Chaos, Fractals & Dynamical Systems
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Lecture No. # 22
Statistics of Chaotic Attractors

There was a question. The question was that supposing we see bifurcation diagram something like this, we have a periodic orbit coming, a period doubling occurring and finally at this point without going to further period doubling suppose it gets into an orbit like this. That was the question. How can that happen? Obviously if you have such an event, there is a sudden expansion of the attractor and that sudden expansion of the attractor can only be caused by which I have already done that by interior crisis.

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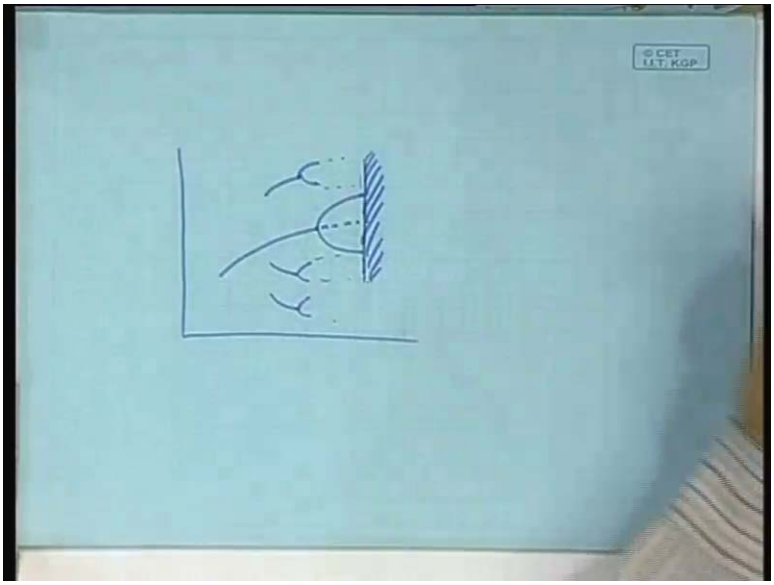

In order for there to be an interior crisis there has to be unstable saddle fixed points, their stable manifolds and then this orbits touching to the manifolds. That is the kind of phenomena that I talked about. His question is where is that unstable fixed point whose stable manifold we are talking about? Obviously at this point there is one unstable fixed point here but it is not this one that we are talking about because it has gone beyond this range, so there is something more that must have happened in this part. Whenever you see this, the immediate conclusion must be that there must be something happening here that created those saddle fixed points whose unstable manifold were involved in this event which means that there must have been some kind of a saddle node bifurcation leading to the creation of another orbits say period three and that went through a succession of period doublings to some extent and then finally at a boundary crisis this fellow went out of stability.

This fellow became unstable but still it continued as the unstable lines and these are the one that has been involved. [Conversation between Student and Professor – Not audible ((00:03:00 min))] No, some part it was stable. It existed as a coexisting attractor and then through a boundary crisis it became unstable. When it becomes unstable still it exists and that is what ultimately manifests itself, when it touches the actual stable periodic orbit as a result of which the whole thing become then stable which in this case is the chaotic orbit. That is a mechanism by which you can see a sudden expansion of an attractor. Today we will do something else. Beside that a chaotic orbit is one in which a large part of the state space is covered that means if it is a one dimensional system, the state space is one dimensional and the chaotic orbit will not be confined to points, it will be covering a set say from here to here.

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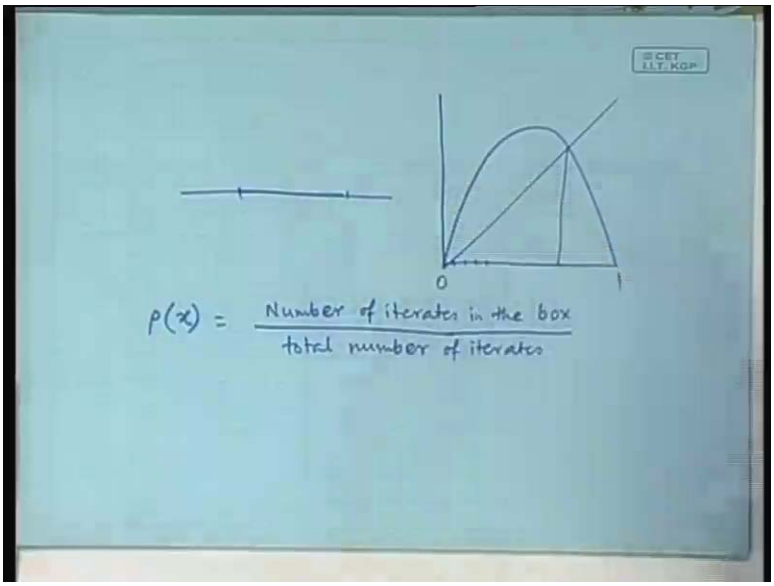

When that happens, that means you are covering a set, it is distinctly different from the periodic orbit in the sense that when it is periodic orbit all the points fall on some specific locations. While if it is chaotic orbits it is distributed. In that case obviously there is no reason to believe that within this range the density of orbits will be the same. It might be different. One of the ways of characterizing a chaotic orbit is to find out how are the points distributed. Means some part might be distributed, some part might be densely populated, some parts might be sparsely populated so that talks about the character of that chaotic orbit. Especially if that state variable represents something for example a voltage or a current then depending on whether the higher part is more populated or the lower part is more populated, you will get a higher voltage or higher current or lower voltage or lower current. That is why it is necessary to have some idea about the distribution of the points.

Now how do you find out the distribution of the points or density of the points? If say you have the normal logistic map distributed between 0 and 1 then the way to find out the density of points falling different parts is to divide the state space in this case, one dimensional state space into bins.

Say this is a bin, this is another bin, this is another bin and so on and so forth and then as you iterate the map, wherever the iterate fall on that particular place a particular bins contents will be incremented by one. If you go on doing that what will happen, the bins will keep on filling up and there will be some bins which will be more populated that means there will be large number of points falling on that. There will be some which will be sparsely populated, less number of points will fall on that and then we can count. You can easily figure out that this can be easily done in a computer that means the state space can be divided into such boxes and you can count the number of iterates falling in each box. Then we will define the density function as row of x. Obviously in order to get something reasonably approximating the actual density of each box, you would have to take a large number of such iterates for example a million of them.

If you take the million that means in the denominator it will be a million and in the numerator for each box there will be certain number, if you divide you get a density. Can you do this exercise for say the logistic map? Simple enough to write on a computer program, yes. Student: We have to start with different initial condition? Here there is a question which initial condition do we start from? Obviously if you start exactly at an unstable periodic orbit say here there is an unstable periodic orbit at this point. If you start exactly at this point where will that be? Million will remain there. So that is very atypical initial condition but since it is unstable, any slight deviation from there will make it go into the chaotic orbit.

There are a finite number of very atypical points from which you will not start but obviously in any computer program, it will be impossible for you to start from there because of the round of errors. When you are actually writing the program, it doesn't really matter but mathematically it does and therefore you will say mathematically they are starting from any typical initial condition. This typical is not an English typical, it's a mathematical typical means these are not atypical special points. Just choose arbitrarily any point and you will land up in a very typical part. That's a definitely important point there. Now you might ask what is the use of this function?

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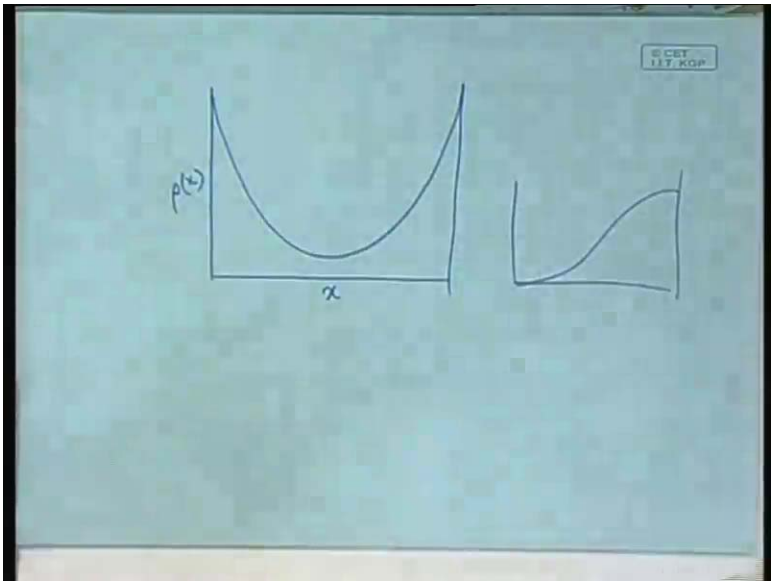

Ultimately you have landed. Suppose in this case, if you do this exercise you will find that you are landing into a function like this. With the logistic map, it will be a function like this. Here is a row of x and here is x, that's a density which means that the density is higher to the two sides near 0 and near 1 and in between it is low. If you actually do this excise you will find it. In other systems, it might be something like this say possible that means there is some kind of a function that has a larger density towards a higher side. So even though it is chaotic, you would easily notice that the average value of that is very well determined by the density. Here the average value would be, since it is a symmetrical but this part, the average value would be something like 0.5 in this case. Here it will not be. Obviously it would be to our interest to find out this invariant density. I have said how to do that numerically. Simply iterate that, count the number of things coming in the box and you can do that. You can also visualize that this can also be done in 2 D.

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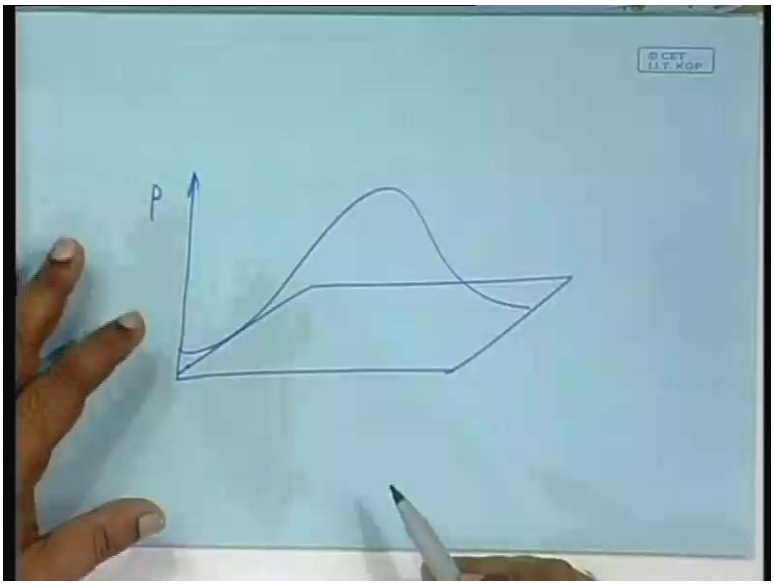

For example if you have a 2 D space and here if you plot the row then the density function will take this form of a surface, some kind of a surface here and that surface will say for this. A particular value of xy what is a probability of finding the state there. It is also related to the probability, the densities are also related to the probability. It will be a nice exercise to do this for example the Henon map where you know that this system goes too chaos beyond certain parameters then to find out by this counting of the boxes, how many fall in each bin, you will find the density. But you might also ask can we do that analytically for some systems at least. Can we derive the density function? There is a very interesting theory for that and that's what will be the matter of discussion today. Yes, it is possible to do that in some classes of systems but before that let's try to understand something a very interesting and important concept.

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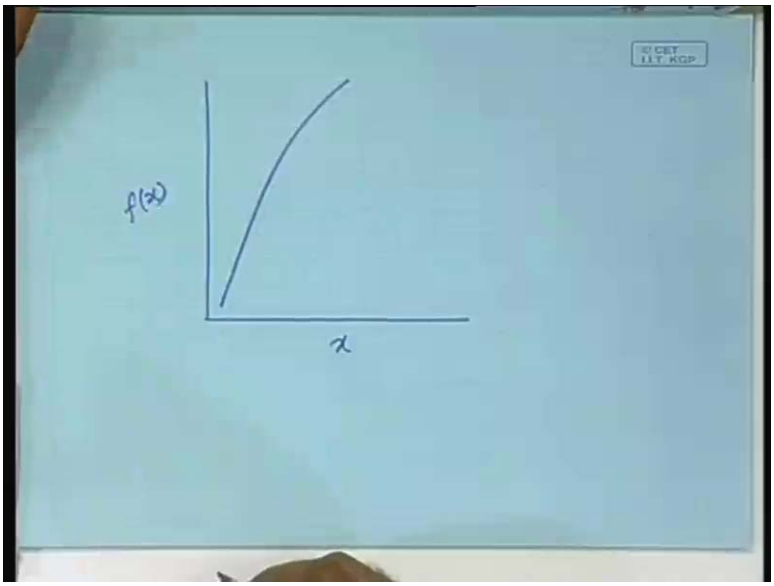

Suppose you have a map something like this, say x here, f (x) here. The point is that I have said how to do this, that you start from an initial condition and go on iterating it. It will fill up the bins and then you ultimately get there. We might also approach problem from a different logic. You might say suppose there are a very large number of initial condition situated on this line, 0 to 1. Take large number of initial condition and apply the map to all of them. Suppose you have started from a uniform density that means there is an equal number of points in each bin and that is where we started. If we apply the map they will be differently distributed again. There will be a different density function.

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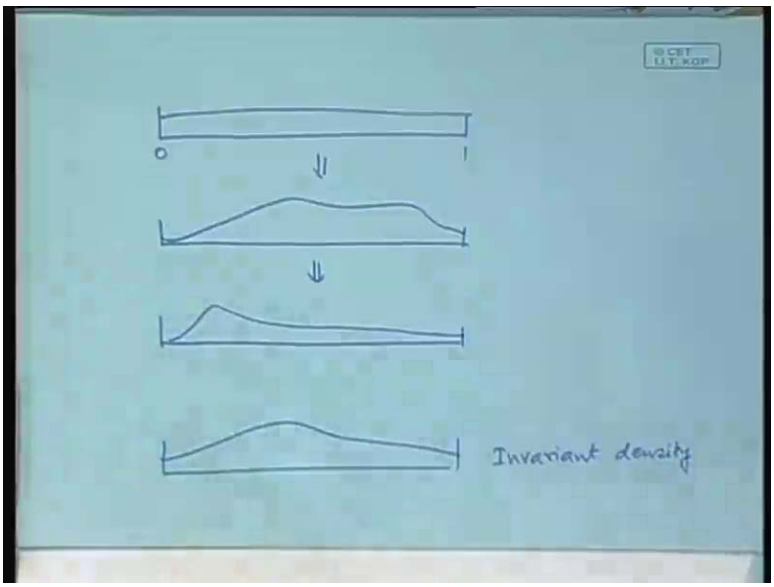

As a result what we are doing is say start from a uniform density and then uniform density like so. It might not start from uniform density but for the sake of logic I said, let us start from uniform density but as a result the application of this function on each point it will then be differently distributed. Again if you apply a same map on each point again, the density will change. Suppose it gets this density, so on and so forth you can go on applying this logic further and further. Ultimately it will converge on to some kind of a density function subject to certain conditions of course provided the iterate do not go to infinity and stuff like that provided the attractor is confined to certain ranges and things like that. These are common sense conditions, so I am not going into the mathematical details of them. Point is that it is logically anticipated that this procedure will ultimately converge on to some kind of a density function.

That density function say it has converged on to say this density function. This density function will have the property that if you apply that map once again on this density function you will get similar density. This is called the invariant density. [Conversation between Student and Professor – Not audible ((00:15:45 min))] Yes, that's my point next. I have then defined two types of density functions. One I started from one initial condition and go on iterating the map that means I have generated a time sequence and then we have found a density of that. In this case it is an ensemble of initial conditions whose densities have evolved and finally homed on to some kind of invariant density and so here it is, the ensemble average that I am talking about and in that case it was time average that I was talking about.

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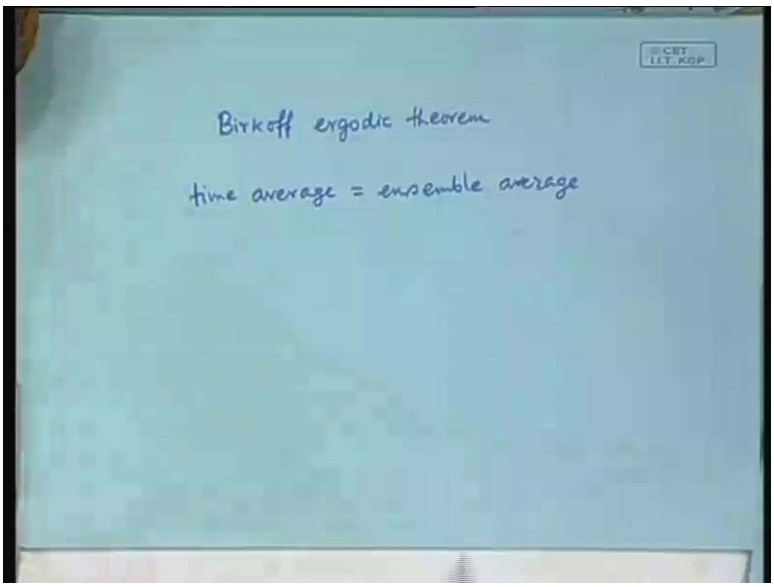

Now there exists a theorem proved by the famous mathematician Birkhoff. I will come to this specific mathematical terms Ergodic and stuff like that later but presently what Birkhoff essentially says is that for Ergodic systems these two are the same. That means the time average is equal to the ensemble average. The mathematical terms I will come to it little later. Essentially you might say that an Ergodic system is where a large part of the state space is covered so that if you start from any initial condition, it will visit the neighborhood of any other part in finite time. Suppose it's a chaotic system like the logistic map.

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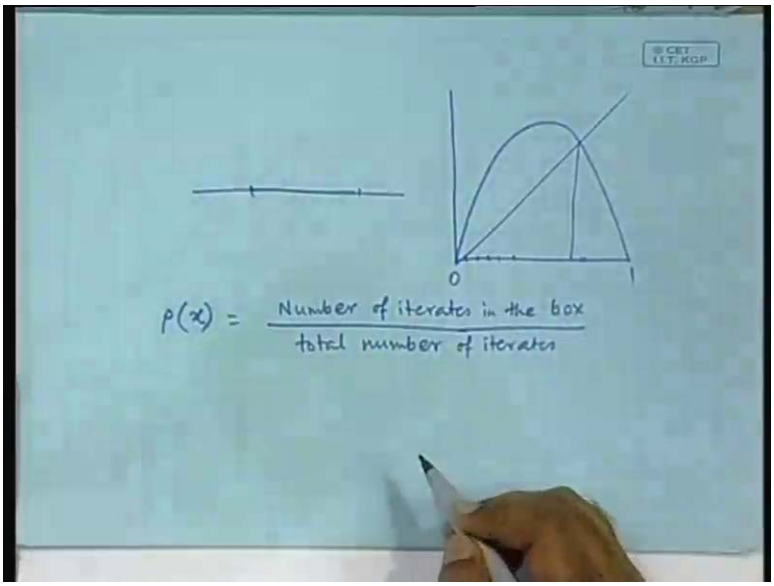

Suppose you have got a bin here and you have started somewhere then you will be able to say that within some finite number of iterates let it be at least one iterate following within this bin. That means every part of this range is visited, so such systems are called Ergodic but rigorous mathematical definitions are not all that necessary for you. The point is that for an Ergodic system, chaotic systems are Ergodic systems then the time average is equal to the ensemble average. Instead of starting from an initial condition and waiting for a long time so that you can measure the time average, you can do it very fast by working with ensemble average because as I said you have to get any proper density with the time averaging, it has to take a long time millions of iterates. Then only you get a more or less accurate description of the density function but with an ensemble average, you can do much faster. That's why we often prefer to do it by ensemble average.

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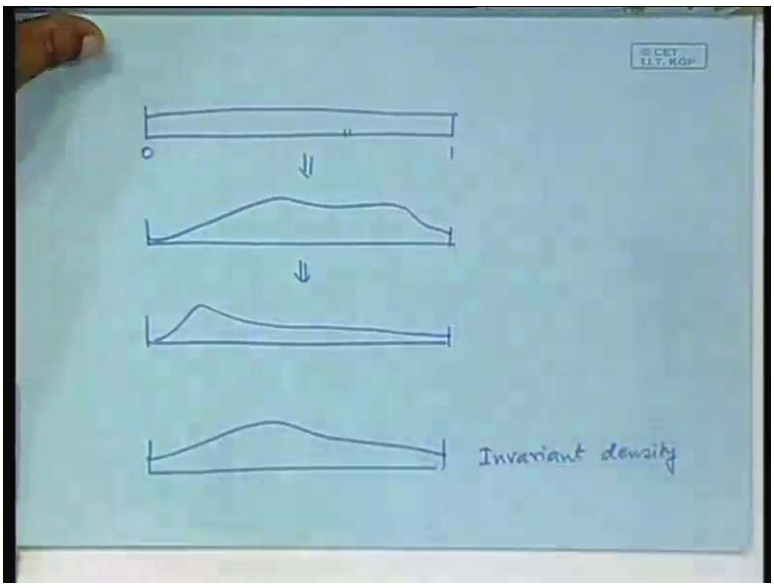

Essentially what we do? We know the function, we know the map, we start from any arbitrary density and then go on iterating it and then finally within some iterates we will see that the density function has converged. We say that it is the invariant density. Notice that something is happening. Here we have some function over the range 0 to 1. In the next iterate we have another function over the range 0 to 1, next iterate we have another function over the range 0 to 1. As we are working with the ensembles and applying the map to it, what is happening? A function is mapping to a function and then we will be able to define a function that operating on a function give this function.

Obviously here we can say that an operator is operating on this function to give me this function. At the base of the operator we have this map, in this case may be the logistic map or whatever but ultimately that map is defining an operator which when applied on a density function will give me a next density function. Again it will give me a next density function and so on and so forth. Ultimately what is the invariant density? The invariant density when you operate on it by that operator, will give me the same invariant density? What does it mean? It is a fixed point of that operator. The invariant density is a fixed point of that operator.

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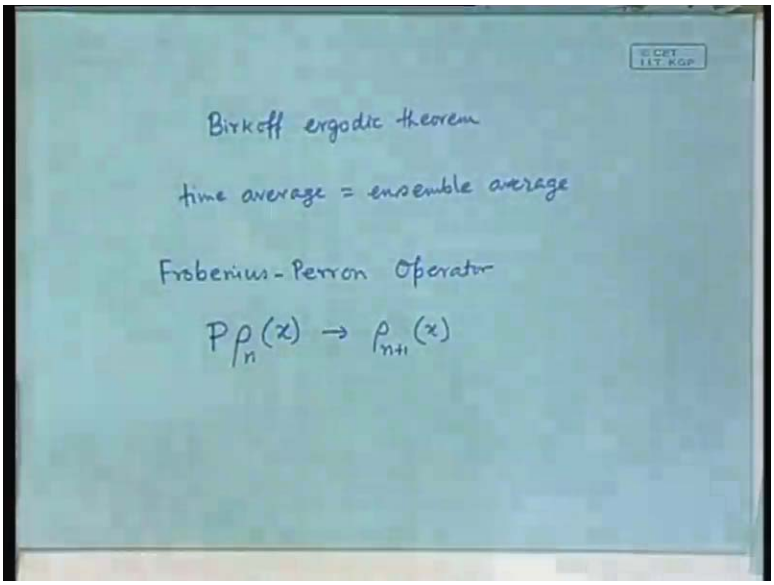

Let me write down this function. This operator is called a Frobenius Perron operator. What is this Frobenius Perron operator? Suppose it is defined by the function P which when operated on row of X in the nth iterate, this is the density in the nth iterate, if you operate this operator P on it, it will give me row n + 1 of x. The invariant density is the fixed point of this operator P. You might ask that conceptually it is fine that there is an operator. Can we really write this operator down? Of course that's a valid question, let's try to address this issue and that exactly why I drew that. Let me get back that.

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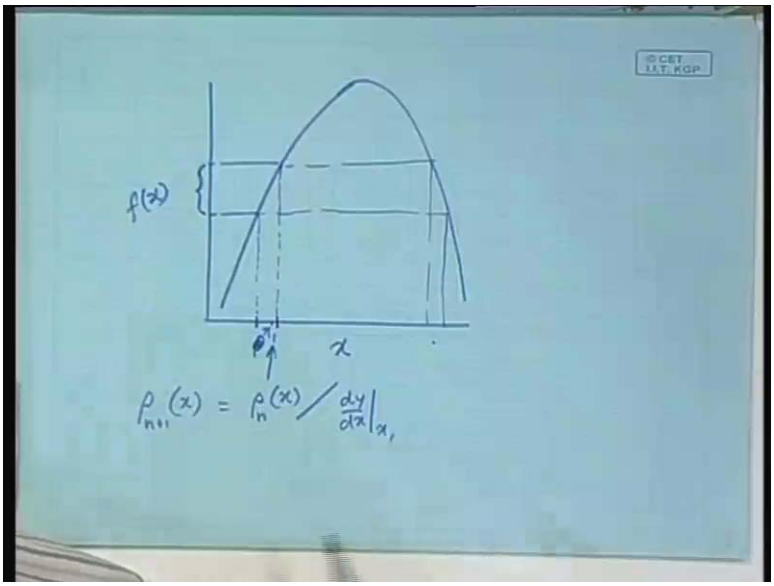

Suppose you have the function like this and suppose you are starting from this range where we have some kind of a row defined. I am trying to find out what will be the row, what will be the density in the next iterate in that range? So whatever was the density in this part, in the next iterate that will be redistributed in this range. Obviously if there are say a thousand points in this range, those thousands points will be distributed in this range and if this range is larger, it will have a lesser density. How much it is less? It is not difficult to see that it is less by the slope. If the slope is unity, it remains the same density. If the slope is greater than one it becomes less. If it is less than one it becomes more so it is this density divided by the slope.

The Frobenius Perron operator is nothing but is taking from here to here and therefore the density here is nothing but the density here divided by slope at this point. How will you write it? Here the density is row of x at the nth iterate then row at the nth iterate here is this, divided by calculated at x_1. Now this is not the whole story. Why? Because the map is like this then the density in this part comes not only from here but also from here because this part also match to the same part. Here it goes to this range, from here also it goes to this range. Since it does so, ultimately when you have to write it, you will have to write it as a summation. So ultimately when you write, it has to be written as a summation.

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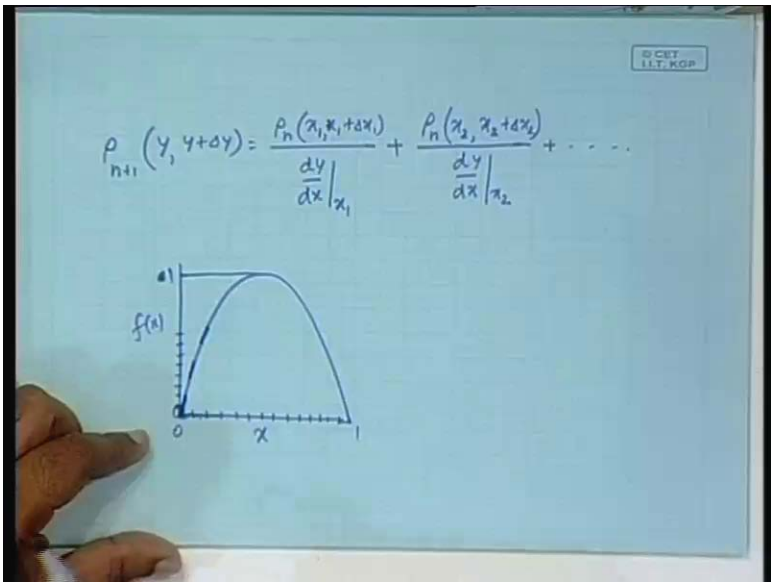

Let me write on a separate piece of paper. So row of n +1 in the range y and y plus delta y is equal to row n suppose this y comes from x_1, x_2, x_3 and all that. You will have to write x_1, x_1 plus delta x_1. X_1 and this is x_1 plus delta x_1 divided by dy by dx calculated at x_1 plus density in the nth iterate in the range x_2 and x_2 plus delta x_2 divided by dy by dx plus so on and so forth. This is the Frobenius Perron operator. In fact you can obtain the invariant density for the logistic map, as I told you both by iterating from the same place starting from one condition and going on iterating, also by this. How will you find the Frobenius Perron operator for the logistic map?

Let's try to work it out. Logistic map it was like this, here is x, here is f (x). Suppose we divided it into a 100 bins. For this bin here is the slope, for this bin here is the slope, for this bin here is the slope (Refer Slide Time: 28:18). All you need to do is to say that here suppose in the first iterate, if I have some kind of a density row over this. That means here there is a row, here there is a row, here there is a row and so on and so forth. Then this row will have to be divided by the slope here plus say notice that here it is 0 to 1, here it is 0 to 1. Then if this is divided into a 100 bins, this is also divided into 100 bins.

Now this bin you will have to ask yourself where does the iterate come from? It can come from either here or here. When you write down the Frobenius Perron operator for this particular bin, you will say that whatever the density was here and here divided by the slope here and the slope there. Similarly here, similarly here and go on to cover the whole thing. Now once we have written down or codify in a program, this Frobenius Perron operator, all we have to do is start from any initial condition and watch as a density involves under the application of the Frobenius Perron operator. It will also converge on to the same density function. Take this as an assignment. This writing again is a very easy thing to do. You cannot really understand or appreciate nonlinear dynamic without actually doing case. [Conversation between Student and Professor – Not audible ((00:30:13 min))] No, there won't be any problem with this symmetric. For example his question is if the curve is not symmetric, will there be a problem? See the curve will be like this okay, it's a non-symmetric curve.

[Conversation between Student and Professor – Not audible ((00:30:31 min))] Yes, let me tell you that. Suppose this is normalized to 0 and 1 and here is the same thing like this. What will happen if it is doesn't reach to 0 and 1? If this is not one what will happen? Nothing will reach that range, suppose this is 0.8 there will be no iterate falling in the range 0.8 to 1 and therefore that part will become empty. Suppose you start from a point between 0.8 and 1, it will come to a smaller range and ultimately that part will become empty. Assume that there is some range, so in that case what will we do is, from here you again divide this range into bins. For this particular bin and in this side, there will be similar bins. For this bin where does it come from? It comes from either here or here, it comes that way. You have to add up the two slopes here and here. That means whatever was the density here divided by its slope plus whatever was the density here divided by its slope.

Then go to the next one, whatever was a density here, you have its slope plus whatever was the density here divided by its slope so on and so forth (Refer Slide Time: 32:09). It's still possible. It is not difficult to do it, if it is non-symmetrical or anything else but there are a finite class of systems where it can be obtained in closed form and they do represent a reasonably available class of practical system that's why this will be of interest. Can the invariant density be derived? Let's address that problem. Of course it cannot be derived in closed form, for just any given system. No, it has to be numerically done. But as I told you there are two types of numerical procedures both leading to the same result. The most convenient would be that which start from a function to slowly apply the Frobenius Perron operator to converge on to the invariant density.

The class of problems in which this can be obtained in closed form… Do you understand what a closed form is? We are going to write down the functional form, which are those piece wise linear that means say something like this, a function x to f (x) and a function is piece wise linear. Not only that, you would notice that here is a break point, here is break point, here is a break point (Refer Slide Time: 34:00). The maps in which break points map to break points, they go by a special name that is the Markov map.

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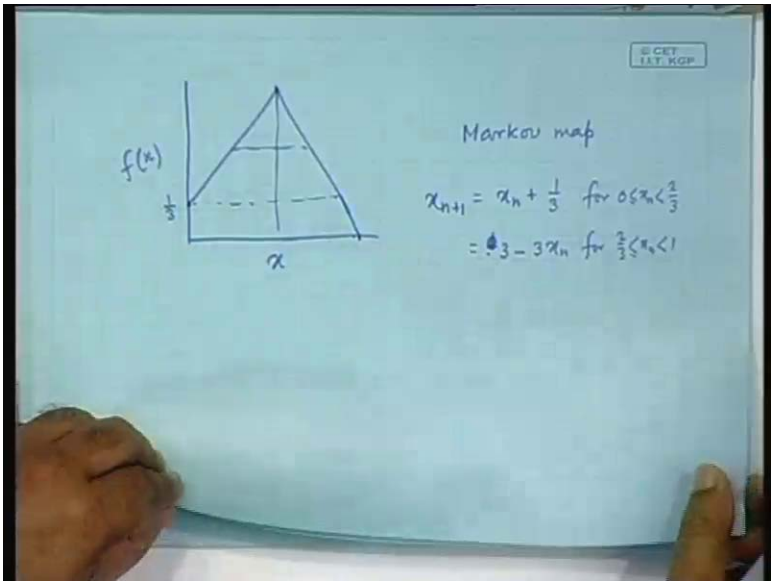

Let's try to understand what you mean by break points mappings to break points. Let's consider this map. It is something like this, the way I have drawn actually. So this is x_n plus one third, so this is one third plus x_n. The slope is unity, it reaches its point. Then what is this part, when x_n is equal to 1 it reaches 0, so it reaches 0. Now you see what do I mean by break points mapping to break points. You might say that this is the break point, where does it map to? 1, maps to 0. If you substitute 1 here because it is in this range, 1 here you get 0. So 0 is break point so it satisfies that condition. 0 maps to one third, one third is a break point. You can say that there is situation where break points map to break points. Now let us try to tackle this problem. In order to tackle this problem properly, actually I need to draw it correctly because this should be two third and this should be one third.

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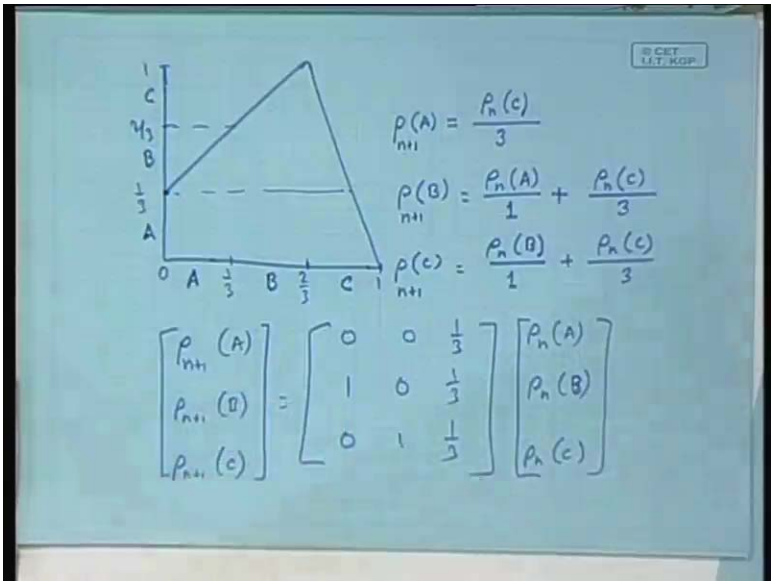

So let me draw it correctly then it will be easier for you to appreciate what I am talking about. It starts from one third goes to this point and then it falls like this. That is a shape of map, this is zero, one third, two third, one third. Does this system harbor an Ergodic behavior that means a chaotic behavior? What is your anticipation in this map? Do you anticipate a chaotic behavior, why? No, it's not because of that. See if you start from here what is the essential thing for there being a chaotic orbit? There has to be exponential divergence of nearby orbits. There has to be expansion. In a logistic map there was expansion. Here also you see if you start from two nearby initial conditions, here is a slope one and therefore it doesn't expand. If anything falls here, it expands because this slope is greater than one.

Now any iterate can fall either here or there, if it falls here the slope is unity. If it falls here the slope is greater than one. So any periodic orbit will be having say a period two orbit. If I ask you what will be the Eigen value, what will be the slope of this second iterate? It cannot have period two orbit with both points here. Why? Because if it falls here, it will go that way it cannot fall in the same part. It has to be one part, another part here. If the slope here is one and the slope here is greater than one, what will be the slope of the period two orbit? Obviously greater than one because the two slopes should be multiplied. What will be the slope of a period three orbit? Again greater than one because it could be either here or there.

The slopes will always be greater than one when multiplied, so obviously any periodic orbit will be unstable. Yet, if you have a bounded orbit it must be chaotic and that is why this orbit is actually chaotic. Here is a chaotic orbit, a system that is Ergodic and therefore we can apply the Ergodic theorem and therefore if you start from any density function, we can apply the Frobenius Perron operator to arrive at the invariant density. Let us write down the Frobenius Perron operator. There are three chunks, say ABC. Here also A B C, so let's first consider row in the chunk A in the n +1 iterate will be… (Refer Slide Time: 40:45). Where will it come from? It will only come from this part, whatever is there, only if any iterate is in this last part C it will go to A. If any iterate is in A or B it will not go to A, so where does it come from? It comes from row n of C divided by the slope here. The slope here is how much? The slope here is minus 3 but in that case I will write it correctly, it is actually the magnitude because we are talking about how much does it stretch. We are not talking about the sign. Sign carries no sense here so three.

Next we come to row of the chunk B in the n +1 iterate. Where does it come from? It comes from A, so anything A goes to B and also anything in C goes to B. So row n of A, what it was divided by its slope. Its slope is one, plus it now comes from two different places row n C by three. Similarly row of C in the n + 1 iterate is equal to, where does it come from? B and C. So row n of B divided by one because the slope here is one plus row n of C divided by 3. [Conversation between Student and Professor – Not audible ((00:42:50 min))] No, it will not be more than one. It has to be normalized at a latest stage but I am not going to add them up directly because if you do so it is simply going to be added up. That's not what I am doing. What I will do next is argue that here we have the next iterate of the Frobenius Perron operator. There was the density function here, applied on it the Frobenius Perron operator gave me this and we can now write it in matrix form as row n plus one of A, row n plus one of B, row n plus one of C. This matrix, this vector is equal to some matrix.

This matrix is 0 0 1/3, 1 0 1/3, 0 1 1/3 times row n A, row n of B, row n of C. See what I am doing. I'm saying that if we start from any arbitrary density in the chunks A, B and C. Here we are taking boxes. Here is a box, here is a box, here is a box and we are not considering that the density within the boxes different. We are considering that situation where we are assuming that the density within the boxes are the same. If these are the densities in some iterate then it will be multiplied by this matrix to give me the density in the next iterate. So this is my Frobenius Perron operator finally. [Conversation between Student and Professor – Not audible ((00:44:56 min))] There is a normalization that we have to do. I will come to that, there is a normalization issue here.

Now notice that ultimately where are you going? Ultimately we are interested in finding the invariant density without really doing the exercise of iterating and iterating. invariant density means in the left side and in the right side it should be the same thing, so ultimately we are talking about a situation where we are talking about row star A, row star B, row star C these are the invariant things is equal to this matrix times row star A, row star B and row star C and this matrix here. So this is 0 0 1/3, 1 0 1/3, 0 1 1/3. Do you notice what is happening here? Very interesting.

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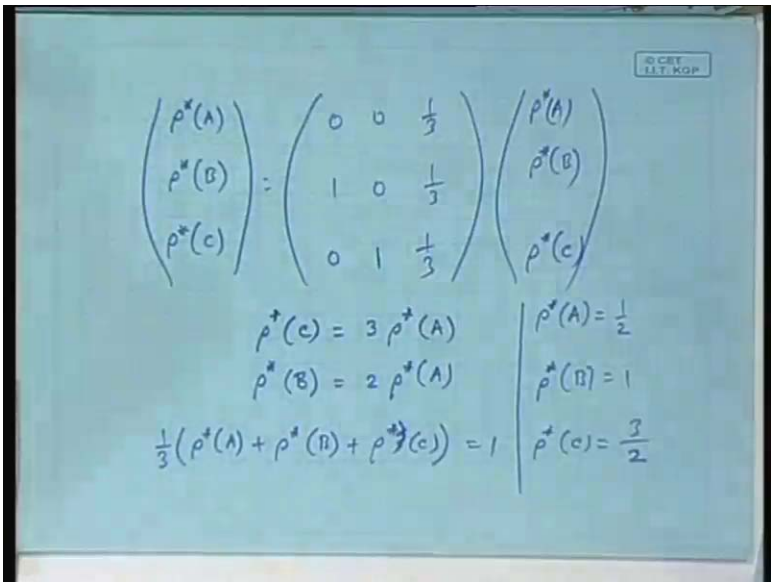

It means that we are interested in finding this. This is nothing but the Eigen vector of this matrix with Eigen value one. This is nothing but the Eigen vector of this matrix with the Eigen value one because this vector when operated on by this matrix, gives me this vector. So if the Eigen value is one then you are getting exactly that. So what is the Eigen vector? It is essentially Eigen vector problem. Can you find out this Eigen vector with Eigen value one? You already have learnt how to find it. In a 3 D map you can find it. No, write down the Eigen vector equation. Eigen vector equations are indeterminate. The point is you have learned that earlier, the Eigen vector equations are indeterminate you will not be able to find it. There will be two equations leading to the same things, same equations. So you cannot really find the Eigen vector like one two three, you can't find it. [Conversation between Student and Professor – Not audible ((00:47:45 min))]

Yes, that is why the normalization come, that summation has to be one because we are talking about densities. [Conversation between Student and Professor – Not audible ((00:47:57 min))] Here there is a little bit different ball game. Though we are talking about the Eigen vectors, Eigen vectors will be indeterminate. Logically Eigen vectors are at certain directions that's all. Any vector along that direction will be an Eigen vector but here we are looking for a specific Eigen vector with the property that the total has to be one. Therefore in addition to this we can write down the equations as row star C is thrice row star of A and row star of B is equal to twice row star of A but that's where we end. What you did was, you assume this is 1 and then 2, 3. That cannot be done, so in addition to that we have to write that one third row star C, this is equal to one.

Once you solve this, in addition to this you get the numbers. What do you get? Three equations you can solve it. Don't tell me that you cannot. You get row star A is half, row star B is equal to one, row star C is equal to 3/2 which means that even though I have not done any iteration, I can usually write down. I can write down the density function as 0, 1/3, 2/3, 1 it will be ½, 1. It will

be a step, it will be a step, it will be a step which means there will be a large, even if you do not do all this you start from that map and start from any initial condition go on iterating it.

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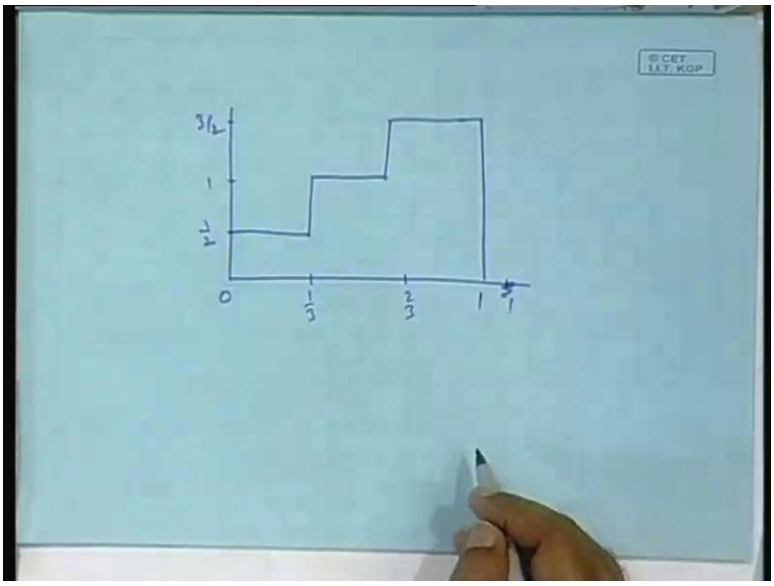

Then there will be a larger number of points falling in this part, a smaller number of points falling in this and a smaller number of points falling in this part. [Conversation between Student and Professor – Not audible ((00:51:14 min))] Here the issue is that their collection put together. Here is the normalization, their collection put together as average to 1. Integration of the row over the range 0 to 1 should be 1. [Conversation between Student and Professor – Not audible ((00:51:55 min))] Let me see where I wrote the definition.

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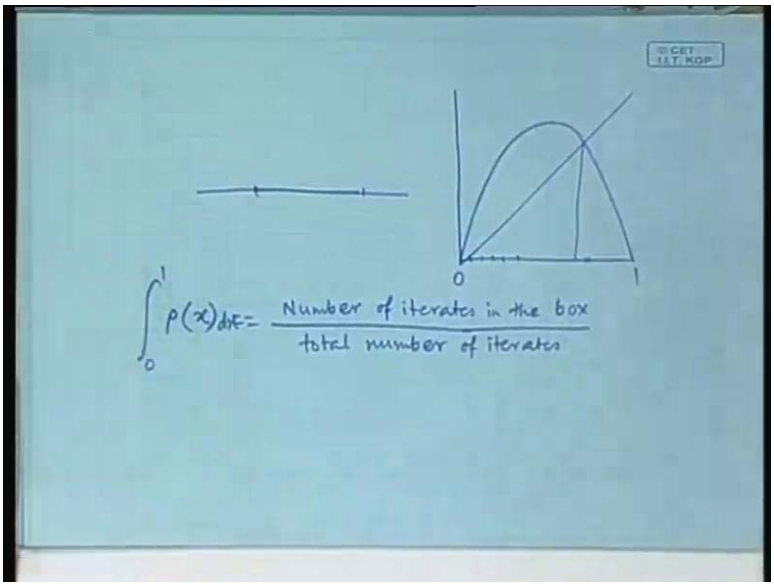

It has to be an integral over the range of x, whatever it is. [Conversation between Student and Professor – Not audible ((00:52:20 min))] This is the right definition basically it has to be integrated over the range of the boxes then that's a right definition. We will end here and will continue in the next class. Remember that it is an assignment to write the program for this, I mean writing the map is trivial. See you might do it for the logistic map, you might do it for this map or you might take instead of 3, you might take 4, everything will be the same.

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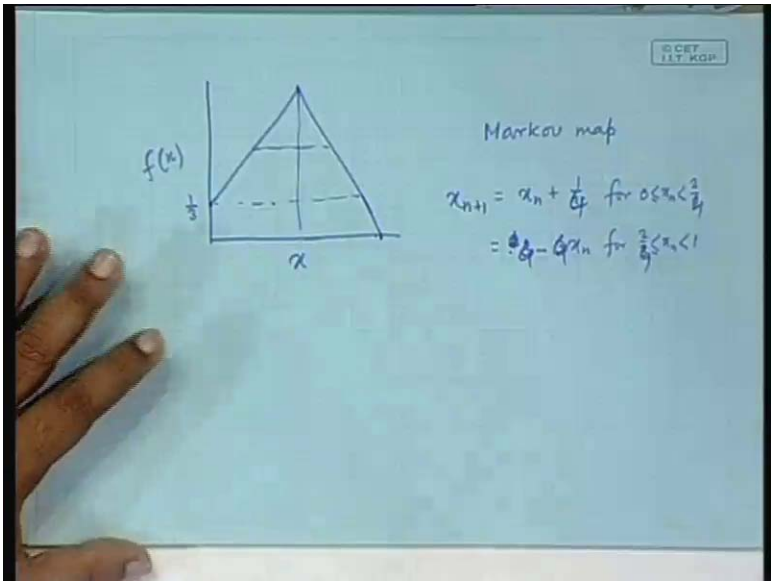

Student professor conversation:

Sir, say that there are 3 ranges. Yes. In which the 3 range is mapped to A B And C that is from 6 by 9 to 7 by 9, 7 by 9 to 8 by 9 and 8 by 9 to 9 by 9 that's one. Now you take say there are 300 points now so 100 points in each range maps to the individual ABC range. Yes, now sir say 100 points map to one range and it is the actual length is increased because of the slope. Yes, you have written row C by 3, isn't it row C by 9. If you take that one I mean. No, not really. Sir we are taking uniform density here.

Yes, that is because of the Markov map property that in the Markov map, you can since break points map to break points, you can divide the range into some finite range of such boxes large chunks. It's not small one by 100 kind of a chunks, large chunks so that boxes map to boxes. That means it's not that some box, a part of it comes from here another part of it come from there, it should not be like that. You can easily and clearly define that this box comes from this box and that box, so that is the property of the Markov map. What is the Markov map property I said? The break point should map to break points and that gives in the advantage that boxes map to boxes.

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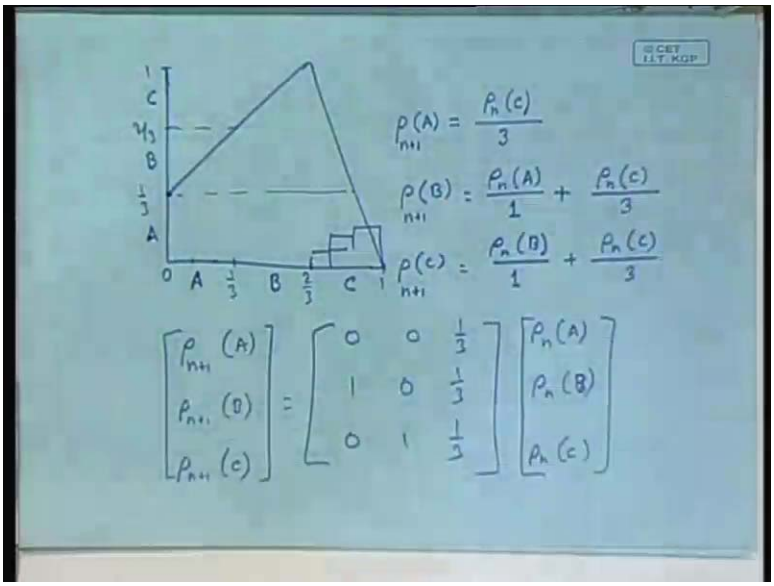

That is exactly why we could say that this box if it has a particular density then in the next iterate it will be redistributed but ultimately your point is if you can divide into different density in this part. That's where your question came from. Yes, that would have been necessary if you could say that the whole of A does not come from B and C or something. Rather the whole of A in order define it, a part of A comes from here another part of comes from there. [Conversation between Student and Professor – Not audible ((00:55:37 min))] Let me clarify it then you come there.

Ultimately the density in the boxes has come from somewhere. Where does it come from? In the previous iterate it came from B and this part of C. If C happens to be uniform, here it will remain uniform. If you start from uniform it will remain uniform, in the next iterate also it will be uniform. That means ultimately the density over C will be uniform. If it is uniform see even if you start from different densities within this three chunks there will be ultimately even out. Why? Because C is coming from B and only one part of this which is uniform. You try that at home simply by writing a code, so that you start from different initial condition, different densities you will find that mysteriously there all even in out. Why? Because of this property of Markov map. Got the point? Let's stop now and we will continue in the next class.