Optimal Control
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Lecture - 43
Frequency Domain Interpretation of LQR Controlled System

So, last class we have discussed the solution of algebraic Riccati equation by using numerical reliable controllable algorithms. That algorithms is the Sehun method we could not complete this more derivation of this one. So, just continue that one and after that will discuss the frequency domain interpretation of L Q R control systems, so if you recollect this one.

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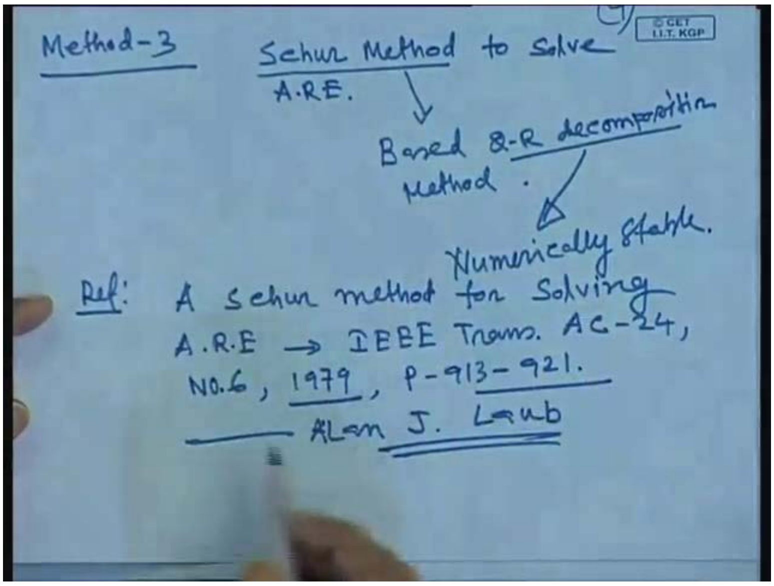

Our method is we have to solve the algebraic Riccati equations by using Sehun method, which is numerically more reliable than any other iterative methods for solution of algebraic Riccati equations. So, this algorithm steps if you recollect this one what we did it.

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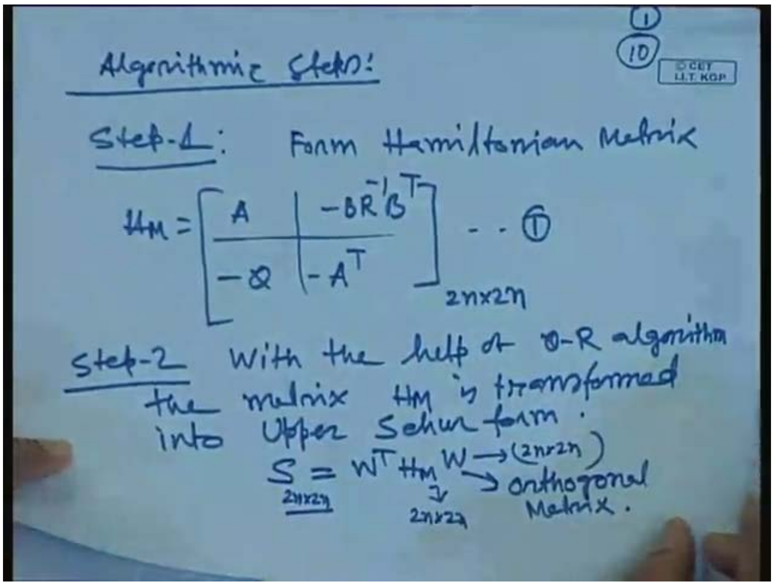

First you form a hamiltonian matrix with the knowledge of a system matrices A B and the performing index matrix ((Refer Time: 01.15)) matrix Q and R, so you can form hamiltonian matrix. Than with the help of what is called Q R decomposition technique or algorithms we convert H M into upper Sehun form. What is upper Sehun form that we have converted H M with a transformation, that is with a transformation W that. So, that W transpose H M W will convert into a new matrix S. Again that S is having a special structure and the structure of S is.

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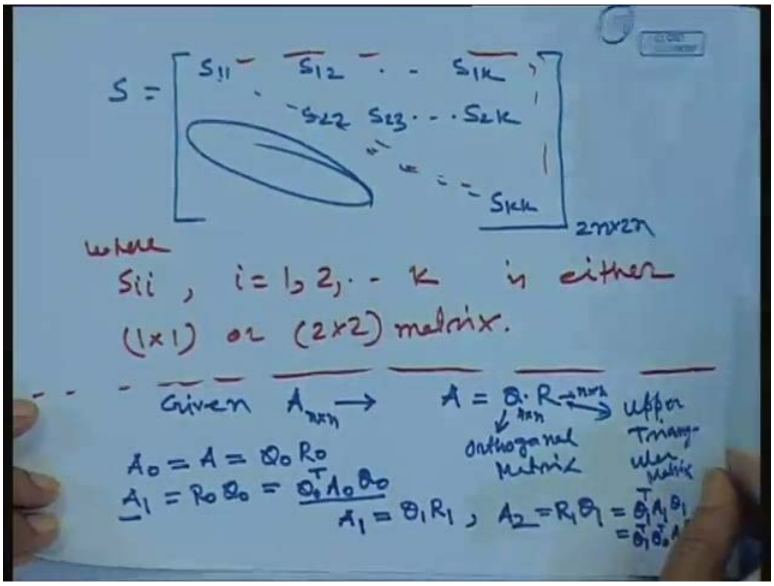

We have discussed that into this form S 1 1 S 1 2 dot dot S 1 K, and we have a such block S 1 1 S 2 2 is a K block. And each block S 1 1 or S 2 2 or S 3 3 block will be either 1 by 1 block scalar quantity or 2 by 2 blocks. And remaining lower blocks or lower parts of this matrix is a 0 all this things. So, it is a upper triangular structure and each diagonal block either 1 by 1 or 2 by 2 matrices. So, this and how to convert this thing using that is Q R decomposition method that we are discussing this one. Suppose any matrix A is given than you assign a matrix as A 0. So, you decompose this thing that Q 0 into R 0 matrix. And Q 0 is orthogonal matrix R 0 is upper triangular matrix than rewrite this equation then A 0 reverse the order R 0 into Q 0 that is not equal to A 0.

So, your writing A1 is equal to R 0 Q 0, which it can be written is in place of R 0 I can write it Q 0 inverse A 0. And since Q 0 is a orthogonal matrix that inverse is nothing but a Q 0 transpose. So, this is Q 0 transpose A 0 is and that A 0 that. Now, A 1 is decomposing this A 1 is decomposing Q 1 into R 1 that is Q R decomposition. Than rewrite in opposite reverse order R 1 and Q 1 that will not be same is equal to A 1. So, A 2 and this R 1 value you put it from this equation, what is R 1 R 1 is nothing but A Q inverse A 1. So, I put the value of Q inverse A 1 Q inverse is orthogonal matrix Q 1 is orthogonal matrix, so that will be Q 1 transpose this one. So, if you proceed like this way if you proceed like this way again you will get at k th iteration is like this way.

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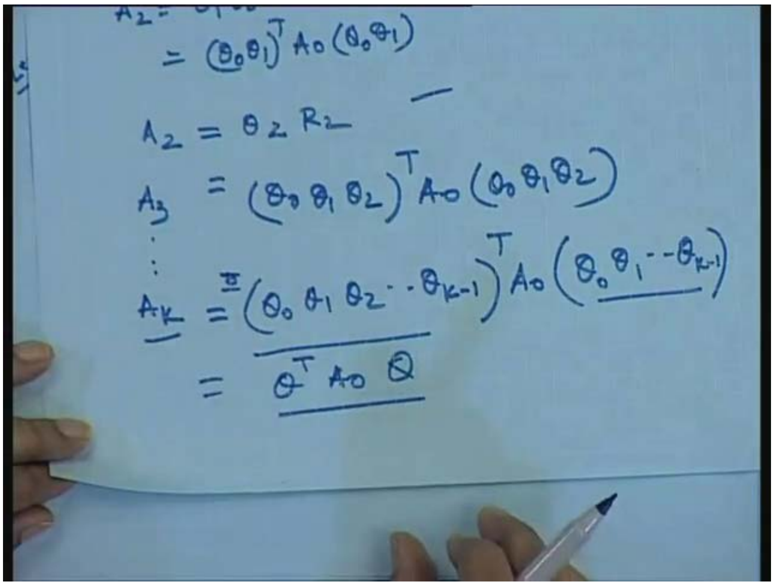

Q 0 is initial 0 th iteration, Q 0 Q 1 dot dot Q n Q k minus 1 whole transpose A 0 Q 0 Q 1 Q k minus 1. Since each is an orthogonal matrix Q 0 Q 1 Q 2 dot dot Q k minus 1, there product also orthogonal matrix. So, return that all products together is Q transpose A 0. And now when it will stop this one until and unless it has converted into a that matrix A is converted into A k is converted into a upper triangular form. Not only upper triangular form, you have to that what is called A 1 1 that final value A k 1 1 A k 2 2 A k 3 3. In this way whatever the matrix will get it each block will be either 1 by 1 matrix or 2 by 2 matrix then will stop it. So, these are converted into a upper Sehun form, so we have discussed up to this in the last class.

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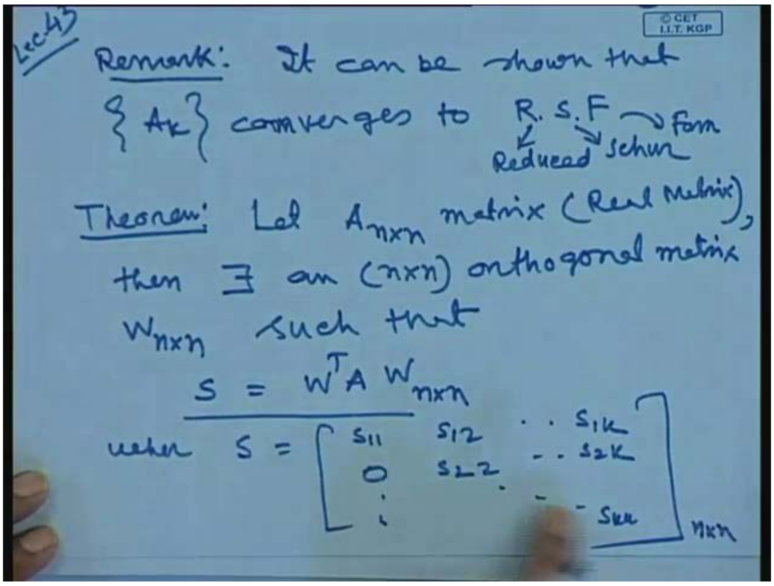

Then will see that is our remark it can be shown that sequence of A k that means A0 A1 A2 in this way by using that A Q R decomposition technique. In this way sequence of A k converges to R.S.F reduced R means Reduced Sehun Form converges to this. So, this we know this one that if you do like this way ultimately that A k will converge to the Sehun form. So, just in short the theorem we can write it now like this way.

Let A is n cross n matrix at real matrix real matrix. Let us call comma then there exist there exist this symbol is exist there exist and n cross n orthogonal matrix orthogonal matrix and n cross n orthogonal matrix W is dimensional done in this, such that such that A S is equal to W transpose A W. And this W is n cross n what is W, if you consider the earlier discussion this W is nothing but A Q, what is Q. Q is the sequence of the orthogonal what you made it Q R decomposition.

Q 0 Q 1and Q 2 dot dot Q k is W k this is done if you say. So, this is where you can do it where S is equal to S 1 1 S 1 2 dot dot S 1 k S 2 2 is zero dot dot S 2 k and this way last will be S k k. So, that dimension is n cross n again this dimension is n cross n. So, this is structure of S 1 1 S 2 2 S k k again is are that 1 by 1 or 2 by 2 matrix and it is a upper triangular form. So, next is that what is our next step our algorithm. First step is from a hamiltonian matrix than convert into a what is call if you see the that is our second step. Is you with the help of Q R decomposition into Sehun form again that Sehun form you know how to do it.

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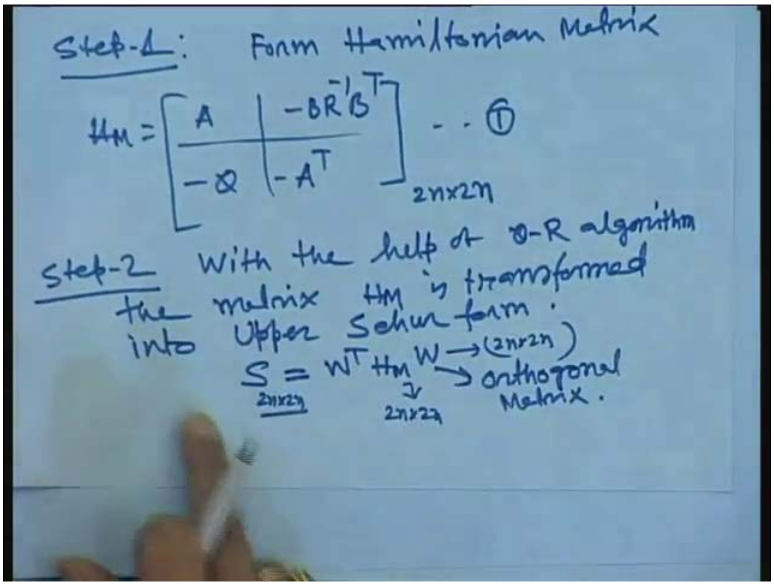

But before that you must know how to decompose a matrix into Q R algorithm again or using is, what is called orthogonal transmission or using a of House holder transmission. House holder transmission is an orthogonal transmission is a special case of orthogonal transmission house holder transmission, so our step 3.

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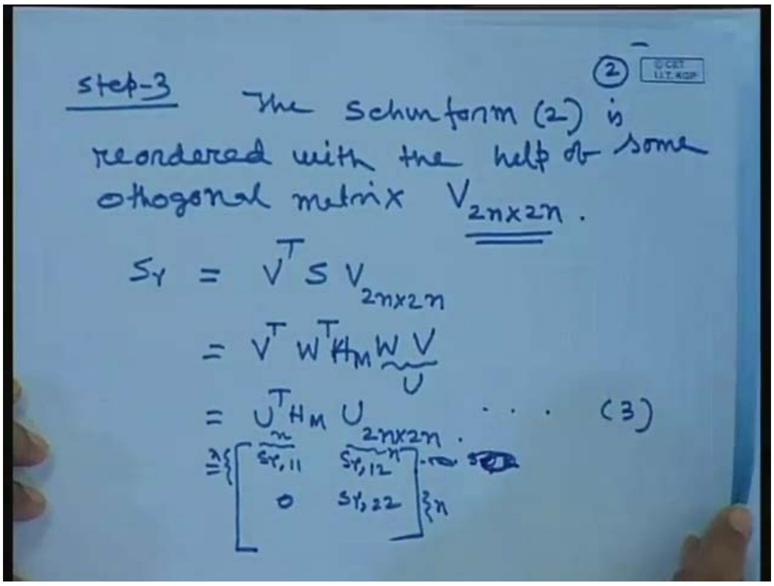

Step 2, that hamiltonian matrix is converted into a Sehun reduced Sehun form and hamiltonian matrix if you see the dimension hamiltonian matrix is 2n cross 2n, n is the order the systems. And they this matrix contains the set of Eigen values in the left of this s plain when, and the set of Eigen values in the bracket of the s plain. And the Eigen values in the both will s plane number of Eigen values. Explain what is called as s plane in the left and half and the number of poles in the s plane the right hand in the number of hours are same. So, and they mirror image of each other, so now next step is the Sehun form. The Sehun form 2 that means this is the our algorithm steps.

If you look one and that equation number 1 and this is the equation number 2, let us call this is the equation number 2. That whatever we got it should form this blocks S 1 1 S 2 2, all these block are either 1 by 1 or 2 by 2, but there Eigen values are may be stable or unstable through block this S 1 1 to S k k. Now, we have to raise the Eigen values first will make all the stable Eigen values of S matrix first. And remaining blocks will be the unstable Eigen values of H M matrix means hamiltonian matrix. So, from the Sehun from 2 is reordered with the help of some orthogonal matrix.

Let us call V which dimension is 2n into 2n, because H is a dimension of H M is dimension 2n cross 2n. And with using the householder transmission some or orthogonal transmission we convert into S. S is nothing but a W transpose into H M into W M. And H M is 2n into 2n dimensional and W is 2n into 2n. Again on S we are doing some transformation orthogonal transformation. So, that the S matrix is converting to a special form, means it will be upper triangular form.

But, only the all Eigen values which are stable that will be put first and the unstable values will put next. So, this using this transformation matrix if you do this one V transpose SV this dimension is 2n ultimately reordering only reordering of assigned S I matrix. That S matrix can be done by using elementary row and column operations, that one can do it into reordering. So, let us call after reordering we got it that, if it is a V transpose than what is S if you recollect S is W H M W then V. You consider this is U, so it is nothing but a U transpose H M U. And this U dimension is 2 cross twice cross as this, so let us call this is equation number 3 and the structure of S r.

If you see the structure of S r will be like this way that S r 1 1 S r 1 2 dot dot S r 1 S r 1 k than. Again let it let it be like this way S r 1 1 S r 2 2. What does mean S r 1 1 block is this is n cross this is n cross. And this is n rows and this is a n columns. So, S r 1 1 contains all the Eigen values of stable Eigen values of H M, which are stable in S r of block, which are n cross n blocks. It will consider and S r 1 block is a upper triangular form block in the upper triangular form. Each block of S r 1 1 is either 1 by 1 or 2 by 2 matrix. And all Eigen values of L S r 1 1 stable Eigen values and S r 2 S r 2 2 all Eigen values whose dimension is n cross n Eigen values are unstable.

But, reverse sign with a stable Eigen values of S r 1 1 is unstable Eigen value. So, we from S r we convert it into S r, so from S we convert it into S r by rearranging that one, so that first stable block n cross n will contains the stable Eigen values. And next n cross n of this block diagonal block is considered the unstable Eigen values of H M. So, that is this one, so just am writing what is S r 1 1.

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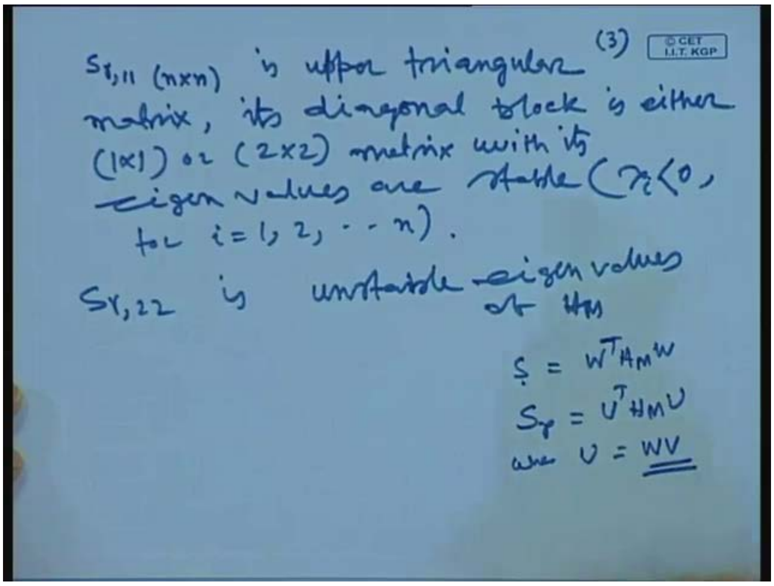

S r 1 1 which dimension is n cross n again is upper triangular matrix. And its diagonal block is either 1 1 or 2 2 block matrix with its Eigen values Eigen values are stable. Means lambda i less than zero for i is equal to 1 2 dot dot n. Whereas S r 2 is unstable Eigen values unstable Eigen values of H M, because how because we know the transformation if we do the transformation. That we have got S W transpose S is a W transpose H M W. The Eigen values of H M is same as the Eigen values of S or in other words you can say Eigen values of S is same as the Eigen values of H M.

Because this is the similarity transformation, like this we are since we have done again a transformation on S. That is S r equal to we got it U transpose H M and U that Eigen values of S r is remaining the same, as Eigen values of H M. Where U is equal to if you see this we have written U is equal to W V this. So, once we done this second third step we have converted into a what is called Sehun form by reordering. Once again we have reordered only this. That is the only difference from the second next step two step three is different from step two. We are been reordered the matrix Sehun form and in S r form. So, that stable Eigen values will come first stable Eigen values and next is unstable Eigen values.

So, from this statement we can write it from equation 3, see the equation 3 that equation this equation 3. So, if you multiply by this if you multiplied by both side by U both side by U. Then what will get it this nothing but a H M U is equal to U S r both side are multiplied by U, U is a orthogonal matrix.

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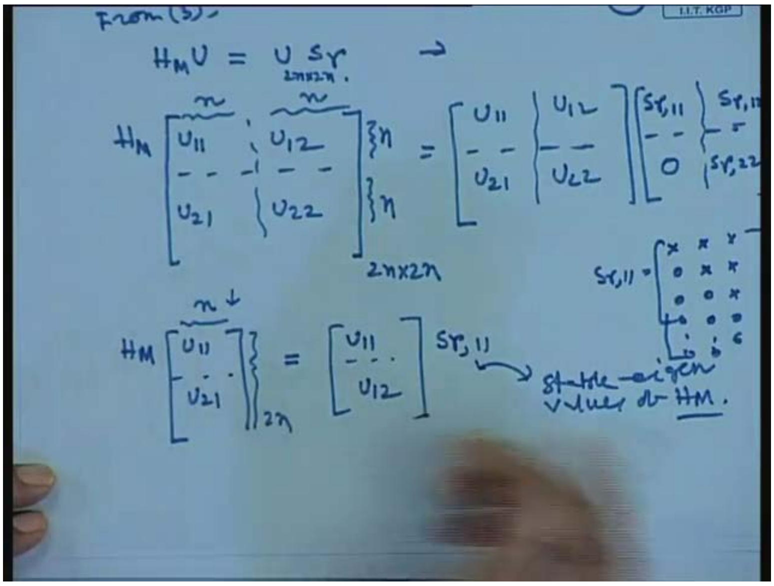

So, U transpose U or U transpose U into U transpose or U into transpose U is equal to identity matrix, so this we can write it that, from 3 you can write it from 3. Than you ((Refer Time: 18:53)) write the detail structure of U, if you recollect the U is a 2n cross 2n matrix. So, I am just putting into that 2 by 2 blocks, U 1 1 U am writing U 1 2 U 2 1 U 2 2. This number of rows n this number of columns n the number of columns n here the number of rows is n, so you know all this things than U 1 1 U 1 2 U 2 1 U 2 2. Similarly, dimension if you partition in this way you can know this one and S r dimension is what S r 1 1 S r 1 2 S r 1 2 then 0 S r 2 2.

Again this is the thing, now you see I just multiplied this into this from than what you will get it this dimension this is 2 cross into 2 cross again. Now, if you multiply by this by this than what will write it H M and multiplied by this block by H M, so it is nothing but a U 1 1 U 2 1. Similarly, I have to multiply this matrix with this block is equal to this block is U 1 1 U 1 2, this multiplied by this this multiplied by this this multiplied by this. So, it is U 1 1 S r 1 1 than this multiplied by this this multiplied by this multiplied by this plus this multiplied by this.

It will be U 2 1, so I can write U r 1 1 of this both side this. So, this this is this this matrix this is a stable Eigen values of H M matrix again. Now, we see this one the Eigen values and Eigen vectors difference say matrix A multiplied by each column. How many columns are there n columns, how many rows are there 2n rows. So, each vector represents the Eigen vector of the matrix Eigen vector of the matrix H M corresponding to Eigen values of S r 1 1. The first element S r 1 1 structure is what is upper angular structure, so if you multiply by this if you multiply by this than it will be coming that one just see this one I multiplied by there is how many columns.

There are n columns are there first column multiplied by H M first column multiplied by H M is equal to that first column multiplied by S r, S r is structure is what S r 1 1 structure is what if you see this is this is this 0 0 0 form. Than this this 0 0 0 form. And similarly, this will be this form 0 0 0 form, and it is something like this structure. So, if you multiply by this than what you will get it that each column corresponding to the suppose first column this first column this one corresponding to the Eigen value of the matrix S r 1 of this is the Eigen values.

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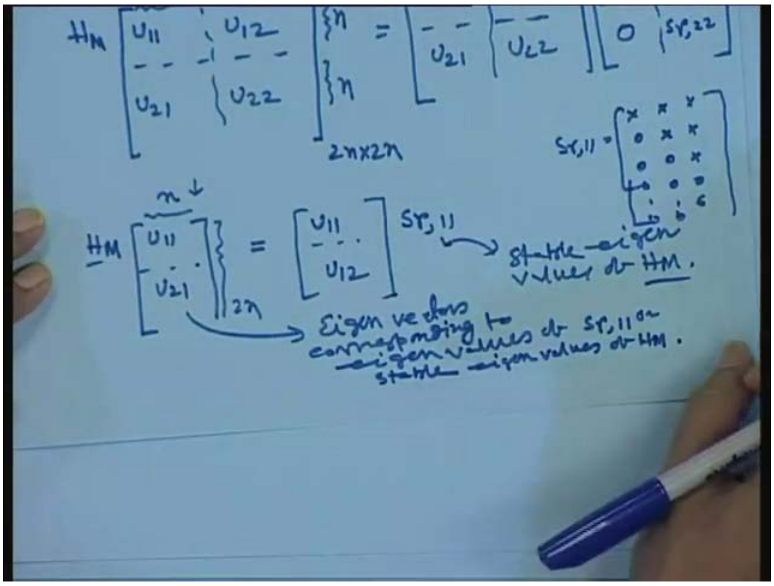

This is the another again Eigen values of this one either 1 by 1 or 2 by 2 Eigen values of the matrix this. So, this vector corresponding to the Eigen vector corresponding to the stable Eigen values of H M matrix. This is the Eigen vectors of the matrix H M for corresponding to the stable Eigen values of H M, that this the Eigen vector. So, you are partition this one and we have proved this one with the knowledge of Eigen values and Eigen vector of the matrix. We can get the solution of algebraic Riccati equation once I know the Eigen vectors of H M matrix. Again then I can find out what is the solution of this one by using the method we have discussed in the method two, so next step so if you want to write it here.

This is the Eigen vector Eigen vectors corresponding to Eigen values of S r 1 1 or the stable Eigen values or the stable Eigen values of H M. Again stable Eigen values of H M, so know step 4 we can write is immediately you can write the step 4 the solution of our Riccati equation.

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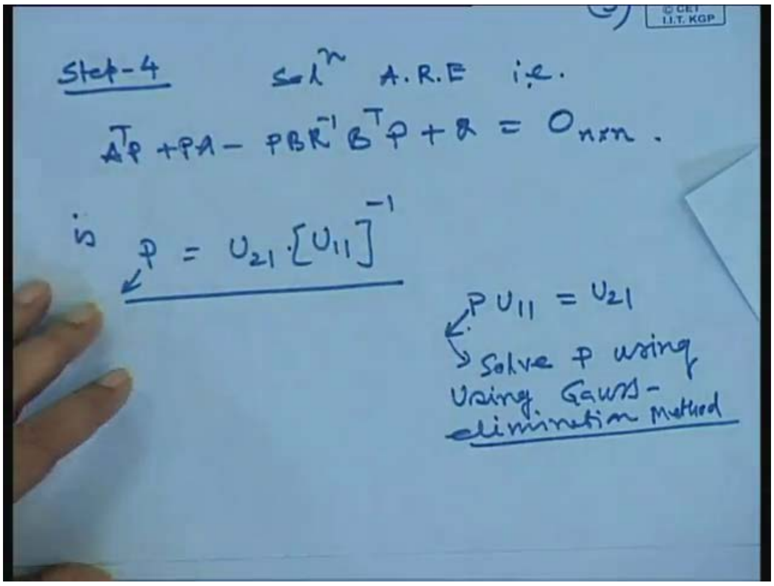

The solution of our algebraic Riccati equation in other words the A transpose P plus P A minus P B are inverse B transpose P plus Q is equal to 0 this is n cross n is P U 2 1 matrix multiplied by U 1 1 whole inverse of that one. Now, see this one this matrix is a vector you got it the dimension of the vector is 2n cross n. So, first n rows you retreat from the vector that is U 1 1 and the remaining vectors of n cross n will be U 2 1. So, you just complete like this way and you will get the solution of that Riccati equation. That what in step 3 is done we have evaluated the Eigen vectors of the matrix H M.

Corresponding to the stable values by Q R decomposition techniques, which is numerically more reliable than what is called the our earlier simple Eigen vector Eigen vector Eigen values Eigen vector methods. So, we are used the different techniques to find out the Eigen vector of the H M matrix corresponding to stable Eigen values numerically reliable algorithm we have used it. So, one can solve this one instead of directly taking the inverse, I can do it like this way that P U 1 1 is equal to U 2 1.

This I can write it like this way that this is unknown, so since this we can solve it the solve P using gauss elimination method. Just to avoid the inversion of this matrix if it is large and dimensional problem, that one should avoid the inversion of this, and solve it by the gauss elimination method, so the advantage of this method.

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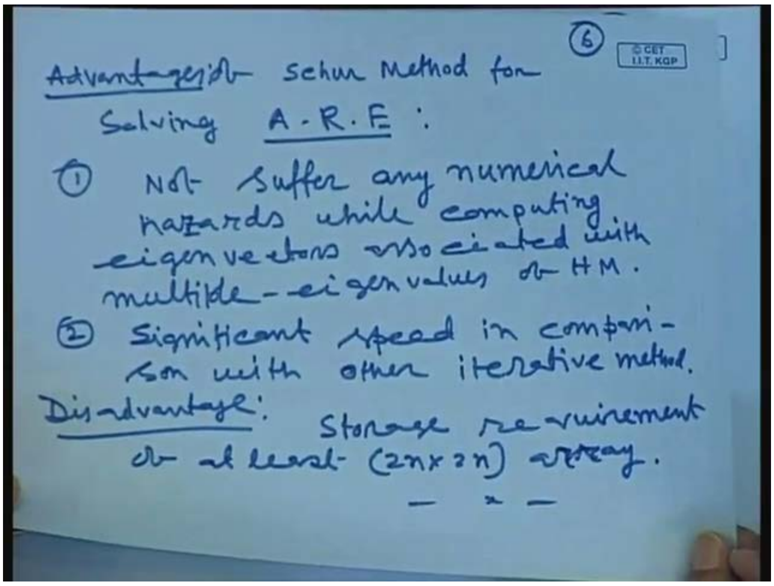

If you see the advantage of this method written just tell the advantages of Sehun method for solving A.R.E. method A.R.E equation. One advantage is they are the numerical hazard is not will not face any numerical hazard. So, not suffer any numerical hazards while computing the Eigen vectors associated with multiple Eigen values of H M 1. Second is significance speed is H is speed of solution of Riccati equation time required for solution of this Riccati equation is significantly improved compared to the any other iterative method.

So, second step is significant speed in comparison with other iterative method iterative method. These are the only disadvantages is computer store is requirement is high because it requires 2n cross 2n. So, next is you compare computer store is disadvantage is the storage requirement of at least 2n cross 2n array this is. So, one can solve the solution of algebraic Riccati equation by Sehun method, one can solve by iterative method. But, the Sehun method is numerically more reliable compared to the iterative method. Because when you have a rather director Eigen value Eigen vector ((Refer Time: 30:29)), when you have a multiple Eigen values are there multiple Eigen values were they are.

So, we have to find out generalized Eigen vectors also this one, so this will create problem if you use the standard Eigen value Eigen vector method. So, this technique that is Q R decomposition technique will avoid such type of problems when you have a multiple Eigen values of a system matrix were. Again next will go for what is called our frequency domain interpretation we have designed the controller. That L Q R controller and we haves studied the L Q R controller. That whether a A minus B k or the close loop system will be stable or not we have seen.

The requirement is Q must be positive definite matrix and R positive semi definite matrix, R must be positive semi definite matrix. That suffice that the closed loop system is stable and. Now, and it of course this before that we have to see that A and B must be controllable at least stabilizable or A and Q must be detectable this.

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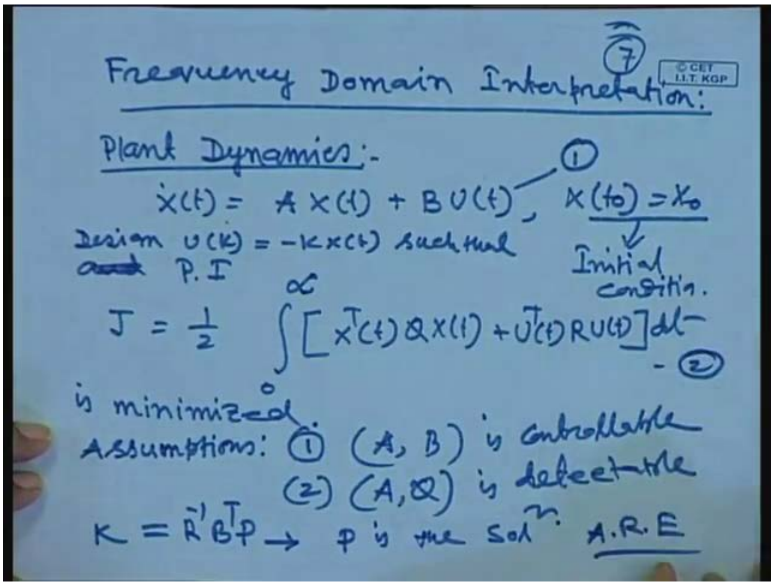

Now, the frequency now this controller designed it, now one has designed it to has to interpretive that results more in depth or details in frequency domain. In the sense in frequency domain you can say what is the face margin and gain margin of the corresponding design controller close loops system design controller. What is that phase and gain margin of this one, so that will investigate or you will study this one that face and gain margin of the L Q R control systems.

So, frequency domain interpretation frequency domain interpretation of L Q R design controlled L Q R control system design problems. So, let us consider the our plant dynamics plant dynamics. X dot is equal to A X t plus B U of t X of t is equal to X of 0 is the Initial condition Initial condition. So, our problem and the corresponding index performing index is J is equal to half 0 to infinity X transpose t this is infinity X transpose t Q X of t plus U transpose of t R U of t dt.

So, our problem is they given the system design a controller of U of A t, such that the performing index is minimized. Again and the perform index this is even the performing index, the performing index is minimized then this problem statement given the plan you design or you can write design a controller U of t is equal to minus k of X t.

Assume all the states are available design k of t, such that the performing index is minimized again. So, before that we have to make an assumption that. Assumptions 1 A and B is controllable or at least stabalizible 2 A and Q is the detectable detectable. These two condition you have to check it, once it is condition than we are we are going for designing a controller U of k, and the k can be found out. And the k can be found out by solving the algebraic Riccati equation again.

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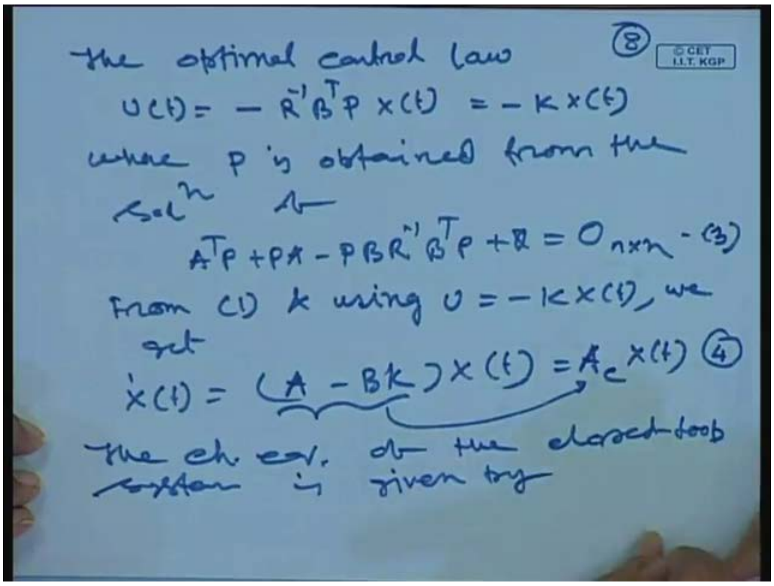

Once you solve the algebraic Riccati equation that the value of P then your k is nothing but a k is nothing but a R inverse B transpose P and this P is the solution of algebraic Riccati equation. That we know all this things, so let us call this is the equation number 1. Number 1 this is the equation number 1 this is the equation number 2 and using the control law in the given system. Then the optimal control law U is equal to U of t is equal to minus R inverse B transpose P into X t.

And which is nothing but a X of k t. Again and P is the solution of Riccati equation or were P is obtained from the from the solution of algebraic Riccati equation, again this solution will be. So, this is let us call the equation number 3, this is the equation number 3 now. So, what is the closed loop system equation in system equation 1 you put U is equal to minus k X from 1. And using U is equal to minus k X we get that X dot is equal to X dot is equal to A minus B k X of t. And this is nothing but a closed loop systems see this denoted by A C again.

So, this is let us call equation number 4 and this is asymptotically stable that we have proved it this A minus B is a asymptotically stable that we have put earlier this one, and its corresponding. So, the characteristic equation of this closed loop system characteristic equation of the closed loop system is given by. What is the closed loop system characteristic equation determinate of let us call.

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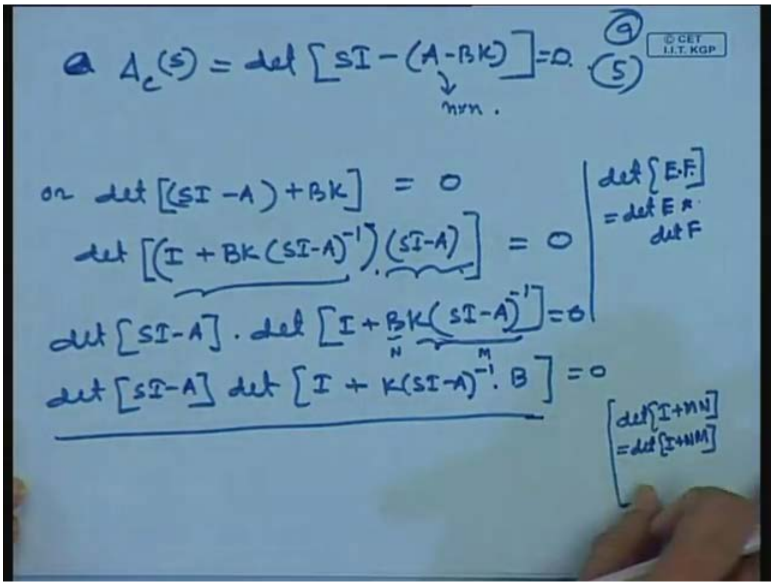

We have denoted by this C is closed by determinate of that a closed loop systems, determinate of S I minus our closed loop system A minus B K this. So, let us call this is equation number 5, so this is the nothing but a if you take the determinate of this one. You will get a polynomial of order n is equal to 0 this the characteristic equation. So, if you take the determinate of this one you will get a polynomial in S of order n, because this dimension of this one is n cross n.

Now, this we can write it or determinate of S I minus A this plus B K of this is equal to 0, which you can write it into this form I plus B K into S I minus A whole inverse this whole bracket into S I minus A is same as this one. So, you push it S I minus A inside, so this will be identity matrix B K S I minus A. This is B K, B K S I minus A inverse S I it is identity matrix so it is B k. And this will be S I minus A as a this will remain same that one.

So, that one so this I can write it now into this form that you know that these properties is the determinate of that, let us call F E. And F E is the matrix F is the matrix with proper dimension is equal to determinate of E into determinate of F. So, that property I will use it here one matrix and this is another matrix. So, you will write a determinate of S I minus A into determinate of I plus B K into S I minus A inverse of that one is equal to 0. Now, we just consider this let us call this is equal to M and this B is equal to N. So, once again I can write it that determinate of S I minus A is same as the determinate of this is same as this plus K S I minus a whole inverse into B.

So, order is reverse, so this is the property of this one determinate of 1 plus I plus MN is same as determinate of I plus NM. Again this we have written is here that property we have used it the reverse that have ordered. This we can consider N, I just reverse the order. Now, we can write it the characteristic equation of the closed loop characteristic equation is nothing but a determinate of this one and determinate of that one we can write it this form again. So, let us see the two that controller what we controller in timed when representation what is the block diagram. We have already seen it now, now will see in frequency domain again.

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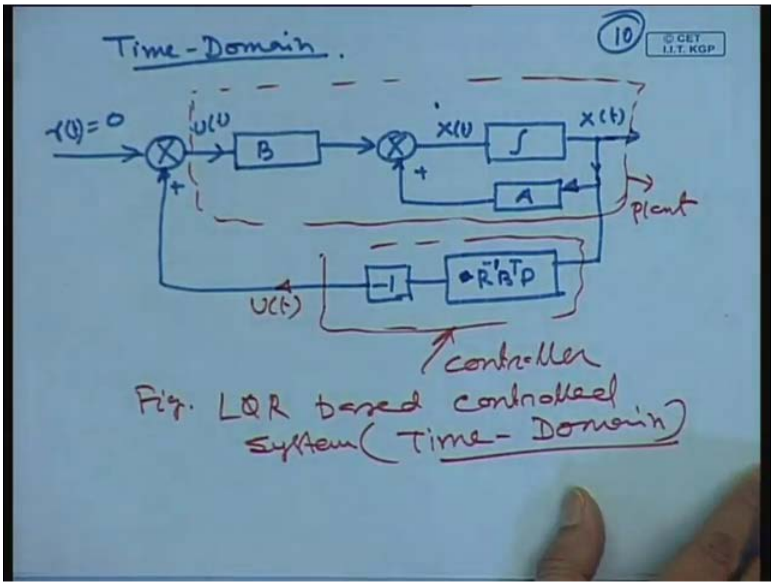

Then what is your block diagram representation of this one in time to mean. So, since it is a regulator problem LQR that is the regulator problem and reference in putting 0. Because it is this centralized design that is in any initial disturbance is there in the state then regulator will take care of to bring that effected state to bring it in its equalization position again, so the controller job is that one only. So, B and this integrator that I have output of X t this is X dot of t than this A than this is your U t. And the controller is designed this is R inverse R inverse B transpose P. This is nothing but a gain controller k minus 1 and that is coming to here plus this plus.

So, this is R is equal to R of t is reference input is 0. This is a regulator problem is that due to the initial disturbance the controller will bring the state to equivalent position by minimizing a performing A index that is. So, this is about the controller X if you see that part this part is our plant this is the plant and this part is the our controller. This is the controller than output of this controller is U t. And this figure if you see more carefully there is a L Q R based control systems. And it is represented in Time-Domain again it is represented in time domain. Now, let us see in the frequency domain it is in the condition of L Q R control systems again frequency.

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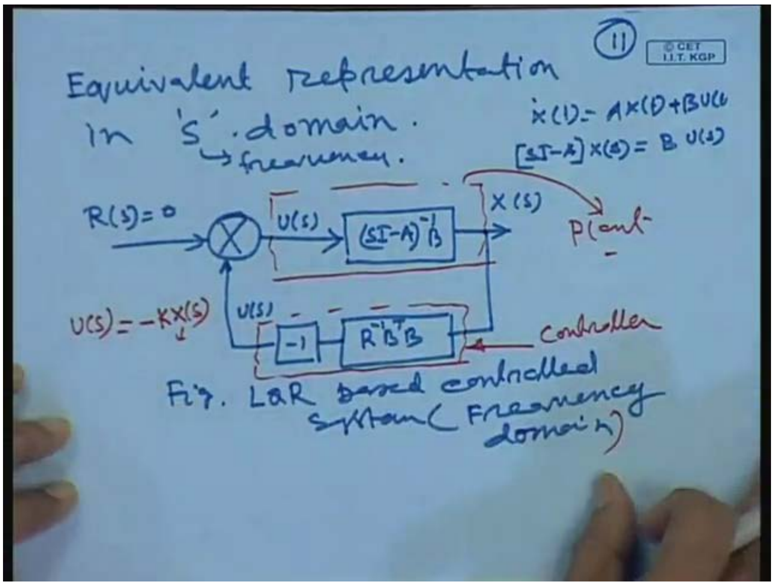

It is in frequency domain the equivalent representation equivalent representation in S domain or frequency domain or you can say frequency domain. So, look at this expression this is nothing but a this block is nothing but a representing X dot is equal to A X plus B U. So, that is nothing but A X dot is equal to A X plus B U time domain and what is the frequency domain representation of that one S I minus take both side Laplace transform. Issuing that initial value is 0, than this is A X of this equal to B U of S that one. So, that am now representing this X of S, so S I minus A whole inverse into B this output is X of S.

So, this the input U is the input and this is the output X of S, so output is this and input is this U of S that output. Then this is going to the controller in S domain our controller is R inverse B transpose P than multiplied by minus 1 and that will come to here. That is our U of S, and this is R of S. Now, say this is our in S domain that this figure is L Q R is based L Q R based control system bracket in Frequency domain.

So, this is our case if you see this is our controller once again if you want to equivalent to this one, this is our controller this is our controller and controller. And controller how it is related U S, U S is equal to nothing but a minus this correspondent A is a constant matrix multiplied by our input is X of S this is the input. And this is output from the controller U S is the output in the controller and X is in the output of the plant to the U S.

So, this is our controller plant plant, so this now. So, let us see this one that now what we are doing it here we are adding and subtracting.

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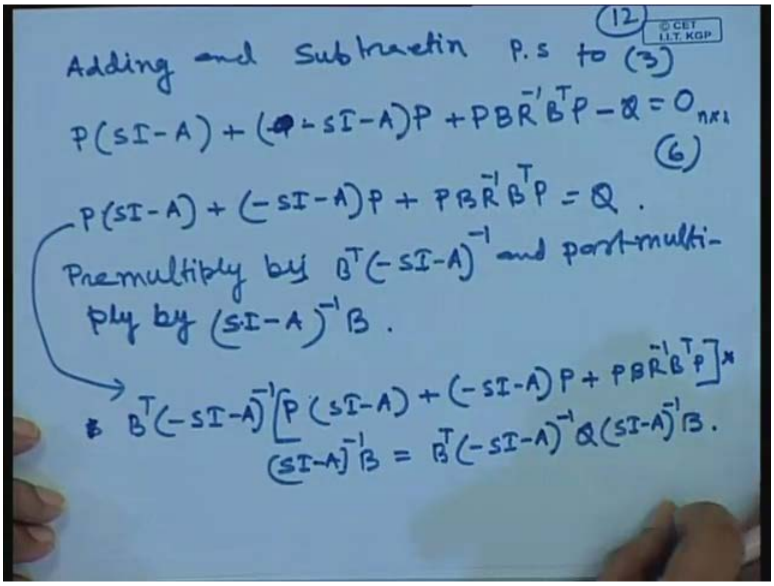

Adding and subtracting P into S P is the that solution of the Riccati equation. That P S is the S domain of P. Again two 3, 3 means equation 3 see this equation number three not that one. So, equation 3 is our algebraic Riccati equation with this equation we are adding P into S and subtracting. So, the resultant expression will remain unchanged, so if you do this one P then S I minus A plus P S are adding. The minus P S minus sorry so minus so minus S I minus A into P plus B are inverse B transpose P minus Q is equal to 0. Across N what I did it if more specifically first that algebraic Riccati equation number 3. And minus by 1 and then I have added P S subtracted P S.

And manipulating we got it this way because this is equation number we are gone up to equation number hmm I think 5, this last equation is 5 again. So, we can see that this is equation number 6. So, this we can write take Q in right hand side I can write it that P S I minus A plus S I minus A P plus P B R inverse P transpose P is equal to Q again. Now, at both side at both side multiplied by both side are multiplied by P multiplied by P multiply P multiplied by that B transpose minus S I minus A whole inverse. And like by S I minus A inverse B this equation this equation I pre multiply by B transpose S I minus inverse and post multiplied by that one.

So, if you P multiply by post multiply that one than what will this come S minus that B transpose. So, B transpose minus S I minus A whole inverse P into that expression am writing now. That S I minus A plus minus S I minus A P plus P B are inverse B transpose P, than this multiplied by post multiplied by S I minus A whole inverse B is equal to B transpose S I minus A S I minus A inverse Q than S I minus A the whole inverse B. So, I multiplied by both the side this one than what we can do it this equation I pre multiply the left hand side and the right hand side. I may pre multiply by this one post multiply by this one and that is what I have written here than after that after simplification of this one.

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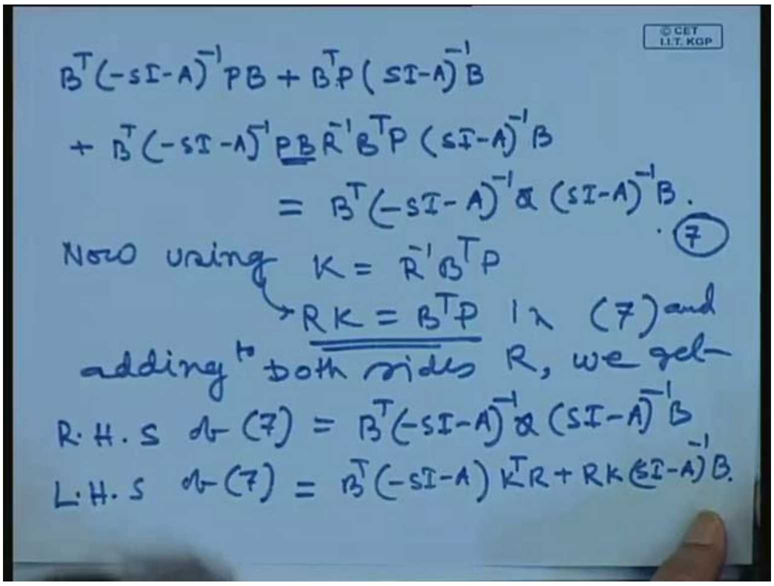

We can write you will be getting that one that B transpose S I minus A whole inverse. See this one if I multiply by this if I pre multiply by this one I post multiply by the you push it this side. So, S I minus A inverse you will be identity matrix, so P B will be there. So, again P B will be there again this will be a P B only plus see that one this multiplied by this. So, S I minus A the whole inverse multiplied by S I minus A, so that will be identity matrix. So, it will be B transpose P into S I minus A inverse B, so B transpose P, P is a symmetric again.

So, B transpose P into S I minus A whole inverse B again this. Than this multiplied by this one and post multiplied by this one multiplied by this of this, we have to write as it is we got it. So, this will be B transpose S I minus A whole inverse P B are inverse B transpose P into S I minus A inverse B is equal to right hand side as it is S I minus A whole inverse Q S I minus A inverse B. Again let us call this equation number 7, so what I did it here I pre multiplied post multiplied by that.

As I mentioned here and then pushing this pre multiplied by part of pushing this expression this is also pushing by than we got it that one. Using now using now using K is equal to R inverse B transpose P, which you can write it R K equal to R B transpose P using this expression in 7 and adding both side adding to both sides R will finally what we will get it what will use it in this expression. Whenever I will get B transpose P ill use R K or P B I will write k transpose R in k transpose R transpose. Since R is a transpose it is R only, so will get it this expression finally, that right hand side will get it as a it is the right hand side of the equation.

The right hand side of 7 will get it B transpose S I minus A whole inverse Q S I minus A whole inverse B. Again and left hand side of S of 7 will get it the in place of this one I will write it B transpose S I minus A. And in place of B it is A transpose R then R K this is will be a into this, where is this one this will be coming R K S I minus A whole inverse B again this. So, I will discuss the, what is called the next class what we have to remaining deduction will do next class again, so will stop it now here.