Chaos, Fractals & Dynamical Systems
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Lecture No. # 39
The Monodromy Matrix and the Saltation Matrix

In the last class we were dealing with a Floquet theory by which one tries to assess the stability of periodic orbits. If you have a periodic orbit like this, the way to understand the stability is start from some initial condition. That means this is x at t_0 and then as you go around this orbit, you end up here at x at capital T. Then the whole period is T minus t_0. The way to understand the stability is to express x at capital T as some function phi times x at t_0 and this phi will be dependent on the initial time, the initial value and the total time, so t, x_0, T.

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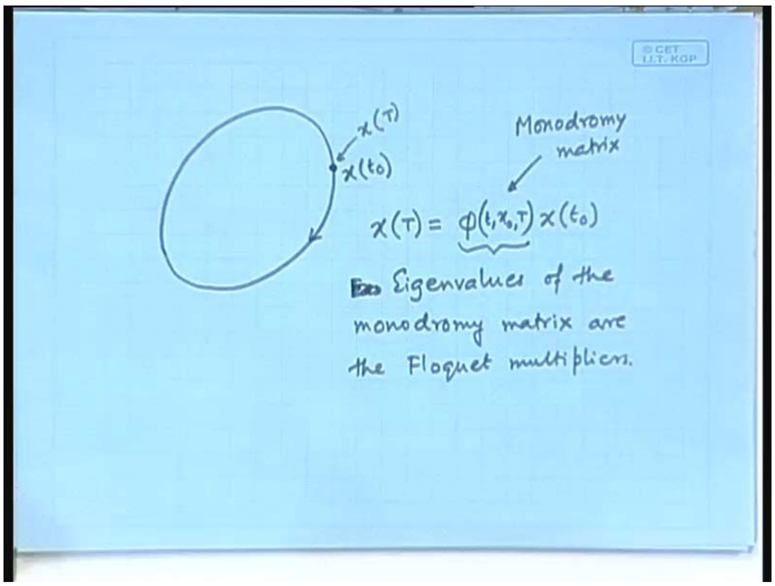

This particular thing is nothing but a matrix because here you have a vector that times a matrix gives the final vector. This matrix as we told is the monodromy matrix. The Eigen values of the monodromy matrix are the Floquet multipliers. Ultimately the job is to identify this matrix and obtain its Eigen values.

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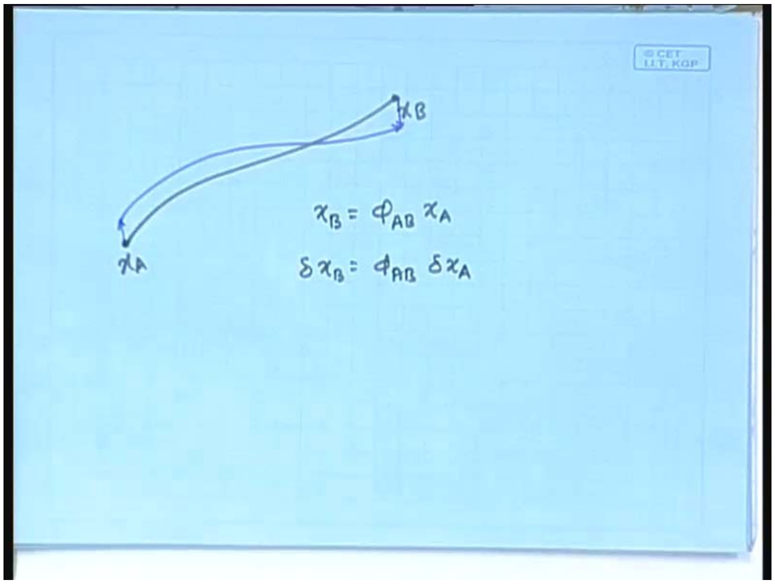

Now as we have said that if we are evolving something from here to this point say I have x_A and here I have x_B then x_B when expressed in terms of x_A is x_A pre multiplied by the state transition matrix. That is how we understand in standard control theory, so x_B is the phi, the state transition matrix times the state x_A. So that is the concept of the state transition matrix then we will write x_B is equal to phi_{AB} times x_A. Now if we write this then it is also true that if you take a perturbation from here and that ends up as a different perturbation say here then this perturbation, the delta x_B can also be expressed as phi_{AB} delta x_A. The way states are related, the perturbations are also related. We will essentially follow this up in order to develop the concept of the monodromy matrix.

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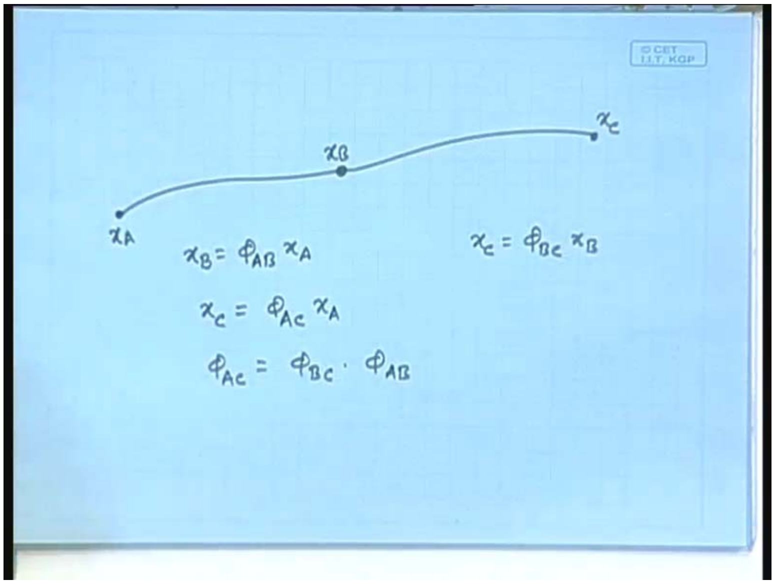

In the last class we had started saying that here if you have a state x_A then suppose it goes to state x_B and then it continues to state x_C or in other words, it goes from x_A to x_C in between a state is x_B. Then you can write as we have just done x_B is equal to phi_{AB} x_A, you can also write x_C is equal to phi_{BC} x_B. These are the individual state transition matrices then you can write x_C is equal to phi_{AC} x_A from here to here and then phi_{AC} will be nothing but phi_{BC} times phi. So simple is if you have them. You have the flight from x_A to x_B to x_C then if you want to obtain the state transition matrix from A to C, it is just a multiplication of the state transition matrix from A to B and B to C. So far so good. This is the general theory that you learned in standard control theory texts. Now what are these?

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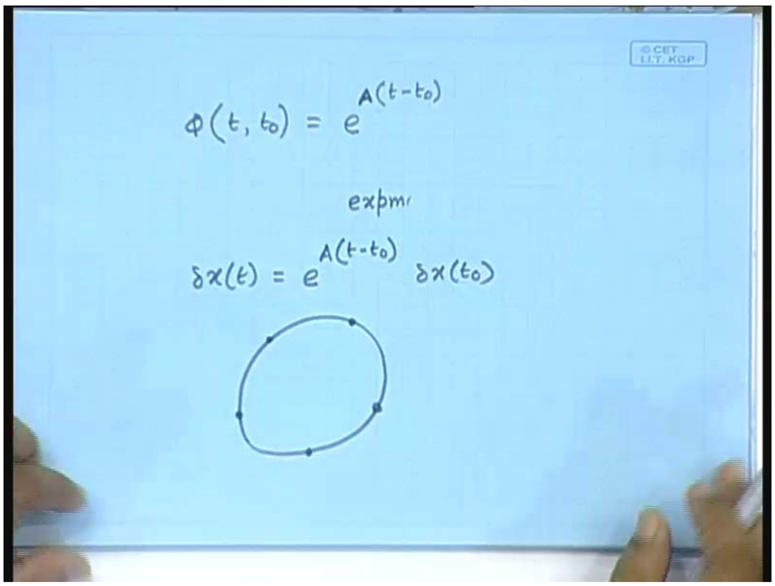

The system is linear time in variant L ti then your phi going from t, starting from t_0 will be… starting from t_0, flight over a time of t will be or t_0 2 t will be exponential matrix A (t - t_0). Here this A is a matrix so this is a matrix exponential. The way the matrix exponential are treated, I do not have the time to discuss it in details here but this can be found in any standard control theory text book. Essentially the way we handle any exponential, we handle the matrix exponential in a same way as a binominal expansion. If you know the matrix A that means if you know the elements of the matrix A and if you know the starting time, the ending time then you can evaluate this just as a number and in MATLAB this matrix exponential evaluation is given by the expm function. So all you have to give is expm within bracket and you have to give whatever is in the exponent.

This is the simple thing and if you know this then you can easily write delta x of t. That means the perturbation at time t is e to the power A (t - t_0) delta x at t_0. This is nice and this tells you that if you have an orbit something like this then if you can start from here, go up to here and if you can calculate the state transition matrix from here to here and then if you can come back then all you need to do is to multiply the same state transition matrix and the state transition matrix to obtain the total state transition matrix of the whole cycle.

Or you can break it up into further segments say five segments. In that case you have to go from here to here, obtain the state transition matrix, go from here to here obtain the state transitions matrix and then multiply all that. No, it doesn't work. It doesn't work if you have non-smoothness in this orbit. Let's come to why.

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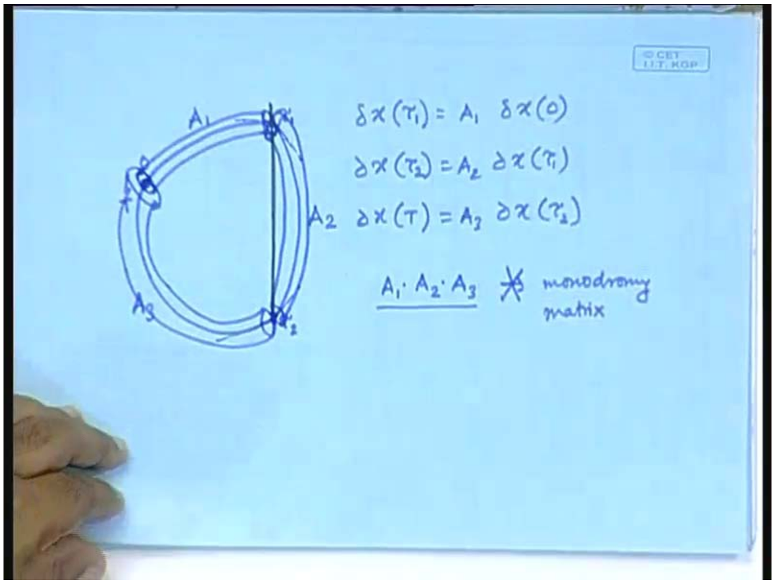

Suppose you have an orbit going something like this. Say here is a switching manifold, it starts from here and it hits like this and then it goes like this and this is different, the vector field is different and then it goes like this. This is the starting point and here you have the vector field is different in the sense that if you draw these two tangents, they will be different. If you draw these two tangents they will be different. In that sense the vector field just before crossing and the vector field just after crossing are different and that is exactly what happens in most non-smooth dynamical systems that we have already seen.

The question is now, if you say that from here to here, my state transition matrix is A_1. From here to here, the state transition matrix is A_2. From here to here, the state transition matrix is A_3 then you cannot say that means you will write delta x of say here it was hitting at tau_1, here is tau_2 starting at 0, ending at T. Then x (tau_1) is A_1 delta x (0), delta x (tau_2) is A_2 delta x (tau_1) and delta x (T) is equal to A_3 of x (tau_2). If this is how the A_1, A_2 and A_3 are defined then you cannot say that the product A_1, A_2, A_3… (Refer Slide Time: 11:47). This is actually not equal to… this is important because most people make a mistake at this point that these are not the same. Why? The reason is something like this.

When you are starting from here, when you are writing this, when you are writing this, delta x_0 you are essentially saying that I am considering a perturbation and then when you are reaching here your saying that this is how my perturbation is flying. Then at this point the original trajectory reaches the switching manifold and the perturbation is something like this. But then after it starts, you see that all the points on the perturbation do not reach the switching manifold at the same time, they reach at different times and therefore after the perturbation when you start, you start from different ellipse. Then you go on, come to this and you get an ellipse. Here also you have a some kind of a different ellipse to start with and then that goes on. When it reaches here, whatever is the size or the shape of the ellipse that is given by the total monodromy matrix. The initial ball means you consider a circle or a sphere and then pre multiply that by the monodromy matrix gives you the final ellipse or ellipsoid and that tells you, whether or not in any particular direction it elongates or shrinks, contracts. If it contracts in all the directions then it is stable. If it does not contracts in all the directions that is unstable so on and so forth. But it's not difficult to see that since all the points, all the perturbed trajectory do not reach the switching manifolds at the same time. Therefore this will not be valid, this will not give the monodromy matrix. That was valid for this case because there was nothing like a switching.

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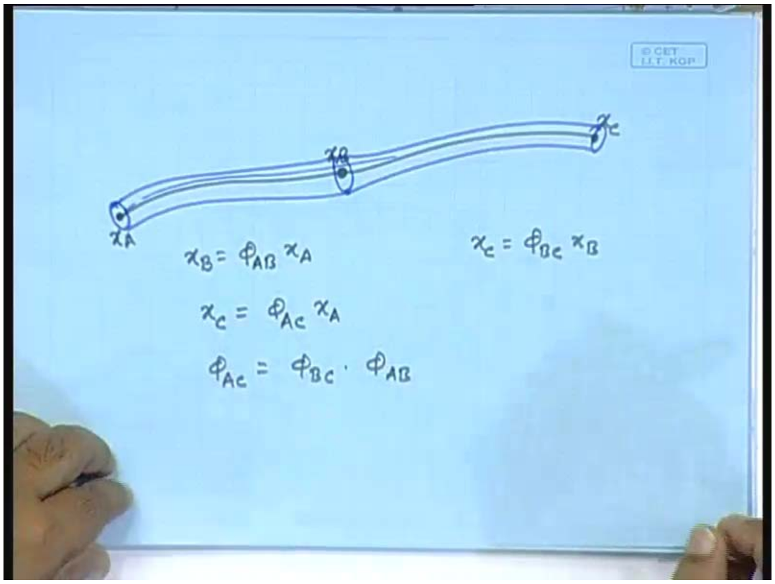

If you consider a perturbation that evolving here and then that continue to evolve here. Essentially it is a same continuous evolution and that is exactly why it worked but in this case it will not work. In that case how to handle it? When we tried to handle it in that situation, essentially let's start in a neighborhood of the switching manifold say here is a switching manifold and here is a part of the orbit that reaches the switching manifold.

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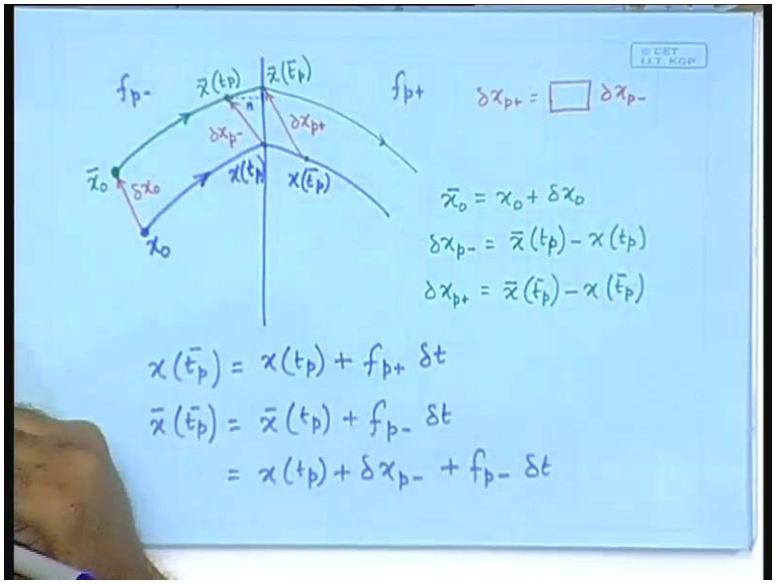

It starts from a point say x_0 and reaches at, I will write it here xt, t_p is the time when it reaches. Zero is the time when it starts and t_p is the time when it reaches and then it goes off. There is a perturbed trajectory that starts from this point and it also goes and reaches and from this point it goes off. The blue one is the original trajectory and the green one is the perturbed trajectory and the perturbation is given by this vector and this vector is delta x_0. The perturbed trajectory is x_0 bar and this is delta x_0. When this fellow has reached the switching manifold, this fellow has not reached and supposing at that time when it is here, this fellow is here. So you can draw the perturbation like this at this point.

What is this particular value? It is x bar which is at the time t_p and then after some time, it reaches the switching manifold and then it goes off. When it reaches a switching manifold, this fellow has already crossed and has come to some distance say here. At that point you can draw the perturbation. Now this perturbation here is delta x, we will call it p minus and this perturbation will call delta x_{p+}. Essentially we are interested in how the perturbation evolves across the switching manifold and you can see that, this perturbation is wholly in this side of the switching manifold and this perturbation is wholly in that side of the switching manifold.

We are trying to find out how this one maps to this one or in other words, we are trying to find a expressions so that we can write delta x_{p+} is equal to something times delta x_{p-}. That is how we are trying to express. What we are trying to express? We are saying delta x_{p+} is something times delta x_{p-}. Once we do that then this is the state transition matrix across the switching surface, this term. We will do that. Now a few things we need to write. What is this? This particular point is green, x bar this is at t_p bar. The t_p bar is a time at which it reaches the switching manifold. So this one is x at t_p bar, so this is the setting in which we are trying to do things. Let us carry on. First thing, what is x_0 bar that means perturbed trajectory? X_0 bar is x_0 plus delta x_0.

Now we need to find out this perturbation and this perturbation. What is this perturbation? Delta x_{p-} is, this minus this. The x bar t_p - xt_p and delta x_{p+} is equal to x bar t_p bar; t_p bar is the time at which this trajectory reaches the switching manifold minus x (t_p) bar. These are the three equations that we can easily write, basically relating these two. Since we are interested in this equation we are interested in the right hand side, how it will ultimately come up. Now you see xt_p bar is this one (Refer Slide Time: 20:30). So xt_{p+}, it has evolved through this. If your vector field in the left hand side is f_{p-} and the right hand side is f_{p+} then it has evolved through the right hand side vector field. This is xt_{p+} in the first order approximation, you will have f_{p+} times the time that is taken. That is delta t so this plus the vector field times the time that is taken. Similarly this point here x bar t_p bar is this x bar t_{p+} f_{p-} and the flight time. X bar t_p is here, so we can substitute. We can write this is equal to xt_{p+} plus delta x_{p-} plus f_{p-}. We have expressed these two. Next step, keep this thing in mind so that we need to refer back to it.

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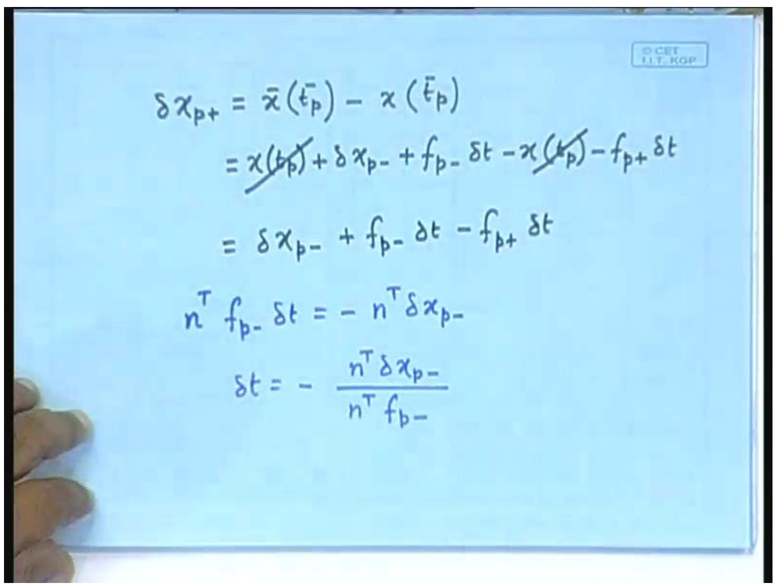

Now we start from delta x_{p+}, this is what we are trying to find out. We are trying to find out this. We have already seen that this is the x bar (t_p bar) minus xt_p bar and the two right hand side that also we know so we will express it as x bar t_p bar is here and xt_p bar is here. We will substitute these two and thus we get x(t_p) plus delta x_{p-} plus f_{p-} delta t minus x (t_p) minus f_{p+} delta t. These two cancel off and so we are left with delta x_{p-} plus f_{p-} delta t minus f_{p+} delta t. Essentially this minus is this times delta t, so far so good. If we drop an orthogonal here then you would notice that delta x_{p-} times the normal to the switching surface is equal to, this is f_{p-} delta t normal times the f_{p-} delta t. If you take its component here and its component here, they are just opposite to each other and the components are obtained by taking the normal to the switching surface.

The normal to the switching surface is n, say n is here, so n times this vector is equal to negative of n times this vector. We can logically write n transpose because n will be a vector, it has to be transposed in order to multiply with it. This times f_{p-} delta t is equal to minus n transpose delta x_{p-}. If you are not convinced, this can also be obtained from the condition that this evolution satisfies the condition for the switching manifold at this point and this evolution satisfies the condition of the switching manifold at this point and switching manifold is beta x equal to 0.

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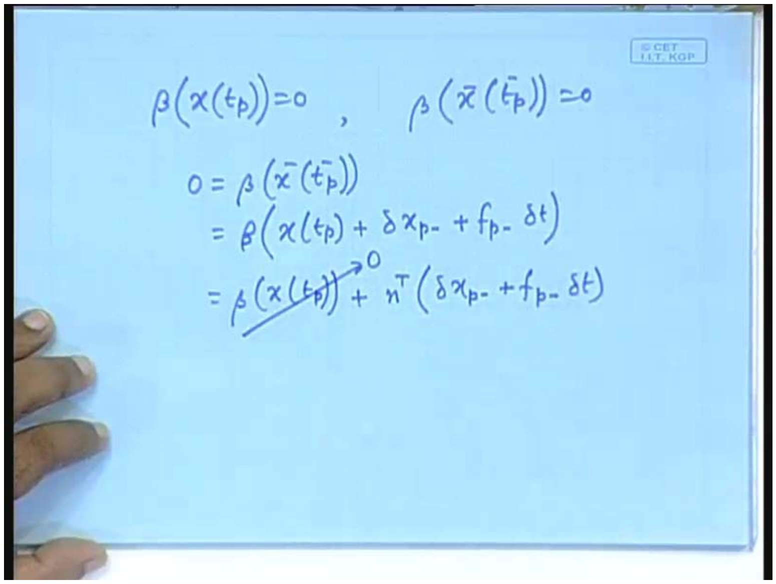

We can write beta of xt_p equal to 0 as the condition, when this is satisfied and beta x bar t_p bar equal to 0, as the condition when this is satisfied (Refer Slide Time: 27:02). Now you can take this and expand this. I will write 0 in the left hand side, 0 equal to beta x bar t_p bar. We will expand it, beta xt_p plus; beta x bar t_p bar is just from here, you already have it; plus delta x_{p-} plus f_{p-} delta t and this can be taken out and this can be written as beta xt_p plus n transpose delta x_{p-} plus f_{p-} delta t. Now this term is 0 because of this, so you get this which is the same as writing this.

At the end of the day, you have delta t expressed as minus n transpose delta x_{p-} divided by n transpose f_{p-}. This is the expression for the additional time taken by the perturbation to reach the switching manifold. Now once we have it, things fall in place because we started with this. We wanted to express this and this was ultimately expressed as here.

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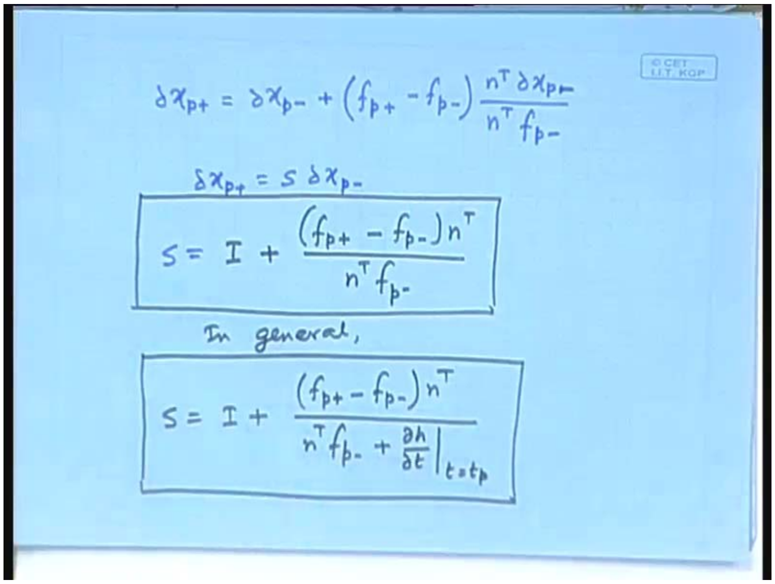

We will start from here, we will write. Just keep this in mind, I will write it here. Delta x_{p+} is equal to delta x_{p-} plus f_{p+} minus f_{p-} times minus delta t. Can you see? If you take delta t out, it is f_{p-} minus f_{p+}, we are putting the negative sign out so that this get cancelled off. So this times nT delta x_{p-} divided by nT f_{p-}. Now what are we driving at? Ultimately we are driving at this and you can see, there we have it really. We are trying to find this which is delta x_{p+} divided by delta x_{p-}. So if I divide now both sides by delta x_{p-}, we have it. We will write this particular thing which is called this saltation matrix. We are actually trying to express it as delta x_{p+} is equal to saltation matrix S times delta x_{p-}. What is the definition of the saltation matrix? It is the state transition matrix across the switching manifold that is called the saltation matrix. Saltation the word means jump, so it is just nothing but a jump matrix. In some literature you will find the word jump matrix.

You can easily see from here then the S will be; this will be divided by x_{p-} so it is I plus x_{p-} has been divided by so this goes off. You will say it is f_{p+} minus f_{p-} bracket times n transpose divided by n transpose f_{p-}. This is the saltation matrix, this is the expression for the saltation matrix. Now in deriving this, so this is important. Keep this in mind. In deriving this, we had assumed that this surface is static. We did not assume any movement of this surface but in general it can move. For example the switching manifold, the switching surface there is no reason to assume that it will be always absolutely static, it is possible for it to move. In case of the impact oscillator, the impacting surface can move. In case of the switching circuits, the condition for switching can be a moving surface so that's all actually not only possible reality.

In that case the expression in general will turn out to be I plus, the numerator remains the same f_{p+} minus f_{p-} n transpose. Here it will be same thing n transpose f_{p-} but additionally there will be one term that relates the rate of change of the switching surface. So will be the partial derivative of the switching surface with respect to time calculated at t is equal to t_p. This is the total final expression for the saltation matrix. Once we have it things will become rather simple because let us come back to the condition that we took. Leave it, let me draw it again.

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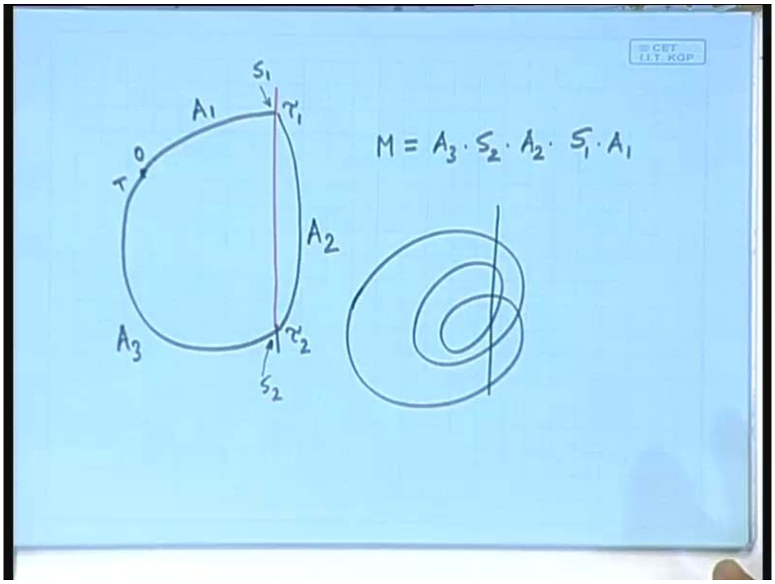

Here is your switching surface and here is your starting point, it go up to this point, come up to this point and say you go up to this point. This is the evolution in that case, this is the starting point 0, this is the ending point t. Suppose your evolution from here to here is given by A_1, from here to here is given by A_2 and from here to here is given by A_3. The state transition matrix from here to here up to the time tau_1 up to the time tau_2, 0 to tau_1 is A_1. Tau_1 to tau_2 is A_2 and tau_2 to capital T is A_3. In that case as I told you cannot say that the total monodromy matrix is A_1 times A_2 times A_3. Here at this point there will be a saltation matrix say let us call it s_1 then there would be another saltation matrix at this point s_2. Then the monodromy matrix M, we will start this one in the right hand side; A_1 times S_1 times A_2 times S_2 times A_3.

Notice the order in which we are writing. It is starting from the right, proceeding to the left as you go around the whole orbit and it is a very simple to do the same thing for an orbit something like this. If it is a complicated orbit, higher periodicity with number of switching's it is nothing but you start from a point and you find out this state transition matrix from here to here and then at this point you again obtain the saltation matrix. Again the next flight, again the saltation matrix, again the next flight and then you simply multiply them. That is how you obtain the monodromy matrix and the Eigen values of the monodromy matrix will be the Floquet multipliers, that's all. If the Floquet multipliers are inside the unit circle, you have a stable periodic orbit. If it is outside the unit circle, you have an unstable periodic orbit.

This is how we actually find out the stability of periodic orbits and in case of the non-smooth dynamical system where there is some kind of a switching. This is the complication ultimately that can easily be solved and ultimately you can obtain the monodromy matrix. Now as I told you, we were considering border collision bifurcations in which the ultimate thing that we need to evaluate is how do the Eigen values change as the fixed point goes across the unit circle. There an orbit like this will be just a fixed point of the corresponding Poincare map and the Eigen values of the fixed point will be same as the Eigen values of this (Refer Slide Time: 37:35).

When such an orbit crosses the switching boundary that means, imagine that there was an orbit something like this.

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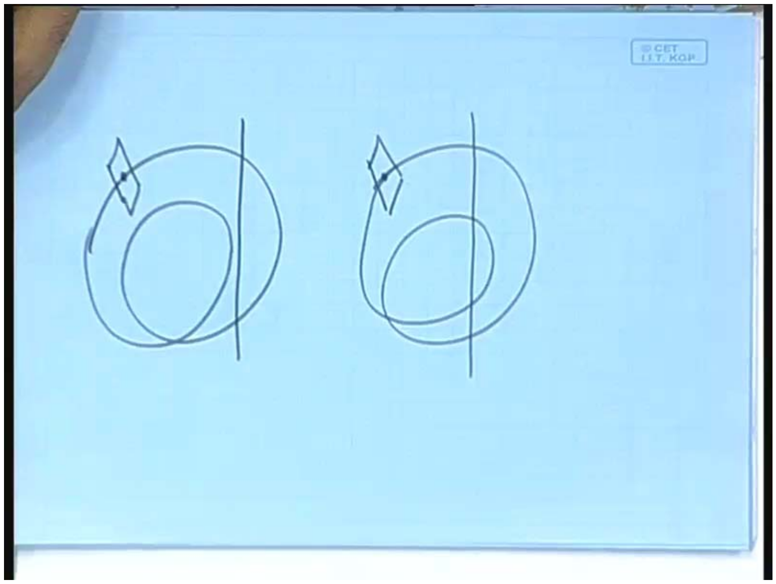

As you change the parameter it crosses and it becomes something like this. Then there has been a change, there was only one excursion to this side earlier. Now there are two excursions here, so in between there has been a border collision event, a greasing event which is nothing but a border collision event. Naturally in order to understand that border collision event, you will have to find out the Eigen values of the fixed point, the fixed point in the discrete map. The discrete map is obtained something like this. You have got a Poincare section and you look at the position here and here you look at the position here and you are interested in the Floquet multipliers. You are interested in the Eigen values of that fixed point which is nothing but what we just obtained.

In one case you will have to obtain the Eigen values of this orbit, its Floquet multipliers and here in this orbit and these two Jacobian matrixes will need to be substituted in the theory that we already presented and there the theory will predict what will be the outcome of this bifurcation. This is how we apply the theory to a particular complete situation of a non-smooth dynamical system. Is that clear?

We have learnt how to handle non-smooth dynamical systems and then let us now go to another topic because we have more or less covered the issue of this in detail. Let us now treat the problem of how to actually do experiments in nonlinear dynamics. Experiment means where you are trying to construct something, you are trying to observe its trajectory in the state space. You are trying to observe how it looks on the Poincare section. You are trying to observe how it looks, when you draw the bifurcation diagram that means some way of experimentally obtaining the bifurcation diagram. We have already treated that in many cases, you will need to reconstruct the dynamics by state space reconstruction technique but that apart. How to actually do the experiments?

Now let’s consider this one by one.

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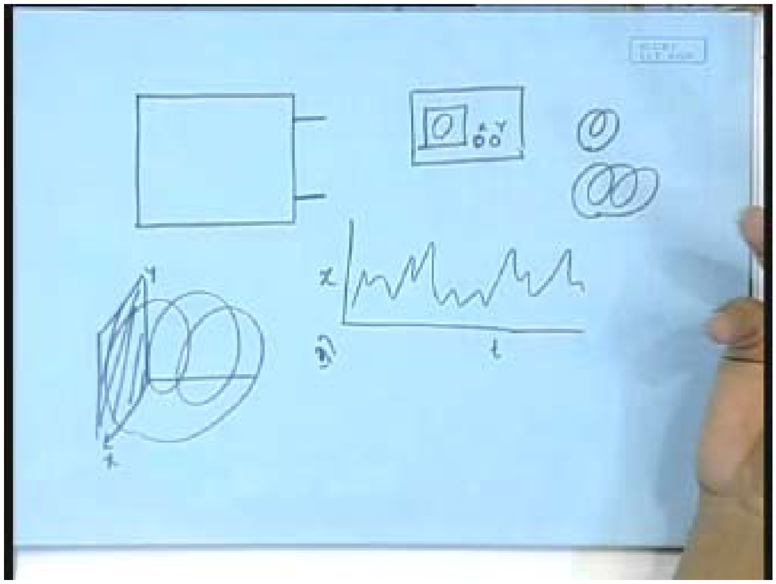

First let us assume some kind of a generic experiment running inside this box. What does it mean? It means that supposing inside you have got a Chua circuit, there is some kind of an oscillation going on in the voltage and the currents. If it is some kind of an electro chemical oscillation, so there is a container in which the constituent components, constituent reagents are changing in the concentration as a dynamical system. If you have mechanical dynamical system, you have got some physical mechanical component moving and ultimately you have got this and you get some kind of a sense of that. What I mean by sense of that is if it is an electrical system, you have some way of measuring the voltages and the currents and the currents are also very often measured as voltages by passing it through a standard resist so that you get a voltage proportional to a current. If you have some kind of a mechanical system represented by say the position, the momentum, the pressure and stuff like that, all these need to be ultimately converted to some kind of a voltage signals. So you need transducers. If you have an electrochemical experiment going on, that also needs to be converted to some kind of an electrical signal in order to actually observe it.

Now let us first illustrate it with reference to an electrical experiment, some kind of electronic circuits in which you are observing this. What are the results of the observation? Results of the observation is some voltage at some points, some voltage at another point and one of this voltage may be a current proportional to a voltage and then we can put that on to the CRO and in the CRO, you will find that there are a few ports, different ports. It is x port and the y port. If you put it in the x port and measure it against time, you will see some kind of oscillation like this. Here is your time, here is your x. There is a knob that you can turn in order to plot it in the xy mode. If you do that and you have to put the other variable also in the y mode, y channel then you get what is actually the orbit in the state space not exactly that. Suppose this is a higher dimensional system but you are observing only two then it is projection of that in the direction of this xy.

You might imagine that it is actually three dimensional system with some kind of a oscillation going on but when you take your say x coordinate and the y coordinate then you are essentially looking from here as if your I is here and you are seeing the projection of it in the xy plane. That is what we will observe here. You will not be able to actually observe the three dimensional thing because for that equates a difference visualization technique. It is possible to do a data acquisition and plot this on a computer in 3 D that’s possible. You have a plot of the phase plane. If you see it is something like this, you know that it is a periodic orbit. If you see something like this you know it is period two orbit, you see something like this you know this is period three orbit and so on and so forth.

Suppose you want to observe it on the Poincare section what do you do? As you know the Poincare section can be obtained in two possible ways. If it is an autonomous system, you will have to place the Poincare section physically that means if it is an autonomous system and you have some kind of orbit like this and you have to choose a particular value at which you place the Poincare section say here.

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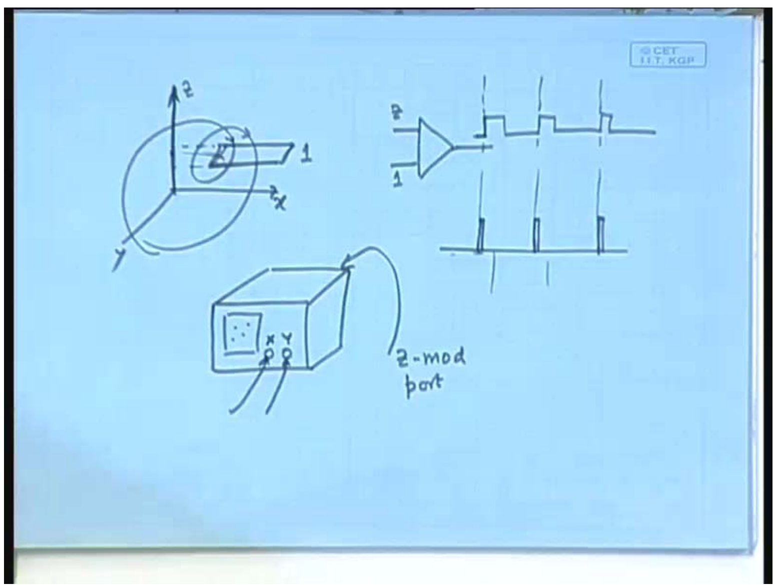

What does it mean? This value corresponds to some particular value of a coordinates say z, so a particular coordinate satisfying a specific value is equivalent to placing the Poincare section. What will you do? You will sense this and these will have to be compared that means the z coordinate value, z coordinate signal has to be compared with a given DC voltage corresponding to this level. You get the point? The point is that here you have the x coordinate, here you have the y coordinate and here also you have the z coordinate and you are placing the Poincare sections such that whenever z is say 1 then I observe it. What does it mean? I will have to take the voltage corresponding to the z coordinate, say this is the voltage corresponding to z coordinate. I will have to compare with a signal one and I have to put a comparator.

Whenever this comparator changes sign, you know that it has been crossed but you are not trying to observe it in both the directions. You are trying to observe it when it crosses from one side which means at this point, you will have a signal something like this out of which you either choose the positive edge or the negative edge but not both. You either choose the positive edge or the negative edge but not both so whenever this is happening, you need to observe the x and the y coordinates. You are actually not trying to observe the z coordinate ultimately, you are trying to observe the x and the y coordinate when a specific condition on the z coordinates is satisfied. So what you do, suppose you are trying to observe it at the positive edge. Then ultimately from here you need something like this. At this point you want to observe, at this point you want to observe, at this point you want to observe. You need a very sharp impulse at these points meaning that I want it to observe at that instant.

How do you obtain sharp impulses from this? It is rather simple. All you need to do is to put a differentiator. If you put a differentiator at this point, it will give a spike. At this point it will give a negative spike, at this point it will give a positive spike, at this point it will give a negative spike and all that. Put a diode, eliminate the negative spikes, you get only the positive spikes at the point where you want to observe it. You have a sequence of spikes, you have a sequence of impulses at the points where you want to observe it now.

In a normal CRO you will find that at the back of the CRO, there is something called a z modulation port where if you put this signal then it will make the CRO observe the x and the y coordinates only at those instances, when it is positive. The simple way of doing it is that in the CRO, here is the screen and here is the x and the y. Put x and the y but also at the back put the signal, this signal at that z mod port. In some CRO's you will find that it observes when it is zero. Some CRO's observe when it is positive so depending on the specific CRO that you are using, you might need to put inverter. That means you will need to make this value 0 when you want to observe it else it is a positive value that depends on the model of the cathode ray oscilloscope that you are taking. So here is the x port and here is the y port, you take the x port and the y port put these things.

So what you do? Ultimately what do you get on the screen? You see dots. Where are the dots? That is when x and y are observed at this instance which means you have done the sampling, you are observing it on the Poincare section. So whatever appears on the CRO screen is the face portrait in the discrete time. This is actually how the observations are done in mechanical or other domains essentially the same procedure has to be followed only you need to get xyz as a voltage signal. Now come to the point how to obtain a bifurcation diagram on a CRO screen. For obtaining the bifurcation diagram say something like this.

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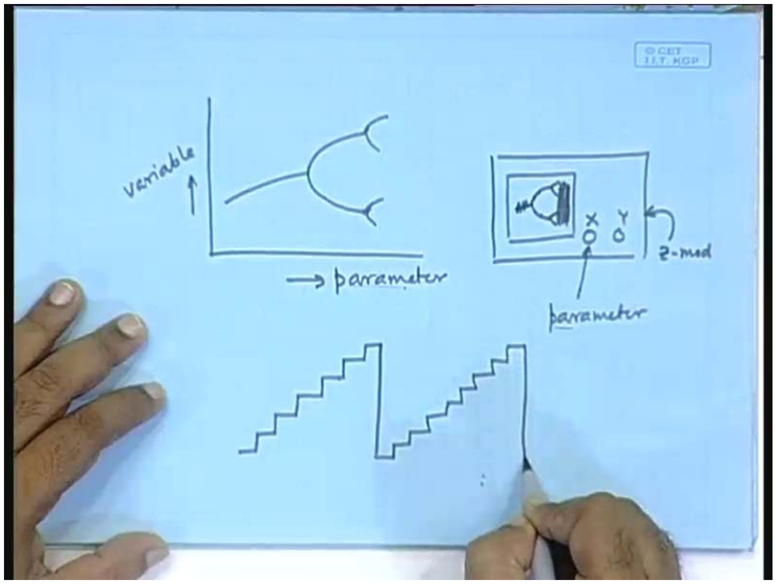

What is your x axis? It is a parameter. What is your x axis? It is a variable so this variable could be xy or z and these are as I told you have to be obtained as a voltage signal. But parameter then also has to be obtained as a voltage signal then only you can put it on the CRO. Now some parameters are naturally a signal parameters. For example if there is a reference signal that you are giving and that itself is being used as a parameter, you can use that or a circuit in which you are applying a input voltage which is variable that is a parameter. So that can simply be taken as a x axis coordinate. Why? Because that is already available as a voltage signal. In the cases where you do not have a parameter directly available as a voltage signal, you have to cook up some way in which it is converted into an equivalent or proportional voltage signal.

Suppose a resistance is varying then how to do that? You will have to use a two limb rheostat, one limb is your load, the other limb varies at the same time and you apply a voltage. You allow a current to flow and depending on the resistance you get the sense of voltage. So there are various ways of actually doing it but the essential message is that ultimately you have to obtain the parameter also as a signal. Now put the parameter, the voltage corresponding to the parameter in the x coordinate of the CRO. So here is a screen, here is a x and the y, x will now be the parameter and put the y coordinate as one of the variables. For example xy or z, you take one of this and put in the y coordinate.

Normally what will you see? For a specific value of the parameter you will get just a point, if the orbit is period one because parameter has a specific value and if it is a period one then the sample value also has a specific value. Sampled, that means here you have already put this particular signal in the z modulation port. That means you are not observing it continuously, you are observing it in the z mod. Then as you change the parameter that means you change the input voltage, change whatever is the parameter then this coordinate will change. As a result this point will move and suppose this orbit goes through a period doubling. Then what will you see? As the parameter reaches a particular value, you will see there are two sampled values for the state y, again two.

You will get something like this. Again if it goes into period four, it will get something like this. If it is chaotic orbit, you will get as much. The whole bifurcation diagram actually appears on the CRO screen but not at the same time. As you move, it actually swifts through, this bifurcation diagram actually swifts through and when it comes here this is no longer there, it is vanished. How to capture the whole bifurcation diagram? In the olden days we used to take a camera, switch everything off, all the lights off. Take the camera and make a very long exposure that means it its kept open so long as it is being given a sweep, so this parameter is given a sweep and it ultimately captures the whole bifurcation diagram on the film. Develop it and you have it.

Now it is you have got the digital storage oscilloscope where you can store as you give the sweep we can store. Ultimately you see the whole bifurcation diagram on the computer screen. This is another nice way of doing it but one word of caution, the digital storage oscilloscopes measure the actual signal and the noise with equal intensity. If the system is noisy then you will see a lot of smudge around this, if it is done with the digital storage oscilloscope. While in analog oscilloscope the actual signal is brighter than the noise signal so you see the bifurcation diagram better with analog oscilloscope. But nowadays it does because it is somewhat cumbersome to switch lights off and put a camera and do that slowly.

The other way of doing it is that if you have a parameter, one way of giving a sweep is to generate a signal that is stepped like this. That means this parameter instead of actually slowly varying it, you gave a stepped input. How do you do that? You can generate a step signal and use that as the parameter and if you do so then what happens is that exactly the same thing that you do in simulation. What you do? You change the parameter and then you observe it, again you change the parameter in another step you observe it, so that is how you do. If you give this kind of stepped signal as the parameter, it automatically observe it and say after sometime it falls and then it goes like this again. Again falls, what will you see on screen? It will be swept, sweep will go on and you will see the image static on the screen. Image of the bifurcation diagram static on the screen. That is another nice way of doing it. We will continue in the next class.