Optimal Control

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Lecture - 9
Solution of Constraint Optimization Problem-Karush-Kuhn Tucker (KKT) Conditions (Contd.)

So, last class we have seen that how to solve a, what is call optimization problem with constraint in equalities. Graphically we have seen how to solve it. Then in general we say, if you have a optimization un unconstraint optimization problem is there, we know how to solve this analytically as well as the numerical techniques that you discussed, so in general if you have a problem optimization problem, in more specifically if it is a minimization problem subject to equality constraint and in inequality constraint. First we can convert them in to a un constraint optimization problem, by introducing some variables known as un specified variables known as the lagrangian multipliers. So, if you recollect this one there are 2 methods one method is the elimination methods.

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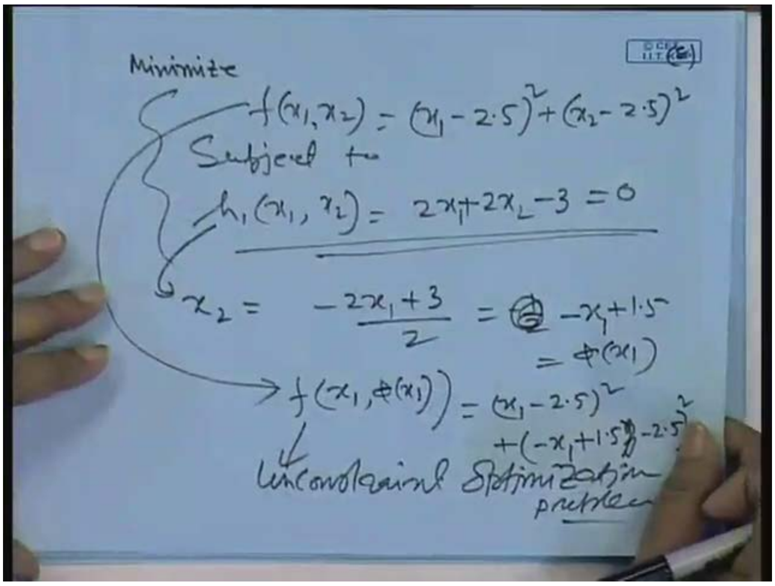

That this problem last class we have discussed that, suppose we have to minimize this function f which is a function of x 1 and x 2 and it is given by this expression. It is quadratic form subject to, let us call it is a equality constraint. If you have a here we have consider only 1 equality constraint. Then what is what will do it who will eliminate the variables, that means in this x expression x 2 is x space in terms of x 1, agree that taking this that side x 1. Then it is a function of x 1 only x 2 is a function of x 1.

So, I have written x f of x which is a function of x 1 and x 2. Now, it is function of x 1 only agree x 2 is replaced by f of x 1. So, this is the expression. Now, this equality constraint is merged in to a all objective function or cost function. So, it is becoming a. Now, un optimization problems. So, this is problem we can solve it by using our standard technique either an analytical method or it is a numerical methods we can do, but this type of elimination process is not easy for variables more than or are numbers of equality constraint.

If it is more several equality constraint are there, then it is not a straight forward case even for 2 equality constraint it becomes more tedious and complex. So, we can do only when that x 2 and x 3. We can express in terms of x 1 expressly then it is easy, but in most of the cases not possible. So, we have look after for another alternative methods and that method is called, that you convert that set of equality constraint or inequality constraint agree in to a what is call un constraint. This equality constraint and inequality constraint will convert in to a what is call, inequality constraint we convert in to equality constraint. Then the overall we will convert in to a un constraint optimization problem. So, let us see how to do that one.

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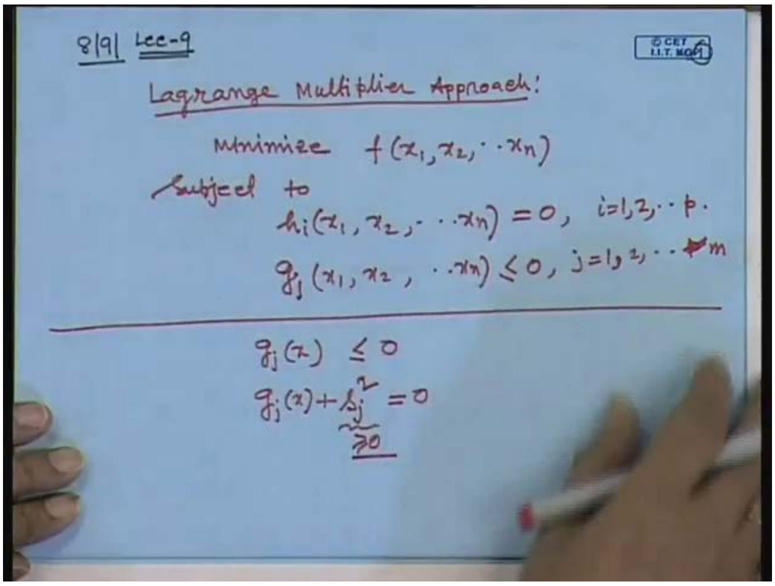

So, that is called basically, it is lagrangian multiplier approach, lagrange multiplier approach, so our in general the optimization problem constraint. Optimization problem is described like this a minimize f of x which is function of x 1 x 2 dot, dot x n subject to equality constraint h i x 1 x 2. In general it is function of x n is equal to 0 and we have how many equality constraint are there. There are p equalities constraint are there and that inequality constraint g j which is a function of x 1 x 2 dot, dot x n is less then equal to 0 for j is equal to 1 2 dot, dot p.

So, our problem is minimize this functions subject to this type of inequality equality an inequality constraint this is a m not p m. So, this problem first we have to convert in to a what is called by un constraint optimization problem wants to convert in to un constraint optimization problem. Then you can solve with by our different techniques, either numerical method or the, what is call analytical methods. Let us say how we can formulate this one. So, this can I just mention you if you recollect that, if you have a inequality constraint are there here, I can convert in to a equality constraint.

Let us call you have a g of x, x is function of this is less than equal to 0 agree. So, that means that g of x which is less than 0 agree would this 1 I have to add some variable. So, that the function g of x can that one would be a what is call 0. So, you can apply g of j x plus a positive term you add it x j square is equal to 0, when x j is equal to 0 that means it is equality constraint of that one it. It as add up to it may be g j of x may be equal to 0 or it will be a less than 0.

When this has a some positive by low this indicate g j is less than 0. So, this inequality constraint i can easily convert in to a equality constraint. This inequality constraint I can convert in to equality constraint by adding what is call sum variables s j square, where this term is always greater then equal to 0. So, this equality constraint, equality constraint this is inequality constraint, I will convert into a equality constraint like this way. Introducing some variable x j as j square, so now you see our problem is re formulated like this. So, this problem is re formulated like this way.

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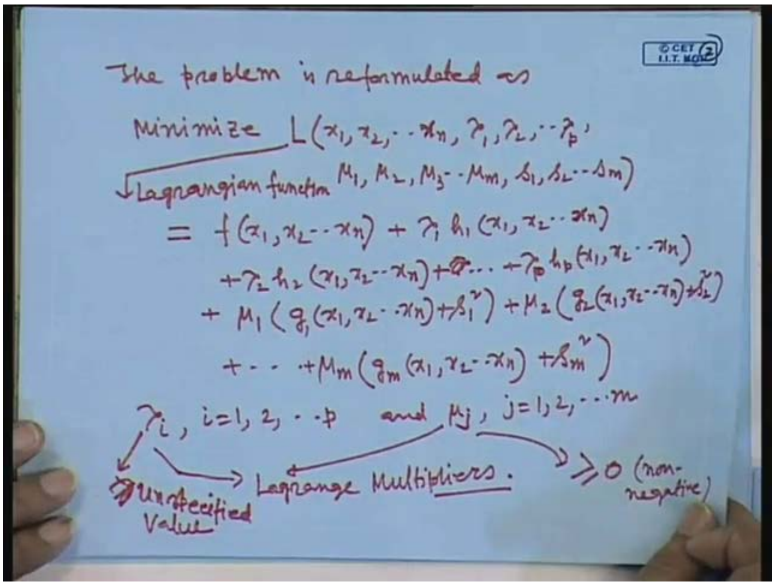

The problem is re formulated, convert it converting all inequality equality constraint agree. Inequality constraint converting inequality constraint and then we are putting this constraint to the objective functions. The problem is re formulated as minimize a function L, new function we introduce which is a function of x 1 x 2 dot dot x n. We have a lamda 1 lamda 2 and lamda p then you have mu 1 mu 2. I will explain what is this mu 3 dot dot mu.

M and there is function s 1 s 2 dot dot s I am sorry this function, I am follow I am up, I will formulate by considering our original objective cost function were objective function. This f of x 1 x 2 dot dot x n, then this inequality constraint agree if I put it here, that objective function will not change, this objective function value will not change when it reach to the what is call the optimum conditions.

So, I can write it lamda 1 h 1 x 1 x 2 dot dot x n plus lamda 2 h 2 x 1 x 2 dot dot x n. We have a such p equality constraint. So, I can write dot dot plus lamda p lamda p h p x 1 x 2 dot dot x n, in addition to that that inequality constraint. I converted in to equality constraint. So, that can be also added to the our new objective functions agree. So, that is is mu 1 in to that inequality constraint. First inequality constraint is converted in to equality constraint, like this way you see g 1 of x 1 x 2 dot dot x n plus s 1 square. This is then mu 2 g 2 x 1 x 2 dot dot x n plus s 2 square and so on.

We have a m e inequality constraint m equality inequality constraint which is converted in to equality constraint by adding s i or else j. So, this is m x 1 x 2 dot dot x n plus s m square. So, idea is that when this function has reach to optimum value, it must satisfy the this condition, this condition and this condition. Which in term it must satisfy this conditions when is it this condition is satisfy then this 0. So, this 0 multiplied by lamda 1 0 by multiplied by this. Similarly, this is also 0 multiplied by mu 1 0 multiplied by this is 0 when it is to the optimum condition it must satisfy the our conditions that means in a what is call.

It must satisfy the our this conditions agree, but in when it is to that the optimum value of this functions, that it must satisfy the our equality and inequality constraint. Now, this I can write if you this is call, this L is call lagrangian functions lagrangian function. So, new objective function is known the lagrangian function. Lamda 1 lamda i where i is equal to 1 2 dot dot your p and mu j and j is equal to 1 2 dot dot m.

This are call the what is call lagrange multiplier. This is called lagrange multipliers agree. So, this and this values lamda i values lamda i values always greater than a unspecified. It is a unspecified value it may be positive negative and 0, unspecified value. This mu i value is always greater than 0, for we you are minimizing the function of that l is greater than is an non negative, non negative. I will explain why it is non negative and this is a any value of that means unspecified value it may be positive negative and 0 this thing i will discuss later.

So, this is the new function. Now, you see what is the our constraint optimization problem is there, that is converted in to un constraint optimization problems agree. So, one can write this expression that is what is call lagrangian function by metrics and vector notation form.

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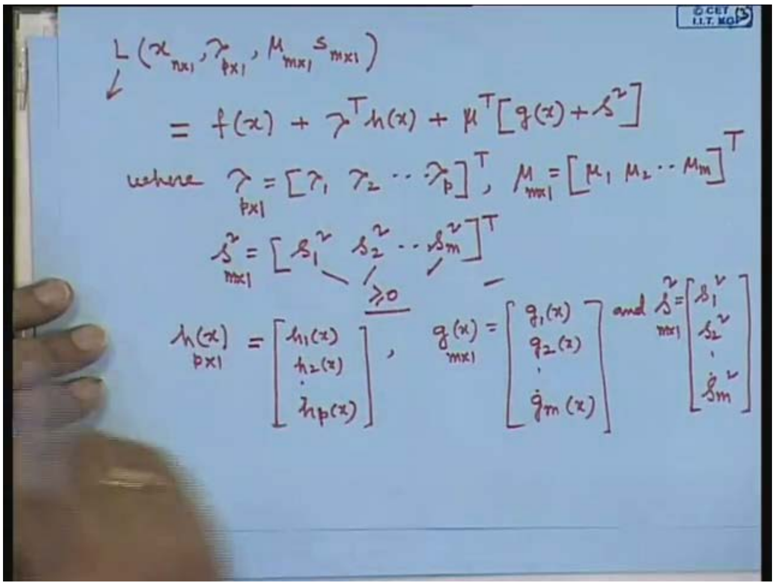

So, this notation one can write this thing in to this notation form. So, lamda 0 L which is a function of x is in n cross 1 function of lamda which is a p cross 1. If function of mu which is a m cross 1 and function of s which is a, I will explain, how you we have defined lamda mu and all this things equals to, I can write f of x this is the f of x. If it is this is the f of x. Then this are write in I am writing in vector form. So, it is a lamda transpose h of x then lamda mu transpose then g of x plus s square.

So, this agree where I am writing lamda is equal to lamda 1 lamda 2 dot lambda p transpose and lamda dimension is 1 p cross 1. Similarly, mu dimensions is equal to mu 1 this is are all lagrange multipliers mu 1 mu 2 dot dot mu m transpose whose dimension is m cross 1. Similarly, this our s, s square that s, s square is see thus 1 is nothing but a s 1 square s 2 square plus dot dot s, m square agree transpose.

This dimension is your again m cause 1 these are the things. I told you this value of s if you recollect this 1 the value of s is here is a positive quantity x is square is a positive quantity. So, these are all greater than equal to 0, when it is equal to 0, that means g of s which is less than equal to 0 when s is s i s 2 is 0 always saying, then it is a equality constraint of g of x is satisfied. When is greater than 0 than it is inequality constraint of g of x is g of j is satisfied agree. So, this are things, then where is h of x.

So, we know what is lamda mu and s agree. So, h of x is equal to your h 1 of x or you write it h 1 of x agree h 1 of x is 2 of x dot dot h p of x agree or if an then g of x is nothing, but a g 1 of x g 2 of x dot dot g m of x an next I will with this dimension is p cross 1. This dimension is m plus 1 and you s square dimension is a s 1 square s 2 square do dot s m square. This dimension is same as that, this g is here g dimension because we are adding with g s s square.

So, that inequality constraint is converted in to a equality constraint. So, this dimension also m cross one. So, keeping all thing is mind that is our mu all this things we have just mentioned it here. This are the all lagrange multiplier lamda and mu are the lagrange multiplier. These are unspecified and this will values are why non negative that is, we have to specify. Now, once I got the what is call constraint optimization problem that instead of minimizing f.

Now, I am minimizing L and we are embedded the our inequality constraint an equality constraint in the objective functions of that. So, then our question call next is we can write it our what is call our k k t condition before that, k k t means necessary conditions and subversion condition in you have to minimize this function. So, this is call lagrangian function you want to minimize that 1 next what problem. So, next is before we discuss then that 1 will just we have seen just.

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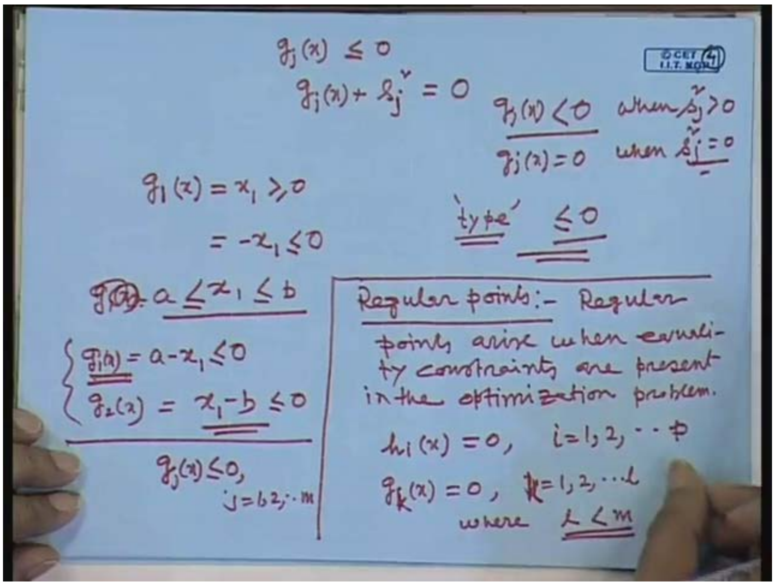

Now, that if you have a x j of x is less than equal to 0 then we can convert in to equality constraint by using a variable s i s j square, positive quantity which will be the leg ranges later on when s l to 0 that mean equality condition is satisfied when as is not equality 0 means positive quantity that equal, in inequality condition is j of x is less than 0, when s j square is greater than 0. This is equality constraint is satisfied, when s j square is equal to 0 agree.

So, let us call suppose, if you have a sum inequality constraint like this way. Let us call g 1 one, one equality constraint I am showing like this a it is equal 0 x 1 an x 1 is greater than equal to 0 then x is the our decision variable or design variables agree and that is given x 1 is greater than 0, means it is a side constraints given that x, variable after doing the what is call optimizations minimizations of the problem x cannot be negative agree non negative.

So, this I can convert in our inequality type less than equal to 0 this 1 how you can make it this is nothing but a minus x 1 this indicate the x 1 is positive agree, non negative this one. Now, if I am put it minus of x 1 that will be less than equal to zero. Now, you see our g 1 is a inequality constraint which is our type that type, our type of inequality we have use less than equal to 0 this type I make it agree. So, any inequality constraint of this type I can convert in to this type our z type less than equal to. We have always we can equality constraint greater than equal to 0 and inequality constraint of type of less than equality zero.

So, suppose you have a variable x 1 and that variable limit is given x 1 is less than equal to s x 1 is greater than equal to a and less than equal to b. That means x 1 has a lower and upper range more bound of x 1 is given. So, let us call this is g 1 of x again this g 1 of x not g 1 this is the bounded is given. So, I can these range I can split up in to a 2 inequality constraint. So, I can write if you see this one I can write a minus. If I a write it a minus x, if you write it a minus x, what will be this one, a minus x is less than equal to 0 both side i minus, minus lagrange both here minus x.

I mean subtracted minus x sep separate it x one. So, this I can consider is same as g 1 of x. Another constraint I can write g 2 of x I can write it that x 1 minus b is less than equal to 0 agree. Now, natural a x 1 is greater than equal to a. So, if you just subtract a from x 1 then it will less than equal to zero. So, this is one equality constraint and this is another equality constraint. If you have a range of decision variable or side variables range is there. So, that can be the compose in to 2 inequality constraint of such type of in a inequality agree.

So, this their mention you the before we discuss that our condition necessary condition. There is important point is there to discuss the what is call k k t conditions, what is k k t will discuss next. So, that is a regular point, regular points. So, regular point arise when equality constraint are present in the problem. So, regularly you can just write regular points arise, when equality constraints, constraints are present in the problem in the optimization problem.

Now, you see if you have a regular point the regular point has a 2 features. One feature is that, if x star is a regular point agree and that regular point must be feasible. Feasible in the sense, that must is must satisfy the equality constraint and inequality constraint is the first feature of regular points. Next is that, this regular points agree that the regular points you will get the what is call that equality constraints. Equality constraints will come from the problem statement it is given equality constraints and some of the equality constraint will come from inequality constraints.

So, we can say the features there are how many equality constraint will get from the optimization problem, that means equality constraint will get h of x equal to 0 that i is equal to 1 2 dot dot p. There are p equality constraint and there is a possibility is there you, if you have a x j of x is equal to 0 for j is equal to 1 2 dot dot small one. Now, our original problem was, if you see j varies from 1 2 or you write k in place of this j, in are to make confusion write k k is eqaul to this agree.

Our L is the because our basic equation of inequality constraints is x j of x is less than equal to 0 for j is equal to one two dot dot small m. Out of this small m inequality constraint of j we have a l equality constraint of g of j. So, total I we have a because where you can say were l is less than m. So, if you see the our original problem we have a p equality constraints agree. There are n inequality constraint, out of m inequality constraint of g there is a small l equality constraints.

So, in all together how many equality constraints we got it p plus small l. So, if you the point regular points, if the regular points that x star. If a x star is a regular point, then it must satisfy that, the gradient of that g gradient of h and gradient of g k for equality constraint all this gradients should be linearly independent. Then that point will be the regular point of this optimization problems. Now, what is this I just whatever just I mention it I will just write it. So, that you can…

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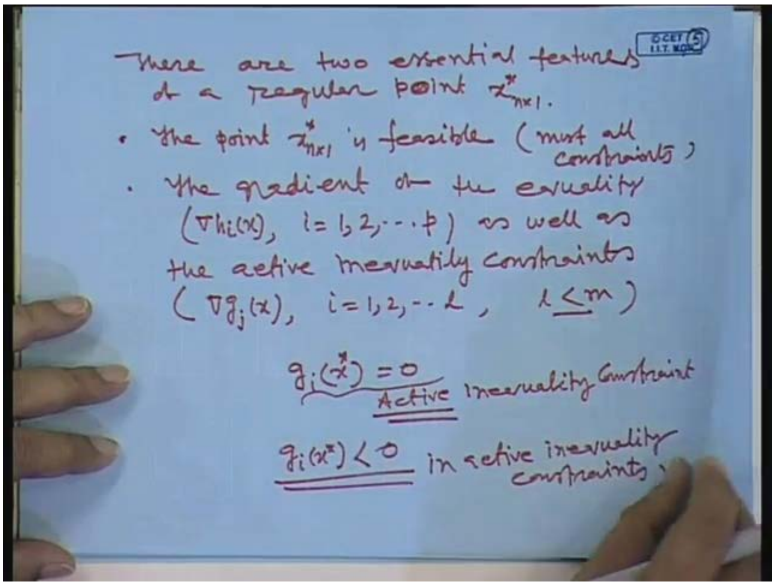

So, there are there are 2 essential features of a regular point x star, whose dimension m plus 1. First point a, this regular point must satisfy is what is call all design constraint. That means it must satisfy all constraint the point x n plus n star which is regular point. If it is a is feasible, it must satisfy. This what is call all constraint, that we must satisfy all equality and inequality constraint, must satisfy all constraints. This is one features, next feature is that, that the gradient of the equality constraint and what is equality constraint gradient g. There gradient of each i of x agree an i is equal to 1 2 dot dot p as well as bracket gradient of equality constraint equality as well as the active inequality constraints.

Constraints that is radiant of g j of x, this is the inequality constraints radiant of this or i is equal to 1 2 dot dot l, where l is less than m. What is an active constraint? Active constraints, active constraints are there for this value of x for this value of x. If it is 0 that means constraint is more stringent strict that conditions. So, that means i is equal to 1 i is equal to 2 i is equal to dot l. This equality 0 equality constraints of there. So, this is called active inequality constraints. This is called active inequality constraints, constraints for the inequality constraints.

This active, an inequality active constraints are which 1 g i of x star, if it is less than 0 then it is called inactive equality inactive inequality constraints. So, we have a gradient of equality as well as active inequality constraints. Active inequality constraints will come from equality constraints for i is equal to 1 2 l and they are i is equal to 1 to p. So, this gradients, that means p plus l gradients of which satisfy this equality constraints of the original constraints equations. If these gradients are linearly independent, then we will say that point x star is a regular point of the optimization problems.

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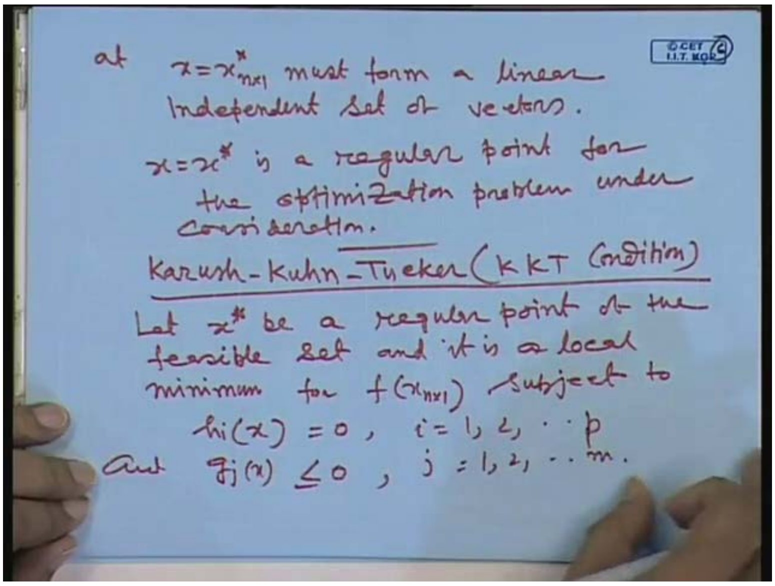

So, constraints that is what you will end, at x is equal to x star n plus 1. You see this, the gradient of equality as well as active inequality constraints for this at x is 0 to this must form a linear independent, independent set of vectors. If these vectors that means in other words vectors are really independent equality constraint and the what is called active inequality constraint that gradient of the vector is, linearly independent then we will say the x star is the independent.

Then x star x is equal to x star is a regular point regular point for the optimization problem, problem under consideration. So, this is the regular point. So, k k t conditions starts with the x star is the regular point, that means the equality. All equality constraints get it into of all equality constraints involved in a s i as well as involved in a s a g j active inequality constraints bring means when g j is equal to g j of x is equal to 0, at that point I studied with the this set of vectors that, this set of gradients, that means gradients of these functions.

If it is linearly independent then we will call it is a point of the optimization problem under consideration. So, this because why this equality of that. That one we will see later this one. So, k k t means Karush Kuhn Tucker it is called k k t condition is nothing but a first is find the necessary condition for the unconstraint what is called constraint optimization problem, which is converted into unconstrained optimization problem using the lagrange multipliers for that function, find out the necessary conditions.

So, let x star be a regular point. Now, regular point we understand if you see the statement of the problem here. Regular problem this for x is equal to for x is equal to x star this condition must verify. This is the regular, all stationary points we know how to wind up the stationary points. All stationary points is not a regular point, but all regular point is a stationary points.

So, if a regular point is there that point must be in what is called visible regiment or it is a design space that x star must be in the design space. This must satisfy all this equality and inequality constraints out of this whichever satisfy the equality constraints. We have a p equality constraint and out if this m inequality constraint. There may be a small one equality constraints may be there. So, this functions, if you find out the gradient of these function and that function at x is equal to x star get it into the function at x is equal to x star. Whatever vector c will get that because is that is a scalar if you differentiate.

This one with respect to the vector than it is get a vector. So, all these vector if these are linearly independent then I will call there x star is a regular point of the optimization problem on that considerations, so that why it starts, x is a regular point, x is a regular point of the feasible set of the feasible set, means if you put the below of the x in the constraint equation. It will satisfy all constraint must satisfy that one, then only we say if it does not satisfy at least one part equality or inequality constraint.

That means our point does not lie in the design space or in the visible space. So, that will not give you an answer of the consent problems. So, this and it is a local minimum, local minimum. It is a local minimum for the function f is a function of n cross 1, subject to our constraint, subject to constants s i hi x is equal to 0 i is equal to 1 2 dot dot p and g j of x is less than or equal to 0 j is equal to 1 2 dot dot m. So, this is a constants. So, how to solve this one we know lagrange function. How to form a lagrange function first step first step. So, that.

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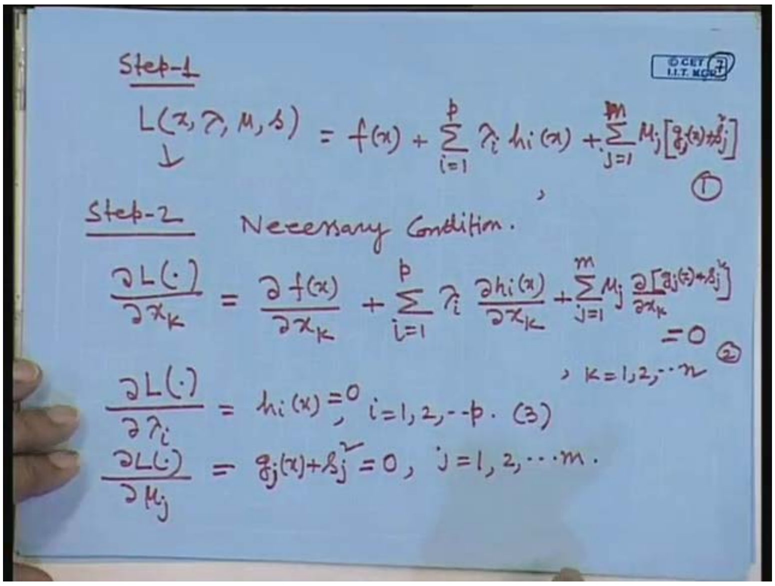

Is a k k t conditions, what is the k k t. First step from lagrange multiplier all l I am not. This is the L function of x lamda mu that we have already discussed that one. This is s this equal to if you see this one f of x plus samisen of i is equal to 1 to p lambda i h i. All functions I am writing in terms of i samisen and this samisen of j is equal to 1 to p. 1 to m then mu j multiplied by g j of x plus s j square. So, this is the case and this is called lagrange function.

So, this function you have to optimize means minimization. Our problem is minimize the functions and we have written what is lamda all this things we have written. So, we will not repeat this one. So, this is a equation this. So, on necessary condition is what if you see the step 2 lagrange function, we have then step 2 is a necessary conditions or to find the stationary conditions necessary conditions. If you see the way you are doing the constant abbreviation problem solve, that means first find out the what is called necessary conditions, that mean gradient of the function s n to zero.

Now, we are writing. So, this now you differentiate this with this is a function of x is a how many components n components x 1 x 2 dot dot x n, similarly lamda 1 lamda 2 lamda n. If you differentiate all this one, with aspect to this mu 1 mu 2 dot dot l a n s is s 1 s 2 dot dot s n. So, this I am not writing respectively dot is the function of this 1 and this is the function of x k x of k, means there is one to n when you write is equal to 1 differentiate lagrange function with aspect to x 1. So, p if you do this one, see this what you will get it. That one both side you differentiate both left and right side differentiate the function to x k and k release from 1 2 dot dot n.

So, this is a samisen i is equal to 1 to n lambda i and differentiation with respect to x. So, lamda is a constraint keeping other variables constant. So, I can write h of i x delta x k similarly, samisen of that 1 j is equal to 1 to m mu j is a constant quantity I can take. Now, it is a function of that one only x, x g is the function of x other is a not a function of x. So, you can write dell of dell x k, then this g that gj of x plus. This term is will not come into picture is you can write it write it.

Finally, it will not come into the picture this and that must be a sign to 0. This is our necessary condition, but necessary condition is not complete. This is we have differentiation, this thing a is equal to 1 2 dot dot n variables of there because we have a this ally is the function of the lamda. Also, again you will write it this equal to one, this then you can write with that function of this. You differentiate with respect to lamda i and if you use this is the lamda I see this 1 lambda into h 1 lambda 2 into h 2 lamda 3 to h 3.

So, I am differencing first with lamda 1. So, it will come h 1 than h 2 h 3 in this way. So, you will get it this h 1 of x and that i is very 1 2 dot dot p. So, let us call this is equation number 2. This is equation number 2 this is equation number 3. So, you have a differential with respect to lamda, then what is left with respect to this with respect to mu j is equal to.

Now, with respect to mu j, if you depend mu 1 i j is equal to 1 mu 1 g 1 s 1 square with differentiate to mu 1. Other terms is not a function of mu so, it will be a g i of x plus x j square g j of x. This equal to 0 because it is a necessary condition, where s n assign to 0. So, this is able to 0 that j is equal to how many 1 2, I will give dot, dot n.

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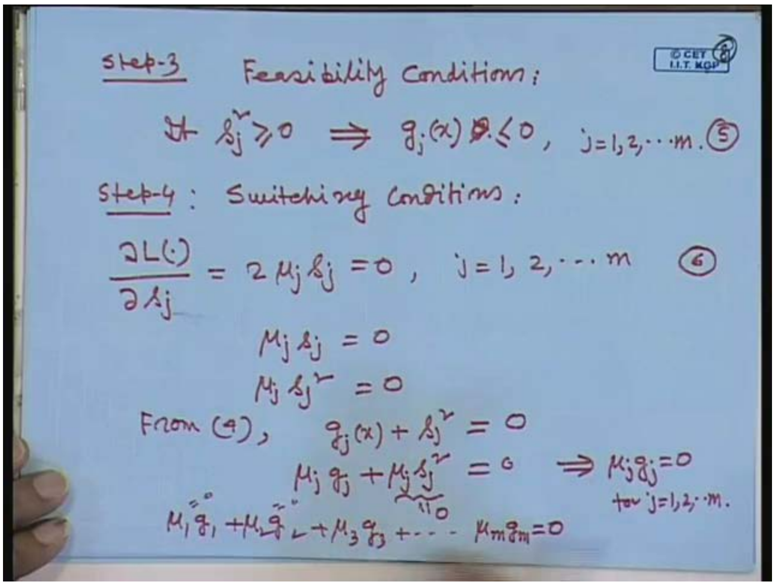

Then you see this is the step 2 and then you see step 3 feasibility check. Step 3 visibility check conditions, if S j square greater than or equal to 0, this implies g j of x explain this in earlier also is less than or equal to 0 and this is j is equal to 1 to dot, dot m. When S j square is 0 then, it is a equality constraint when s j better than 0 means positive then, this equality constraint less than 0 is satisfied.

So, this we can write it. So, we have given equation number 1, 2, 3 this is equation number 4 then, this is equation number 5 there is equation number 5. Then we say that equation number 5 then step 4 because if you see this one we have still left 1, that means we have to do the find the partial differentiation with l with respect to s 1 s 2 dot, dot s n. So, that is switching condition, why it is a switching condition? You will see later that by clubbing the equation 4 and at I will write it switching conditions.

Now, this condition is as this differentiation of this with aspect to x j because it function of l of s this is up to s. See if you differentiate this with respect to s mu 1 j is equal to mu 1 s 1 square. So, you have to differentiate with the respect to s j. So, this is constant when you are differentiating with this j. Only this term will be there this differentiation will come twice mu j s j that is equal to 0 that is j is equal to 1 2 dot, dot m. So, let us call this is equation number 6.

So, this equation I can write it this are all are necessary conditions equation number 2 3 and 4 equation. So, if you see the equation number 4, which is differentiable differentiate with the respect to mu j and step 5 or equation 6 partial difference l lagrange of the difference with respect to x, these two equation we can combine together and I will say resultant conditions one. So, let us say 1 that one what we can do. So, you multiply by sides of this equation that is this equation is mu j s j is equal to 0, both sides multiplied by s j to s j square is equal to 0.

Now, recall this equation 4, recall this equation 4. If you from 4 we can write this from equation 4 for equal equation 4. If you see this one g j of x that means if you do the partial difference of l with aspect to mu and getting that 1 s j square is equal to 0. Multiplied by both sides that secular point mu j g j mu s j, s j square is equal to 0 and this quantity is 0 from here, this quantity is 0. So, you see we are merge this 2 equations that is 4 and 6, which in term that the equality constraint of g j, which is converging to a equality constraint that a g j plus s square.

This two conditions we are merged together. Actually separately if you see dell l of dell s mu j mu j s j is equal to 0 and if you see dell l lagrange function with respect to mu j that is we have seen that equation. So, this equation and this equation we have merged together and ultimately we are getting this equal to mu j g j is equal to 0 for j is equal to 1 2 dot, dot m, this is the condition.

So, whatever from now on wards, if you want to solve this one necessary condition, you will write it dell i dell l with respect to x dell l with respect to lamda dell l with respect to what is called s and with respect to called mu, which march together I will write these conditions. This is a condition we have to write it. And these if you see write this all these equality constraints then i can write it mu 1 g 1 mu 2 g 2 mu 3 g 3 and dot, dot mu, mu what is called m gm is equal to because component is equal to 0, which is not an individually. This I can write it if you see mu 1 mu 2 dot, dot mu m. This transpose this is nothing but a we can write it as g 1 g 2 dot, dot gm and this equal to 0.

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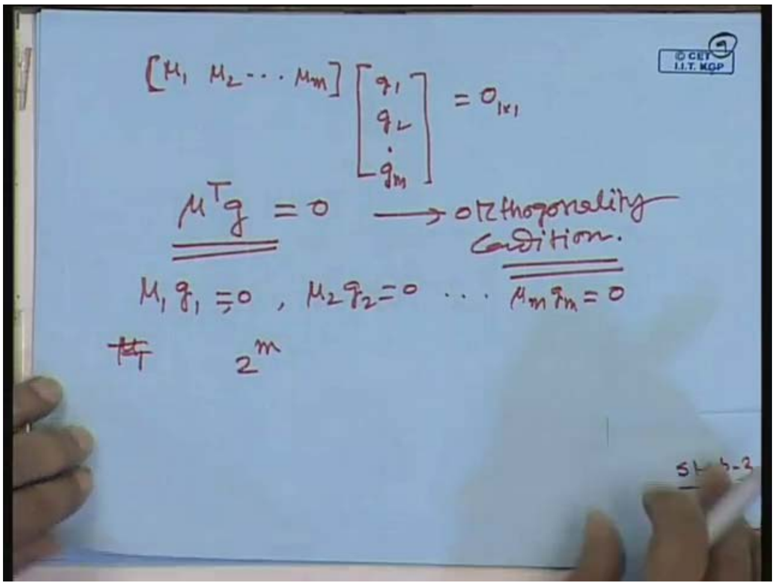

This is a scalar quantity 1 cross 1 see here. This is a scalar quantity of 1.1. So, if you multiply by this is in the vector form, we can write mu transpose g is equal to 0. So, this indicated switching conditions it is nothing but a orthogonality conditions also you write it, this is a orthogonality conditions. So, now look at this if you look carefully this one this let us call i is equal to 1 g 1 g 2 g 1 mu 1 is equal to 0.

There are two possibilities they are mu 1 and g 1 is 0, I can consider mu 1 is 0 when g 1 is not equal to 0 that this condition is satisfy. It can be also another conditions maybe g 1 is zero, but mu 1 is not equal to 0. So, if you have a what is m such conditions how many possibilities are there to make this conditions 0. If you have a two are there let us call only two equations are there, two inequality conditions.

One is this equal to 0 and this is this able to 0, I can consider mu 1 is 0 when g is not equal to 0 than this condition is satisfied and I can consider g 1 is 0 mu 1 is not 0. Similarly, here mu 2 is 0 g 2 is not 0 when g 2 is not 0, it indicates it is a inactive conditions. I mean g 2 of x is less than 0 and maybe g 2 is 0 when mu 2 is not zero. So, if you have a such type of m conditions are there in term, we will get 2 to the power of m conditions cases 2 to the power m cases we will get it while we solve the what is called necessary conditions problem.

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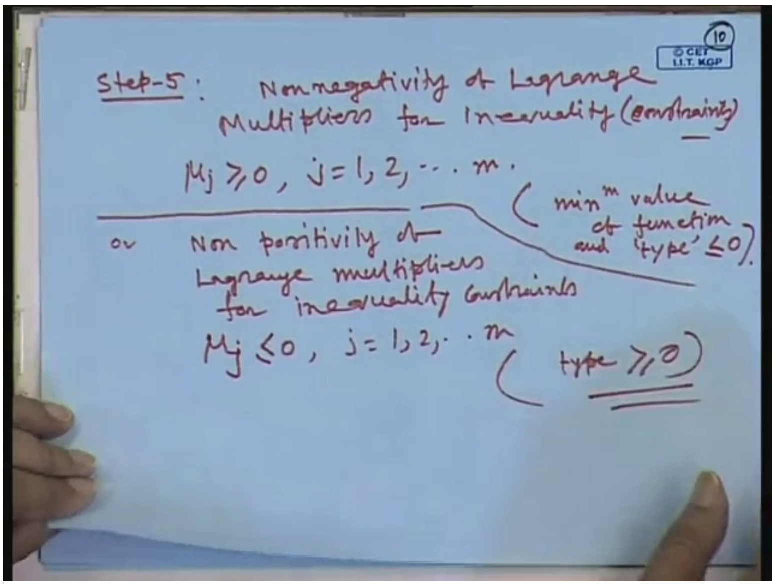

So, next is your step 5 non negativity. Non negativity of lagrange multiplier for inequality constraints. Look the beginning what is just that we have considered that lamda is a lagrange multiplier, which was necessity to the equality constraints. If you see this 1 our first slide, we have made a lagrangian function where lambda lambda 1 lambda 2 dot lambda p all are associated with the equality constraints where, as mu 1 mu 2 mu 3 this is also lagrangian, but there are associated with the inequality constraints g 1.

So, next is this one mu j is greater than or equal to 0 j is equal to 1 2 dot, dot m. This will be this condition must satisfy when we are finding out the minimum valve of the function and the this type of inequality is used. And for this is the non negativity or we can get it non positivity of lagrange multiplier for inequality constraints, that means mu j is less than 0 for i is equal to 1 2 this. When you will use minimum value of function, but type is greater than or equal to 0. Anyway we will this is the most general way we will always take less than 0. Last check is your step 6 is a regularity check.

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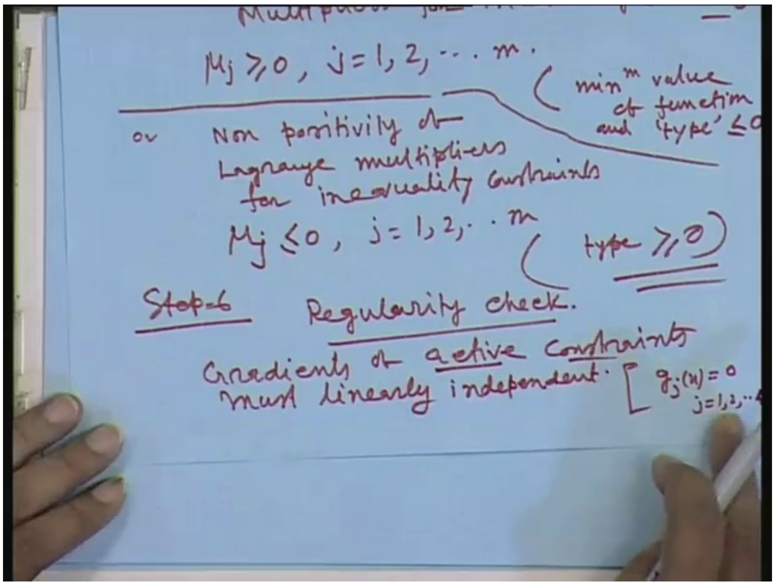

Regularity check is no, once you find out that optimal point x star then, you check whether the all equality constraints involve in the optimation problems whatever you got it gradient of that one is nearly independent or not. If it is a linear independent, you can say that you are obtained the minimum value of the functions. So, you have to check gradients of active constraints must be linearly independent means, this is you will see for which g j of x is equal to 0 j is equal to 1 2 dot, dot l and this is the active constraints. It is small n not n out of n inequality constraint one equality is equality constraints is there so, that gradient must be linearly independent. So, that in short you know how to test how to test linearly independent test.

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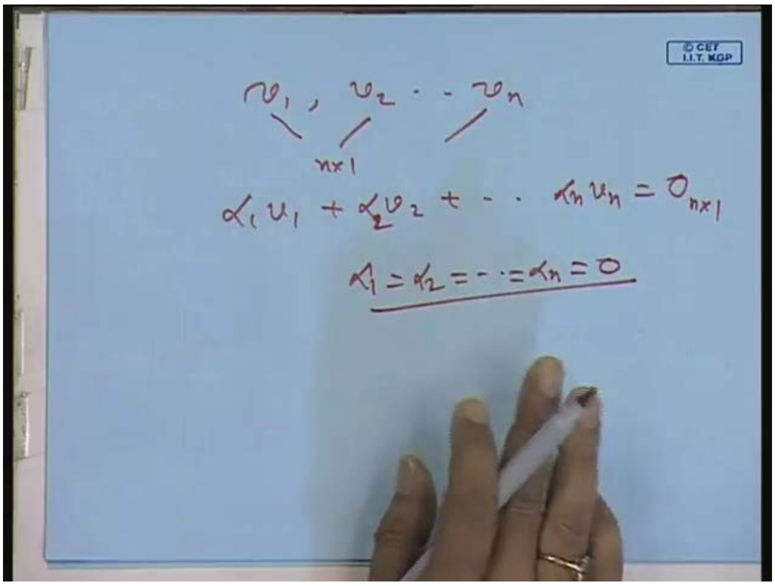

If you ever set of vectors are there let us call v 1 v 2 and v n and that dimension of these dimension is m cross 1. If alpha 1 v 1 plus alpha 2 v 2 alpha 2 v 2 plus alpha n v n is equal to non vector this. If all the coefficient's of associate to v 1 v 2 is equal to dot, dot alpha n equal to 0 then, that means it indicates the set of vectors alpha 1 v 1 v 2 v n dot v n are linearly independent. If any one of these is non 0, this set of vectors you can express linear combination of any vectors then, we call set of vectors are linearly independent. So, next we will just work out some problem tell how to do and also see the sufficient conditions of the problems. Necessary we have done, next is a sufficient conditions.

Thank you.