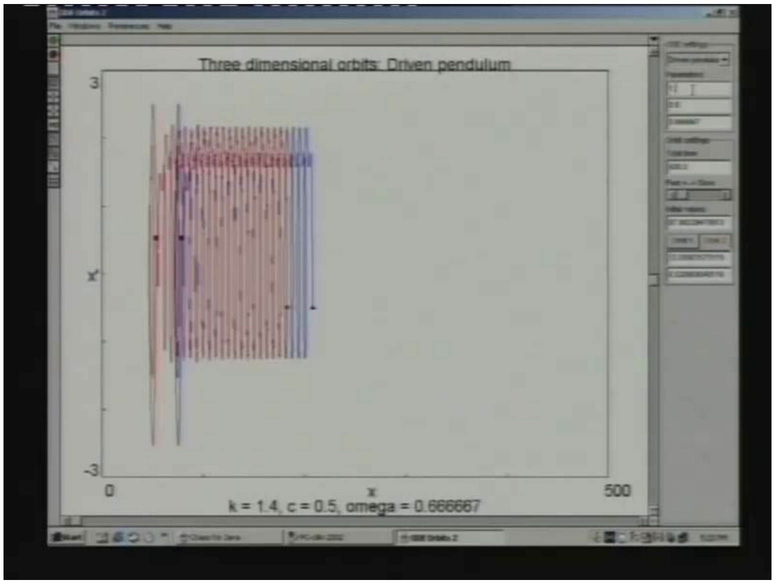

I have changed it too much let’s do it slowly. You will not be able to follow what is happening.

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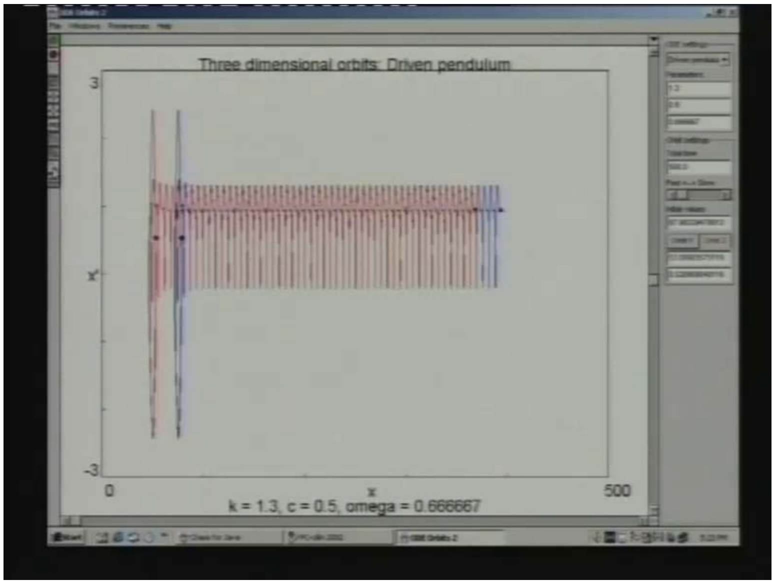

No it is going up in time.

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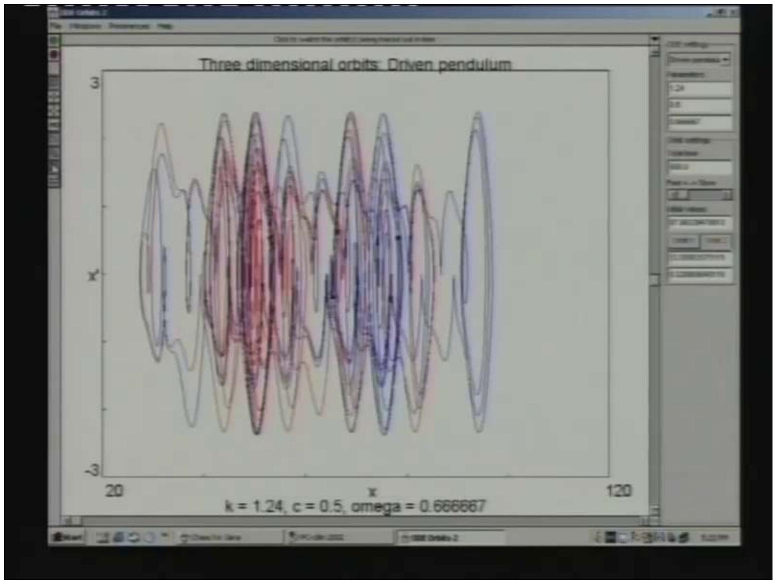

See the blue parts and the red parts are getting mixed up. They are no longer separated, they are getting mixed up. So beyond a certain parameter value starting from different initial conditions their behaviors at different always, their exact positions at different all right but essentially they tend to behave in a qualitative way in the same manner. They visit all parts of this attractor which means that the two initial conditions, if you have two D initial condition the state space that they visit they sort of mix up. This is called a mixing in the state space. If it is a periodic orbit with two different periodic attractors then there is no mixing. Start from different initial condition they go to different parts of the state space.

While in this case you have a mixing of the behavior starting from different initial conditions and this behavior is really chaotic. How do we make sure? In order to make sure let us start the 2 orbits from very close positions. For the red one it would be 0.00001 and 0. Now let us start it.

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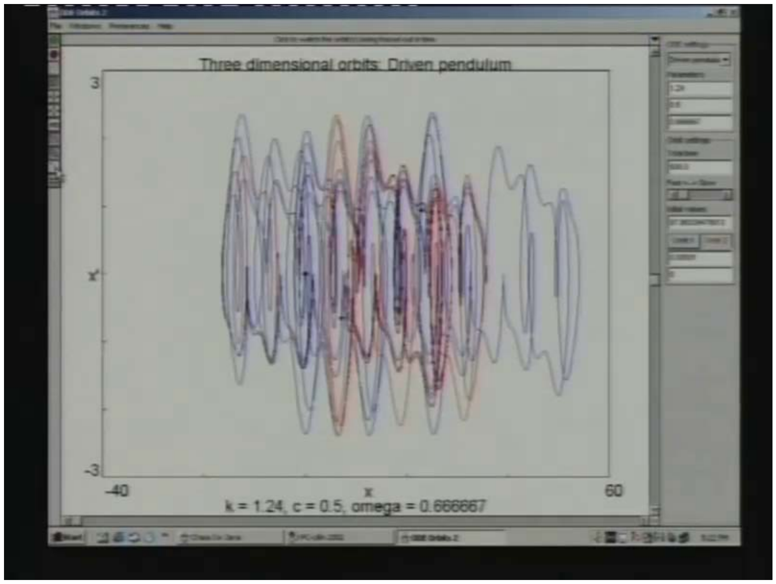

You cannot distinguish them. Now you can, this is the hallmark of the sensitive dependents on initial condition. We have started from very close initial conditions but ultimately they got separated out which means that its behavior is a chaotic behavior. Just to recapitulate what is the technical connotation of the term chaos. One it is the bounded behavior, two it is a periodic behavior and three there is sensitive dependence on initial condition and in all three systems that you are studied we saw this property that there is these three characteristic features. Now you see the last day's one of you had commented that if you have a chaotic orbit, I had said that then prediction fails.

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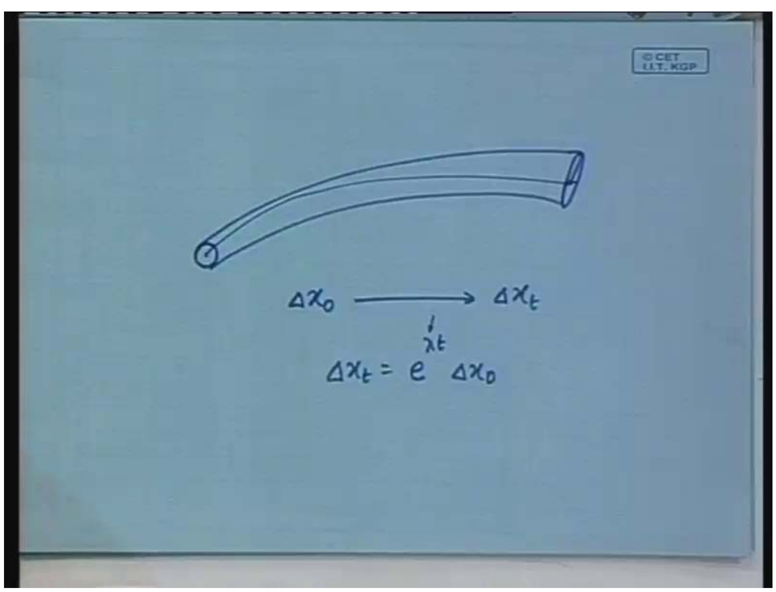

Why because in order to do the prediction you will have to start from initial condition and solve the set of equations to come somewhere there and if there is even a minischool (00:39:58) error in the initial estimation of the state then obviously there will be a large error in the estimation of the state after some time, for due to this particular reason that two extremely close initial conditions evolve completely differently. They exponentially separate out and that's why prediction fails. prediction fails not only because of our instrumental errors. Instrumental errors means if I want to measure the initial condition of any physical system, there will be some instrumental errors which I represented in the last class in terms of an error ball and we had argued that if this error ball reduces then the prediction is successful. If this error ball, I am not talking just of the error ball expanding in size but it is distorting in such a way that in some direction the distance increases then the prediction fails.

Under what condition would the error ball increase in size? If you take the Jacobian, his determinant is greater than one. Then it will be increasing in size, non dissipative system there is a energy input due to which things can blow up but here we are mostly considering dissipative system which means that this error ball will shrink in size but even though it shrinks in size, it may sort of flatten out in such a way that in one direction it grows. In other direction it shrinks and the growing direction that will go to infinity unless something is done to it and that something is where it is folded. That is a mechanism we talked about. Now this failure of prediction does not happen only because of your instrumental errors.

Suppose due to some way by some means, you have accurately estimated the state. Still how will you convey to the computer as a real number? The moment you want to convey to the computer as a real number, computer obviously has a finite length of any number. So it will have to be truncated somewhere. You might argue that I may specify the initial condition by means of a rational number which requires really a finite number of bits. You might argue that but the fact is that in the real number space, the irrational numbers are dense on the real number space which means that if you want to to estimate it in its neighborhood there will always be a some irrational numbers which means that this argument will fail. Ultimately in order to specify that you will need to write down a infinite number of digits in order to specify that, which is not possible and therefore both due to the instrumental error as well as due to specification error there will be this error ball and therefore there will always be an error in the prediction of the future state.

Now this as one of you commented in the last class you can still make a prediction to sometime in the future and how long the time is can be estimated on the basis of how far it goes off. These two initial conditions will go off but there can be the question how far it goes off or how slow it goes off. Two initial conditions will exponentially diverge from each other but that rate of divergence can be different for different systems and depending on that we can confidently say that okay for weather three days hence I can predict with confidence but not more.

For another system, for the given pendulum I can say that 0.5 second hence I can predict with confidence. Obviously that depends on, if this error ball is a round shape and then when it reaches here it takes this kind of shape. Then it went something like this and as a result you can say that here there is an expanding direction and here there is the contracting direction. In general so long as you can represent it as an ellipse I can easily identify the expanding direction as the major axis and the contracting direction as the minor axis.

Then we can say that the separation between two points, let us write delta x_0 the separation between two points and that goes into delta x at some time t. Now if these two points happen to be in this direction then you can write it as delta x_t is e to the power… So here is the term that comes in sort of a linear approximation we are making here. You might argue quite logically that there is no guarantee that this lambda, it really exponentially increases. Yes, may not really exponentially increase but nevertheless if at every point you assume that there is a local linear representation then in that local linear neighborhood, you can see that if there are two real eigenvalues then the immediate evolution will be in terms of e to the power of lambda t and then goes here then again e to the power of lambda t, goes here again e to the power of lambda t. So that this will go on as more or less like this. Only thing is that at every point this lambda may not be the same.

The answer to that question about how long can I predict essentially depends on this value of lambda. If this lambda is positive, we know that there will be exponential separation increasing. If it is negative it will decrease. If it is positive then it will increase, if it is negative then it will decrease. So for a chaotic system we can confidently say that one of the signatures of chaos would be that the lambda will be positive but nevertheless we can still ask the question how chaotic. Some kind of a major of chaos that will be given by this number here. The question is how can we estimate this number in a physical system? Let us consider this issue.

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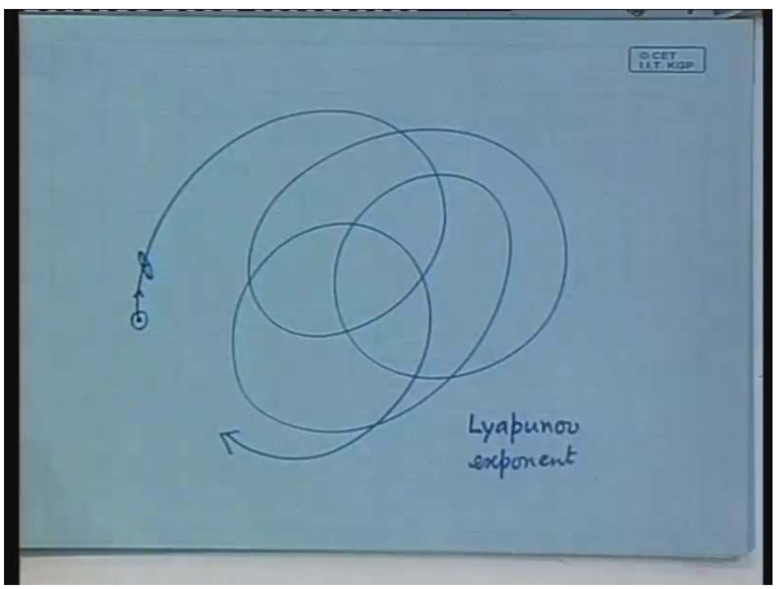

So let us talk in terms of two different representation. Suppose there is an orbit that going like this in the state space. So at every point I can locally linearize it that means at every point I can write down the Jacobian, obtain the eigenvalues and make a reasonable estimate of how this ball is going to change its shape. If it is a saddle kind of orbit I know that it will increase in some direction and it will shrink in some direction so that I can easily say that as it goes, after sometimes it will take shape something like this and the way it evolve depended on the eigenvalues.

So we can confidently say that the unstable eigenvector direction will be somewhere in this direction and the stable eigenvector direction will be somewhere in this direction. So for small stretches of the evolution we can represent the rate of divergence in terms of the maximum, the bigger of the two eigenvalues. If it is negative then we know that it sort of shrinks, if it is positive it expands and therefore if there are 2 or 3 eigenvalues the bigger one will represent the rate of divergence. So we can do one thing. A program can be, suppose we simply look at the Lorenz system like this.

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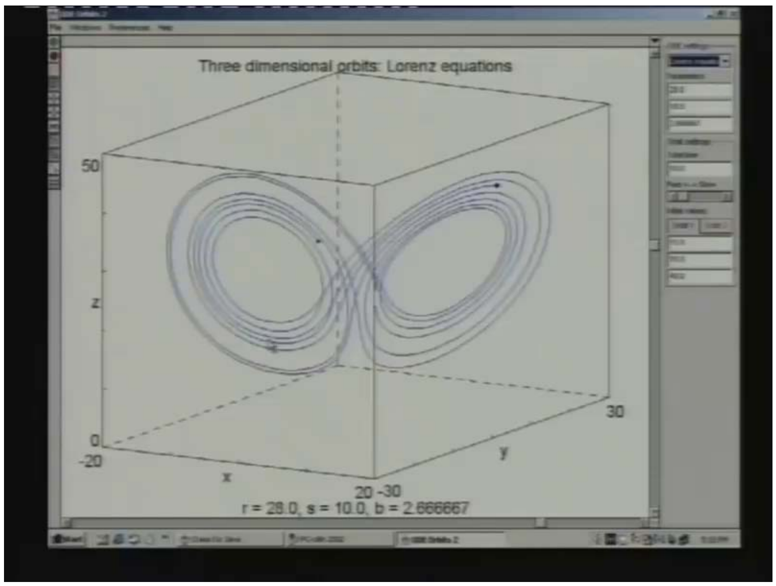

Start from this point and go on, as it goes on at every point I am calculating the Jacobian, calculating the maximum eigenvalue and since this is a non linear system that will not remain the same. At every point the values might be a bit different. So we keep on keeping track of the maximal eigenvalue and then average it around over the whole orbit. Can you do that? Yes so that we get an estimate of this number lambda, average about the whole orbit you can do that. That number is called the Lyapunov exponent. So the concept is that as the orbit goes at every point in the orbit, not at every point in the state space but along the orbit. Why because if you look at this state space, this is an attractor which means that start from any initial condition outside, it will converge on to this orbit which means that if you start from two initial conditions outside their distance will reduce, as it goes homes on to this attractor. So it is not just anywhere in the state space, it is not average over the state space. It is averaged over the orbit.

So as you go along this orbit at every point you locally linearize the Jacobian. You obtain the Jacobian, obtain the eigenvalues and keep track of the bigger eigenvalue, the one that is positive. So you keep on calculating that over the whole cycle as it goes on and average it out. Had it be a linear system that would be necessary? Because then the Jacobian will be same every where but since it is the non linear system this is necessary. That would represent the exponential rate of divergence of nearby orbits that bigger eigenvalue. Now the actual problem is not this, this is trivial.

Given a set of equations you can always do that but the actual problem is that often you have an experimental situation. From the experiment you have got some kind of an orbit because you can measure at successful instants, the value of the position and the moment and the stuff like that or in case of electrical circuits, you can measure the currents and the voltages and thereby you can plot the orbit on CRO screen. You can see the attractor, chaos and everything can be seen on the CRO screen. Now suppose there is an experimental situation like that, how do you going to measure the Lyapunov exponent? In other words suppose I have got a set of equation that you are not very comfortable with linearize at every point.

Let's take it like this. Can we take another route which will give us some clue about doing it on an experimental data set? Let's try to understand this question. What question am I asking? Essentially thing is that at every point I have to local linearize and get the eigenvalues but of course I cannot do that unless I have a hold on the equation set. So let us try to formulate some other means of doing so which can be applied also to experimental data where you do not have the equations. So in that case what we will do?

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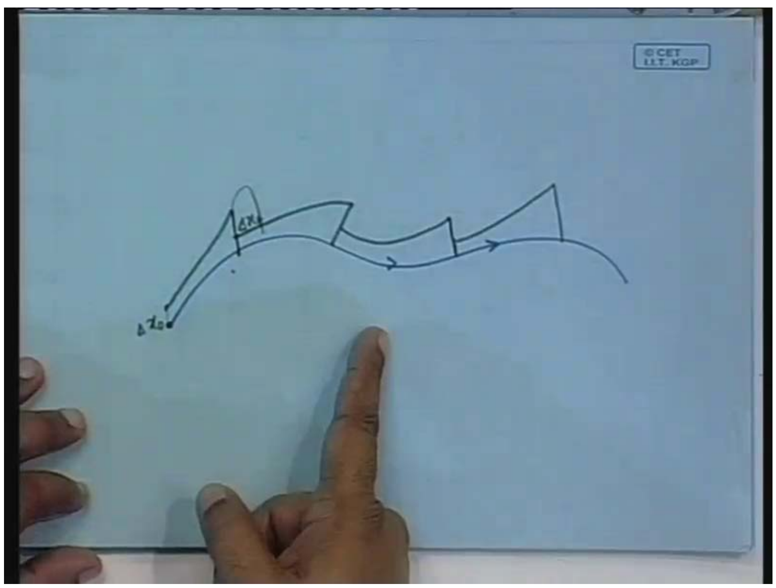

We will start from an initial condition which will sort of evolve in the state space. We are trying to find out how nearby initial conditions diverge and therefore we will have to take another initial condition say we will take so much here. So this orbit, the blue one is the natural orbit or I shouldn't say natural orbit, every orbit is natural. It is the orbit that we have calculated initially and then we perturb it and then recalculate the orbit. From here as a result of which its distance will go on increasing after some time stop the procedure and calculate how far it has moved. So this is delta x_0 and this is my… I can now express it in that term since I know the time. I can express it that delta x_t is equal to e to the power of lambda t time's delta x_0 but this is only about this much. Can we do that over the whole? No we cannot, why because this expanding direction will go on changing. As we have already learned it will fold. So after some time this expanding if it folds, it will come back to a position near to it. Even though there is a stretching in the state space, the point might actually come closer. Will it not?

Suppose you start from another initial condition it goes on in this orbit, suppose after some time this fellow go this way and that fellow goes that way. After some time both come to one of the lobes. Will the distance reduces? You would not infer from there that the exponential divergence is stopped, no it's still there. So what we will you do? We will then have to rescale this difference, after some time we have to stop it and again we have to come to a point that is closer to this one. Again we have to evolve, we come to this point. Again we have to come to a point that is closer again we have to evolve this point, again we have to find where the difference is so we have to go this way. Why because it actually folds frequently that is a character of the state space of such systems?

So periodically you let it evolve two trajectories. Let it evolve, one of them is what is known as fiducial trajectory, the trajectory is the nominal trajectory and the other is part of trajectory. So the part of trajectory you allow to evolve for some time and then you rescale it in that direction. Do you understand that? Why do you have to put it in that same direction, why can't you start here from another position? Because this has already aligned, this line has already aligned in the direction of the expanding direction. So you take another point closer to this initial condition, along the same expanded direction let it evolve. The expanding direction moves, changes again come here, take another point so do you understand the algorithm the way you have to write it. From here to here the evolution is calculated by the set of equations by the runge-Kutta method and these are also calculated by the same things but depending on periodic rescaling and again you have to evolve by the same method. Today let us stop here. In the next day we will see how this can be also taken to the experimental situations where you do not have the equations to evolve where you have just a data set coming from the experimental set up. So that's all for today.