Optimal Control
Prof. Dr. Goshaidas Ray
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 44
Gain and Phase Margin of LQR Controlled System

So, last class we are discussing about the frequency domain representation of LQR control systems.

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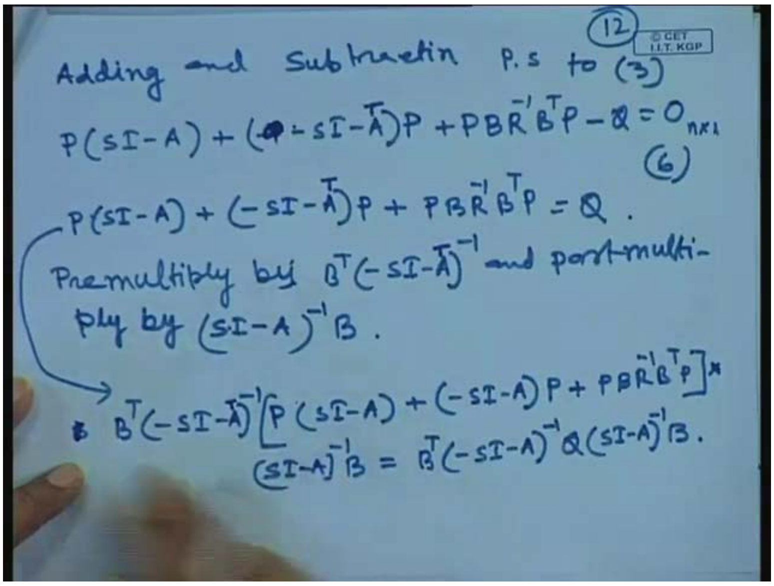

So, when you are discussing this one class we just recall that our basic Riccati equation algebraic Riccati equation, which is refer to equation number three in our earlier lectures and that algebraic Riccati equation we added P S term and subtracted from this one. If you add and subtract, then this is a P S I minus P S a P so it is cancelled that one, now this is P A minus P A and this is minus that this is A, if you see this one S 1 minus P A. So, that that will be minus here, it should be A transpose, because our basic equation is A transpose here is A transpose. Then P B R inverse B transpose minus Q because our Riccati equation is A transpose P plus P A minus P B R inverse B transpose minus Q.

So, both sides I multiplied by minus, it does not matter, so you got it this one, now what we did it this one. So, this will be our transpose, then pre multiplied by both pre multiplied and post multiplied by the factor this one and this should be A transpose both side and pre multiplied by and post multiplied by this one. If I pre multiplied by B transpose minus S I minus A transpose whole inverse, then we have here is A transpose and post multiplied by that one both sides.

So, here also will be A transpose, so after multiplying by both side by B transpose, we got it this expression. So, if you see this expression B transpose this S I minus A transpose this S I minus, this and this is post multiplied by this. So, this this quantity and this quantity will be identity matrix, so P B, so here only the correction is there A transpose.

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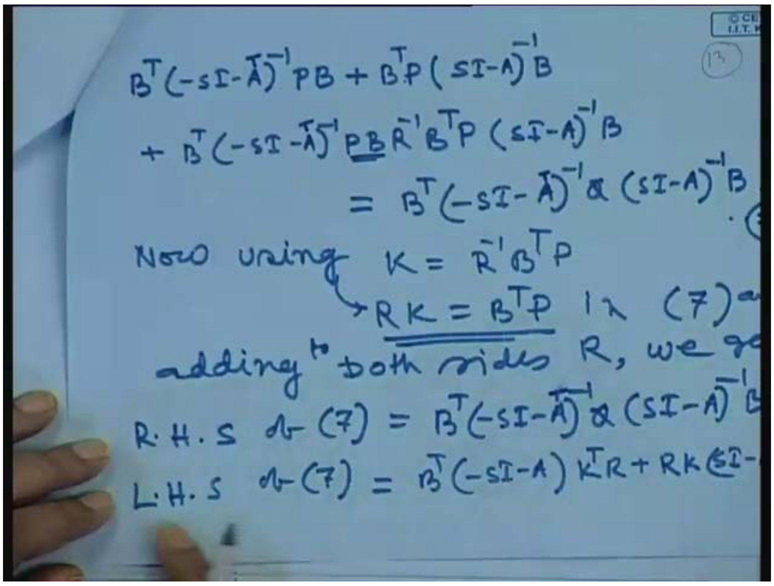

It is A, this term you see S I minus K S I minus this, if you push it in this this side, so this and this will be A transpose, this as A transpose. So, this this will be identity matrix so B transpose P multiplied by S I minus a whole inverse B. So, B transpose P S I minus a whole inverse B, then we have A, this term you see both side I am multiplying by this one. So, this term as it is pre multiplied by this and post multiplied by this, so here will be A transpose is that and most by is this one this is all left hand side and right hand side.

Since we have multiplied by Q pre multiplied by this one post multiplied by this one, so this transpose is missing, so this is that now what we have did it from equation seven. We have, we know the our controller gain is nothing but A R inverse B transpose P, so if you pre multiplied by R, so K R is will B transpose P.

So, in this equation whenever we are getting B transpose P will replace by R K, if you replace by this one the right hand side of this equation that B transpose S I minus this is transpose. Here, Q S I minus this and left hand side of this one P B, P B is replaced by what is call by K transpose R actually K transpose R transpose.

Since R is a symmetric matrix so R transpose is equal to R, so this will be A, this one and for P B I am writing is K transpose R K transpose R. Then B transpose P, I am replacing by R K, so this is S I minus this, so this will be A transpose that one. So, this is A transpose here, so if you do this one then after that we proceed like this way. So, last class we have come up to this point, then we proceed like this way, so see the left hand side, what we can write it this portion, what we are writing next.

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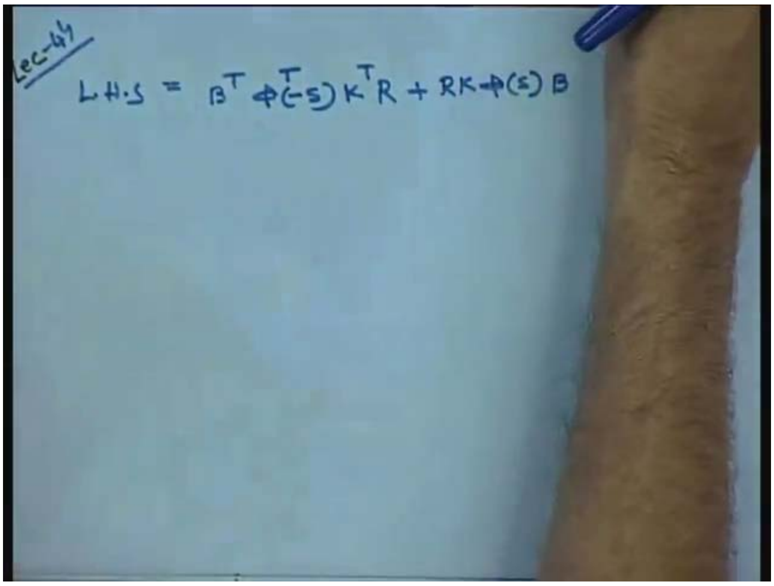

So, the left hand side we can write B transpose and denoting by this one this by 5 S 5 S that S I minus a whole inverse is 5 S. So, then I can write it B transpose, then 5 minus S whole transpose see this one S is replaced by minus S. So, it will B A 5 minus S, then whole transpose in place of this is our 5 S, then if you take that whole transpose that will be S I minus A transpose. Then you would K transpose R then K transpose R, the next term is R K as it is you see R K, it is this term, I will replace by 5 S, so this will be R K. Then I am replacing this is 5 S that means S I minus say whole inverse is a 5 S into B, so this quantity is the left hand side here, you see I have just written this part, but this parts are also left it in left hand side.

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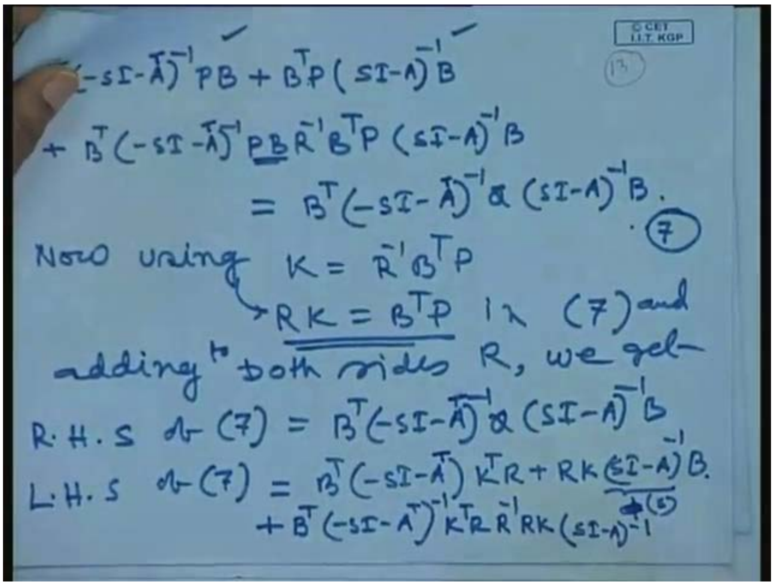

So, that I just missed it here, so I will write it this is B transpose then S I minus A transpose whole inverse then P B, P B is what K transpose R K transpose R that P B on left hand side. We have this term plus this term and this term is replaced by this one replacing P B and this term is replacing by this one. This also there B transpose this as it is P B is now replaced by K transpose R, then we have a R inverse, then B transpose P is a R K into this term into this term. So, this is S I minus a whole inverse, now the third term is coming here, if you see it is a B transpose 5 minus S transpose.

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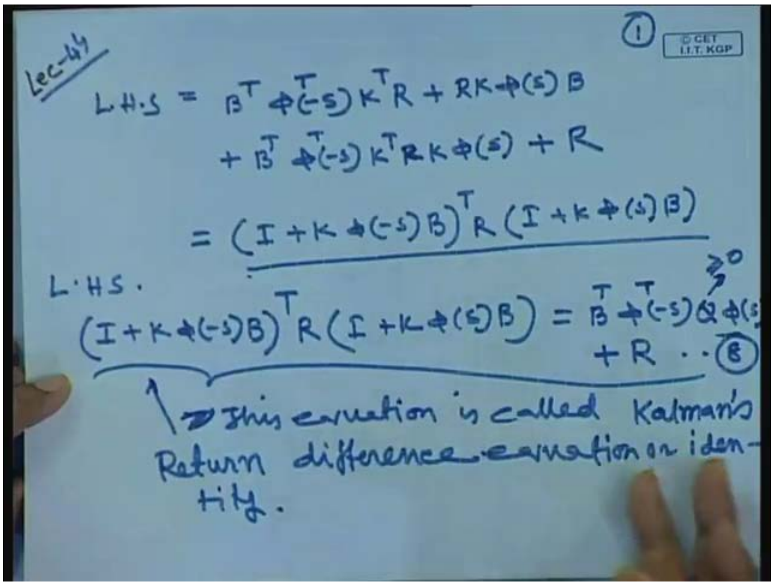

Then K transpose, R K K transpose you see this is as it is this identity matrix, this is identity matrix. So, K transpose R K K transpose R K plus into 5 K 5 S plus this we have written like this, so that this plus this, so this we can write. So, this we can if you factorise that one, you will get it plus R K from where it is coming R K. So, A transpose our basic equation A transpose B P B R inverse B transpose R inverse B transpose P plus that one into plus Q. So, I just both side I added R here and right hand side also I in the right hand side of the S equation seven, also I added R.

So, this adding is the seven equation I and R here and R, R is at both side, so it will not change the expression. So, I have written the left hand side is this one and right hand side is this plus R, so this is our basic equation of the left hand side. So, this can be factorise like this way I plus K five minus S B whole transpose R into I plus K 5 S 5 S into B K 5 S minus 5 S B whole transpose R this. So, this can be factorise like this way, you just expand this one you will get it this one.

So, if you just tell you briefly we are started from the Riccati equation and every algebraic Riccati equation, then both side I have added that in the that it as algebraic Riccati equation I added P S term and separated P S term. Then then after that both side I multiplied by A, this expression the pre multiplied by B transpose minus S I minus A transpose inverse and post multiplied by this. After multiplying an after multiplying both side pre multiply where post multiply and plus R is added left hand side and right hand side.

Then, ultimately we got left hand side equation is this one and the right hand side of this one, we got it that one, this one, now this one from equation. I can write it left hand side I K 4 S plus B whole transpose R plus I plus K five S B. This is the left hand side is equal to right hand side right hand side what we have got it we got it B transpose 5 minus S transpose Q five of S plus R. So, let us call this equation number eight, now look at this one Q is a Q is a positive semi definite matrix Q is positive semi definite. This quantity will be always positive non negative number for any value of frequency, so I can write it.

This equation is called that whole equation, this equation, this equation is called the Kalman's return difference equation or identity. So, this equation since Q is greater than equal to 0 means positive definite may this quantity for any frequency. If you swift from

0 to infinity that quantity will be non negative number, so I can write it this quantity is always greater than equal to than capital R. So, I can write it this quantity is always greater than equal to then R.

So, this is the represent the algebraic from algebraic Riccati equation, we got this expression and algebraic Riccati equation. We need it to solve what is call get gain of the controller to get gain of the controller because we need the value of P and P is the solution of algebraic Riccati equation. So, after getting this one, we will now define the gain margin and phase margin of the LQR control systems, so we can write it this this this one that equation eight this equation eight from equation eight.

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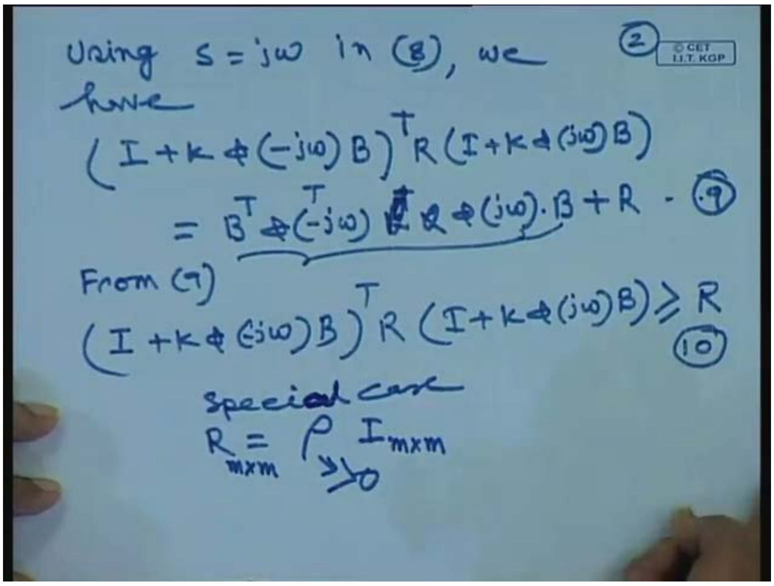

Using S is equal to in frequency domain S is equal to j omega in eight, we have now I plus K 5 S is replaced by j omega. So, it is with j omega then B whole transpose where ever the earlier expression eight equation number eight S is there S is replaced by j omega. This into R plus I plus K 5 j omega B, this is equal to B transpose agree is equal to B transpose 5 minus j omega transpose K transpose. Then this equal to not K it is a Q, then you have a this 5 j omega into B plus R.

So, this is equation number nine that you see this equation I have written it here putting the value of S is equal to j omega in frequency domain. So, what is the Kalman written difference equation different equates and differential equation, so this just implies the S is equal to j omega. So, from 9 putting that reason that since Q is a positive definite matrix positive semi definite matrix for any value of frequency from 0 to infinity. If you swift that, this this whole expression will be greater than equal to 0 non negative number because it is pre multiplied and post multiplied by a matrix which is transpose.

If you consider this is a matrix of m m Q and m transpose and since any Q is a positive definite matrix whole matrix after transformation, it I will be a positive definite positive semi definite matrix. That means this is non negative number, so from 9, we can write it that I plus K 5 j omega minus j omega B whole transpose R is equal I K 5 j omega B, this bracket is greater than equal to R, so let us call this is equation number ten.

This expression is valid for whether it is a multi input system or single input systems, this expression is valid for all cases. Now, consider this situation special case consider a special case special case that R is equal to we consider R is a weight age in the input vector. So, this dimension is m cross m and this is equal row which is this quantity is any positive number this equal to I into m. So, we have consider R is diagonal matrix of each diagonal element is row which is greater than 0, so from this 10, now we can write from 10.

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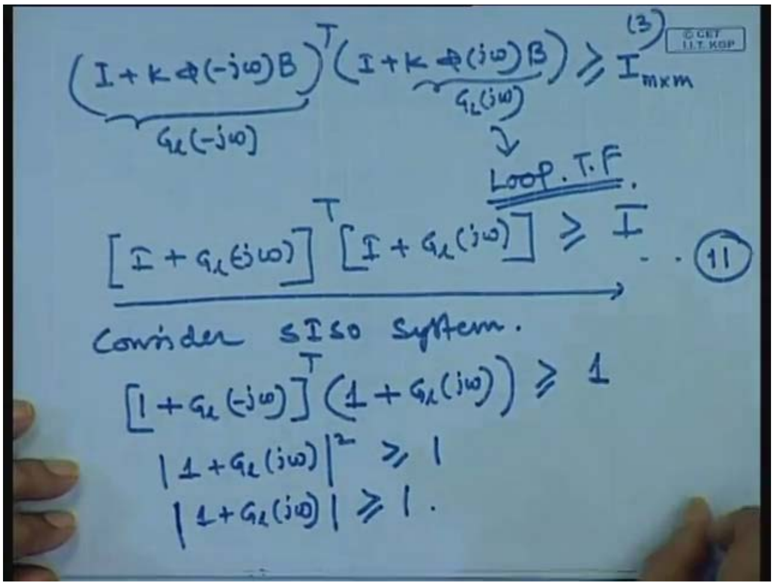

We can write from 10, we can write it I plus K 5 minus j omega B whole transpose into because R is replace by row I, row is will be there and right hand side. Row will be there, it will be cancelled, so it will be I K 5 j omega B, it will be actually row into I R is replaced by row into I. So, right hand side also row into I, so row, row cancel, so it will be a simply I into the matrix of dimension m cross m. So, this equation we have observed of this one and if you see we have discuss earlier this quantity.

This quantity is nothing but A, our loop gain if you recollect our previous formulation that what we last class we have discuss see this one this is nothing but a loop gain that this whole thing is nothing but a loop gain of the systems. This G if you consider the G S this is h S G S h S is the loop gain, so it is this thing is nothing but A K, so K into a S I minus A inverse whole inverse which is I denoted 5 of S into B is a loop gain of this one. So, if it is a five S this will be nothing but the whole thing will be G l minus j omega l this whole can be and its transpose, so this will be a five l j omega S is equal replace by j omega when it is a minus sign.

So, this that means we have j l transpose into j l loop gain this is greater than equal to identity matrix, so this is nothing but a loop gain loop gain or loop transformations. So, you can write it, now this I plus G l j omega minus whole transpose into I plus G l j omega is greater than equal to identity matrix. So, this is those expression we got it for LQR system in frequency when we got it 1 plus G S and then you take the conjugate transpose of this one multiplied by these. This will be always greater than equal to 1, let us call this equation is important equation is representation is very significant.

We will see later then this equation number 11, so from 11, we can write it now from this one, we can write let us call consider a special case means consider it is a single input. Single output system agree this is a single input single output systems though output will not come into the picture in throughout this one. It is our single input system, let us call that means our R is equal to 1 and in in this case this dimension will be 1 plus 1. So, I can write now 1 plus G l j omega whole transpose whole transpose into 1 plus G l j omega this is greater than equal to 1.

Since this is a single input case this is a ratio of two pole inner j l e, we will get it ultimately if you take the absolute values of that one what is the absolute value of this one and what is the absolute value of this one both absolute value will be same. So, I can write it one plus G l j omega absolute value square is greater than equal to 1 if this quantity square is greater than equal to 1. That means absolute value of 1 plus G l j omega must be greater than equal to 1, so this is 1, ultimately you know the expression for G l loop transformations and our loop transformation.

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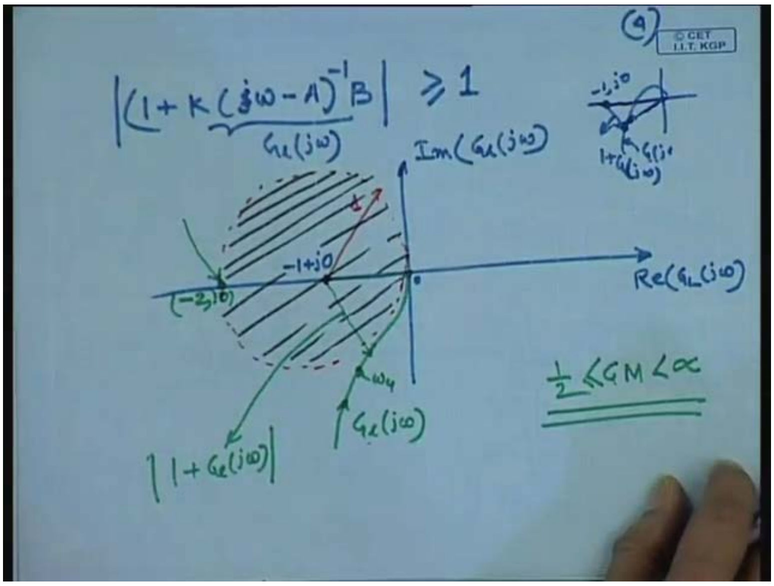

We could write it mod of 1 plus K S I or in frequency remain S that means j omega, then a whole inverse B mod is greater than equal to 1, then what is the it representation of that one, it is nothing but a loop transformation of that is G l j omega. So, let us see is representation in polar plot form, so I am just plot plotting in this a real of real of G l j omega part and here is I am y coordinate is imaginary G l j omega I am plotting. This is the our origin and this is the point which is minus 1 plus j 0, so I draw a circle of unit radius with a centred is minus 1 comma j 0, then we have A.

This one and this radius is one I have drawn with circle, then what is this condition we are getting basically from LQR design problem. We have to find out the controller law controller gain K is R inverse B transpose P, P is the solution of Riccati equation algebraic Riccati equation. That Riccati equation we have manipulated, ultimately we got a condition like this way, so if it is you now plot it that what is called Nyquist plot of G l j omega, let us call G l j omega. Let us call this is some plot is that G omega, this is minus so what is the distance from this point to any point on the polar plot of G polar plot of G is nothing but A one plus this is minus 1.

So, this is 1 and this this distance is mod of j omega what is call G j omega, so this distance plus this distance would be 1 plus G l j omega values, so that that one is nothing but A one plus G l G j omega. Similarly, here if you plot it that what is call our loop transformation Nyquist plot, it must not cross, it will not cross that unit circle that unit circle is I am just represent by shaded and this is the plot for G l j omega polar plot of G of j omega. Any point on this curve that means this indicates that omega is increasing it is approaching to the 0 because it is what call real transformation in the sense that it is a low pass plant of this one.

So, this will approach to the 0 of this one h omega tens to 0 this 1 and you see what is this from here to here. This quantity is one plus G l j omega mod of this one this is mod of this one is less than that this is nothing but 1 plus G l a. This is less than unit circle, this condition you have to satisfy this condition this curve then G l j omega. That means loop transformation of the system when we have used LQR controller that should not cross what is call unit circle.

In the other words it will not enter to the shaded portions by anywhere, this one, so look at this expression, it does not cross the real axis this one. So, what is it, it cross there what is our gain margin of this one our gain margin of this one is in infinity. You can what is the present gain is there from there you can this is the present gain what is there this is the lookers of that one. This is the look my polar plot of that loop transformation based on I repeat that this loop transformation is formed based on the LQR controller design because K is involved in this one the loop transformation.

It will never cross the shaded region, so our gain we can here hope from the present gain we can increase the gain up to infinity. So, use can say whatever the gain present gain is here we found out by solving what is call LQR problem and what is the present gain, we got it. That gain has you can change the gain up to form a factor of infinity that means you can increase the gain up to infinity still the system will be stable. You want this is the present gain we can increase from the present gain what factor we can reduced it.

So, the system still maintain is stability, now you see this point if it is A 1, this point is minus 2 minus 2 comma j 0. This point this point agree this point is minus 2, this one, so this gain whatever gain present gain is there at our situation. It can loop transformation can touch this one, so we can reduce the gain by a factor of half, so it will touch the minus one point whenever increase in and decrease in that gain of the present gain of this one phase angle will not change. It will not alter anything, only it will change these magnitude that a particular frequency different frequency magnitude to only will change suppose this frequency at this frequency.

Let us call this is zoom frequency omega is equal to 4, this frequency what is the magnitude this is the magnitude of this one what is the angular. So, set of this one, so what is the present gain is there you increase the gain twice, so its magnitudes only increase by twice, but phase angle will be remain same. So, some polar plot of loop transformation if it is touching here, but it is not entered in here. It is just touching, it cannot enter because it is based on the LQR design polar plot, and he cannot enter this one, then after that we can decrease the present gain by a factor half.

So, this present loop transformation Nyquist plot will just touch the minus 1 point till then the system is stable. So, we can say they our gain margin gain margin of the LQR design controller base control system. We can increase the gain up to infinity and decrease the gain at least that half, so our gain margin you see from infinity to half what is the present gain is there. Let us call present gain what I have got it the present gain of this system, we got it let us call 5.

So, you increase the gain 5 up to infinity by factor of infinity by if you want to decrease the gain due to some reason the gain is decreasing, if you decrease the gain by factor of this half, I am telling if the gain is reduced by a factor half.

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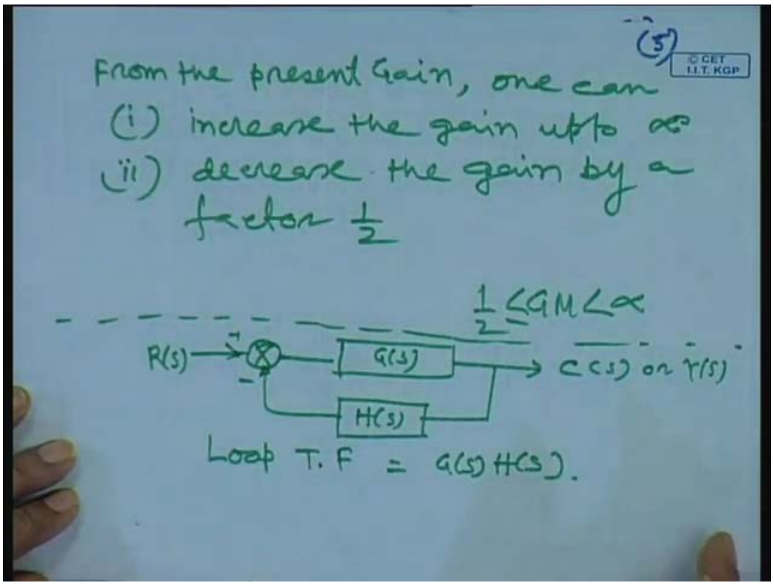

Then, I can give you guarantee the system will remain stable, it will not unstable, but below this one has to further investigate, but this is the range of gain margin. So, I just write it this that one the gain, if you summarise this design the gain from the from the present gain.

Now, one can one increase the gain up to infinity, one can decrease the gain by a factor half, so our gain margin is minus infinity by of this one. So, equation now you see this is our equation number what is the equation number any way. So, let us call this equation number you give it 12, here we have given up to up to 11, we have given. So, this equation number we have given is twelve, so from 12 from 12 one can say that what is called this conclusion is made from equation number 12. So, before that we study the gain margin those I will just gain margin we have done it phase margin. We have to do it just how to study the stability of a control system based on Nyquist stability area that I will briefly discuss here. So, it will be easy for you to understand this one, so let us call we have a close loop system like this way.

So, our G S agree our G S is this one and our h S is a feedback five transformation is that one and this is our c S and y S on polar also write in y S this is our R S. So, our loop transformation is what you see loop transformation is nothing but A, from here to you just complete the loop and what is the transformation. You get it there is loop h of S, so if you see how to study the stability of this system by using what is call Nyquist stability criteria, the first thing is we have to see the what is the our Nyquist path.

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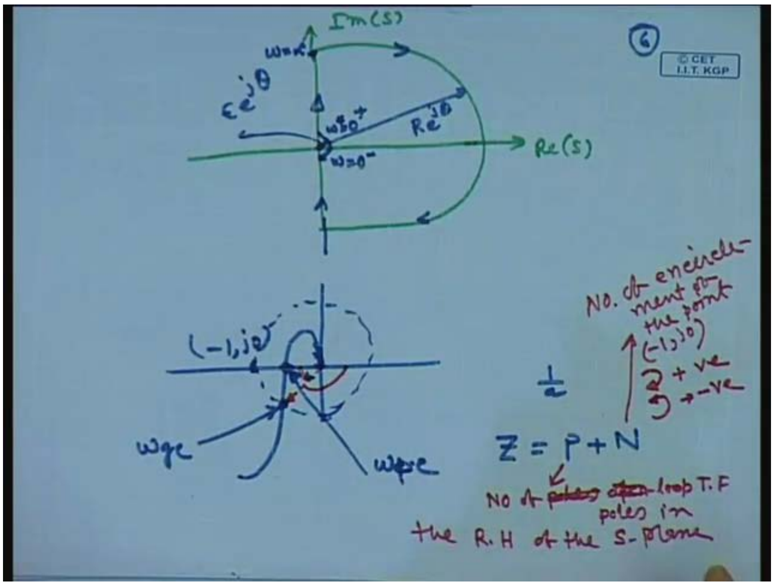

Here, Nyquist path, our Nyquist path is the whole right of the S plane whole right up of the S plane and this is our real S this is our imaginary S and if you have any pole at the origin. Then you bend this curve like this way either left hand side right hand side or left hand side bend it. So, that you should not you should not be value of the function the transformation should not be infinite because you see one if S, S divided by S, S plus A numerator divided by S into S plus B when S stand to 0 as say 0. This becomes a infinite this one so this infinite but a we just make it limit in this one there is a small.

This quantity is the e to the power of j theta, when will just put limiting value and evaluate the function value along the Nyquist path and this is the Nyquist path agree this is the Nyquist path. Then omega is equal to 0 omega is equal to 0 omega is equal to we will just add PSI loam theta of this one very small value is just go on this one. So, you can write it omega plus omega is equal to 0 plus this is this is omega is equal to 0 minus this. So, that R is a PSI loam it is for j omega going like this way and this is the omega is equal to infinity agree and this omega is equal to minus infinity.

So, this is R it is the power of j theta and R is tends to infinity this one, so we will find out through loop transformation Nyquist plot loop transformation,m Nyquist plot while we move along this Nyquist path from 0 to omega 0 to infinity. We swift once you know from the Nyquist plot of loop transformation from 0 to infinity, then minus infinity 0 is the mirror image of that polar plot. We can do it this one, so our Nyquist stability criteria says that that let us call for the given system our Nyquist stability and Nyquist what is call polar plot of that one is like this way just touching this one.

This is minus 1 j 0 point and it curves is here and if you see this one with a unit circle if you draw a circle of this one and this is the gain cross over frequency omega. This is the gain cross over frequency G c and this is the phase copper cause cross over frequency phase cross over frequency. So, our gain margin if you recants, here then reciprocal of this distance will be the our gain margin that means we can increase the gain by a factor what is call reciprocal of that one. Suppose, this is a, we increase the gain by factor 1 by A, so it is a gain margin of this one and if you assume that no poles open loop poles are right up of the S plane.

Then, we can say gain can be increase this one, so in general a z is equal to that P plus n agree and what is this phase gain cross over frequency. We will get it what is call phase margin this our magnetite of the polar plot and this is the phase angle. So, we can introduce additional phase this much before the system becomes unstable, so we have a this criteria we have to check it this is the you can say number of poles number of poles number of which poles open loop. Other loop transformation loop transformation poles loop transformation poles agree in the right hand of the S plane this is n.

This n is the this n is the number of number of encirclement of the point of the critical point minus 1 j 0 number of encircle of the critical point by the polar plot of the loop transformations. I mean to suppose there is no poles in the right up of the S plane open loop transformation poles, there is no poles thus P is 0 then n is n if n does not encircle minus one point. Then P 0 n also 0 it does not encircle minus 1 point is encircle 0 so P 0, n 0, so z is 0, z 0 means it the system there is no close loop poles in the right up of the S plane that figure z number figure.

What we will get is z is equal to let us call there we got it 0 that means no close loop poles of the system lies in the right up of the S plane. That means system is stable suppose there may be system open loop system is a unstable loop transformation is unstable. We assume we have seen there is a two poles are in the right up of the S plane of the loop transformation or open loop poles the two poles on the right up. So, P value is to come, so in here to become the systems two that n must be 2 so that this is the encirclement is clock wise if the polar plot encircle the minus 1 point in clock wise.

Then, this is sign will be consider as a positive if it is enclose the minus 1 point anti clock wise that it sign will be consider negative. That means this if the polar plot encircle the minus 1 point clock wise j is on twice, then we will write 2 plus 2, if it encircle the minus 1 point counter clock wise twice. Then we will write it that 2 minus 2 sign is minus will be consider, so in this way we can study the stability of the close loop system even this system is a non minimum phase system.

That means poles in the right up of the S plane if it is there open loop poles that you can study the stability of this one. Now, we will see this one that what is call phase margin, so what is the gain margin what we have seen if you design a controller based on the LQR controller. That controller has a gain margin up to infinity and that what is the present gain is there of the LQR controller design present gain is there. That gain you can reduced by a factor half, so gain margin varies from infinity to half of factor.

I am talking about the factor what is the present gain is there that gain you can multiplied by infinity very large number of that factor. That means infinity or you can multiplied that present gain of the LQR controller by a factor half before it becomes unstable. So, this is very robust even though controller parameter change, it is guarantees the system is stable this one.

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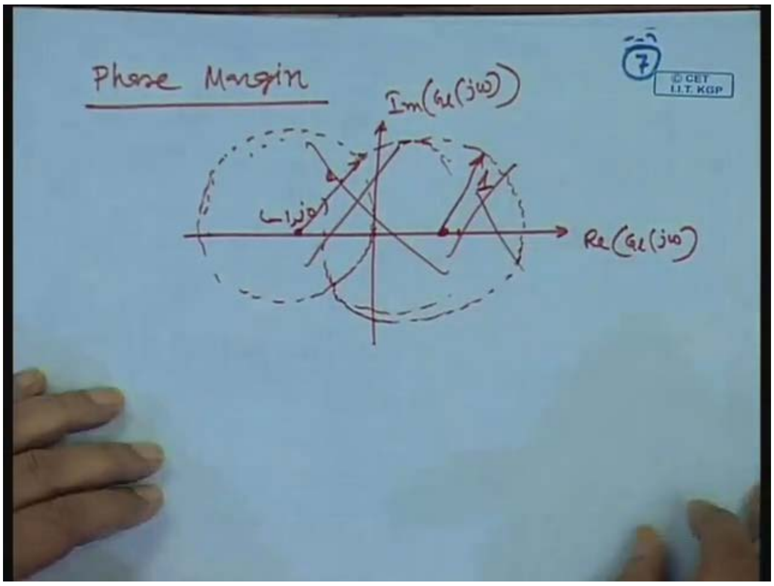

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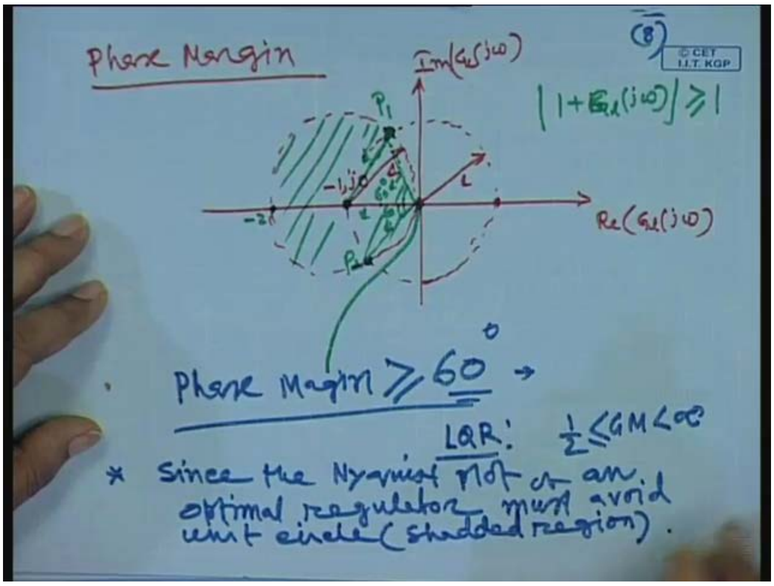

So, let us see is phase margin, so let us plot it this this one, so minus 1 j 0, this point, so I am just plotting this. So, this is a 1, then this I am just plotting real of G l j omega, this is imaginary of G l j omega. Then another circle I am drawing you to this one centre of one. I am drawing the centre, this is also proper.

Just once again I draw it that that one phase margin, so this is a minus 1 j 0 and in this I am real of G l j omega in this and imaginary of G l j omega. I am just trying a circle with a centre this of one agree and this is with a centre 0 with a radius 1. I am drawing another circle, this is also 1 and this is also 1, let us call 1, now this is minus 2, this one is minus 2. Now, see according to the LQR design problem, we have shown it that if you the loop transformation plot is like this way, it will never enter to the region of that one shaded region that is by condition, we got it.

So, it may touch it here this thing this thing proper it may touch it may never entered to this one again it may touch this one. So, if you just add this point and add this point you see this radius is a though it is not drawn properly, but this radius is 1 this also radius is 1 and if you just add with a draw a line from here to this is also one this is also one. From this triangle, if you see this is one because one with this centre with the origin is the centre we have drawn a circle of radius 1.

So, when the two circles are cross this point let us call this is P one point and this is the P two point P 1 point from this point to this point this because we have drawn a circle with a inner centred is minus 1 j 0. So, this this this are all is same length, so this value will be 60 degree, similarly, this length this length and this length 1, 1, 1 A, so this will be a 60 degree. So, one can come to conclusion, since this curve will not enter to this what is call the shaded region it can touch because why it can touch you see 1 plus K or 1 plus G l j omega mod will be greater than equal to 1 because it is when equal to 1.

It will touch this this is the condition this one, so if it in since it cannot enter into this point with the present gain of the controller. So, maximum or at least the phase angle will be what 60 because it can touch it here this this curve can be touch it here. Then what is this one, if it is this, then this is the phase angle of this, suppose it is this one phase angle of this this this this one that agree and that this will be 60. That means additional 60 degree phase can be introduce into the system when the controller is design what is call based on LQR at least 60.

It can introduce because you can introduce further more than 60 degree, so our phase margin phase margin is phase margin is greater than equal to 60. So, it will be a 60 and it will be more than 60 because it can touch only it cannot enter in this one. If you can enter here, then it bailers our condition, which we derive from the algebraic Riccati equation coming from the design of the controller, LQR controller. So, our phase margin will be greater than equal to 60 at least 60, it may be more than 60, so this this the phase margin of this one. So, in a short if you design LQR controller, it ensures that our gain margin is infinity and half.

That means what does it means we have design the controller K based on LQR and that gain can be increase by a factor. I say increase by a factor up to infinity that means gain what was the gain is gain can go up to infinity and similarly, then system will remains stable. Similarly, the gain can be reduced by a factor half what is the present gain is there, let us call present gain is 10, I can go reduce the gain up to 5 up to 5, I can tell the system is stable. So, if there is a gain variation is there after designing the controller, then it is the system will stable in this region.

That means it is a robust more robust and not only this it also ensure that it a good what is into phase margin also retain that means 60 degree is minimum may be more than 60 degree of this. It will be more than 60, so let us take the or I just if I write the since Nyquist plot since the Nyquist plot of an optimal regulator must avoid the unit circle shaded region.

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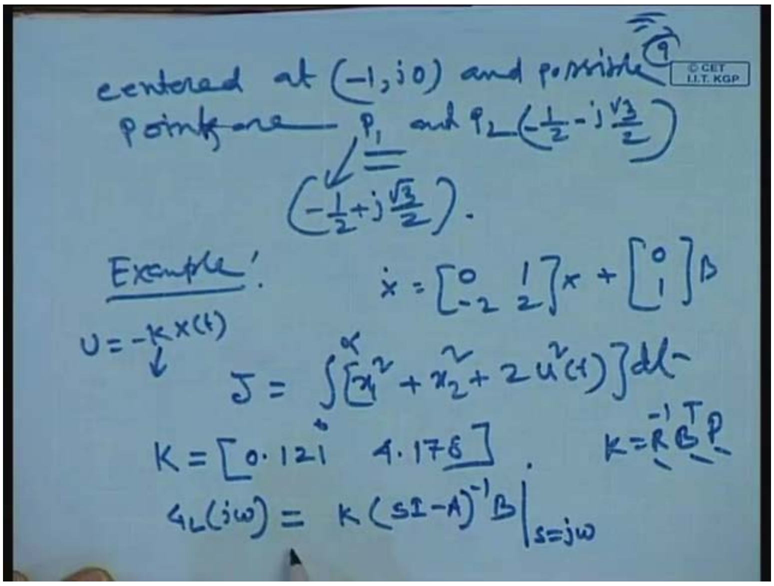

So, our possible points possible points are that we have shown it our possible points are this one P 1 and the P 2, this is the two possible points are there points are points are P one and P 2 and P 1 P 2 if you see the P 2 is 60 degree. So, one that means it is coordinate is minus half minus j root 3 by 2 and this coordinate is minus half plus j root 3 by 2. So, this the our ultimate conclusion if you recollect our earlier example, let us see what is our gain margin and phase margin what we will achieve after is implementing through what is call LQR controller.

If you recollect our earlier example that is x is equal to 0 1 minus 2 2 x, then B 0 1 that B and our performing index is half 0 to infinity x 1 square plus x 3 square plus twice u square of this d t and you have to put in proper what is call expression format. Then you apply the LQR controller that we have solve it earlier you just recall the earlier example and our problem is to design a controller u is equal to minus x t where x is the controller gain this turn the LQR design technique.

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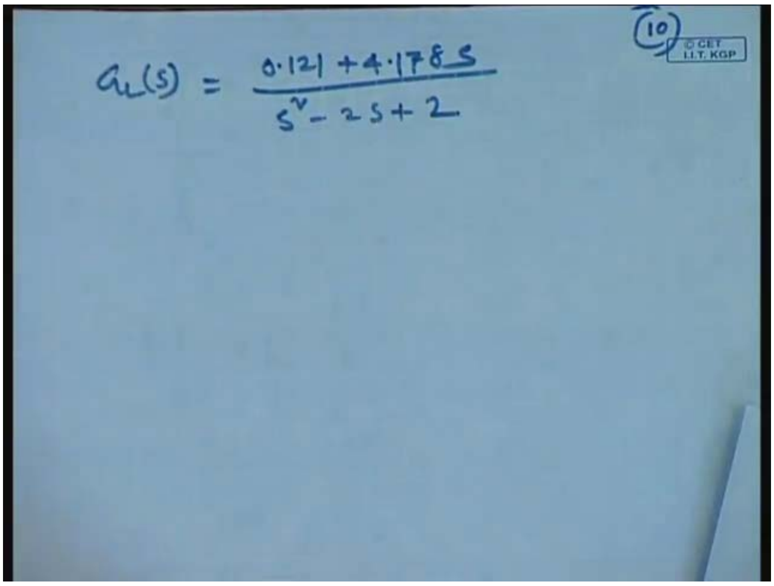

We got the controller gain if you recollect we got the controller gain point 1 2 1 and four 0.178 the controller gain if you realise then and what is our loop, once you get the controller gain how you got it we have solve the Riccati equation. Then we got it K is equal to R inverse B transpose P P, you know B you know R, you know from this expression. So, once I know this one I can find out G l j omega G l j omega will be coming if you do this one K S I minus a S I minus a whole inverse B. This is our, what is call G l and put S is equal to j omega and if you put it this one G this G l j G l s.

I am getting it 0.1 to 1 plus 4.178 divided by S square minus twice S plus 2 if you do this operation you will get it that one. So, you know you plot the Nyquist plot of that one if you do the Nyquist plot of this one what you will get it you will see this one, I will just you know how to plot the Nyquist plot.

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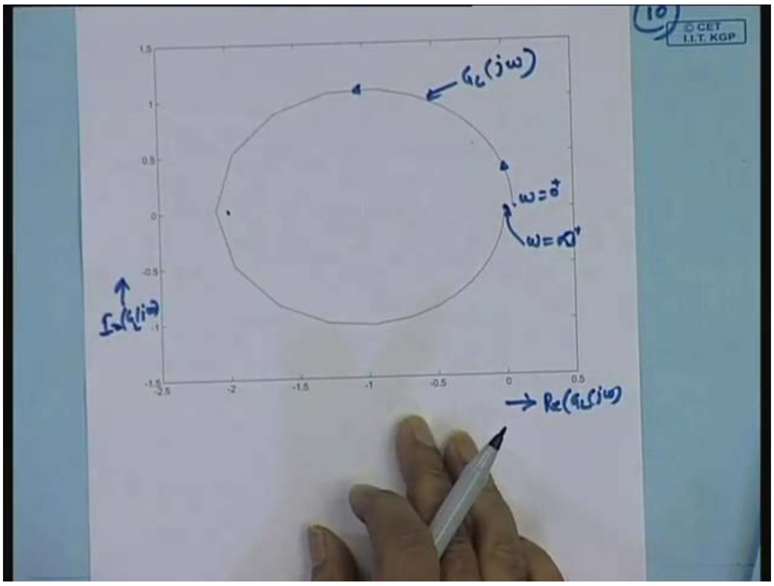

This figure Nyquist plot this is the in this x is real of G l j omega and in this x is the imaginary of G l j omega G l j omega plot it. Then it will see it will start from here as omega is equal to 0 it is here when omega is increasing this G l plot this is the G l j omega plot. The omega is equal to 0 plus and this is the origin 0 0 and this is the omega is equal to infinity omega is equal to infinity plus this. You got it this one, now see this one our two point minus two points is here, this one.

It is outside this it is not entering to this one the Nyquist plot of that one agree is not entering to this one if you draw a circle pi with a minus 1 circle of that one you will not enter this 1 minus 1 of that one. So, you can find out it is the gain margin gain margin is infinity you see it will not cut here as here you increase the gain of this one it will be infinity the gain margin from the present gain. If you decrease the gain by a factor half still, then this will be a stable of this one, so at least half it may be a less than half also it will be stable this one, then if you see the both way plot of this one both way plot of G l j omega.

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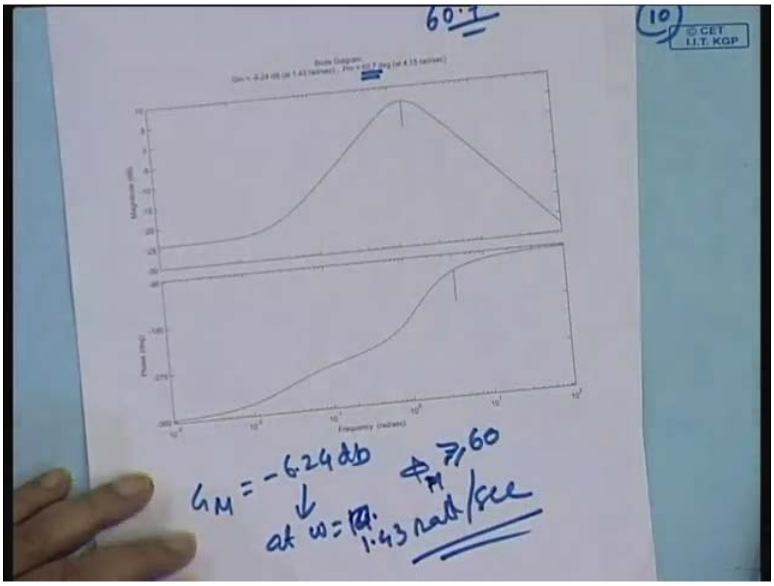

This is the thing and from the both way plot of this one the gain margin the phase margin you see 60 degree 60, you are getting the phase margin is 60.7 degree and we according to theorem. It is told at least 60 degree that phase margin phase margin is phase margin P m is 60 degree greater than equal to 60 degree. We are getting 60.7 and our phase margin and gain margin is you see our gain margin is equal to minus 6.2 for dB minus 6.24 dB.

You can easily find out what is what factor is there, you can reduce the present gain of the controller and it occurs at omega is equal to 14.1, 1.43 rad per second. So, this very, very verified our, what is call statement? We made it in LQR controller phase margin is greater than equal to 60, and the gain margin is gain is merit infinity and by a factor of half also, we stop it here today.