Optimal Control
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Lecture - 8
Solution of Constraint Optimization Problem-Karush-Kuhn Tucker (KKT) Conditions

So, last class we are discussing the solution of unconstraint optimization problem using Newton's method and we have seen there are two drawbacks. One drawback is that you have each iteration; you have to find out the Hessian matrix or second derivative of the cost function or objective function. That must be at each iteration must be a positive definite matrix and this is first drawback and because this condition must be satisfied because we are going what the descent direction so that each iteration from k th iteration to k plus 1 th iteration. The function value should decrease that is the condition, so Hessian matrix must be positive definite.

Another drawback is there, in each iteration, you have to do the inverse of this matrix, whose dimension is n cross n that inversion Hessian matrix inversion. You need it at each iteration and n is the small n is the number of variables involved in the objective functions. Then, we have seen that this problem can be overcome by using quasi Newton's method, it is similar to Newton's method, only the inverse of the Hessian matrix at each iteration is replaced by some matrix as suffix k k th iteration. That matrix is updated at each iteration that you have to update, then how you have to update that one we have explained this one, but now we will derive that expressions if you recollect this one.

We want to replace in the Newton's method this inversion that means Hessian matrix inversion by a positive definite matrix s of k suffix k. Then, next is s of k, how we have what is called updated this one, we have seen that this is the updated expression, we have shown you that s k plus 1, if you see this one, s k plus 1 updated s k.

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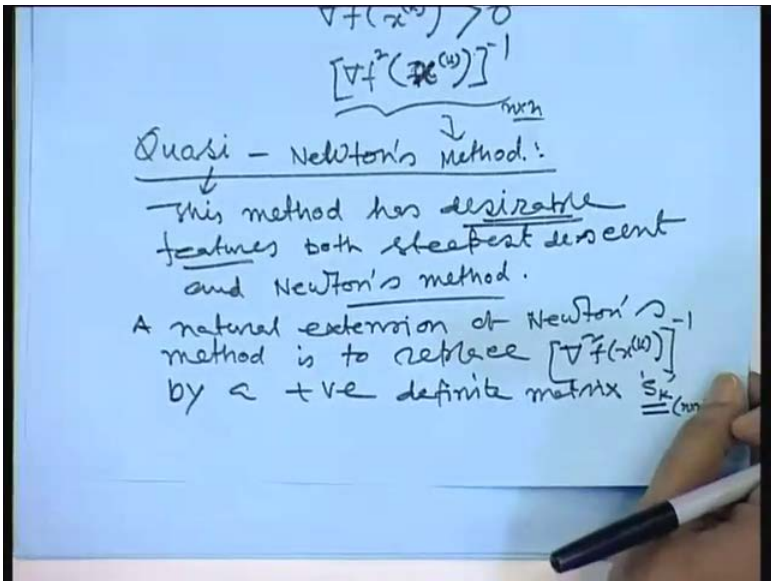

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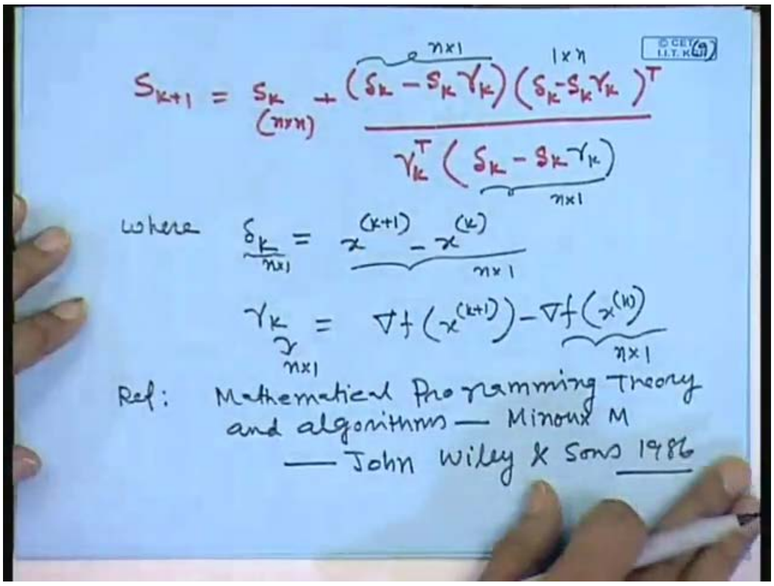

Then, these things you have to calculate delta k is nothing but that successive two iteration that x difference of s values gamma k is the successive two iterations. The gradient of objective functions difference of gradient of objective function, this way you have to update how we got this expression that we are we are going to discuss now and the derivation is based on that our concept like this way that the function value.

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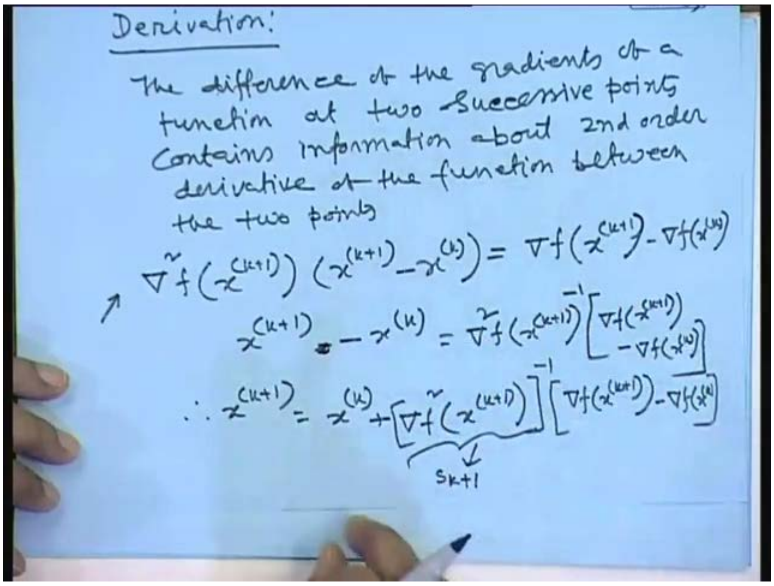

Let us call that f is a function which is a function of x and variable the function difference of function value at two successive points carries the information of derivative of the functions. Similarly, the next second derivative that gradient of the function at two successive points carries the information of the second derivative of the function. That means Hessian matrix information carries based on this one, we have written that one, the Hessian matrix of the function at k plus 1 th instant or the iteration is equal. The difference of two difference of two gradients at successive points two successive points x k plus 1 and x is equal to x super script k.

This difference carries the information of the second derivative of this function that means Hessian matrix information. If you multiply it by that, change in that is the variable change in difference in that variable change. If you multiply, then this is equal to a expression you can write it from there. I rearranged it if you see x k plus 1 minus x super script k is equal to this matrix inversion.

Then, Hessian matrix inversion, so I mentioned earlier that the Newton Raphson method, Newton's method for solving the unconstraint optimization problem. This is creating problems first thing delta square f, Hessian matrix must be positive definite that is next is this inversion is always exist when the number of variables is very large. The competition burden involved here is much, so this one we are not taking the inverse, we are replacing this one by a matrix s k plus 1.

So, this we are replacing s k plus 1, now see this one how this s k plus 1 is that s of k each time it is up updating, so that is we will discuss now. Now, look this let us call this expression that is what we will write it this expression, you write this equation number one, this is equation number 1, now question is given s k.

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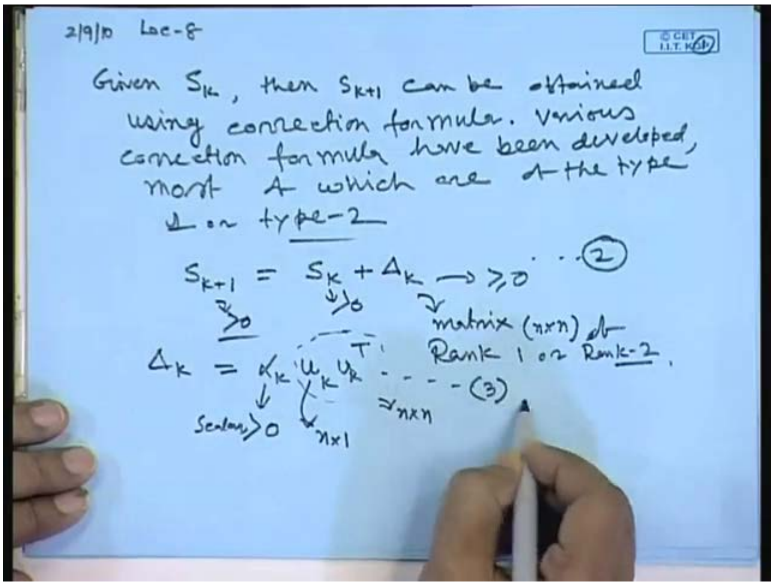

Then, s k plus 1 s k plus 1 can be obtained can be obtained using correction formula correction formula and that correction formula is based on the rank one type or rank two type correction formula. So, various type of correction formula is available that means one maybe a correction type of order one, rank one, rank two. So, you can write various correction formula have been developed in the literature have been developed, most of which are of the type one or type two, so let us see.

So, now we can write s k plus 1 is equal to s k, so this now I am updating s k, I know s k plus 1 at k th iteration, what is the Hessian matrix inversion at k plus 1 th iteration that I am finding out by s k plus adding a correction term w of delta of suffix k. Here, this delta of k is the matrix of same dimension of s k and s k dimension as you know it is n by n this matrix also n cross n matrix.

That matrix is a positive definite matrix, so that matrix of rank 1 or rank 2 or rank 2, the simplest choice of delta k 1 can take like this way. The inner product of the outer product of any two vectors which is which is not a null vector, so I can define delta k as a special choice of delta k like this way alpha k u k and u k transpose. This is the inner product of vector u k at k th iteration u k any vector, you have considered multiply it by u k transpose. So, this product of this outer product of the vector u k and this same vector outer product this is matrix of dimension n cross n, where the u k dimension is n row, one column agree and alpha k is a scalar quantity whose value is greater than 0.

That is scalar quantity scalar greater than 0, so this is the choice of this one and this matrix if you see this is always positive semi definite matrix. So, you have added with a positive definite matrix, another positive semi definite matrix results will be a at most it will be a positive semi definite matrix of that one, sorry this result will be a positive definite matrix of that one.

Now, a positive definite matrix add with a delta k which delta k is positive semi definite matrix the results will be a positive definite matrix, this matrix is positive definite matrix and this is positive semi definite matrix. That results will be positive definite matrix, so this with this one, let us call this equation is I am considering this is equation number two and this is the equation number, this equation is equation number three.

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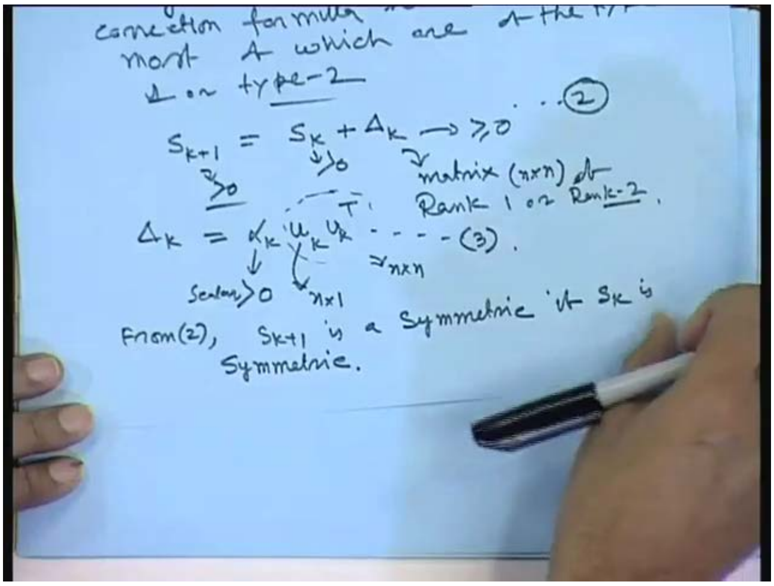

From two, one can write it from 2 s k plus 1, what I mention is a symmetric matrix, symmetry if s k is symmetric and see when you take the outer product of two same vectors, these results is you will get a symmetric matrix. So, symmetry matrix plus symmetric matrix is symmetric matrix of this one that is from the two, we can see this one.

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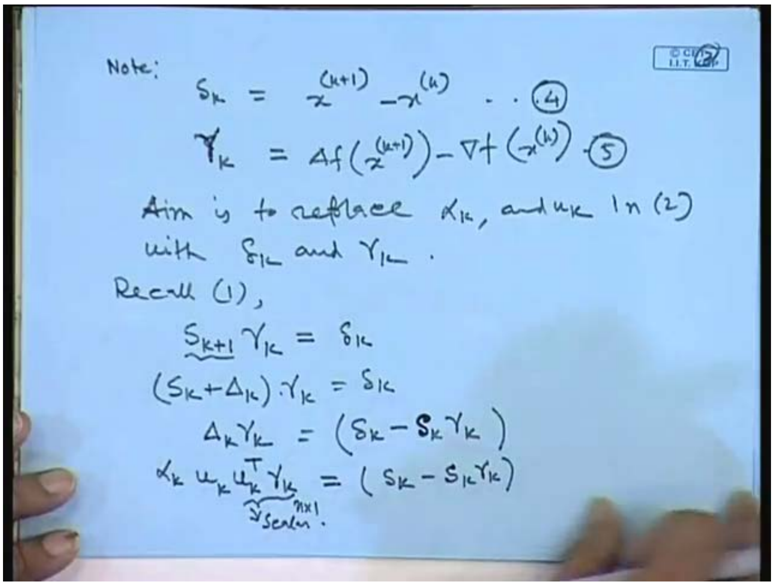

Then, note what we have defined delta k is nothing but the change in decision variable value at k plus 1 th instant minus at k th instant. So, that is we are denote it by delta k symbol similarly, gamma k this gamma k is the difference of gradient at two successive points, what is the successive point delta f x is equal to super script k plus 1 minus delta f at super script of k this given. So, let us call this is equation number last equation, we have given the number three, so this is four and this is five, our main job is here. Now, replace that u k and alpha k this u k and alpha k replace in terms of the information delta k and gamma k.

That is our next step, how to replace it aim is to replace is to replace that our alpha k and u k in equation two with delta k and gamma k. So, how we will do this one, now see this equation number one, this equation number recall the equation number one, this equation number one, if you recall equation number one, then I can write it s super s sub script k. This is what we have, denote it by this one gamma k, the change in what is called gradient of the function at two successive point changes.

We have denoted it by gamma k, so I can write this is gamma k is equal to delta k see this one, if you take this if you take this side it is a delta k, so I can write this expression from equation number one. So, now what is s k plus 1 this I can write it s k plus delta k that is the what we have considered s k plus 1, we have considered like this way into gamma k is equal to delta k. Now, I can this if I take it that side s k gamma k in right hand side, so I can write it delta k gamma k is equal to delta k minus s k into gamma k this is s k.

So, that one you can write and one can write it this one, because you know what is delta k i have considered, delta k is nothing but a alpha k product, inner outer product of same matrix u k. So, this I can write alpha k, u k, u k transpose, this is delta k into gamma k is equal to delta k minus s k gamma k and note this one, what is gamma k, gamma k is nothing but a difference of the gradient value at two successive points gamma k. So, it is a vector of dimension n cross 1 and this is also dimension u k transpose dimension 1 cross n, so this quantity this two product, take it as scalar quantity scalar quantity.

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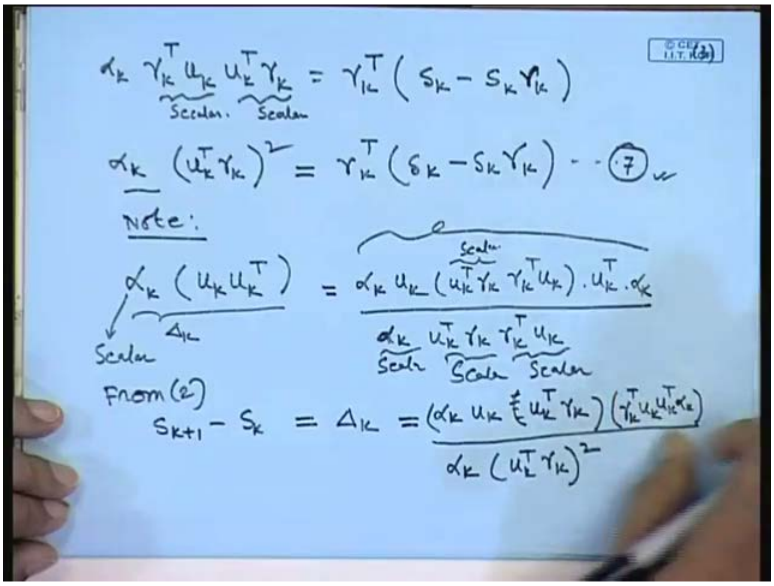

Now, both side I multiply by gamma k transpose and alpha k is also scalar, so both side you multiply it by gamma k, then left hand side alpha k. Then, gamma k transpose u of k, then u transpose of gamma k is equal to because both side I multiply it by gamma k, this equal to delta k minus s of k into gamma of k gamma of k, this is gamma of k. Now, this you see this is a scalar and inverse of the scalar is that one, this is also scalar, so what is this value is, let us call it is five. That is also five if you take inverse of this scalar quantity scalar, so I can write square, so alpha k, I can write u transpose, u k transpose gamma k whole square to scalar quantity square.

I can write is equal to gamma k transpose delta k minus s k gamma k, so the let us write it this is equation number last we have gone up to equation five, then you can recall that one is this, you can consider equation number six. Then, you can call this is the equation number seven that one agree and that is the identity. You can see one can write this identity without any problem, note this identity, what is this identity, alpha k, u k, u k transpose and this the whole thing is nothing but a delta k. We have considered and alpha k is the scalar quantity that is what I mean is a scalar quantity, if you recollect that, our alpha k definition what we have just considered alpha k is a scalar one.

We have defined your that is delta k this is the alpha k is the scalar and this is the matrix, so delta k is this, we can write it the same thing, I can write it alpha k, this we can write it, then we can write it u k that one as it is. Then, you are writing u k transpose gamma k then gamma k transpose u k, then you are writing u k transpose, you get transpose this one into alpha k alpha k divided by alpha k u k transpose gamma k. So, this you see I told you gamma u k transpose, gamma k is a scalar quantity, this is a scalar quantity scalar, so this is a scalar, so this is this scalar, this scalar, same thing, same quantity I have written.

So, we can cancel it, so this alpha and this alpha you can cancel it, so what is left alpha k u k u k transpose the same thing, but we have written into a this and this are the identity. You write this one or you write this, the same thing, but this quantity whole because I can divide this is the matrix right hand side. This quantity from here to here must be a matrix and this divided by a scalar quantity, you see this is a scalar, this is a scalar and alpha we have considered alpha is a scalar, so I divided it by matrix by a scalar quantity so these are the identity.

So, now from what is the equation number two, if you just see the equation number two, here this equation, what how to update that s k plus 1 k plus 1 th iteration. So, two if you take that side s k plus 1 minus s k, it will be equal to delta k, so I am writing s k plus 1 minus s k and s k, you know what is the inverse of Hessian matrix at k th iteration at k two successive points k plus 1 th iteration. The inverse of Hessian matrix and this is the k th iteration inverse of the Hessian matrix, the difference of this one is written by you see that that one equation number two, this difference of this delta k and delta k.

So, I can write it that the value of delta k, now here what is the value of delta k from equation, I can write it the whole expression, what is because this is nothing but a that one. Now, we have shown it, so you can write it alpha k, u k, then u k, u k transpose, this

I am writing into gamma k. So, this four quantities one, two, three, four quantities together I am writing this up to this remaining is I am writing gamma k transpose, u k, u k transpose alpha k.

So, you can see here this is nothing but a transpose of that one, that quantity is transpose of this quantity, this quantity is transpose of that one or vice versa. So, divided by that scalar quantity and what is this quantity, this is a scalar, what were the value of this one and this value are same. So, I can write it square, so alpha k, u k transpose, gamma k whole square because what are the value of gamma k transpose, u k, the same value of u k transpose because transpose of scalar quantity is same quantity. So, it is a scalar quantity and this quantity you see this is what you can easily verify this one.

This is a scalar quantity and this is also scalar quantity, these two scalar quantity is a scalar quantity. Here, u k is a row vector, sorry column vector u k is the column vector whose dimension is n row, one column and this is the transpose of this one that will be a row vector. So, row vector column vector multiply it by post multiply it by same matrix with a row vector that results is a matrix, so I am getting this one.

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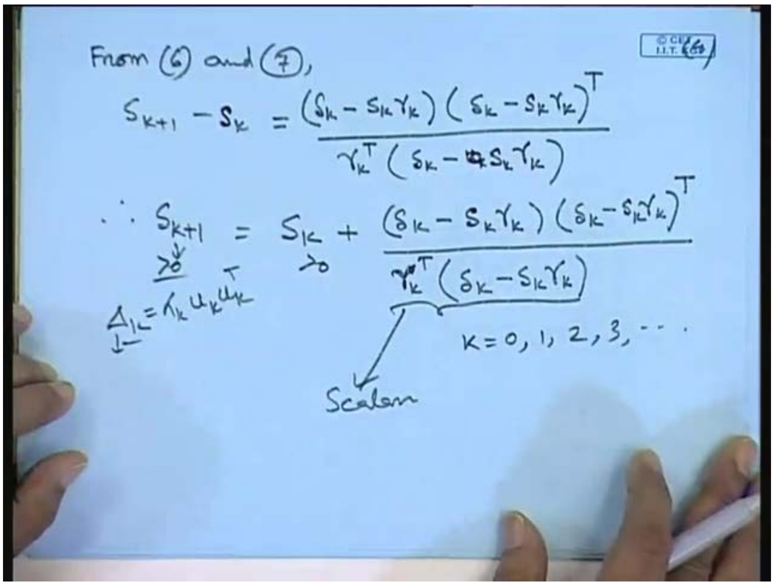

So, from six and seven that seven expression you see this one, alpha k, u k, gamma k square, this quantity I can write by replace by the this one denominator expression. I can replace by that one agree or i can replace by that equation number seven, so using equation number six alpha k, u k, u k transpose, gamma k, alpha k, u k, u k transpose, gamma k, I will replace by that quantity.

So, using six equation number six and seven, using equation number six and seven or from six and seven, we can write s k plus 1 minus s of k s this is s k is equal. I am replacing from six and seven delta k minus s k gamma k into delta k minus s k, this is s k gamma k whole transpose divided by gamma k transpose, delta k minus u k and this is gamma k. So, that is this value, I just see this one this quantity gamma k transpose s k, this is delta k minus this, I am replacing equation see this one. Here, alpha k, u k transpose, gamma k equation gamma k transpose delta k minus s k, then gamma k, s k, gamma k, so this is s k delta k minus s k, s k into gamma k.

So, vector minus this vector and multiply it by another row vector, so it is a scalar quantity, so this I can write it, therefore I can write s k plus 1 is equal to s k plus delta k minus s k gamma k. That transpose delta k minus s k gamma k whole transpose divided by gamma k, this is gamma k transpose delta k minus s k gamma k, this quantity.

So, you see each of the iteration, now we have derived at each of the iteration, the inversion that is Hessian matrix inversion is replaced by a matrix equivalent matrix s k and next iteration s k plus 1. That means inversion of Hessian matrix at k plus 1 th iteration we can obtain knowing the previous value of s k and compute that one and this one. You know delta k is the difference of the values of the decision variables x, x value difference at two successive point k plus 1 and k th 1 and gamma k you know the difference in gradient of a objective function at two successive points difference. So, I can compute iteratively this one for k is equal to 0, 1, 2, 3 in this way.

So, each iteration one value computed recursively like this way, I am avoiding the inversion of what is called that Hessian matrix inversion when we will compute the solution of optimization problem using the Newton’s method. When you are replacing by a matrix this and recursively, we are calculating s k plus 1 in place of a inversion of a matrix. Then, we will call it is a quasi Newton’s method, this is like Newton’s method only the different way we are calculating. So, one of the disadvantage is look at this point, this is a scalar quantity, this whole thing is a scalar one must be careful that denominator part should not be very small or 0.

If it is 0 or very small, this quantity will blow very large value will come, so that is the one disadvantage of this rank one, this problem can be this is this derivation is based on the rank one. If you recollect the delta k, we have added with a s k by choosing the delta k matrix, like this way a vector outer product of same vectors u k, u k transpose and that rank is one on that basis we have derived. So, if this is very small or very small or 0, then we cannot apply these things, so the alternative way is you go for rank two for choosing the delta k.

So, this is the one disadvantage, advantage is there, you do not need to check the gradient direction because the direction that is what we are checking the gradient direction, which in turn we got that what is called Hessian matrix must be positive definite matrix. This is not necessary in this case, because it is automatically once you have you see this one once, you have selected at k is equal to 0 s of 0 is positive definite matrix, any positive definite matrix of that one and this I told you delta k that is what this is nothing but a delta k. The delta k expression we can write it like this way, this is always a positive semi definite matrix and in turn that matrix is that matrix is a positive definite matrix, you will get.

So, the results always it is grant is that this is a positive definite matrix, so you need not to check the what is called descent direction of a function at each iterations. So, that is the one disadvantage advantage of this Newton's method and due to this one, it is widely used for optimization, what is called unconstraint optimization problem, that method quasi Newton method is widely used due to this advantage. So, next we will see so far we have discussed that how to solve that what is called unconstraint optimization problem by using the numerical techniques or directly you can solve it. Let us call unconstraint optimization, how we will solve it, first the gradient of this vector you assign 0 and solve it, find out the next what is called stationery points.

Then, you find out the second Hessian matrix of this one and check whether the Hessian matrix is positive definite or negative definite or not, and you will be able to conclude that function is minimum or maximum. If the Hessian matrix is positive definite, then the function is a minimum function value of the function, you achieve the minimum at that stationery point. If the Hessian matrix is negative definite, by analytically if you solve, you get it negative definite, this means at the stationery point to the stationery point the function value you will get the maximum.

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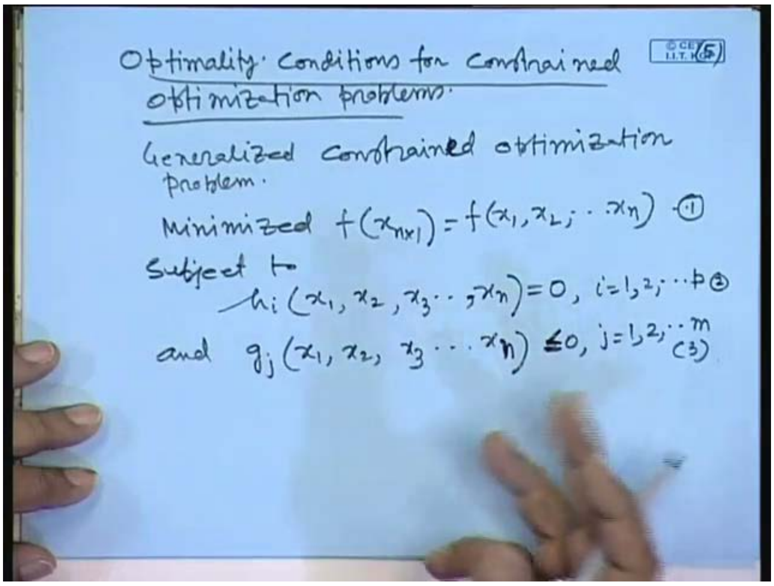

So, let us go next is that constant optimization problems, how to solve the constant optimization problems. So, next is your optimality and once you see knowing these algorithms and one thing I just forgot to tell you that what is called method the convergence rate, convergence rate is linear means order one, whereas in conjugate gradient method. The convergence rate is between 1 and 2, order one means what it is a linearly it is approaching to what is error is decreasing or it is approaching to the function value. It is approaching to the minimum value of this function linearly, whereas in Newton's method, the convergence rate is quadratic.

In other words, the convergence rate order is two and similarly that is what is called quasi, what is called Newton's method is order the convergence rate is much faster than the method and this order is two. So, next we will consider the optimal optimality condition for constant optimization problem, optimality conditions for constrained optimization problems. So, let us recollect our first lecture we have discussed that what is the basic structure and mathematical formulation of optimization problems.

Now, we are going for constant optimization problem, basic structure if you see generalised constrained optimization problem, most of the practical problems you see there must be constant real time real world problems. There must be constraints and not only constraints, then equality constraint means what is called inequality constraints. In addition to this one, there is a side constraints, some of the variables cannot be positive, negative all this things that we have discussed at length in our first lecture or second lecture.

So, let us call what is the generalised optimization problems that minimize, if you recollect minimize f of x and x dimension is n cross 1, which you can write it each as x as a n variables. So, our decision variables are n variables are there, we have to minimize this functions, so let us call this functions equation number one. So, subject to this, I just recollect our problem that subject to equality constraint, so we have a h, which is a function of all decision variables may not be somehow function, maybe if you decision variables x 2, x 3 dot x n and this are the all equality constraints and this i varies from 1, 2 dot p.

So, there are p equality constraints is there with the optimization problems subject to how many i is equal to 1, h 1 is equal to 0, h 2 is equal to 0 dot h p is 0. So, we have a p equality constraint and also inequality constraints g j, which is a function of x 1 x 2, x 3, dot x n, x n is equal to 0 and that we have a x 1, 2 dot m small m. So, we have a m inequality constraints, sorry this is inequality we have a m inequality constraints are there. So, our problem is minimize this function subject to this constraints, so this is equation number two and this is equation number three.

If the equation number two, three, one, two, three are all function of linear functions, then we will call linear programming optimization problems. Otherwise, it is a non linear optimization problems, anyone of this is non linear that we have discussed earlier also. So, we have a linear optimization problem maybe non linear optimization problem, maybe let us see about the constraints get some idea of this constraints. Suppose, we have a problem like this way minimize example just to explain the constraint of minimize f of x and x is a we know in our case two variables dimension. So, it is a x 1 minus 2.5 whole square plus s 2 minus 2.5 whole square.

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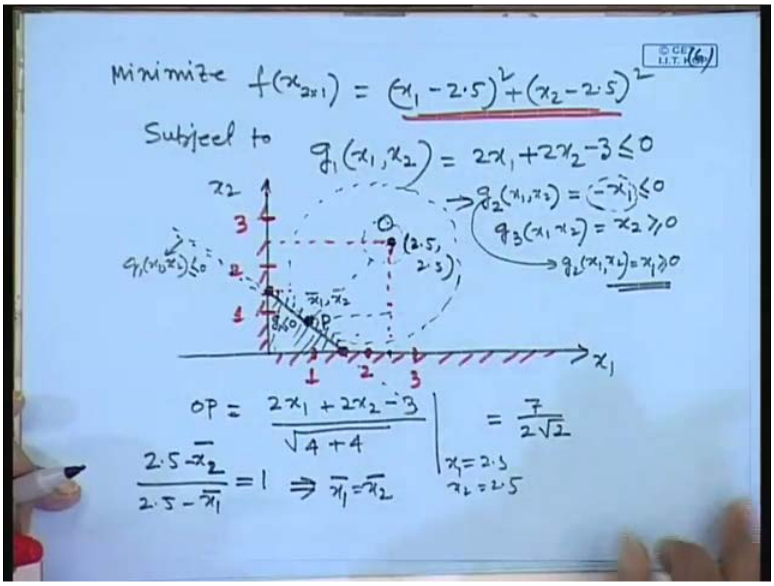

Our problem is minimize this quadratic functions subject to g 1 of x which is a function of x 1 and x 2 and is equal to 2 x 1 plus 2, x 2 minus 3 is less than equal to 0. Let us call we have a only one inequality constraint value, please note that when there is a equality constraint is there, we are using h, small h, when there is a inequality constraints are there. We are using the small g that is we will use this notation throughout our discussion. So, this is the problem and this is you have to optimize this one and if you see graphically this problem of that one and another constant.

Also, you can see g 2, which is a function of x 1 and x 2 is equal to minus x 1 is less than equal to 0, another is g 3, x 1, x 2 is equal to x 2 greater than equal to 0, just we can see this one. So, this is this one, let us see this point g 2 inequality constraints minus x 1 is less than equal to 0. That means it indicates the x 1 value is always positive, so this in other way I can write it this one, I can write it also g 2 of x 1 and x 2, I can write it is equal to x 1 greater than equal to 0.

This expression is showing that this quantity always less than 0 when it is possible when x 1 is greater than 0 greater than equal to 0, so this I indirectly or in other words I can write this one. So, let us see that constant g 2 in place in place of this one I am writing that one x 1 is greater than equal to 0. That means this indicates where which region that x 1 is always positive, so I can see that our region of x 1 is that side. So, if you consider this is our x 1 and this is our x 2, so this indicates that x 1 greater than 0. So, x 2 greater than 0, the third inequality condition g 3 means x 2 value is always positive or 0.

So, this indicates that region that region, so the first quadrant g 2 and g 3 indicates the first quadrant, our design space or design variables must lie in the first quadrant come g 2 and g 3. I can say this, now what is this equation that first constraint inequality constraint that you just draw the line that 2 x 1 plus 2 x 2 minus 3 equal to 0 draw the line first. Then, I will decide less than 0 in which side of the line greater than 0, which side of the line. So, if you draw equal to then x 1 is if x 1 is 0, then x 2 is becoming x 1 is 0 2, x 2 minus 3 is equal to 0 means x 2 is 1.5. Let us call this is 1, this is 2, this is 3, so 1.5 is here, this point this is the equation of this straight line, then when x 2 is 0, then x 1 is again 1.5.

This is 1, 2, 3 and 1.5 is here, so this is equation of straight line, so I can draw it, this one is I cannot extend because it does not valid because x 1, x 2 value is greater than 0 means in first quadrant. So, you cannot go like this way the valid up to this region design space is only first quadrant, so this indicates that equal to 0, g 1 that means g 1 is equal to 0, any point on the line, but we have given that g 1 is less than equal to 0 less than equal to 0. That means which region the g 1 less than equal to 0 that should be what is called in this region is g 1 less than 0 less than equal to 0.

When this is equal to g 1 equal to 0 that is on the line when g 1 less than 0 that means in this region the whole region, if you see just straight line, if you extend it, the whole region g 1. The whole region of that one, the whole region g 1 of x, x 1, x 2 less than equal to 0 either on the line or this, but our g 2 and g 3 are telling that our design variable x 1, x 2 lies on this first quadrant. So, you cannot go outside this first quadrant, so our design space or design variables must lie within in this triangle. Now, what is our problem, our problem is to minimize that one, so if you see this one this is nothing but a equation of a circle whose centre is 2.5.

This is our centre of the circle, now it is obvious you see minimum value of this, suppose this constraints are not there what are the minimum value of the function at centre means 0, but we have a constraint. Now, if you increase the size of this that is that with the centre if you just the function value you have to increase it. Suppose, I have put it a function value this is some value of function, but this does not satisfy our constraint.

These three constraint combined we have shown this is our space where this our x 1, x 2 must lie. So, now if you go on increasing, so our minimum value of the function from geometrical point of view, one can say lies on the straight line.

From there, you draw a circle which will touch this straight line and beyond that if you want to increase the size of the circle it lies, no doubt on the design space or the design space, but function value is increased. This is not the minimum of will be what which will touch this line, let us call that one, so this is point, this is p point here. So, now you can easily find out what is this point at which this circle will touch and that point you will get the minimum value of the function. It is a simply geometric concept, so when it will touch this, one can find out which form geometric point of view from this point on this line, what is the perpendicular distance.

You can find out once you know the perpendicular distance of this one, you can find out what is the coordinate of that one by using some geometry of this one. So, once you know this coordinate, immediately you know what the function value is, put the value of x 1 and x 2. So, I will just give you this hints that you can find out if you write the perpendicular of distance, this one by coordinate geometry distance 2 x 1. You see this equation plus 2 x 2 minus 3 divided by the coefficient of a x plus b y plus c is equal to you know the distance from any point is that value is 2 square means 4 plus 2 square 4, put the value at x is equal to x 1 is equal to 2.5.

From this point, we are drawing and x 2 is 2.5, if you find out this one that value will come approximately not approximately, it will come this one. Now, this line this o p and this straight line are perpendicular to each other and you will see you can actually find out the slope also. If you like the slope of that one also agree, what is the slope that means if you draw this one, this coordinate you do not know. Let us call it is x 1, x 1 bar and then x 2 bar this point, so immediately you can find out this is from here to here is x 1 bar why x 2 bar. This is you know 2.5, so it is 1.5 divided by this is you know 2.5, this is 2.5 this is x 2, x 1, so 2.5 minus x 1.

So, this two ratio this by this ratio is same means one, so it is a slope I know this one from there, you will get it one expression x 1 bar and x 2 bar expression. So, I can write it if you see 2.5 minus x 1, x 2 bar minus 2.5 minus x 1 bar is equal to 1 that implies x 1 bar is equal to x 2 bar. So, this coordinate x 1, x 2 are same, next what we will do this coordinate must satisfy this circle equation and what is the circle equation, if you see the circle equation.

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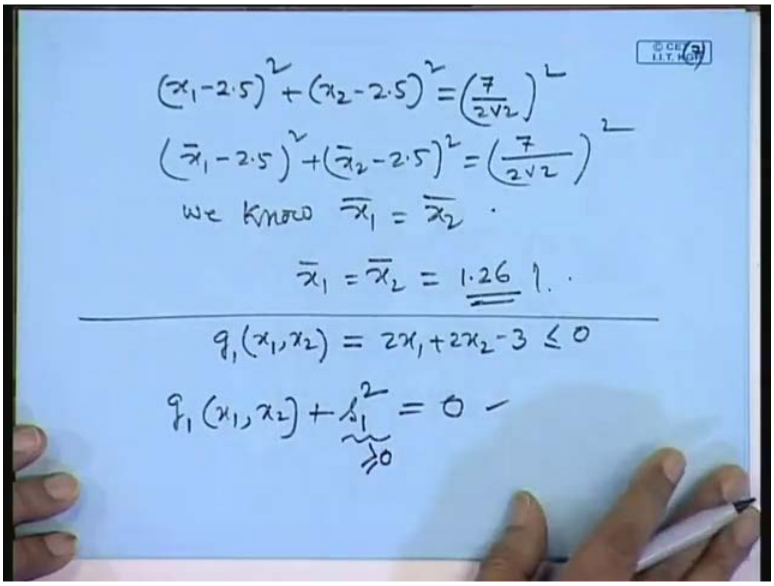

Here x 1 minus 2.5 whole square plus x 2 minus 2.5 whole square is equal to the distance, we got it if we recollect this one, o p distance is 7 by 2 root 2 7 by 2 7 by 7 by 2 root 2 7 by 2 root 2 whole square. So, put this value x is equal to because it satisfy at x is equal to x bar and x 1 bar x 2, so if you put this value in this case, so it will be x 1 bar minus 2.5 whole square plus x 2 bar minus 2.5 whole square is equal to this quantity. So, solve this two equations, I know x 1 is equal to x 2, so x 1 we know x 1 is equal to x 1 2 is equal to x 2 bar. Then, we can find out x 1 bar from this one, you can find out x 1 bar is equal to x 2 is equal to that.

This value will come x 1 bar will become 1.26, please check it just in this expression, if you put it you will get it. So, we know physically this is the what is called this three equations inequality constraints, I have just drawn in a graph and shown without solving any what is called optimization technique. All these things physical geometric concept of b y, I find out what is the point at what point the function value will be minimum that is, but next we have to do it through what is called by analytically. Now, look at this expression if you recollect this one in general, let us call we have a one expression is there x 1 x 2 that expression and that expression is what in your for this specific problem see that one, 2 x 1 plus 2 x 2 minus 3 is less than equal to 0.

So, this quantity I can always write in equality constraint form, but what does that mean this quantity is less than 0. That means if I add some positive quantity with this one, then I can make it this is equal to 0, so I am writing now that our inequality constraint inequality constraint is now converted into equality constraint. This one plus I am adding one positive term, because this is this is less than 0 means negative term, I have to add some positive term to make this is equal to 0. So, any inequality constraints are there, I can always write it into this plus some positive quantity. You have to add this quantity that is what quantity is greater than 0 or equal.

Now, look at this one when s is equal to 0 that means this satisfy the equality constraint agree that satisfies the equality constraints. So, any inequality constraint, now I can convert into a equality constraint by adding s 1 square which is a greater than equal to 0 and it cannot be negative. If it is a negative s 1 square is negative that means it indicates this quantity is g 1 is always is negative this quantity and if it is coming negative that means this this violates the our constraints given that. Now, question is that we know at this moment how to solve by unconstraint optimization problems. So, if you have a what is called general problem that we have mentioned it minimize, the function subject to equality constraint and inequality constraint and inequality constraint.

We know how to convert into a equality constraint, now next question is if we can convert the unconstraint, sorry constraint optimization problem with equality and inequality constraints. We can convert into equality constraints, then the problem is if you can transform that unconstraint optimization problem into what is called constraint optimization problem into unconstraint optimization problem. Then, we can solve our problem as per we have considered earlier because unconstraint optimization problem, you know how to solve it either numerically or what is called by analytical method.

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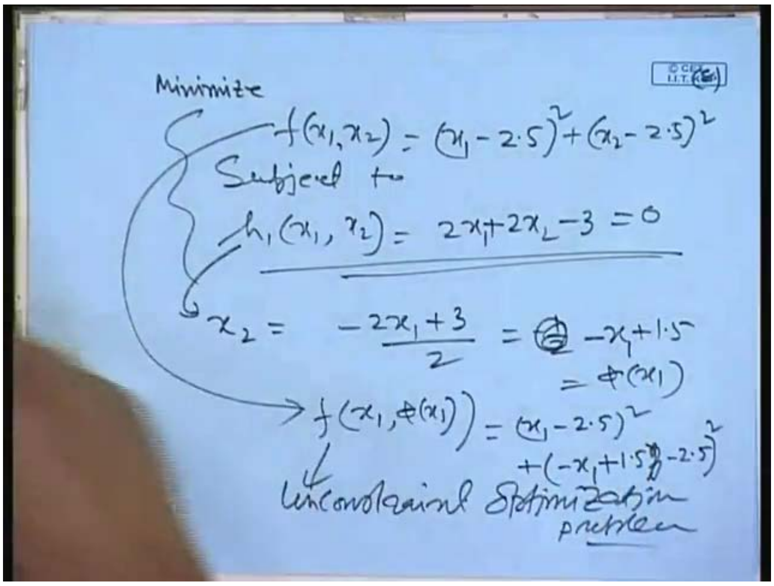

So, let us see this one how to con convert one way conversion is quickly, if you see this one minimize same problem f of x 1 x 2 that x 1 minus 2.5 whole square x 2 minus 2.5 whole square subject to h of 1. Only one equality constraint is there is equal to minus 2 x 1 plus 2 x 2 minus 3 is equal to 0, so only one equality constraint is there. So, how to convert this, this, what is called constraint optimization problem how to transform into a unconstraint optimization problem, one can do like this way. I will find out x from here x 2 is equal to what minus 2 x 1 plus 3, if you take this divide it by 2 is nothing but a half this 2 cancel it is a minus x 1 plus 1.5.

If you put this value of x 2 put the value of x 2 in this objective function, now it is a function of x 1 and x 2 is a function of I can write this is a function of x 1. So, it is a function of x 1 which I can write it, now if you just put this value of x 1 minus 2.5 whole square then in place of x 2, I will write it minus x 1 plus 1.5 minus 2.5, 1.5 minus 2.5 whole square. Now, you see this constant I have forced the equality constraint, I am forcing objective functions, now it is become a unconstraint optimization problem.

This is unconstraint optimization problem optimization problem, so this problem you can solve what we have discussed earlier this one. Similarly, inequality constraint is there, I have just mentioned how to convert into a equality constraint and then proceed in the same manner, so next class we will discuss more general in inequality constraint, also along with the equality constraint, how to convert what is called equality constraint optimization problem into unconstraint optimization problems.

Thank you.