Chaos Fractals & Dynamical Systems
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Lecture No # 21
Boundary Crisis and Interior Crisis

In the last class we were talking about the boundary crisis. Just to briefly recapitulate where we were, we said that if there is an attractor that could be a periodic attractor or a chaotic attractor but supposing there is not only one attractor but there are more than one. Also we had considered the attractor at infinity, the instability condition that means the condition under which the system will collapse. There are states that will run to infinity that's also an attractor.

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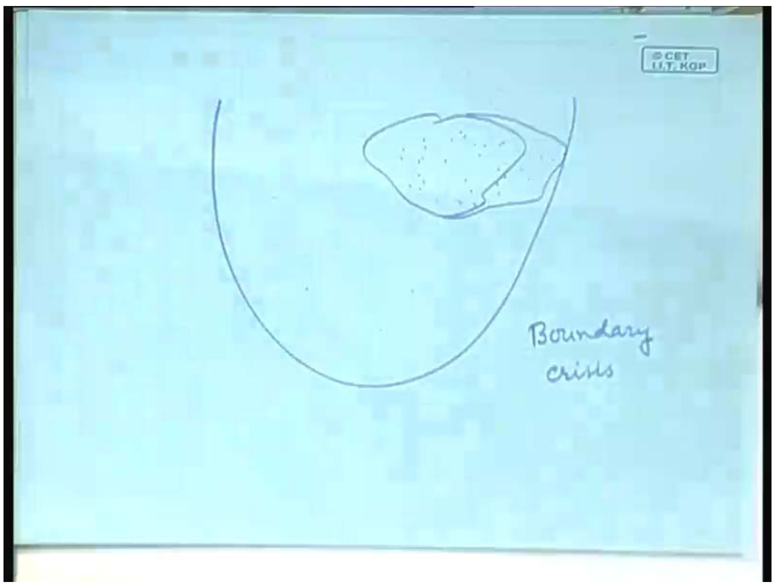

If that is so then there would be some kind of a basin boundary. Suppose there is an attractor at infinity and suppose here is some kind of a stable attractor. That stable attractor could be a stable chaotic attractor also. If with the change in the parameter, this chaotic attractor moves in position, also the basin boundary would move in position and if it's so happens that the chaotic attractor makes contact with the basin boundary then what will happen. Suppose it comes somewhere say here. Then if you start an initial condition somewhere here, it will still go there. It will still go on making its iterates within this but at some point of time it should fall here, a point that is outside the basin of attractor of this attractor and then it will go to infinity.

Under that condition what would you see in that system? You would see very long chaotic transient and it's a transient that means after a system has under gone a boundary crisis, the chaotic attractor will become unstable. Notice earlier when we were talking about the stability or instability of a fixed point then we could locally analyze that means we could locally take the Jacobian, we could locally look at the Eigen values and stuff like that. We could conclude about its stability but obviously we cannot do that regarding the stability of chaotic attractors because it’s spread over a region. I cannot talk about the stability of a particular point. Therefore local stability does not really apply for this stability of chaotic attractors. It has to be some kind of global stability concept and it is this then. That means there would be a basin of attraction and if that basin of attraction somehow touches the chaotic orbit then its stability is lost. What produces the basin of attraction, what produces the basin boundary? The stable manifold of a saddle fixed point sitting on the basin boundary. So there should be a saddle fixed point sitting on the basin boundary and its stable manifold should be making this fellow.

So essentially it is a question of the contact between the attractor and the stable manifold of another fixed point. What is the name? It’s called the boundary crisis. It is you can see that this phenomenon is there for the handy work of manifolds. The attractor itself is sitting on the unstable manifold of some saddle fixed point and here is another stable manifold and they come into contact that is what we making the issue. So these are the effects of stable and the unstable manifolds. So whenever we talk about the stability of a chaotic orbit, after all you can easily understand that chaotic orbit is unstable at every point that is why it is chaotic. If it is stable at any point, that point itself will become stable. it’s not so. Within the chaotic orbit it is unstable at every point but it’s globally stable. that global stability is indicated by this and that global stability may be lost because of the boundary crisis.

Let us now try to understand another type of interaction between the stable and unstable manifolds. As we have already said attractors are always sitting on the unstable manifold. All attractors and therefore chaotic attractors also sitting on the unstable manifolds. periodic attractors also sitting on the unstable manifolds but the unstable manifold is not itself the attractor. The attractor must be there on the unstable manifold because unstable manifold has the attracting property but the actual attractor may be a subset of the unstable manifold. So there would be a large part of the unfold manifold that is not in the attractor. The attractor will only be a subset. Let us see an example.

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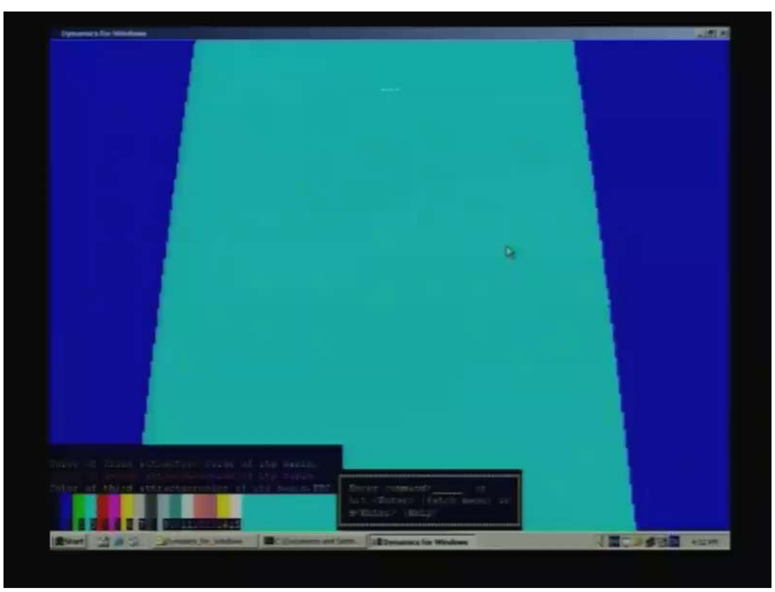

For example here we have the Henon map. On the screen I am displaying the basins of attraction of the Henon map but in this case let me see the parameters that I have taken. In this case your B is being varied, B is constant so this is minus 0.3, earlier we have taken 0.4 and A is being varied. Right now this parameter value A is 1.0 and we have got a nice basin of attraction. The moment you see the basin of attraction, you would say that there must be a saddle fixed point sitting on the basin boundary. Where is it? Again for this system the fixed points sit on the 45 degree line because y_{n+1} is equal to x_n because of that property. So it should be somewhere on this 45 degree line.

So in this program how do we locate the fixed points? There was a comment II in which if we initialize the point, its image is also shown and you have to pull them so that they converge. So I have the small cross which I am moving and there is a big cross which is moving as a result. Can you see and I anticipate that fixed point to be somewhere in the left hand corner so I will bring it down and then will go to the left. Let me go to the left hand, as a result of which the image has taken a short turn and moving fast. Can you see that and moving quite fast, we have done well. We are more or less at it, yes we are reasonably close. Somehow the resolution of this computer is not matching. So let me bring the resolution little more down that's all. I will change the resolution to… so I am sitting there. Now I want to plot the unstable manifold to the right.

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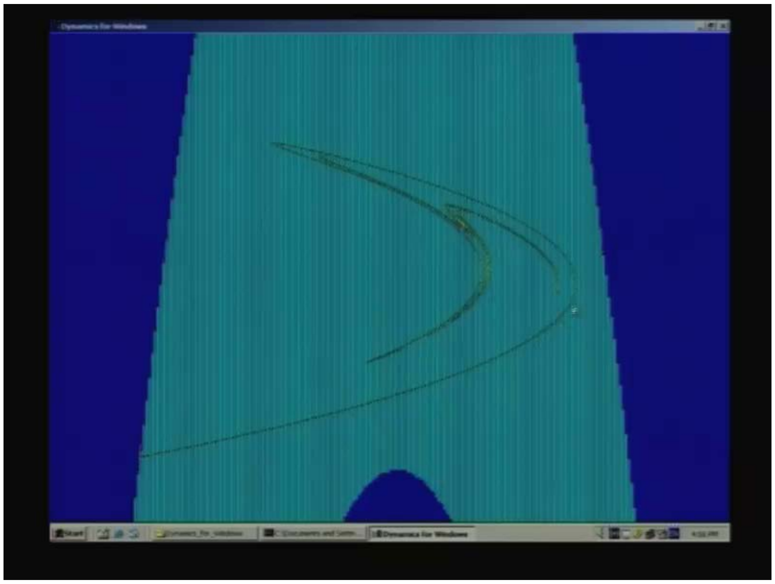

So can you see the structure of the unstable manifold? It starts from here, can you see that and it goes, turns round and the fixed point is sitting somewhere here. So you see here is a turn that is coming very close to this (Refer Slide Time: 10:14). This is what stable manifold. It is not difficult to anticipate that as you change the parameter further, at some point of time it might intersect. Now what would your anticipation say, what will happen then? What did we say. If the stable and the unstable manifold intersect once, it must intersect at infinite number of times. You have proved that so let us see what is the result of that. As we change the row Further I will come to 1.3 may be.

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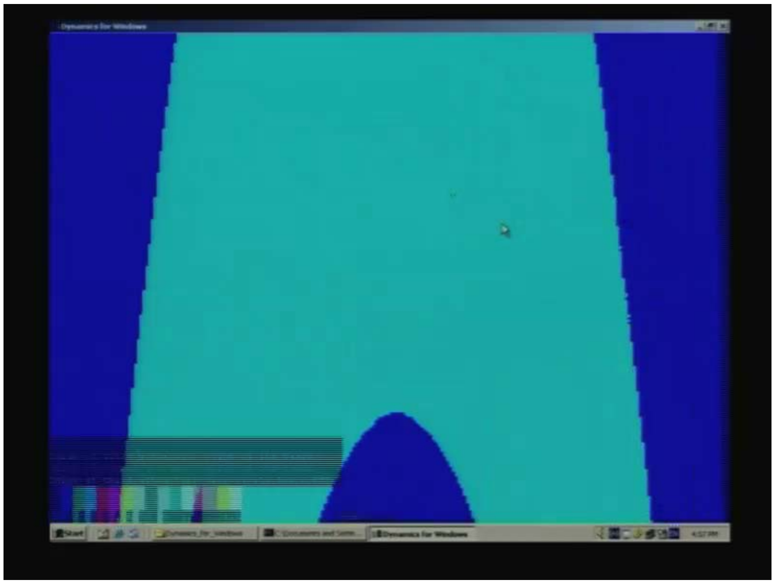

So you got see the fixed point is sitting here, there is a period two orbit sitting here and now let me recalculate the position of the fixed point. It must move a little because I change the parameter.

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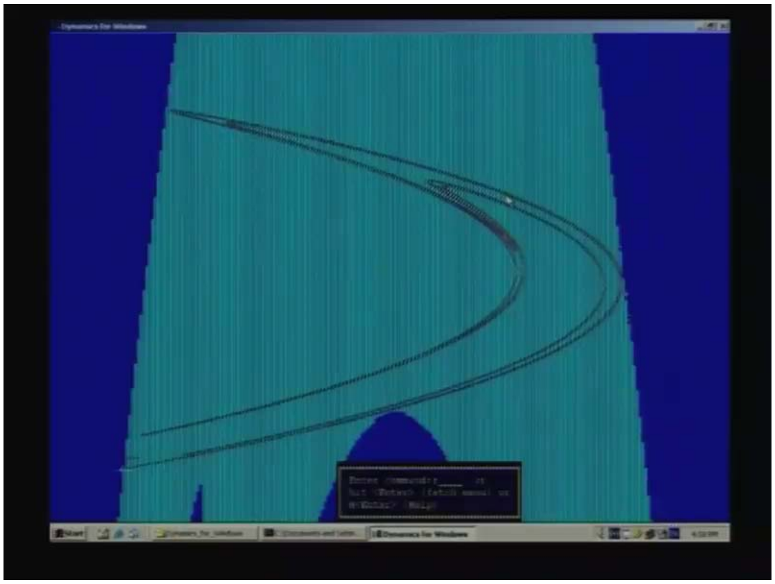

So I am here (Refer Slide Time: 11:36) let me now go to the unstable manifold to the right. Can you see the structure started from here but has come rectories close to the basin boundary but it is still a periodic orbit. We saw that it is a period two orbit sitting here. Now let us increase the parameter slightly it was 1.3, let me make it 1.35 a small increase.

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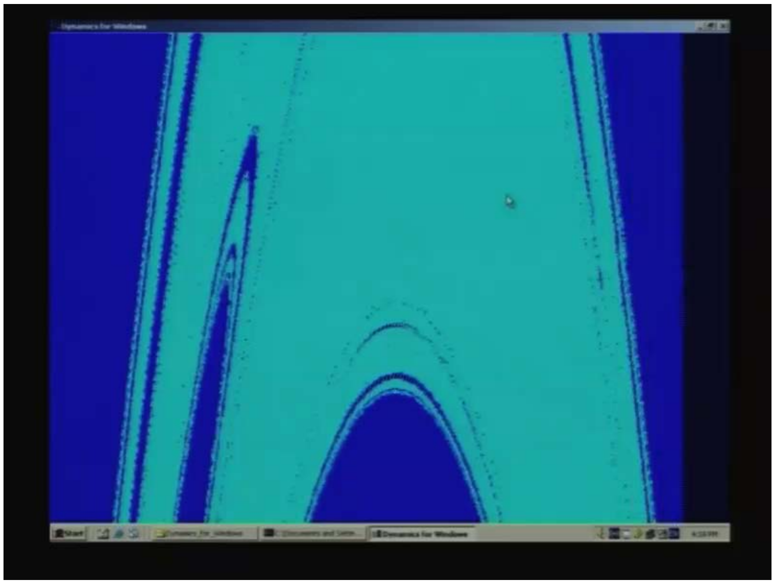

Let me increase the basin resolution. Let me make it little larger. Basin resolution let me increase because without the high resolution, it will be problem. See it has become fractal structure. How did it come about? Let us locate the fixed point again. Now I will draw the unstable manifold to the right.

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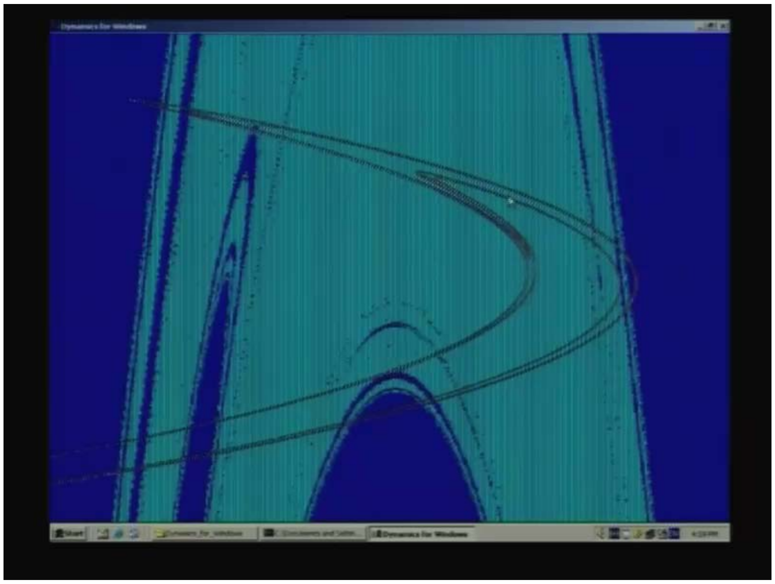

Can you see it has intersected and as a result it must have intersected at infinite number of points. So as these intersection happens, you can easily see that immediately the structure of the basin boundary becomes fractal. So this is another kind of very interesting nonlinear phenomenon that is caused by the interaction of stable and unstable manifolds and these things have interesting implications in engineering systems also. For example there is a group in UK, who showed that there are situations where people who work on naval architecture, they try to study the ship rocking motion in the sea. So what they did was, if you have a ship perfectly there in the sea then it is more or less similar to a pendulum because its center of gravity is down there and point of suspension can be assumed to be somewhere up there and suppose it is being continuously hit by waves and waves can be a sinusoidal excitation. So you can easily approximate it by a sinusoidal excitation. So it is nothing but the problem of a simple pendulum being excited by a sinusoidal excitation.

They found that if the magnitude of excitation that means A sin omega t, that A term if it is smoothly increased then normally there is a periodic orbit, it will drop like that and it will have a basin boundary and then if you keep on increasing the wave intensity then from the basin boundary there are fractals fingers that get in. So that while rocking in the wave either state somehow hits the basin boundary, the ship captures it. So it is not a very simple action, even in the simple system there is lot of nonlinearity because if you write the equation it will be… Can you imagine what the equation would be? x double dot plus damping term x dot plus some constant is equal to A sin omega t that's it. So even in this kind of simple system interesting behavior can be seen.

Essentially the point is that the basin boundaries, if they are fractal then that has a lot of implication in the engineering system also because we want to ensure that state does not hit the basin boundary. If the basin boundary becomes fractal what is the concept of the basin that if I start from here, I will land into the actual attractor. If it becomes fractal then what? Then if you look at my cursor, if you say some where here, see you are normally inside the basin of attraction of the attractor. Remember this is not the attractor, this is the unstable manifold. The attractor is the periodic attractor sitting somewhere here. So if you are somewhere here, you would expect that normally I would go to the attractor but you can see there is a finger of the attractor at infinity that has come inside here and if it is a fractal structure then if you say that my initial condition somewhere here.

Obviously you cannot specify the initial condition accurately. There will be some kind of error and with an error ball, there will be a finger of the basin of attraction of the other attractor coming into that region. So even if you believe that I am safe, you are not safe. these kind of very typical nominal phenomenon happen in such in nonlinear system. This is a typical nonlinear phenomenon. Completely you cannot really understand this kind of phenomenon, this kind of events using linear systems here. It is typically nonlinear effects. There has been lot of work trying to find out supposing there is an error ball depending on the error ball there will be people who proved that there should be some points of both the attracters in that error ball.

What is the probability of finding your state really within the basin of attraction of the attractor that you want to go to? Those studies have been done but essentially the message is that near the boundary where the structure is fractal, it is extremely difficult to place your initial condition in such a way that you can guarantee that you will reach that attractor. Now let us try to understand another phenomenon. Another phenomenon called interior crisis which is caused by the interaction of the stable and unstable manifolds. Let us start. Supposing there is an attractor somewhere here.

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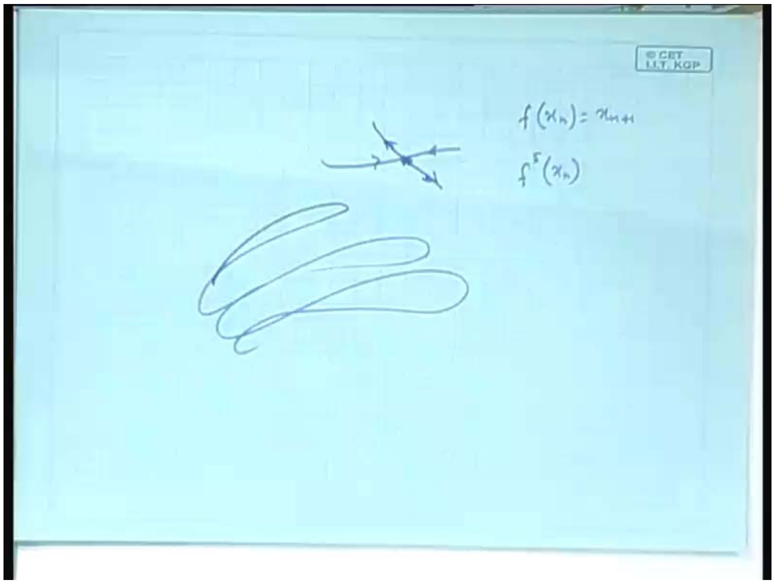

I mean arbitrarily I am drawing a chaotic attractor but nevertheless you can assume that and then there is a saddle fixed point sitting here that saddle fixed point could be a period one saddle, could be period two saddle, could be period three saddle, could be period five saddle whatever but a saddle fixed point sitting here. What do you mean by period five saddle? If it is f (x_n) is x_{n+1}, if you take the fifth iterate that means f 5 of (x_n) then it is a saddle means this function will have an Eigen value that is less than minus 1 or greater then plus 1. So here is an attractor, here is an attractor and here is a saddle fixed point which must have its stable and unstable manifolds. Keep this picture in mind and with that let us try to understand a theorem called a lemma, it's called lambda lemma.

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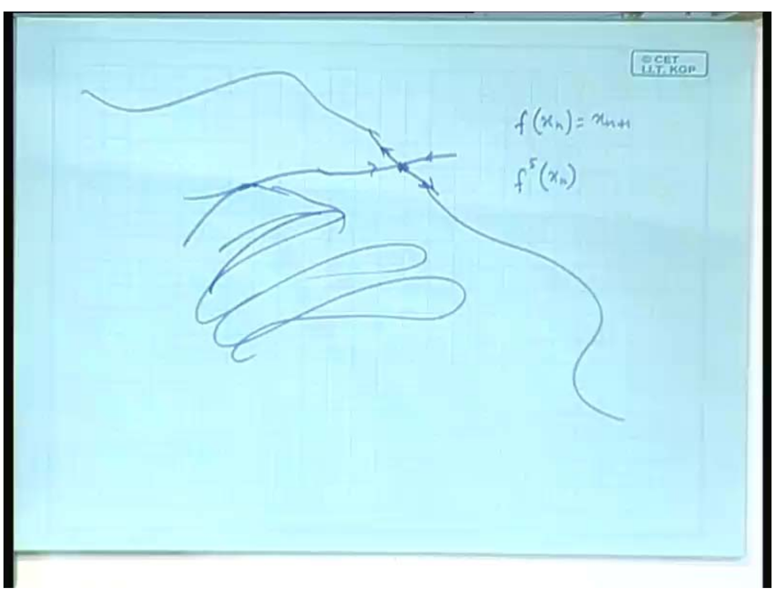

That lemma says that if you have a saddle fixed point and it is unstable manifold and the stable manifold like this and suppose let me extend a little bit. Suppose some line L intersect the stable manifold, some line whatever it is. Then if you iterate this line with the map that means this is the stable and unstable manifold, the fixed point of a map. If you take the same map and iterate all the points on this line then ultimately the line L will converge on to the unstable manifold of the fixed point. So what will happen? In the next iterate it will go somewhere here, it will go somewhere here and ultimately it can come here (Refer Slide Time: 22:20). Why? Because of the action of the stable manifold. Because there is a stable manifold anything that intersects the stable manifold will come under the influence of the stable manifold and as a result, forward iterates of L will make it converge on to the unstable manifold. So you say the f n of L limit n tends to infinity will converge on to that.

Now what is the implication? Let us get back here. Here is a stable manifold of this and here was an attractor. Suppose at some point of time this attractor moves as a change of parameter and makes contact with the stable manifold then what will happen? Here is an attractor, imagine it taking the position of L that line. So as it makes contact, what will it immediately imply? It will immediately imply that forward iterates starting from this orbit will converge on to the unstable manifold of this fellow. That means where ever this unstable manifold goes, the attracter will immediately go there which means that earlier, the attractor was small only this much and suddenly you will find that beyond a critical parameter value what are actually happened you don’t see that because you don’t see the stable and unstable manifold all the time.

These are hidden in the dynamics but under line phenomenon is this that attractor has made contact somewhere here. It has made contact with the stable manifold and as a result of which the whole of the unstable manifold of this saddle fixed becomes part of the attractor. The attractor on forward iterate converges on to the unstable manifold and therefore the resulting attractor becomes much larger in size. This phenomenon is actually very common and you will find that in many situation this happens. Suddenly beyond the certain parameter value, you will find a huge attractor suddenly being born. Why? Because of this. If you want to see an example let us get back to another system that is the Ikeda map.
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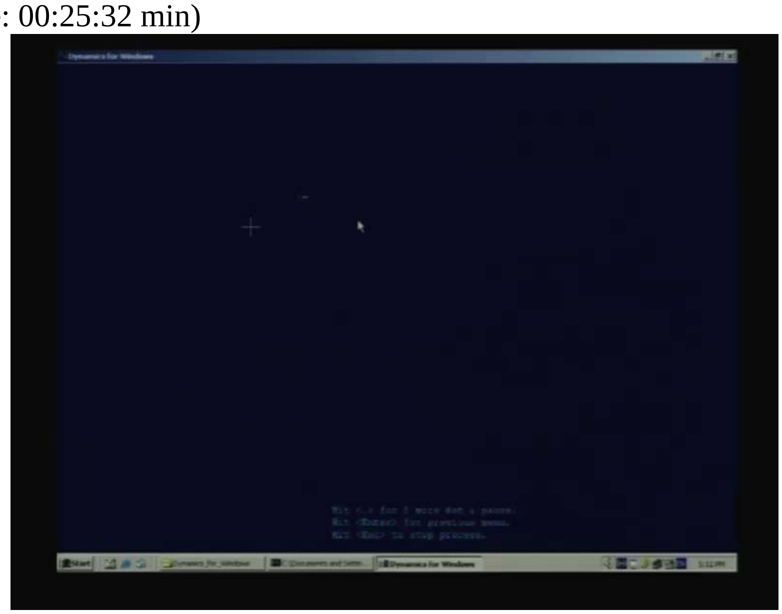

Here is what is known as Ikeda map. It is a map where the variables is complex variables so real axis and imaginary axis that are been plotted but in any case essentially thing is that it has defined a 2 D dynamical system that can be plotted and as you can see that starting from initial condition, it is ultimately converging on to a fixed point. For a parameter value the row was 0.5. What system is you don’t really need to know, you can easily find out because it is a bit longish definition of the system if you want I can write it but nevertheless.

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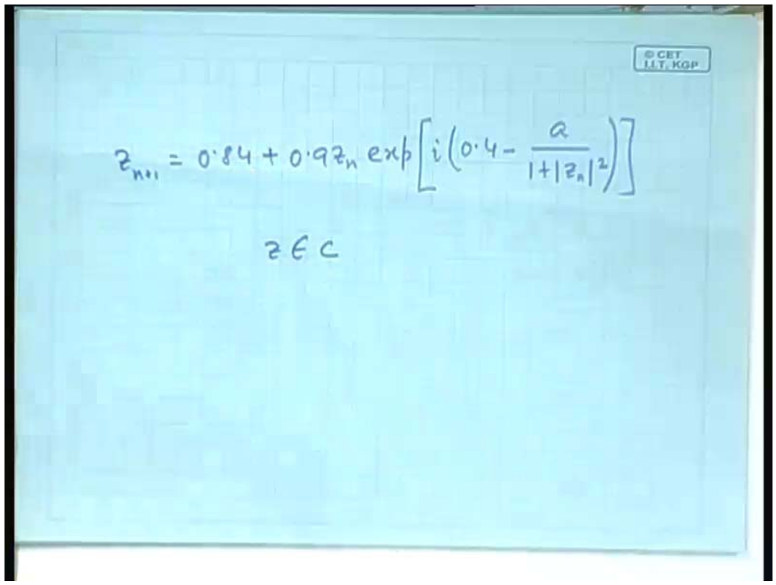

That is z_{n+1} is equal to 0.84 + 0.9 z_n exponential [i (0.4 minus a divided by 1 plus mod of z_n whole square)]. This is the essential expression of the Eked map where z is a complex number and we are plotting the x versus y coordinate, the real verses imaginary coordinate. This is not all that important because you do not immediately see ways of analyzing that. There are ways of analyzing that but I do not want you to get into this, look at the screen. So here we have the parameter a is sitting here, so we are just changing it.

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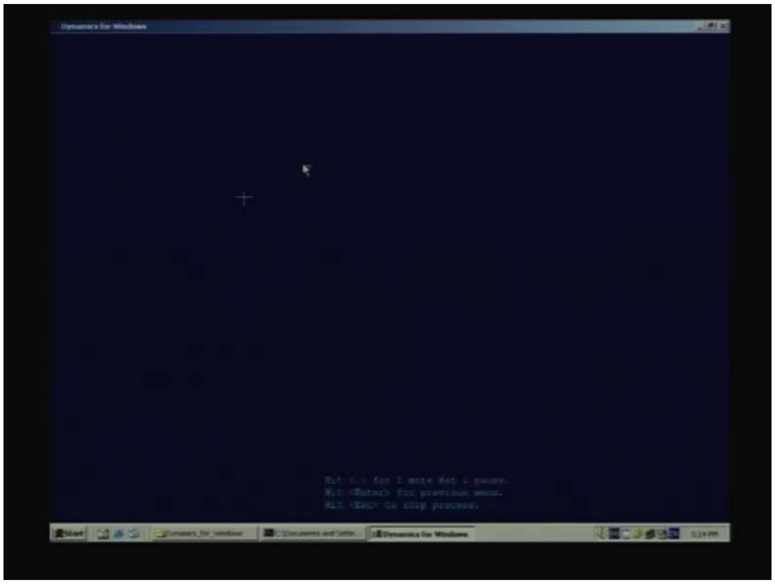

Make it 0.6, it just moves. It has moved to this nothing happened, 0.7.

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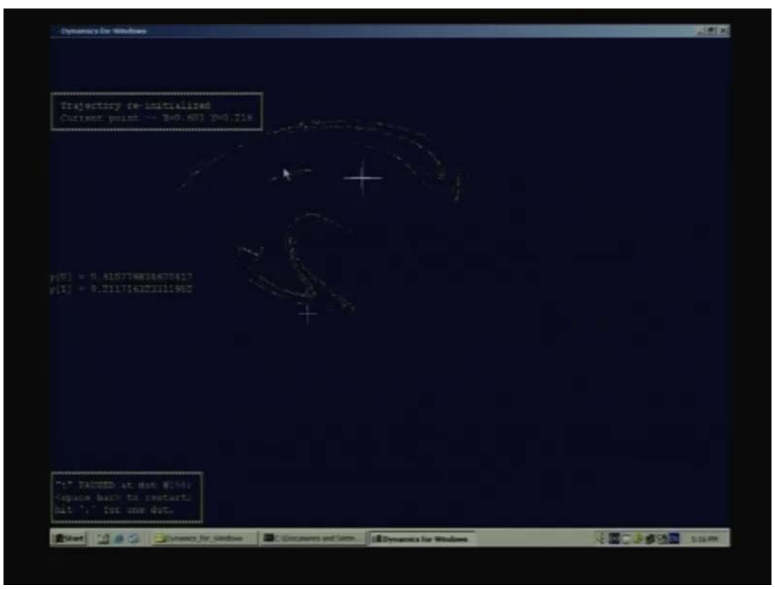

If you want to see now what happens? Let us start from a point that is here and let us go to T, T is the trajectory plotting. Can you see the cross, it is making all sorts of jumps but finally can you see that it is jumping everywhere, some kind of a structure is emerging, a ghostly structure. I am just keeping on iterating the map. You see it is still not converging on to anything; it is going on making its rounds on a very specific structure. Has it sufficiently immerged. If I now move the initial condition somewhere out of there and say here see now what is happening? It's a period two orbit.

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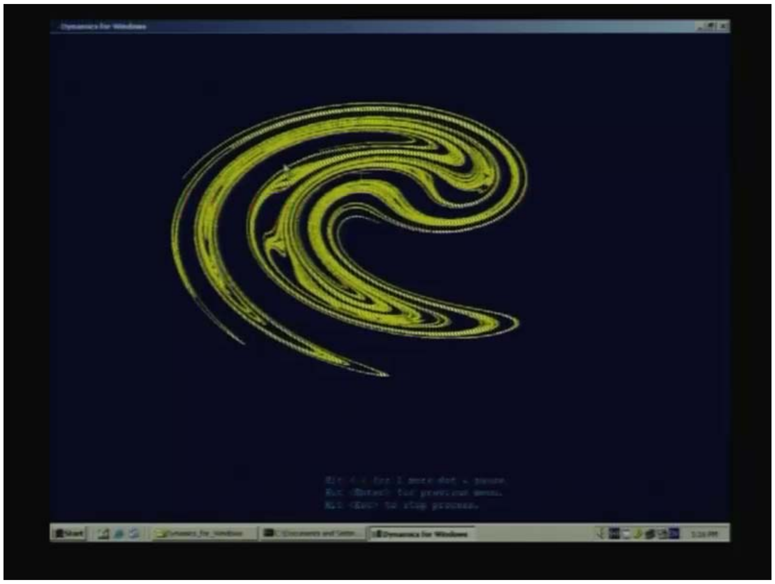

So actually the period two orbit is stable and there is nothing out there but you can still see a structure. Why? That why will become clear, the moment we change the parameter slightly further, as yet nothing. So there was this thing that suddenly appeared. It is a chaotic orbit obviously that appeared to have suddenly appeared. It was a period one orbit then you used to see a period two orbit and then suddenly this. How did this come? It obviously did not go to the usual sequence that I have already talked of like period doubling cascade and all that. It didn’t go through that. It suddenly went into a large chaotic orbit but last time when we are doing it, I deliberately started the initial condition somewhere else not very close to the actual fixed points and we saw that it was jumping for a long time. What does it mean? It was actually doing chaotic transient. At the certain parameter value suddenly it appeared. Why? What actually happened was that here there was a periodic orbit but this orbit was at that time unstable, it was an unstable chaotic orbit.

At some parameter value it made contact, there is an unstable saddle fixed point sitting here and the whole thing is on the unstable manifold of that saddle fixed point. At some parameter value that attractor which was a period two attractor at that time made contact with this stable manifold. The moment that happens the forward iterate will contain the whole of the unstable manifold that is what has happened. If you want to see the bifurcation diagram, if you want to be convinced that really there is no period doubling cascade, just look at it. What was the parameter value we chose? It varied between 0.7 and 0.9 something. Let me change the 0.7 to 0.9.

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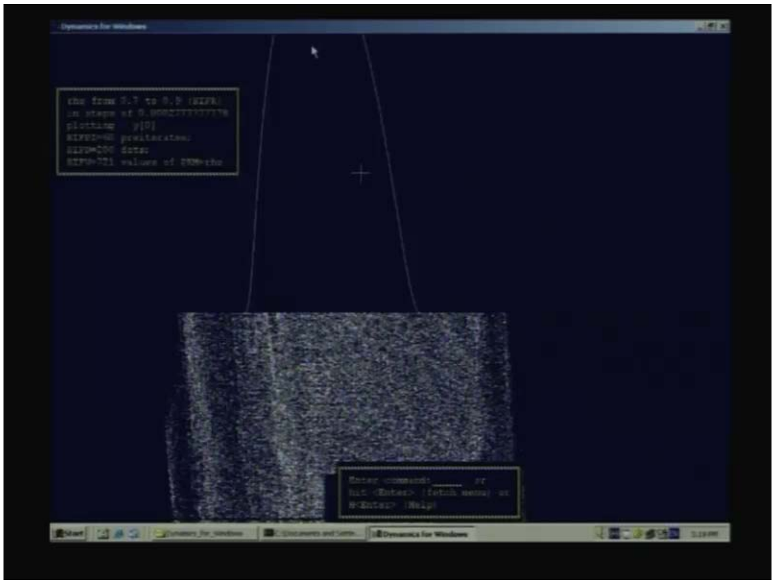

See there was periodical orbit coming, it is from the top to bottom. It is a period two orbit the parameter is being varied like this, being varied in this direction. It was a periodic two orbit and suddenly it becomes chaotic orbit. Why? Because of this phenomenon. You might ask where this chaotic orbit came from? Obviously that is a question where did it come from? Obviously it must have come from the kind of phenomena that we have talked about but ultimately it become unstable. In this part it was unstable (Refer Slide Time: 33:52). Why was it unstable? Because of the phenomena that we have already talked of, boundary crisis. It underwent a boundary crisis thereby it was unstable and at this point of time, at this particular parameter value what happen was, the period two orbit make contact with the stable manifold and therefore the whole orbit became visible.
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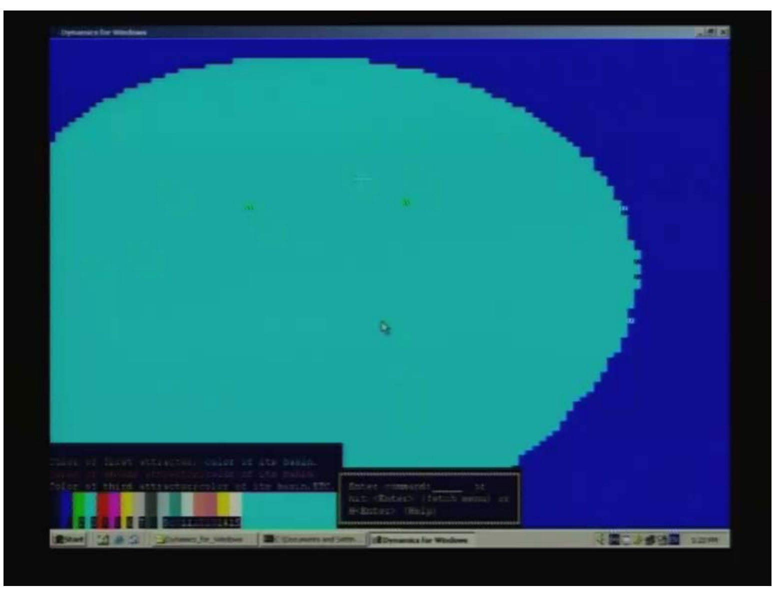

Let us chose a parameter somewhere in between. Say row 0.8 and just plot the basin of attraction. See the basin of attraction is fine. This is the basin of attraction of the period two orbit. Do you see any chaotic orbit? No, so there is no chaotic orbit. It has a nice basin of an attraction so imagine that this is a practical engineering system whose behavior, a practical designer has designed and therefore everything is fine. One is happy that my basin boundary is far off, yet something is larking very close. See it was 0.8, I make 0.85.

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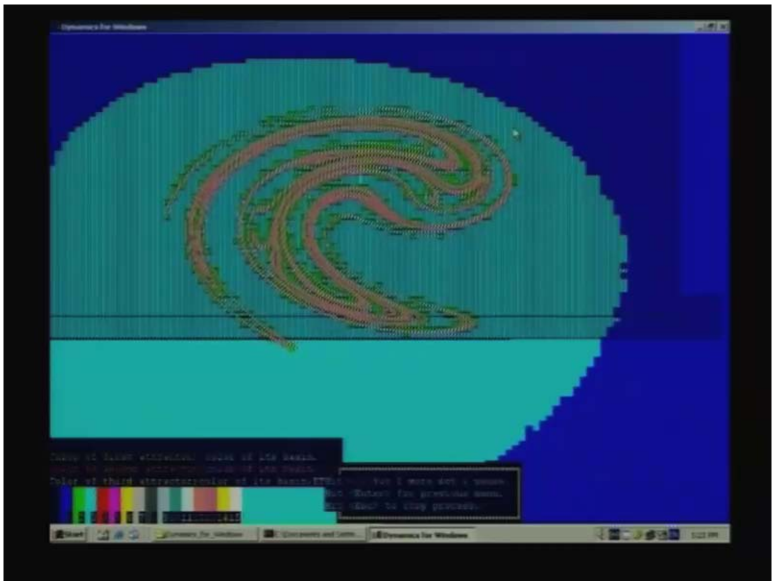

Suddenly becomes chaotic, if you want to see the behavior, see slight change in the parameter it becomes chaotic. The basin boundary remains the same because the basin boundary is created by a saddle fixed point. It is still there and its stable manifold has not changed much, something else is happened here. That is an interior crisis. If you want see a simple understanding of this particular phenomenon let us take a look at the simple logistics map.

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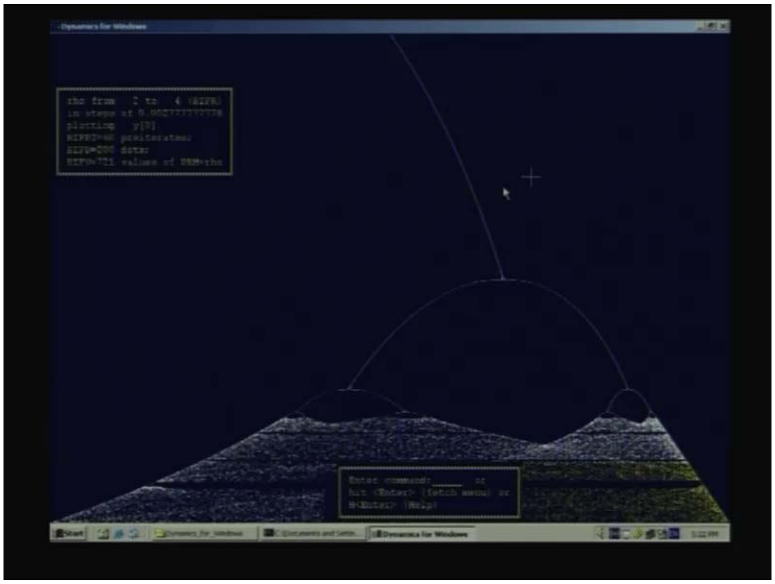

Here is the logistic map. This is the logistics map, this one I suppose you all know pretty well but I will make it little more darker so that you can see it clearly.

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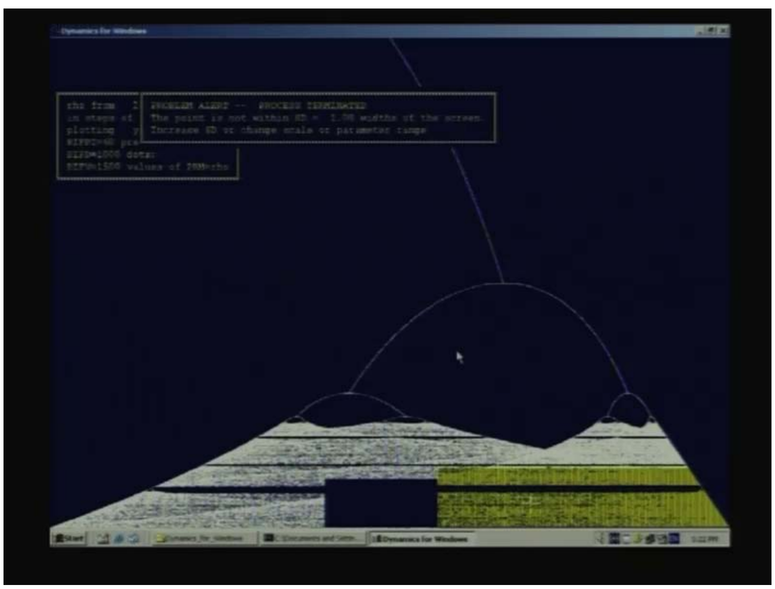

Can you see now? Yes, now if you see this part what happened here you remember? It was a period three window created by what? A saddle node bifurcation. We know that it is a saddle node bifurcation. So let us blow up this part. What was a parameter value? We have done that earlier.

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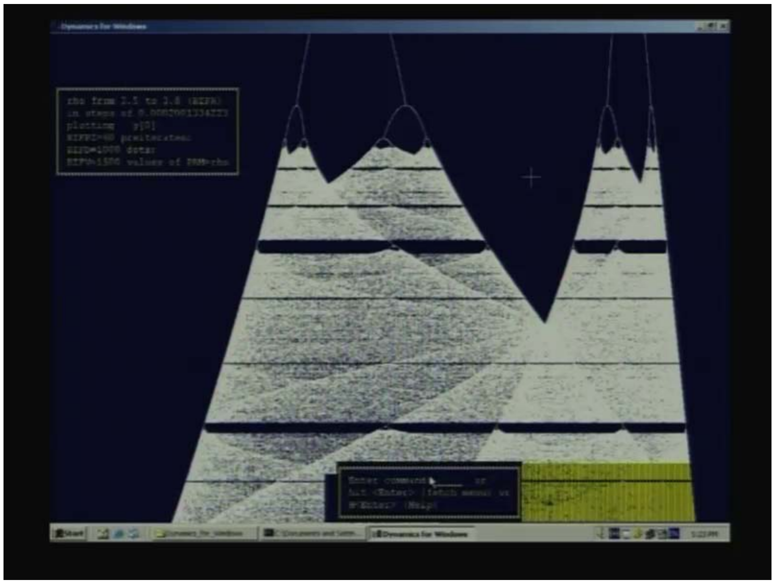

Let me see 3.52 and let me increase to this range 3.8. It is farther 3.8 to 3.9.

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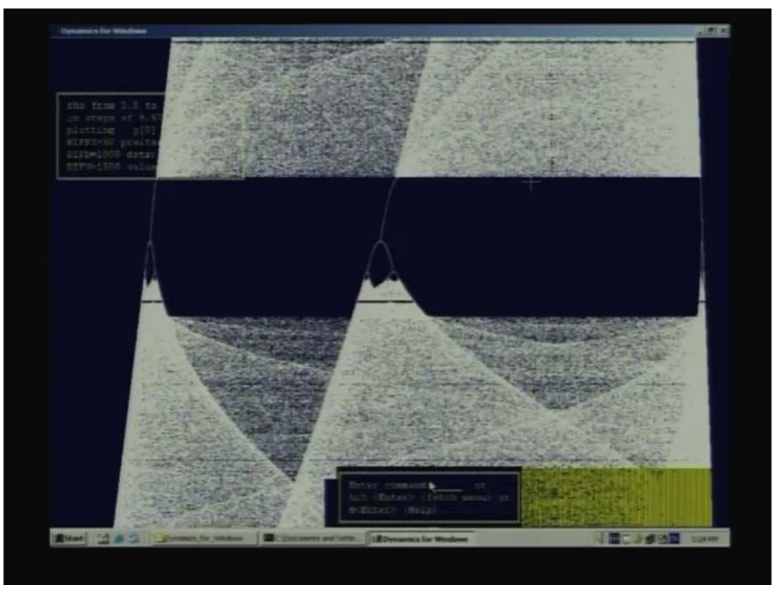

Chaotic orbit and a period three orbit is borne. Are you clear about this structure? Now see this particular event, we had already explained. it was the birth of a period three orbit through a saddle node bifurcation so we already done that and this orbit also goes through the similar period doubling cascade resulting in chaos but it's a chaos with a small spread. Can you see that? Suddenly at this point what happened? Can you see that just before this point, it was a chaotic orbit nevertheless but it was distributed into three chunks, one chunk here and other chunk here and third chunk here with small spread. Suddenly at this point it became hole.

What actually happened was this is also interior crisis. What happened was here at this point though we can see one of these lines, this line represented the node line. Here is a saddle node, there is a one saddle bond and the node bond and this fellow was the node. There was obviously another saddle also bond that saddle came like this and at this point that saddle contacted the chaotic orbit. When that happens what do you expect? The whole of the unstable manifold of the saddle will become part of the attractor and that exactly what happens. But in one D what is the unstable manifold? Of course we don't have unstable manifold there. This is a one dimensional system and in the one dimensional system you don't expect to see unstable manifold but then it becomes easier to understand.

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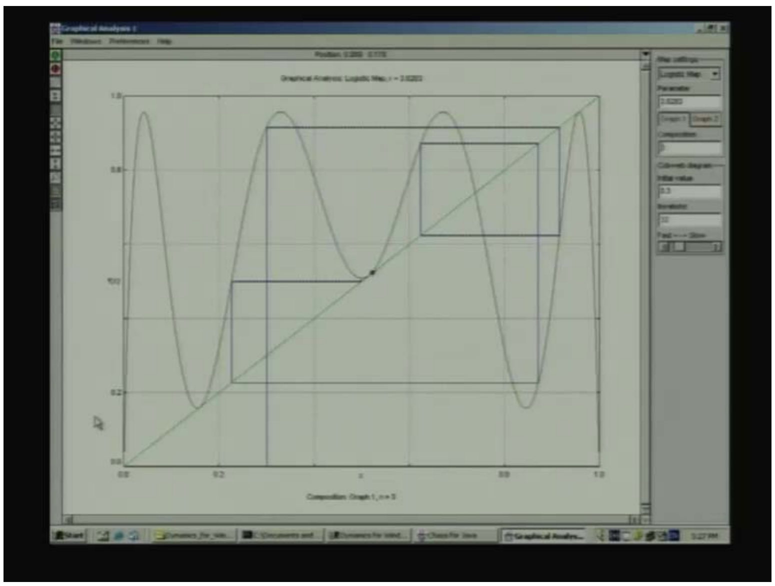

Why? Because then what happens? Let us make it small and let us see. It will be easier to see here. (Refer Slide Time: 00:40:40) graphical analysis. So logistic map, what is the parameter value corresponding to that? This is the path of the periodic orbit, so I will draw composition three. So this is the path of the period three orbit I have drawn x_{n+3} versus x_n. How many contacts there are now with the 45 degrees line? one two three four five. Actually there are two contacts here and two contacts here. If you change the parameter slightly further you see here is that two contacts, these are crossed.

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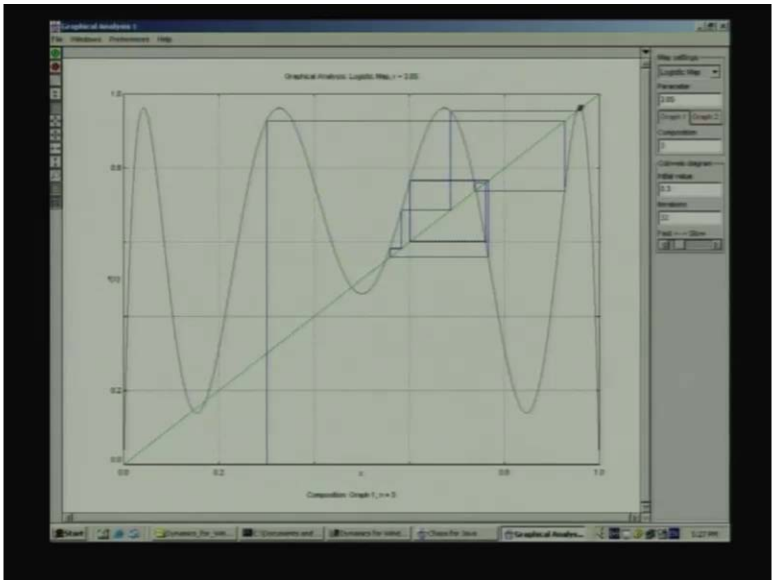

Now what happens is that because of this, here there is a fixed point that was initially a stable one (Refer Slide Time: 41:53). Beyond the certain parameter value that becomes unstable. As a result you have a chaotic orbit bound here. This is a saddle that was bound. When this particular chunk cross the 45 degree line, this for this node and this was the saddle. This could not be seen, that could be seen but this underwent the period doubling cascade and became chaotic. When that happen let me draw a larger number iterates, it will be clearer. See it is now jumping there, it will get locked there. Now let me increase the parameter little further.

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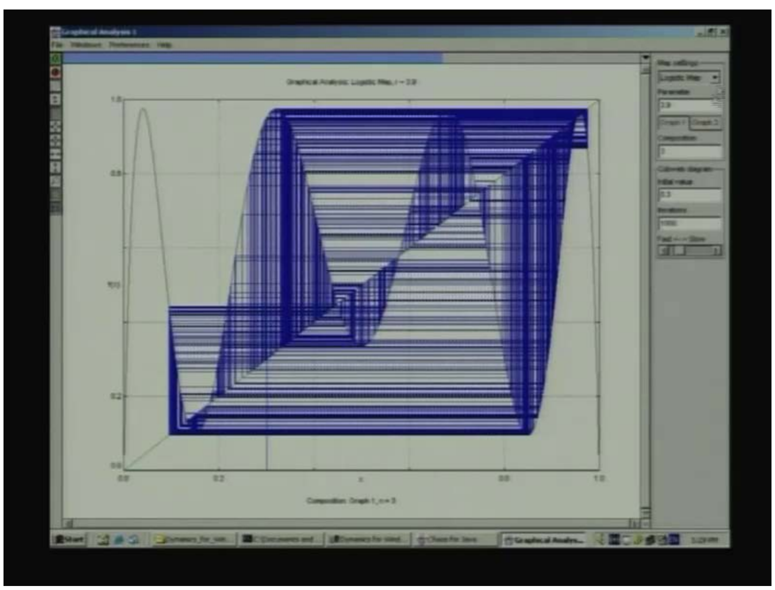

See now the whole place becomes covered. Why? Because there was a chaotic orbit created because of this but the chaotic orbit made contact with this line at this point. As a result anything that goes to this side that gets thrown off to the rest of the area. So that earlier the chaotic orbit was contained within this. Let me reduce the parameter then it will be clearer say 8, 7 or something like that. Now it still enlarge right see it gets struck here. To start off with another initial condition it will be stuck here, say if the initial condition is say 0.5.

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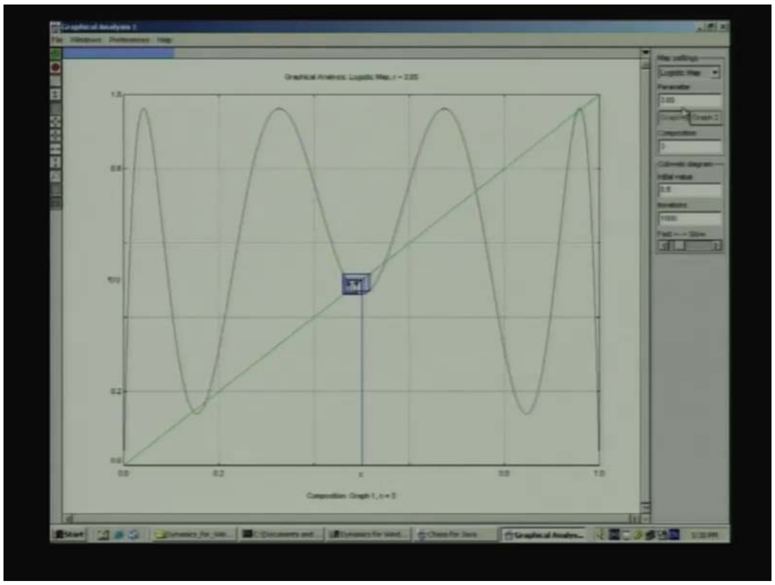

It gets stuck only within this part. Increase the parameter very slightly it doesn’t get stuck. Why? Because this saddle point has now covered the orbit. So because it has this slope larger than forty five degree line anything that goes to this side, goes to the rest of the orbit. This particular phenomenon, when put it to two D can be seen as the fact that the whole of the unstable manifold of a saddle fixed point becomes part of the attractor. So let us now concentrate on the sheet. What we have learnt? We have learnt three particular things that are caused by interplay of stable and unstable manifolds.

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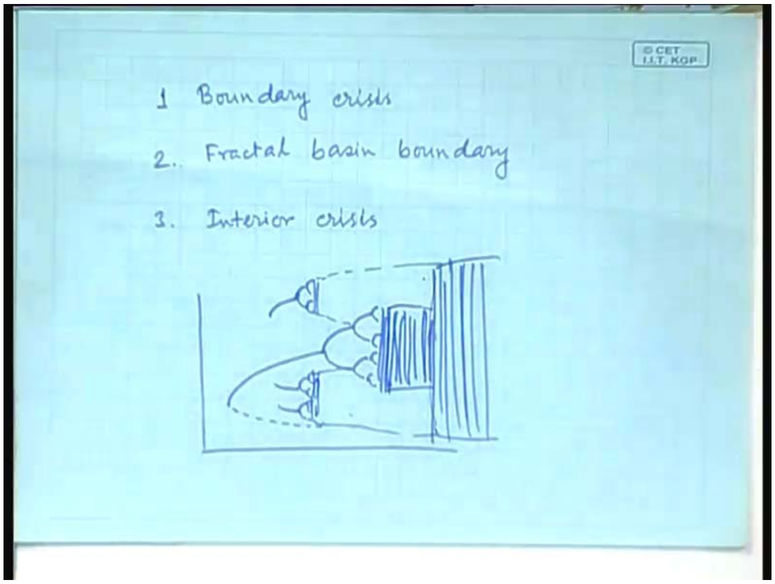

One, we have learnt the boundary crisis, two we have learnt the fractal basin boundary and three we have learnt interior crisis. Now mainly this two taken together explains much of the nonlinear phenomenon that you see in practical dynamical system. Take together with it, what you have already been talked off that means the normal local bifurcations like the period doubling saddle node and stuff, these are local bifurcations and this a global phenomenon. These are called crisis because as you can see that there was a stable manifold, there was an unstable manifold. They come close at some point, they may contact. So all this phenomenon are because of that because something made contact to something else. It is not to be understood in terms of some local instability like taking the local linear neighborhood of the point and stuff like that. It cannot be explained by that. It is basically something making contact with something else in all this.

Now very common situation in practical dynamical systems is if you should draw the bifurcation diagram, you see that there was a stable periodic orbit bond at certain time. While it is bond what you can expect? Also an unstable periodic orbit being bond, they are always being bond together then you have a period doubling cascade and stuff like that. While it proceeds, there can be coexisting attractors being bond. How can it be bond, how can a coexistence attractor come in to existence. Through a saddle node bifurcation that means at this point there was a saddle node bifurcation occurring in the third iterate of the system. As a result this fellow is bond, as you increase the parameter further this will also undergo period doublings and so on and so forth.

Very common thing, not uncommon at all but then when this additional periodic orbit is bond obviously in this system there are two periodic orbits at the same time coexisting periodic orbits that means they will have the own individual basin of attraction. There would be own individual basins of attraction and then beyond a certain parameter value if one of these attractors say this axial area or secondary attractor makes contact with the basin boundary then what happens this fellow simply goes off. It no longer remains stable. So in the bifurcation diagram you will see a small chunk and then finally it vanishes, extinguishes. You won't see it any further.

Now normally a person not equipped with proper knowledge in dynamics would say that i don't need to worry about it further, this fellow has gone out of existence. No, it has become unstable but it is there. That is a chaotic orbit, this is a chaotic orbit which is become unstable because it has undergone a boundary crisis but it is there and it may again regain existence, regain stability through an interior crisis. So it might happen that beyond the certain parameter value, you suddenly find that is enlarging to this size and it will baffle you normally but the moment you know that this fellow were actually existing you would say that my orbit was actually going on like this. This was the unstable part of it, the chaotic orbit became unstable but it still existed and at this parameter value, the actual existing and stable chaotic orbit made contact with the unstable one. As a result of it, the full orbit becomes much larger, these are very common phenomenon.

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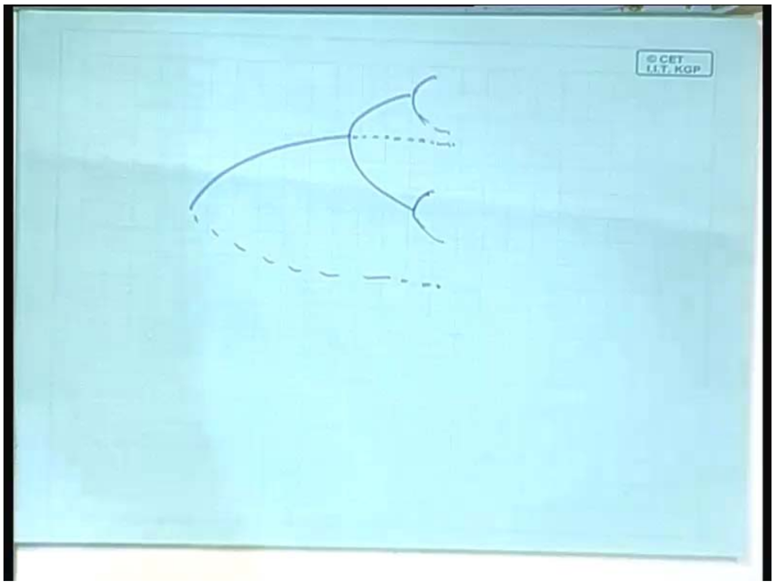

So two things to remember; one whenever we draw a bifurcation diagram like this, most people forget that there is also another fellow coming like this. What is this? The unstable fixed point, in case of two D is a saddle coming like this and when this orbit will make contact with the stable manifold of this one, you do expect a saddle enlargement of that orbit. Not only here, the moment this fellow became unstable it continued as an unstable periodic orbit that one also would have the same effect. When these branches may contact with this one, we do expect some enlargement of the orbit. So this sudden enlargement of the orbits are rather common, you do expect that to happen very frequently.

Let us summarize the boundary crisis makes an orbit not to go out of existence, go out of stability. Normally you would understand stability in terms of a continuous time system, in terms of the Eigen value going to the right of plane, in terms of the descriptive system, in terms of the Eigen value going out of unit circle, all this fail. So the stability of chaotic orbits are understood by whether or not that has undergone a boundary crisis.

When something undergoes a boundary crisis remember that the fellow is still existing which is manifested the way I showed. That is you start from an initial condition not on that factor, it goes on accelerating for a long time, in that unstable chaotic attractor it is there, very long chaotic transient and I have seen such chaotic transient persisting for millions of iterates. That means you cannot simply say that I have eliminated the transient for about the thousand iterates. If I get it, it is there. No, there are situations were such chaotic transient persist for millions of iterates and that chaotic transient becomes stable whenever there is an interior crisis. When the actual existing stable orbit makes contact with the unstable chaotic orbit that unstable chaotic orbit is sitting on the unstable manifold of some saddle fixed point. Finally how would you actually draw a bifurcation diagram like this?

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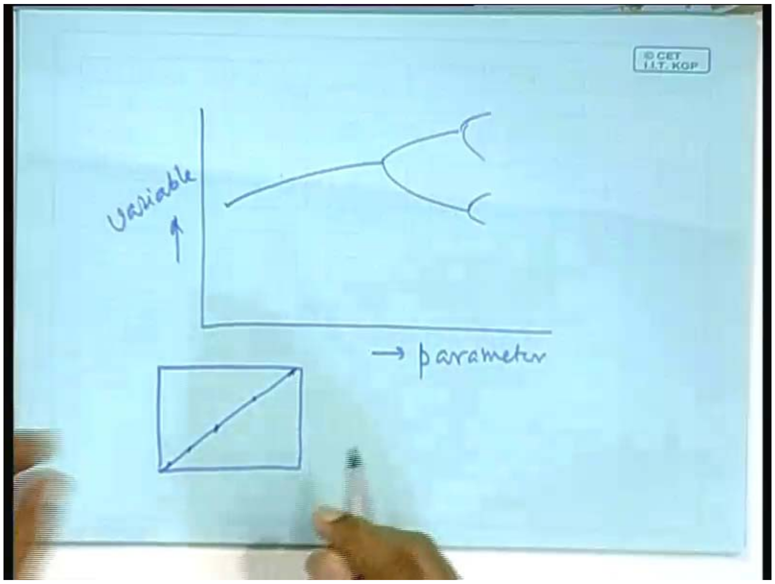

The way to draw a bifurcation diagram normally is that you start from an initial condition. Here is the parameter axis and here is a variable. Start from a parameter and start from an initial condition. Whatever initial condition is, you only need to ensure that the initial condition is inside the basin of attraction of whatever attractor there is. So if you now iterate for a longtime say a thousand iterates and eliminate the first 900 iterates, there is a high probability. I am not saying that it is guaranteed but there is high probability that would have reached the periodic orbit or whatever orbit there is. Then plot the last one hundred points. If it is a periodic orbit, all the one hundred points fall in the same place, go to the next parameter value do the same procedure then you get the picture but it is a stupid procedure. Why? Because every time you have to eliminate the initial transient for every parameter value.

As a result the computation time will be large which is unnecessary. A most smarter procedure would be to make the final condition of one parameter value, the initial parameter for the next parameter value that means when you shift the parameter to the next change, use the final variable of the last parameter value, the initial variable for the next parameter value. So that in the case since you are changed the parameter only slightly, you can expect the attractor plane move only slightly and therefore you are now slightly away from the attractor.

So only with a small amount of transient, these are called pre iterates. Pre iterates means the transient that you have to ultimately eliminate that becomes less. So from the next one you could simply eliminate only 10 or so, becomes faster. But this way you will be locked into one periodic orbit that means if it is like this, you will be able to see this nicely but if there is another one being bond you will not be able to see it. Why? Because your initial condition will lock to this one. So though that procedure is smart because it is faster but that cannot detect the co-existing attractor. In order to do that you have to start with the many initial conditions. So if say it is a two dimensional system then take a rectangle where first find out this rectangle, so that all attractors will be within this rectangle.

Then you need to take the initial conditions everywhere but if you take initial conditions everywhere and do this procedure, it will take ages. So a smarter procedure is take initial condition on a diagonal because if this part is divided into basins of attractions, there is always a high probability that some initial conditions say you have taken a five initial conditions, at least one will fall in the basin of attraction of one of them. This is how we detect those co existing attractors. Thank you, that’s all for today.