Chaos Fractals and Dynamical System
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Lecture No. # 36
Multiple Attractor Bifurcation and Dangerous Bifurcation

So far we have been studying the non-smooth bifurcation phenomenon that will occur in one dimension or two dimensional maps. We will go to more details of it but before going to that stage let us take stock of where we are. Essentially I said that there are a large number of physical and engineering systems which when modeled in a discrete way gives rise to piecewise smooth map. Therefore in order to understand the bifurcations occurring in them, you will have to understand the bifurcation that can occur in a piecewise smooth map and then we said that in the neighborhood of the border collision event we have… if you want to understand what exactly happens we can locally linearize the state space around that border crossing point and thereby we obtain the normal form and then we went on analyzing that normal form and we have gone on and on. Let us come back and then figure out where we are. Our objective was that we should be able to explain the bifurcations occurring in such systems. Based on whatever we have learnt regarding the character of the bifurcations, the different types of border collision bifurcations that can take place and so on and so forth. Can we now explain mathematically the bifurcation that we actually observe in systems?

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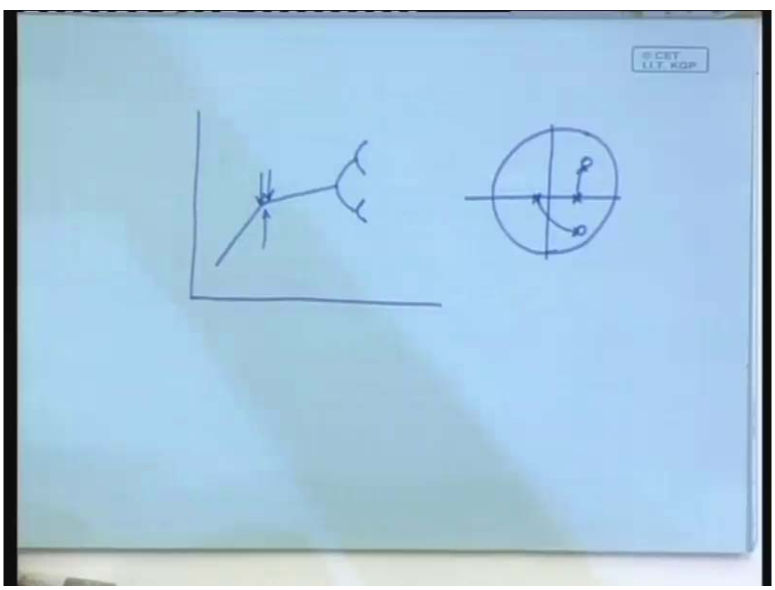

For example if you observe that in a physical system, I am not talking about the normal form now. Suppose a bifurcation diagram is observed which goes like this. Then based on what we have learnt, can we infer anything about what is happening here. Obviously we can see that there is a sharp bend in the bifurcation diagram and if the map is smooth, we don’t expect this kind of a bend.

If we encounter such a bend where before the bend and after the bend, the behavior is period one or some other periodicity but nevertheless there is a bend. Then based on what we have learnt we would say that no, at this point there has been some non-smooth phenomenon occurring and that should fall under the category of the border collision bifurcation where a stable periodic fixed point hits the border and still remains a stable periodic fixed point. In order to explain that we will have to obtain the Eigen value just before and just after this event. With these two arrows I am showing that at these two points, just before and just after we will analyze the fixed point.

We will obtain the Eigen values and then if you do so, we will notice that the Eigen values have jumped. But again from inside the unit circle to inside the unit circle. It might be something like this, I am just schematically showing it but it could be anything else also. Say initially it was something like this, these two fixed points and they jump to these two positions, still its stable. This kind of phenomenon then if you encounter, we will be able to explain in terms of the physics that you have learnt.

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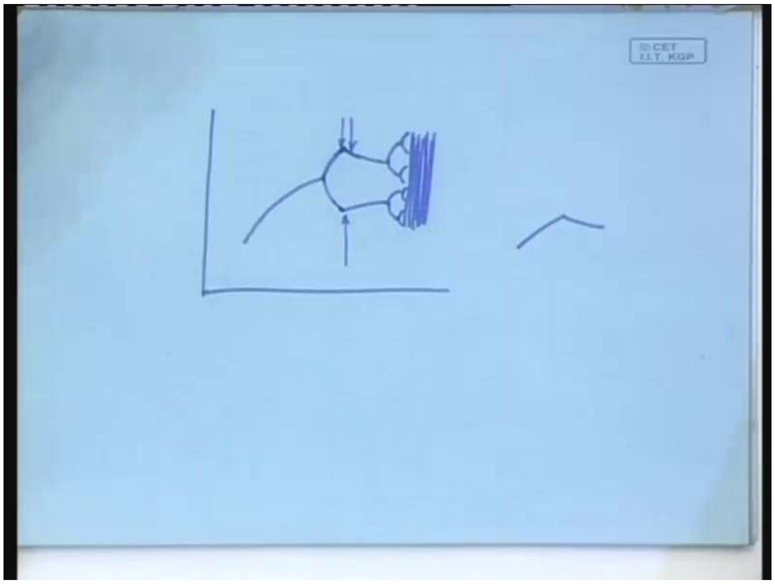

What if you encounter something like this and so on and so forth? What will we say about this? Here as you can see there was again a bend, rewind the period two orbit. In order to explain that we will need to take the second iterate of the map that means from the original system, we have obtained the map. Now we will take a second iterate and apply the same logic. If you take the second iterate then it is simply something like this. This will be second iterate remember then this will be period one. So you will see just one bend and then you will have to obtain the Eigen values at these two points and if you obtain the Eigen values of the second iterate then you will again encounter the same kind of situation that the Eigen values initially were inside the unit circle. They jumped discretely, they changed discretely but still they remain inside the unit circle. But that logic will now be applicable to the second iterate of the map, not the first iterate. You might do that by simply obtaining the Eigen values of the fixed point that is here, another fixed point that is here, the Jacobian matrices, multiply the Jacobian matrices, you get the Jacobian matrix of the second iterate. If you analyze that you will get this behavior.

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If you see the computer screen, I am drawing a bifurcation diagram and that is the bifurcation diagram of the boost convertor. Can you see what is happening? You might see that here this is again drawn by dynamics. It is the bifurcation diagram drawn from the top to bottom. Can you see the mouse? Yes. From here a period one orbit starts and the parameter, in this case the load resistance has been varied from 100 ohm to total 600 ohm. So up to say something like 300 ohm. It was a period one orbit, so there is a range of the parameter, load resistance for which the periodic orbit remains.

At this point what has happened? It is a smooth period doubling, a normal smooth period doubling so just by the appearance of it, you will be able to infer that this is an event where the Eigen value of the period one orbit became minus 1. Following this event, the two branches of the period two orbit diverts from each other and then something very peculiar happens here.

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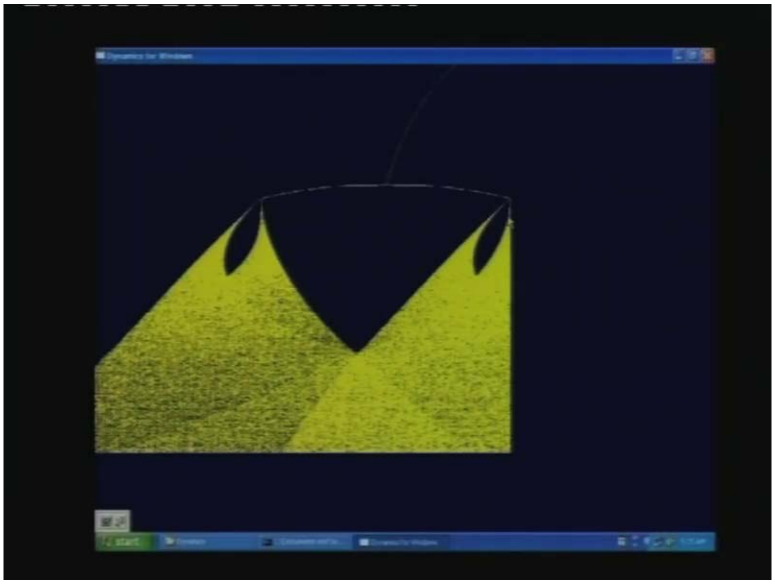

Can you see this? At this point and at this point, something very peculiar happens because now the orbit suddenly becomes chaotic. Let me draw this because this is too narrow line. Let me draw this here which will make it clearer.

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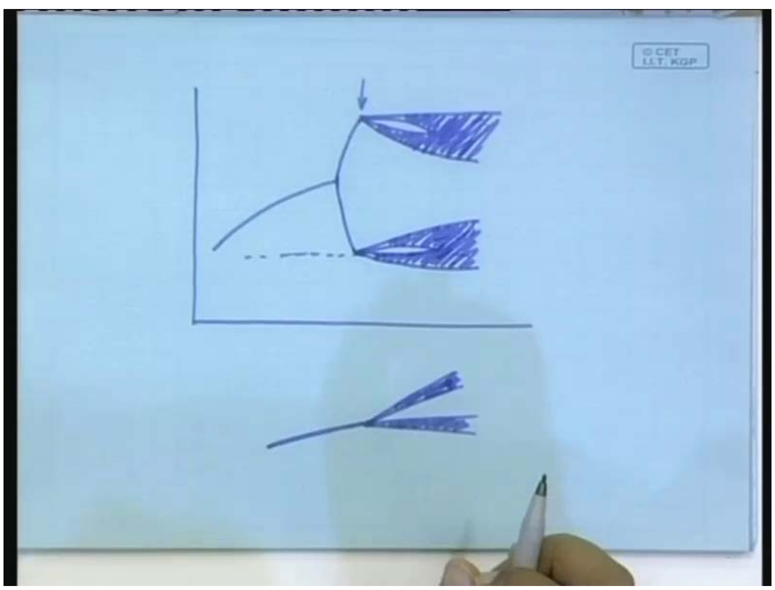

It was like this and then it underwent normal period doubling bifurcation but at this point it became chaotic. How would we explain this event? Now you can see that here is a period two orbit that suddenly underwent a transformation and obviously this event you can easily see that cannot be a smooth event, it cannot be a smooth bifurcation so that as to be a border collision event.

But what actually happened? At this point a period two orbit hit the border. In order to explain it, we will have to consider the second iterate of the map and one of the points of the second iterate map hit the border. Imagine that the border was here and it was hit. So following that I can see that it went into a two piece chaotic orbit and then after sometime this two pieces merged. If you look at it from the point of view of the second iterate map, you would be able to see something like this. If you start in a normal form then it would be something like this that a period one orbit hit the border and then it became a two piece chaotic orbit. In general it is a transition from period two orbit to a chaotic orbit. When did that happen, under what condition did that happen?

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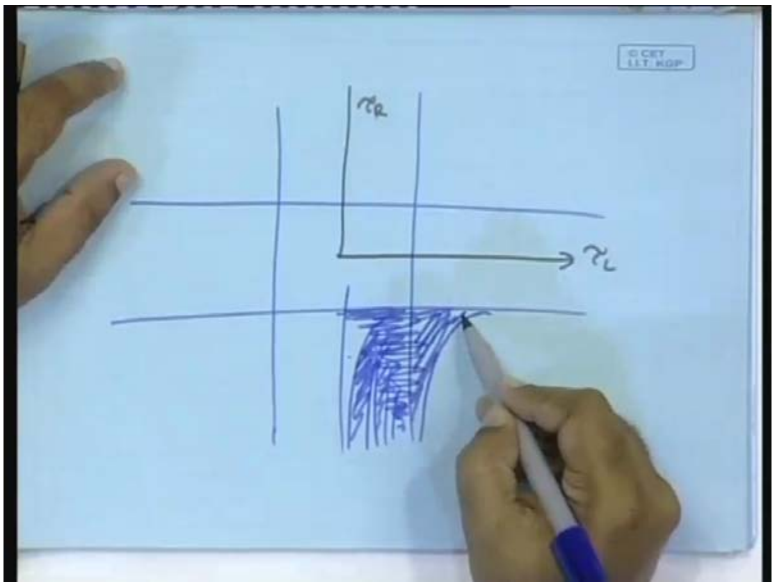

This is our tau$_L$ axis and this is our tau$_R$ axis and we found that happens somewhere here. In this part there is a transit from periodic orbit to chaotic orbit, not in this part because here it is a transition from no fixed point to a chaotic orbit, so somewhere here (Refer Slide Time: 11:27). We have seen that in this part we have the period two orbit and then you have period three orbit and period four orbit, in between you have the chaotic orbits. In this case if you try to analyze it then it should be explainable by means of that theory, if you want to analyze it. But how would you analyze it?

Essentially in order to analyze what you have to do is to take the second iterate of the map, obtain the Jacobian matrix of the fixed point just before the border collision happened. But after the border collision happened, what happens? You don't have the period two orbit any further, it is a chaotic orbit. So whose Jacobian will you find out? In fact inside this chaotic orbit, there will be an unstable period two orbit. The essential point is that in this case also when we are trying to study we studied the Eigen values of the period two orbit before and after the border collusion event. Here also even though you do not see in a salient way the period two orbit, you should know that it must be there and your job is to find out the Jacobian of the period two orbit. Only then you will be able to fit into this theory. The way it is done is that you will obtain the Jacobian of the period two orbit before and after. How to locate that unstable periodic orbit? I will come to that later.

You will need to locate that unstable periodic orbit and then on that, you will have to calculate the Jacobian. Once you have calculated the Jacobian, what are we doing? We are obtaining the trace and the determinant. Now from the original system we have obtained the trace and the determinant but the strength of this theory is that these two quantities, the trace and the determinant are invariant under coordinate transformation.

When we transform it into the normal form, these two quantities don't change. From the original system if we obtain the trace and the determinant, we can thumb the table and say that I have got the right trace and the determinant for the normal form also. Then we will say that for the particular parameter that I have chosen, it falls here and from a theory we can see that it must lead to the transition from a periodic orbit to a chaotic orbit and that is exactly what happens. Let us give you one more example to illustrate what we mean.

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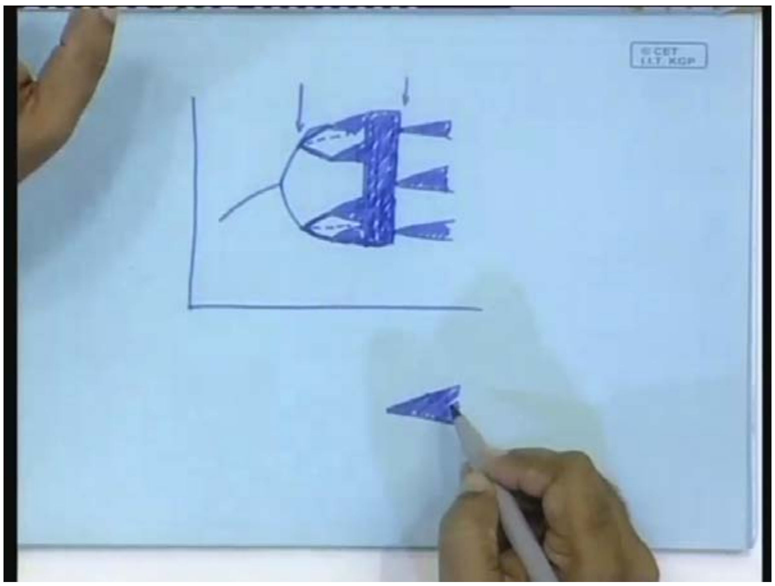

Say you encounter a bifurcation diagram something like this. Say it goes to another set of … (Refer Slide Time: 00:14:41) I wish to draw it like this that means acute angles and then and finally it becomes chaotic. At this point what happened? There was a period doubling but the branches that immersed out of it, went in acute angles which are not the characteristic of smooth systems. By looking at it you should be able to infer that it is a non-smooth period doubling. In order to analyze this, what will you do? You will obtain the Eigen values just before the border collision. Whose Eigen values? Period two orbit that means the second iterate of the map. After the border collision, whose Eigen values you will find out? Now it is period four. No, you will not obtain the Eigen values of period four map. You will obtain the Eigen values of the same iterate. That means earlier it was the second iterate, now also you will have to locate the period two orbit which will be going like this unstable period two orbit. You will have to obtain the Eigen values of that. Then once you have done that, you will be able to show that it satisfies the condition where we have already shown that there will be a transition from a periodic orbit to a period two orbit, higher periodic orbit.

Let’s go one step further. Suppose after this you have encountered that in a smooth system, after it goes into chaotic, there are periodic windows. You have encountered that. Now in this case also, in non-smooth system also there can be periodic windows but suppose you find that the periodic windows are like this. Not really periodic windows, most of you who study literature in this field will come across bifurcation diagrams looking like this. What will you say about that? We know that such periodic windows are normally created in a smooth map by saddle node bifurcations where a new periodic orbit is born. Now why we were studying the non-smooth systems?

We also learnt that a non-smooth saddle node bifurcation can also lead to a situation where both are unstable. If both are unstable then there is a possibility that the resulting orbit will be chaotic. You should immediately realize that’s exactly what is happening. Here there has been a birth of a periodic orbit, not only one but a pair of periodic orbits but both unstable. As a result around that there has been a new chaotic orbit that is been born and because of the birth of this new chaotic orbit, you see this window which is not a periodic window. It is also a chaotic window but a different chaotic window than this.

But remember, here there are three bands that are going. At three points I can see that it has been born. What does it mean? It means that this is the event happening in the third iterate of the map. You will have to look at the third iterate of the map and if you do so, you will find that there must be two fixed points. I mean the theory says that there must be, by closing your eyes and thumbing the table you can say that there must be two different unstable period three orbits. Two different unstable period three orbits that has been born at this point. The way to analyze it, locate this period three orbits, find out their Eigen values and then fit into the theory. Where do we expect it to be? You expect it to be somewhere here, in this part where the bifurcation is from no orbit to a chaotic orbit.

So seen in the third iterate, it will be no orbit to a chaotic orbit, it will be like this. There was nothing before that. At this point two periodic orbits were born both unstable but what is the different between them? One a regular saddle and another flip saddle. So even though both are unstable you can easily figure out what kind of orbits are they. One will be a regular saddle another will be a flip saddle and then the bifurcation diagram individually seen from the point of view of the normal form will be like this and seen from the point of view of the actual system, it will be like this. The explanation of this kind of an event will have to be obtained by the theory of the non-smooth maps.

What happens here is that you have the birth of a two periodic unstable orbit, one regular saddle another flip saddle but the chaotic orbit is not stable. That is what happens here. In this part it is stable, in this part it is not stable because in between there has been a boundary crisis so this fellow is unstable. You would not say that nothing happens here. You would say that if the parameter range is here then it leads to the creation of an unstable chaotic orbit. Now unstable chaotic orbit had been created, you might argue that what if it is not there anyway. No, it’s not true that it is not there.

The point is that these unstable chaotic orbits may later in the bifurcation diagram have its own influence because at some of point of time, it may take part in any interior crisis so that suddenly you will find the attractors enlarging into something you don’t know what. But the point is that orbit was created some times back in this kind of a bifurcation which you did notice because it was not observable. But theory will prompt you to say that no in such a system, it is possible to have the birth of unstable chaotic orbit which will have its own influence on the dynamics, later if it somehow encounters an interior crisis. Just to recall when interior crisis happens. When the existing stable periodic orbit somehow contacts with that unstable chaotic orbit then the whole chaotic orbit becomes stable. Under that condition you will find that suddenly something that was not there appearing into the picture and often we are very hard put to explain where did this fellow come from. This actually came from this kind of an event. So we have more or less understood.

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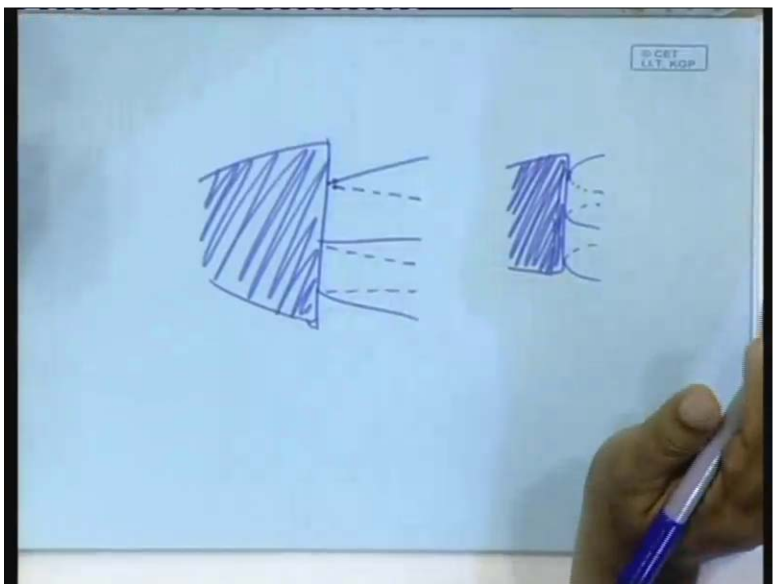

What will you say if you have an ongoing chaotic behavior say at this point, it terminated and we have this in a piece wise smooth system. What will you say? This event since it is a piece wise smooth system, this could be related to a smooth saddle node bifurcation or it could be a non-smooth bifurcation where the parameters are here. If they are here then what is the character? It is like a saddle node bifurcation were a pair of stable and unstable fixed points are born. Will the stable fixed point be visible? If you have something like this, you would said that similar thing is happening in the third iterate but no you will have to test that. Why? Because such thing could be as in the logistic map, you have the period three window created by a saddle node bifurcation. Here also it could be so.

How do you distinguish between them? You will have to say that at this point, not only this fellow has been born but another fellow has been born which is unstable. It cannot have this orbit otherwise. Just following this, you obtain the Eigen values of these two orbits, a period three stable orbit and locate a period three unstable orbit because you know from theory that it must be there. Locate the period three unstable orbit and find out their Eigen values.

Now what happens in a smooth system? In a smooth system if you assume that this is the chaotic orbit and at this point there is a birth then it will be born like this and then the other orbit will be going like this. Here these two are going at this point in an orthogonal direction but here it is an acute angle but this is sometime difficult to identify. So as you go closer and closer, these two fixed points come closer and closer and then at this point they have the same Eigen values. Not here.

Here as you go closer and closer, these fellows Eigen values will remain different. That is the characteristic of the non-smooth saddle node bifurcation where two orbits have been born but you would notice that one with the Eigen value placed such that the tau_L is greater than 1 plus $delta_L$ and the other with the tau_R between plus 1 plus $delta_R$ and minus 1 plus $delta_R$. They are different at this point itself. By examining that we would be able to infer whether it's a smooth saddle node bifurcation or a non-smooth saddle node kind bifurcation where a parameter is placed here. Basically my point is that, all this theory based on the normal form is fine but do not be carried to think that it is just a mathematical exercise. It is essentially an exercise by which we would try to explain phenomena that actually happen in physical systems.

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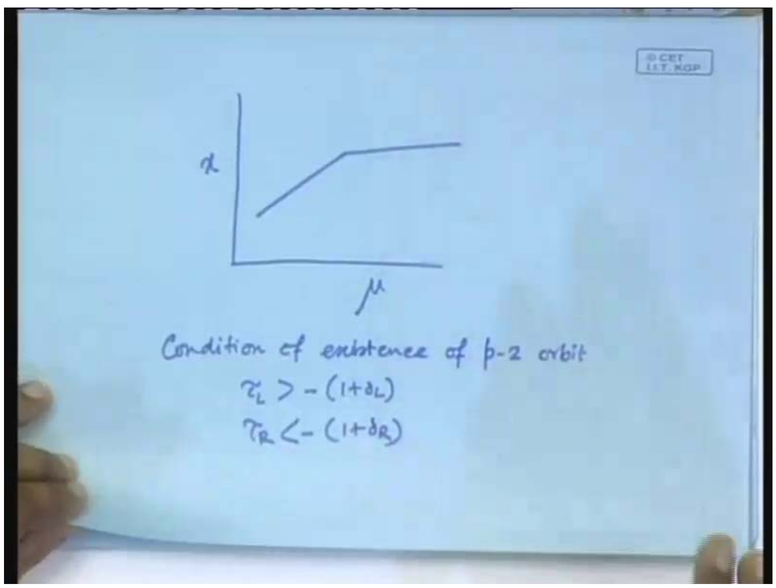

Now let us come to the middle part which I said may show some surprise where we had concluded that it is a bifurcation where a period one orbit remains stable, like it hits the border and remains stable as a period one orbit. So this is mu and this is x. There are complications because the existence of the period one orbit does not prevent the coexistence of high periodic orbits.

Let us consider. Will any period two orbit exist here? In the last class we have seen that the condition for existence of period two orbit is that it is below this and it is above this. We have seen that I am not repeating. The condition of existence of period two orbit is that tau_L is greater than minus 1 plus $delta_L$ and tau_R less than minus. That is the condition we have already seen that and since we have seen that I am not going into the details of it.

We do not expect the period one orbit to exist here because it exists outside this range. But that's not true for the period three orbit. That's not true for the period four orbit. We will need to consider the existence of these orbits. How do you probe whether or not a period three orbit will exist?

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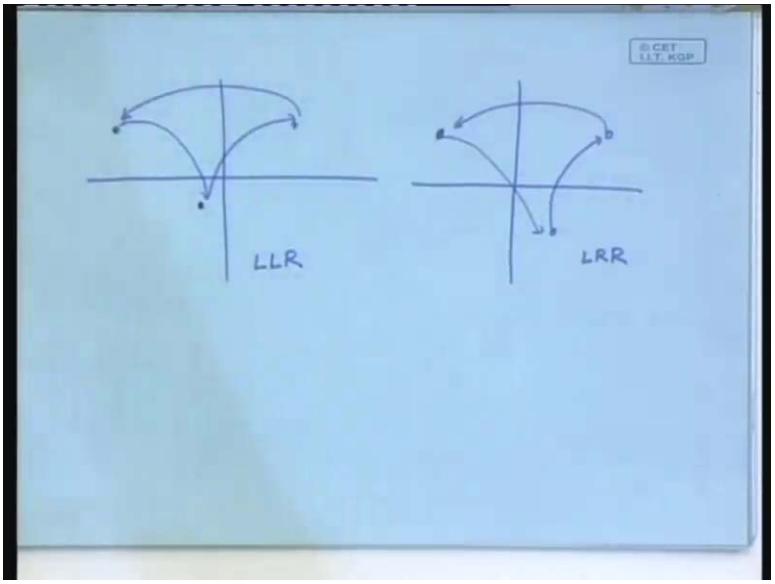

The period three orbits existence will be given by, suppose this is the state space and there will be one point here another point here and third point here. This point will map to this point, this point will map to this point and this point will map to this point. This can be a period three orbit or this point will map to this point, this point map to this point (Refer Slide Time: 29:10). What is the distinction? This orbit is having two points to the left and one to the right, we will be calling it LLR orbit and this orbit will be having two points to the right and one point in the left, so it is called LRR.

When we talk about the period three orbit, it could be either of the two and we will need to talk about both. We will have to find out where both will occur. How will you do that? That is algebraically pretty straight forward but you might need to do some algebraic manipulation in order to actually obtain it. What will you do? You will start from this point, apply the left hand map once, apply the left hand map another time and apply the right hand map again to ultimately come to x_3. Then you would say that $x_3 = x_1$, so you will then solve the equations in order to obtain the position here, similarly here, similarly here.

Similarly here you will apply the left hand map once, right hand map again, right hand map again, come back here solve it. That way you can solve for each of these points. Once you do so what you have obtained? You have obtained the positions of these individual points. Out of that one you can ignore because it will be immaterial. For example this point in the right hand side that will always be in the right hand side. That cannot go to the left hand side. If it does then all the points of a period three orbit will be left hand side which is impossible because the left hand side is a linear side.

In the linear map you cannot have a high periodic orbit. Similarly this point cannot go to the right hand side so it essentially boils down to finding the locations of these two and imposing the condition that both these x coordinates should be negative in this case and both these x coordinates should be positive in this case. If we do that then we can find out the range of parameters where this will exist and this will exist. Similarly we can obtain a period four orbit and so on and so forth. It may so happen that some of this will occur somewhere here. That means just imagine suppose a period three is occurring here. What does it mean? Suppose a period three is occurring for mu greater than 0. If you actually do this, you will find a large number of such regions where other high period orbits will occur. But just try to imagine what will happen, what will we see when we change the parameter mu.

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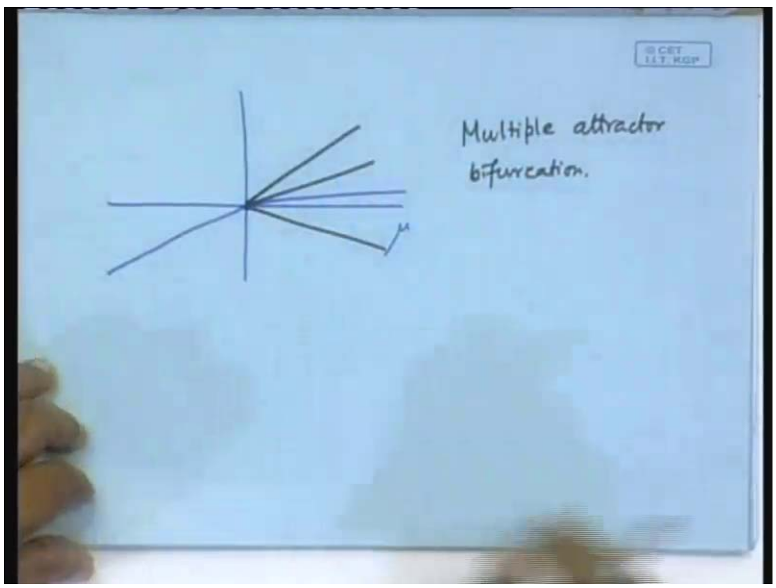

We are changing the parameter mu and there was a periodic orbit that came and hit the border. In the other side of the border, the period one orbit remains stable and so we will draw it something like this. But at the same time, the other period three orbit will exist in this side that means this fellow will be something like this. Now imagine such a thing is happening in a practical dynamical system and you are changing the parameter like say in an electrical circuit of voltage, in a mechanical circuit some applied force or something like that. Something representing the parameter mu. You are changing it and its moving like this and finally it hits the border. Just following it what happens? There are two orbits, one period three, one period one. If there are two stable orbits they should have their own basins of attractions. So each one will have its own basins of attraction but you see the character is that the more closer to the value of mu is equal to 0 you go, the closer this points are and at mu is equal to 0 they are actually at the same point, physically. Then they move away from each other and they have their own basin of attraction.

Now every physical system has some ambient noise, you can’t help it. Every physical system will have some noise. As a result of that noise it will be slightly moving, not exactly following this particular small line but slightly moving around it. Now here it encounters a situation that there are two basins of attraction but they are arbitrarily close to each other and the noise is stronger than that. What will happen? What happens is that there is no guarantee in which region will it be, in which basin of attraction will it be. It is moving arbitrarily, randomly and in that random motion it is intermittently going to this basin of attraction and intermittently going to that basin of attraction and while it does so, this parameter is changing.

These basins of attractions are going away from each other but while they were arbitrarily close to each other, this fellow was randomly moving. What does it mean? It means that ultimately when they move away from each other, these basins of attraction, this state point will be in either of the basins but there is absolutely no guarantee which one will it be. This is another fundamental source of uncertainty. There are uncertainties in various ways occurring in a chaotic system but this is not a chaotic system, periodic system but still it has a fundamental source of uncertainty. As you change the parameter, once same system it will lock into a period one orbit here. Another time same system you do the bifurcation diagram, it will go in to the period three orbit without any difference made in the actual system configuration, no change in the parameters. This is called a multiple attractor bifurcation and these are considered to be somewhat undesirable because we never know which orbit it will ultimately go in. In any engineering system you would like to predict that if I make this change in the parameter it should behave like this but here is a case where you cannot predict, it either goes there or goes here.

Now how to actually find the parameter range in which this multiple attractor bifurcation occurs. You see it will be either a period one with a period three orbit, period one with a period four orbit, period one with a high periodic orbit, period one with a chaotic orbit whatever it is but period one plus something. I will illustrate how to obtain the range in which the period one orbit will exist with the period three orbit because that is something that can be done. Following the same logic you can find out in which parameter range will the period one orbit coexist with the period four orbit, five orbit, six orbit and all that. As I said, suppose I am dealing with this LRR orbit and I am trying to find out its value. We can solve the equation and we can obtain the value. I will write, you can easily check but it will be advisable if you redo the thing so that you can obtain it independent of my writing in this paper.

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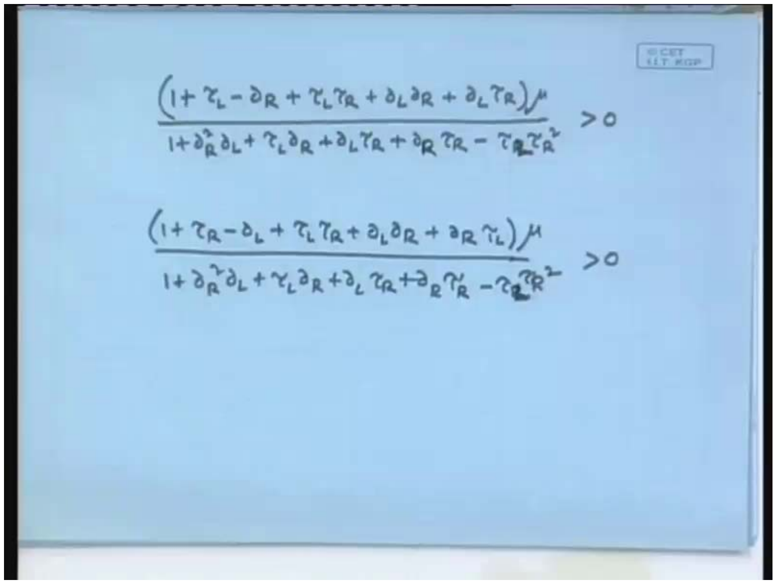

For the LRR orbit, there will be two equations, given two conditions. One this fellow become equal to 0 and another this x coordinate become equal to 0, these are the two conditions. If you write it will be like this, 1 plus tau$_L$ minus delta$_R$ plus tau$_L$ tau$_R$ plus delta$_L$ delta$_R$ plus delta$_L$ tau$_R$ whole with mu divided by 1 plus delta$_R$ square delta$_L$ plus tau$_L$ delta$_R$ plus delta$_L$ tau$_R$ plus delta$_R$ tau$_R$ minus tau$_L$ tau$_R$ square. It may look formidably large and complicated but don't get worried about it. Obviously there will be another equation which will be similar, it is 1 plus tau$_R$ minus delta$_L$ plus, this is symmetrical plus, this symmetrical plus delta$_R$ tau$_L$ whole mu divided by 1 plus delta$_R$ square delta$_L$ plus tau$_L$ delta$_R$ plus delta$_L$ tau$_R$ plus delta$_R$. No, this is tau$_L$ tau$_R$ square, this is greater than 0.

Obviously the range of existence of this orbit, LRR orbit will be delimited by these two conditions. Now here is a number in the numerator and here is a number in the denominator and both are equal to 0 for the condition or greater than 0 for the condition that it will exist. Obviously this will be positive, if both these are positive or both these are negative. Similarly this will be positive when this and that, both are positive or both are negative. This immediately tells you that this condition, this condition and this condition, notice the two denominators are the same; these two lines, this condition is equal to 0, this condition equal to 0 and this condition equal to 0, they delineate the range of the parameter space. So this will give an equation of a line, this will give a line and the denominator will give another line.

These three if you draw, you simply get some kind of an area. You can say that this area will be representing the range in which this fellow exists. If you place the initial condition there or the parameter there then you know that in this parameter range, I can expect a multiple attractor bifurcation. If say you have obtained a bifurcation diagram looking something like this, you are happy that it was a periodic orbit and it remained a periodic orbit. But as I told, you have to obtain the Jacobian matrices and then you have to feed to the theory and after having put this trace and the determinant into that picture, if you find that it falls somewhere here, you know that even though I have not detected it in the bifurcation diagram, this fellow exists. Locks somewhere, so I am in a dangerous situation. That is the power of the theory that from the theory we can anticipate there no, even though I don't see it, I am in a dangerous situation.

Similarly you can obtain the condition for existence of the period four orbit, period five orbit, six orbit and all that and all these will represent the multiple attractor bifurcation conditions. Now these are understood to be problematic but there is another situation that is understood to be dangerous. That is when there are two attractors, here we are considering one period one attractor another period three attractor but suppose it is period one attractor and another attractor infinity. That means it is another situation where this period three attractor is not there. It is an attractor infinity but the same multiple attractor bifurcation occurs. That means as you go arbitrarily close to this point, there is another attractor at infinity and its basin of attraction comes arbitrarily close to the fixed point.

Even though if you just look at the fixed point and keep on obtaining or looking at it Jacobian, you will find the fixed point is stable, you are happy. Yet its basin of attraction collapses to 0 at the point of bifurcation. That is called a dangerous bifurcation because simply, normally in control theory the Eigen values are understood to be the indicator of stability. Here is a situation where the Eigen values give no indication on the stability. If you obtain the Eigen values you will find them they are stable, yet the system collapses because there is always some ambient noise and it has a finite probability that it will go into the basin of attraction of the attractor at infinity, it will collapse.

It is necessary to understand why that happens and where that happens because even though this middle part as I told you is, many people think this is a safe part because it was a period one, the remaining period one it is not really the safe one. Let us try to understand clearly. You have seen that these two conditions will actually give three conditions because the numerator here, the numerator here and the two denominators are the same. These two individually are the conditions.

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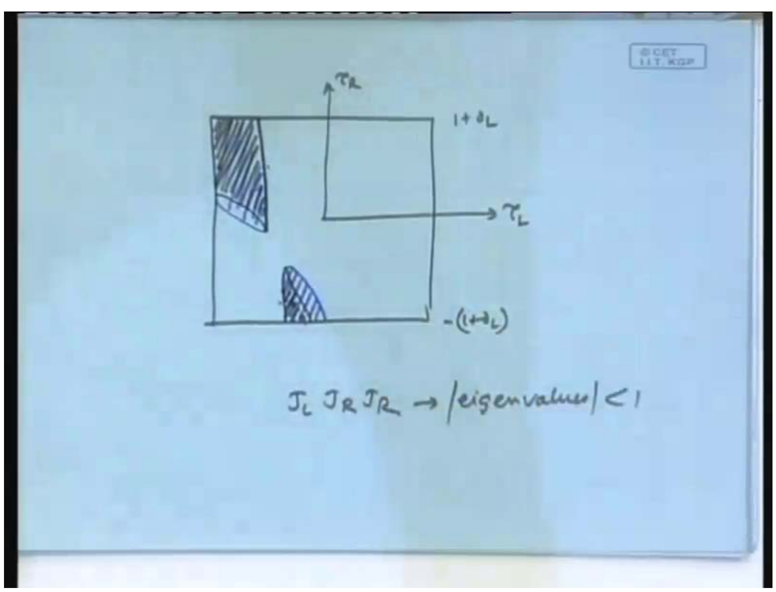

If you plot them, you will get say I am plotting in that middle part, so this is tau_L. So what is this? This is 1 plus $delta_L$ minus 1 plus $delta_L$ and so on and so forth. This condition will turn out to be like this and so out of that one is this condition which is here, another is this condition which is here and there is the denominator condition which is there. It is actually single line. You can say that fine I have understood that the LRR orbit will occur in this parameter range. Will it be stable? The stability condition is a bit different because the stability condition, how do you obtain? The stability condition is that the Jacobian of the left hand side times the Jacobian of the right hand side times the Jacobian of the right hand side, so J_L J_R J_R. Its Eigen values should be less than 1, that is the condition.

Now if you obtain that then you will see that in one of the condition, Eigen values can be less than one means it could be plus 1 or minus 1. The plus one condition is exactly the same as the denominator. You can easily do that, it will turn out to be exactly the same as the denominator. This line is also the condition for its stability, this side it will be stable. We understand that it could be stable. The moment I say that it could be stable, I understand that it is stable in addition to the period one fixed point. In addition to the period one fixed point, it's a LRR type period three orbit is stable. The moment you tell me that I will ask who forms the basin boundary. If there are two stable orbits, there has to be a basin of attraction. Immediately I will ask who forms the basin boundary and you know that the basin boundary can be formed only by the stable manifold of a regular saddle fixed point. Who is that regular saddle, where is that regular saddle?

In fact all these orbits will have a complementary orbit. Complementary orbit means if it is a LRR orbit then take one of the symbols to the left then whatever orbit you get, it is a complementary orbit. So LLR orbit is a complementary orbit. First suspect is that, is there a LLR orbit? Yes, in fact there is a LLR orbit, if you really investigate you find that there is a LLR orbit and that is what forms the basin boundaries. Naturally we will try to find out where does this orbit exist? The moment we try to do that we find that this fellow exists over a range, you will do exactly by the same procedure that means for the LLR orbit again you will find similar conditions. I am not writing that, it is not necessary for me to write all the time but if you do that you will find that exists over a condition that is like this and namely over a larger range.

Notice I have said that there is no reason to think that the LLR orbit in LRR orbit will occur for the same range because these conditions will turn out to be different. If the conditions turn out to be different, obviously the orbits region of existence will be different. Because you are solving different equations and the difference is manifested mainly because the stability conditions would be the same. Here it will be a different number, different range, different line. What does it mean? It means that if I place the parameter here then I have the coexistence of the period three stable point and the unstable point because if LLR orbit at that point is unstable. I have the condition, the LLR orbit is a regular saddle. Its stable manifold is forming the basin boundary but have you understood the condition.

This condition is where period one orbits is there, period three stable orbit is there but at the same time period three unstable orbit is there which is a regular saddle and at that point it will be unstable. So that unstable regular saddle points stable manifold will form the basin boundary, simple case.

What happens here? Notice if I place the parameter here then the period one orbit is there, the period three unstable orbit is there but the period three stable orbit is not there because the existence condition has not been satisfied yet. You will be satisfied when you move here but here it is not there. What does it mean? It means that there is only the existence of a period one stable point and a period three unstable point but the unstable point is now a regular saddle, it will have a stable manifold and all stable manifolds of regular saddles form basin boundaries. Basin boundaries between what and what? Yes, one is at the period one attractor, the other will be at infinity. So under that condition it will lead to that situation that I talked off that there is a coexistence of a period one attractor with an attractor at infinity. If that happens then the system will inevitably collapse at the bifurcation point. Even though the periodic orbit remains stable because its basin of attraction shrinks to zero size. What you will actually see is something like this.

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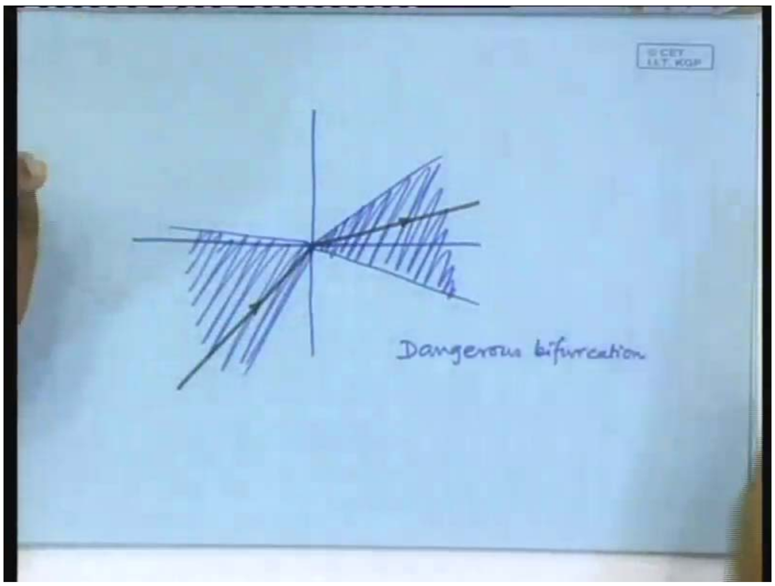

Let me draw it with this. It was stable, it remains stable but this stable fellow had a basin of attraction say something like this which also scales with mu and therefore that shrinks to zero size. Here also there was a basin of attraction, I am not drawing a chaotic orbit it is just a basin of attraction that also scales with mu and that also shrinks to zero side at this point. So at this point the basin of attraction has size zero. That is called a dangerous bifurcation that has no equal, no parallel in smooth systems. In today’s lecture we have essentially tried to understand how to apply this knowledge to actual physical systems and we have in addition come across with two possibilities.

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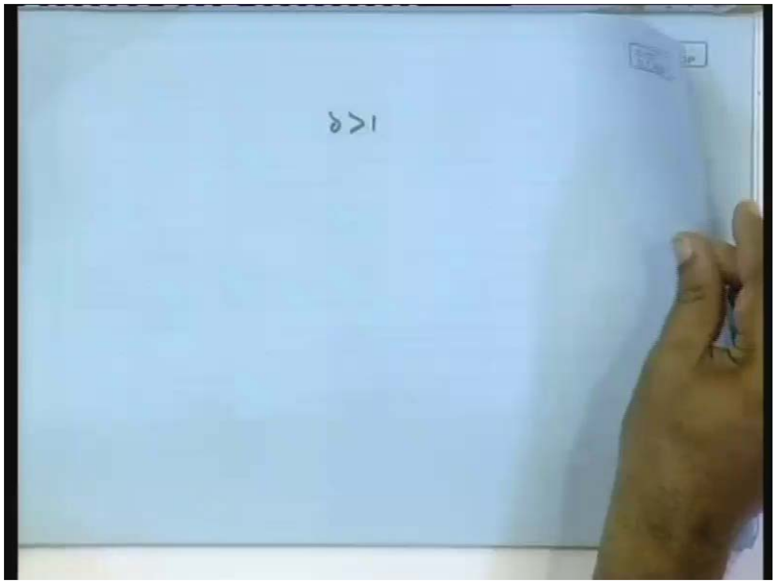

In brief so far I have not discussed one possibility. That is if you have the determinant greater than one then what? So far we have not talked about that. We have considered only dissipated systems but if the determinant is greater than one then what? This condition, since I am running out of time today, I will come to that in the next class. This condition what happens where the determinant is greater than one? That’s all for today.
Thank you.

Preview of next lecture

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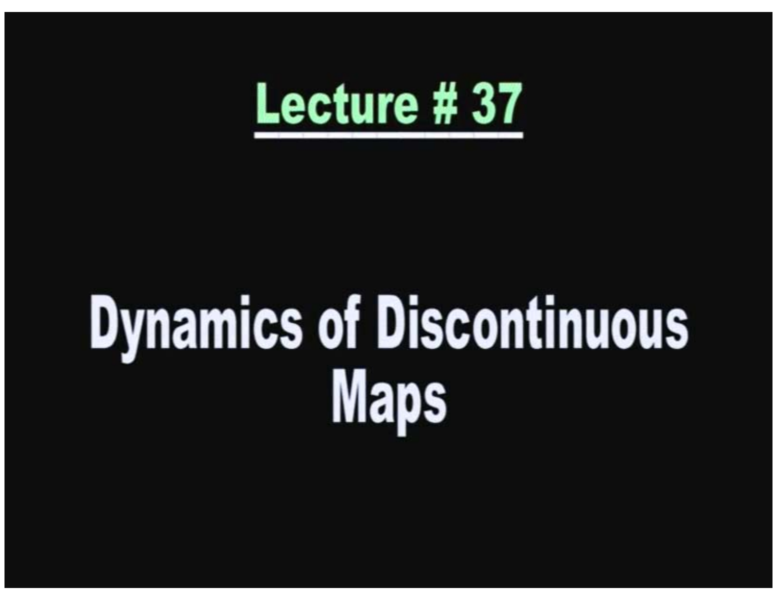

So far we have been considering the situation where the determinant in both the sides of the piecewise smooth map were less than unity. What does it physically imply? It implies that if you take some area then in successive iterations of the map, the area will shrink. Essentially that is the meaning of the determinant being less than unity. The determinant being negative means, the area flips to the other side. It’s like a dish, it flips. But in general we have been considering the situation where you have modulus of the determinant less than unity. Just recall that in case the determinant is negative then it does not allow a complex conjugate pair of Eigen values. If the determinant is negative, the Eigen values are always real either it can be a regular saddle or a flip saddle or attractors, regular attractor. Is regular attractor possible? No, probably only flip attractor is possible. We have more or less dealt with these possibilities.

Now let us come to the situation where which admits the possibility of the determinant becoming greater than one. Now obviously if the determinant is greater than one in both the sides then it is sort of expanding in both the sides. Take any unit area, it will expand in all the subsequent iterates and as a result the attractor will not exist. It will go to infinity. But supposing the system is having one side in which the determinant is positive. Under what condition can it happen? For example, in most power electronic circuits there is a switching. The switch is turned on or off and that is what gives the piecewise smooth character of the system. We were trying to figure out the physical meaning of the determinant becoming greater than unity. Is it possible in a physical system? Just imagine the situation where the switch remains on perpetually. What happens then?

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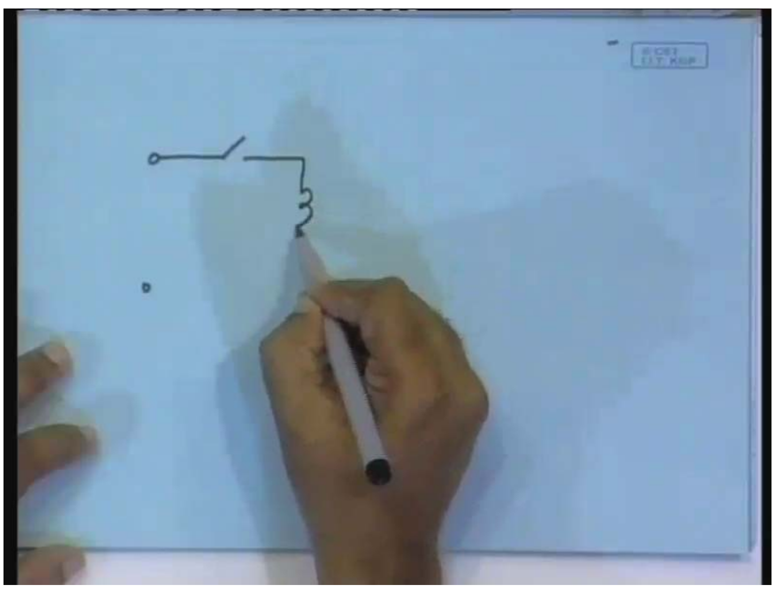

Obviously if the switch remains on and there is only an inductor connected across it. For example a circuit like this, the power supply is here, there is a switch and then you have got an inductor here.

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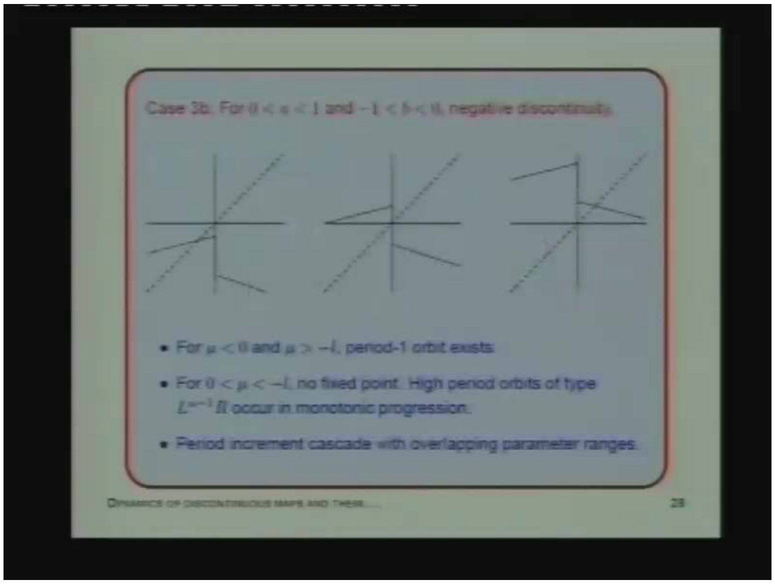

Why? Because since this side is negative. Its implication is that if there is any initial condition in the right hand side, in the next iterate it will go to the left hand side because of the flipping behavior. It has a flipping behavior.

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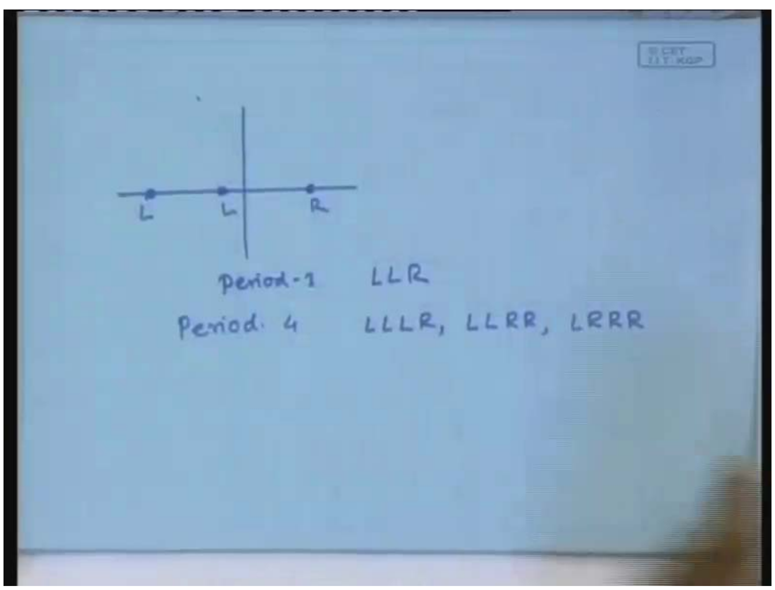

Since any iteration in the right hand side goes to the left hand side in the next iterate, it means that you can only have this orbit but not this orbit. Period three, we can only have LLR which means that there will be one range for period three, one range for period four, one range for period five and all that. That you can calculate from those equations.

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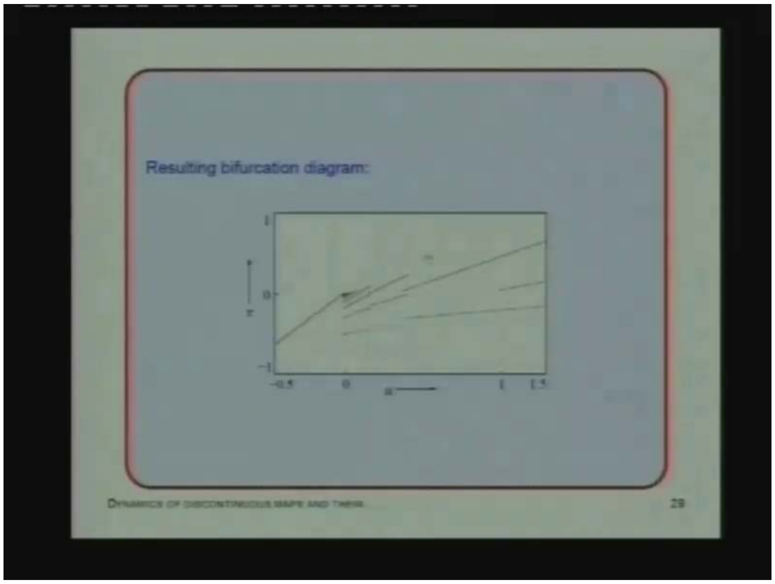

The resulting behavior is like this. There is one range for period two, one range for period three, one range for period four and so on and so forth but that will ultimately add up to an infinite periodicity here. It is a period adding cascade which will add up to infinite periodicity but here, infinite periodicity will not imply a chaotic orbit. Is that clear? It's a bit tricky because we had earlier said that a chaos is nothing but an aperiodic orbit. This is also aperiodic orbit, infinity periodicity means aperiodic orbit yet, it is not a chaotic orbit. Why? Because since every part of the state space is contractive, the Lyapunov exponent must be less than 0. If it is less than 0, this fellow even though it is a high periodic orbit or infinite periodic orbit, it is not a chaotic orbit. So with the details, we will continue in the next class.

Thank you.