Optical Control

Prof. Dr. Goshaidas Ray
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 42

Numerical Example and Methods for Solution of A.R.E (Contd.)

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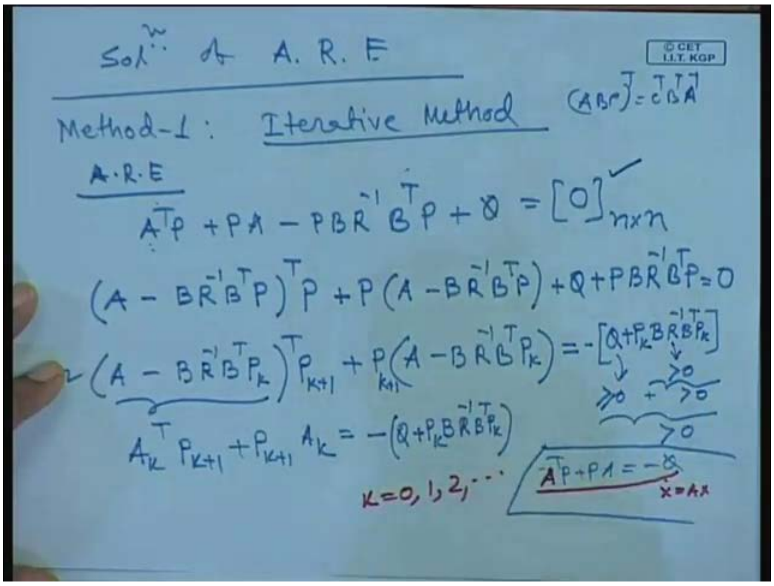

So, last class we have discussed the methods, so last method different methods there, so the first method is iterative method. So, you have given what is called the algebraic Riccati equation that algebraic Riccati equation will arise in case of infinite time regulator problem again. So, this is the algebraic equation, so this term and this term will club together again P into A this one P into B R inverse B transverse P.

There is A another one additional term is added both in negative sign, so with positive sign we have used here P B R transverse P. So, this and this constant, so this will remain as same as the previous equation, so this we have written it this way. Now, we introduce some P we have introduced P of suffix A means value of the P matrix at k th iteration and outside this bracket is this P k plus 1 value of P at k plus 1 k th iteration. Knowing the value of P at k th iteration we can compute the value of P at k plus k th iteration, so we have written into like this way. So, in order to solve this equation it is a something like Lyapunov function that we come across in the solution of stability analysis of the autonomous systems.

This means suppose you have a x dot is equal to A x and we want to study the stability of the system based on the Lyapunov function. Then, we consider an energy function B in quadric form x transpose P x, then this energy function is a positive definite function positive semi definite function. We have to show for the system A to be stable provided B dot means rate of change of energy should decrease with time that we have to show it and that condition is that that one A transpose P plus P A is equal to minus U. So, this is called Lyapunov function while we will study the stabilization of A autonomous systems, how to solve this one.

That you assign what is called Q is positive matrix, then solve a transpose P A, again for P you solve it if P is a positive matrix it indicates A matrix is a stable one based on the liability of function we write this condition. So, here also we initial value let us call a is equal to 0 so P of 0 you choose in such a way that a minus B are inverse B transpose P it is nothing but a B k if you recollect a is R inverse B transpose P. So, it is B k that I told you have to select P of 0 k is equal to 0 in such a way this matrix is stable, so a transpose and A C transpose you can write a k is nothing but the system matrix at k th iteration.

So, it is A k transpose P k plus 1 A k, this is A k is equal to Q P k P B R inverse B transpose then P k. So, I know P of 0 we assign initially in such a way that a minus B are inverse B k transpose 0, sorry stable, so since this is a positive matrix U is a positive semi definite matrix result will be positive matrix. If the result is positive definite matrix that we know a minus B are inverse B transpose P k P is the stable one, so if you solve this one then will get P k plus 1 k plus 1. Then, P value will be positive definite and next I know from the knowledge of P 0 I can compute P 1, then use this expression a minus B are inverse B transpose P is P 1. Now, you find out P of 2 by finding that earlier equation in this process, you go on solving recursively until and unless the P converges to a some constant value.

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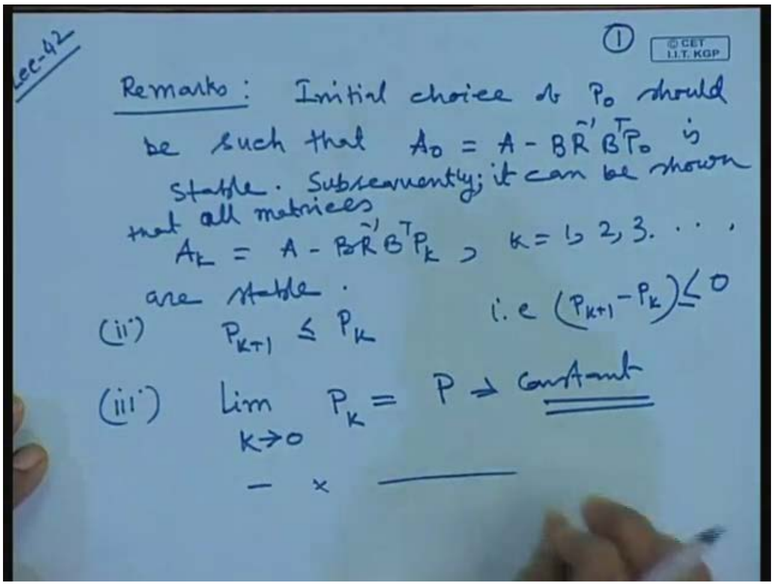

So, next is remarks we can just write it the initial shares of P if you recollect that what I told you that initial choice of P 0 should be such that A of 0 is equal to a minus B are inverse B transpose P of 0 is stable again at any iteration k th iteration. I can write k of a you see with j th iteration A 0 at j th iteration a j is equal to A minus B are inverse B transpose P k and k is equal to 1, 2, 3 dot iteration. So, it is stable initial stables and other stables, subsequently you can write it there, subsequently it can be shown it can be shown that all matrices is for A is equal to 1 to A k are stable. Next iteration, we can see it P k plus 1 is less than equal to P k that is this P k minus, in other words you can say P k plus minus P of a this matrix is equal to less than equal to 0.

This is a negative definite C matrix that means in other words I can say the P k plus 1 is less positive definite than the P k again. Then, I can write the third one is limit the k tends to infinity that if you write go on increasing that were you reversibly solve it the equation. Then, you will see P of k will be equal to P which is a constant it converts to a constant matrix, so only things is when you start the iteration you have to select the P of 0 in such a way that A of 0 is stable. Then reversibly solving this one until and unless that P k plus 1 and minus P of k is less than equal to norm of this one equal to some P is values these small values are there or when P converts to some constant value.

So, this is the method in this way, solve the Riccati equation, algebraic Riccati equation that is called integrity method. One disadvantage of this method is there if U initial gas is not appropriate, then it will take longer time to converse the solution of Riccati equation to a constant value. This is the one disadvantage of method, so next method is Eigen value and Eigen vector method that means based on the concept of Eigen value and Eigen vector, one can find out solution of algebraic Riccati equations again.

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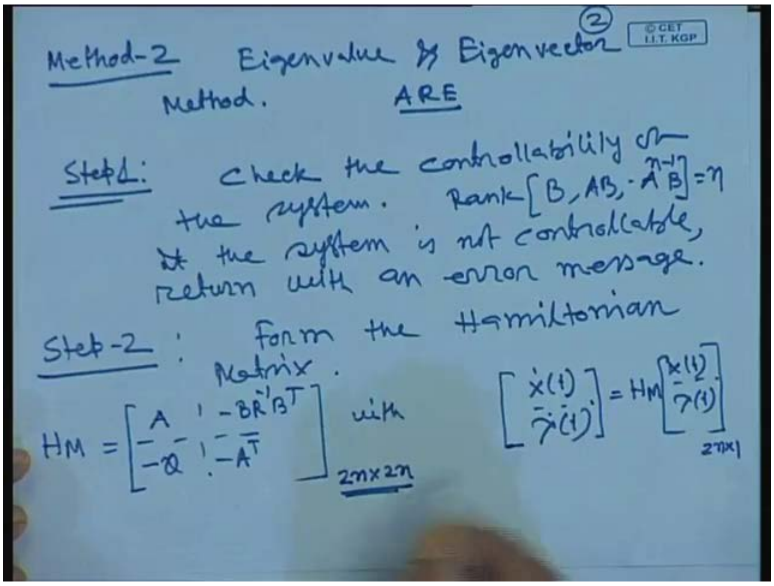

So, method two method two is based on the Eigen value and Eigen vector method that means solution of the algebraic equation based on Eigen value Eigen vector in vector method of Hamiltonians matrix. So, first step, first all designing values then first check it is there you have to see whether the system is controllable or not. So, the controllable test we have already mentioned it here, so just mention first check the controllability of the system again. That means if you rank of B A B dot n minus 1 B must be equal to n, n is the number of the states or order of the states the system and B is the input matrix and A is the system matrix the rank of the matrix must be n.

Then, we call it as system is controllable all the states will be form to initial state will be able to write the state at derived state to a finite control actions, again in a finite time, this is the thing if this fills than this program will show the error message. That means will not be able to design the require problems if the system is not controllable return with n error message the error. This message will be there that will not be able to design a l 2 R control R for this systems step two, so from the Hamiltonian matrix if we recollect we need the information a matrix B matrix q matrix and R matrix. So, A minus

B are inverse B transpose, then Q minus a transpose and this is the Hamiltonians matrix and this Hamiltonian with how you form it.

If you recollect that x dot of T and lambda dot of T this is the cost effecter and this is the vector, this equal to H M into x of T lambda of T. So, this is the augmented state vector with the state vector and the acute state vector and each dimension this dimension is 2 n cross 1. If you see in this one, now this we form the Hamiltonians matrix this Hamiltonians matrix it is dimension 2 n cross 2 n, so this is the n.

So, this matrix this have how many Eigen values are there, two Eigen values are there, we show it analytically that this matrix having a set of Eigen values that n Eigen values are the left of the i in the left of the slain. You can say n Eigen values are with negative parts and n Eigen values are right with the positive sign with a real positive sign. That means n Eigen values are stable Eigen values and n Eigen values are unstable values and they are symmetric about the imaginary axis.

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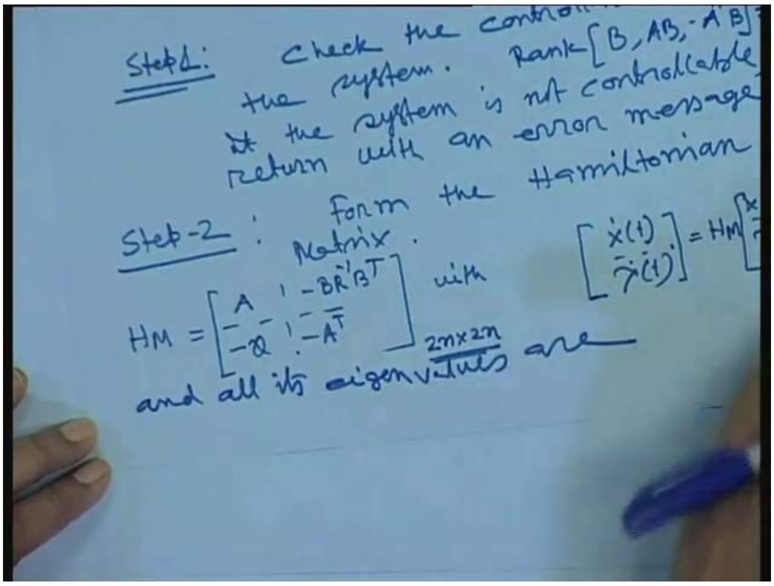

So, these are the Hamiltonians matrix and all its Eigen values.

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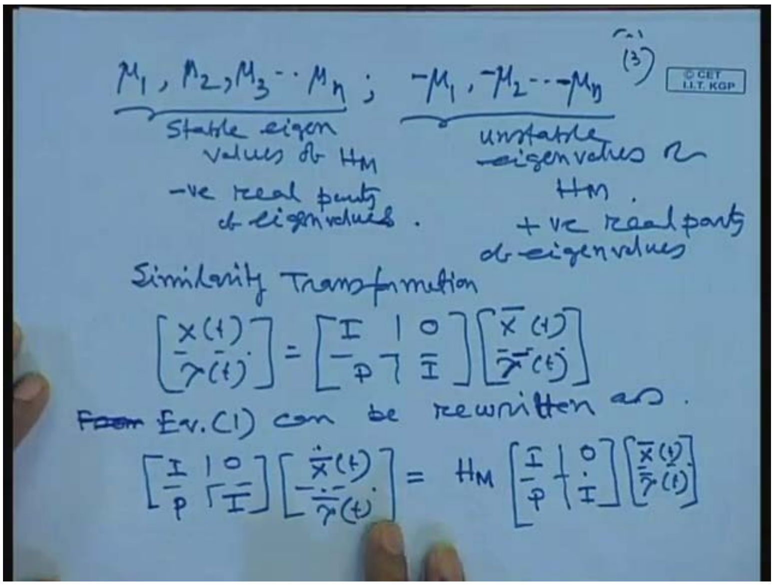

Let us call you have defined the Eigen values are mu 1 mu 2 mu 3 dot mu n and another set of Eigen values will be minus mu 1 minus mu 2 dot mu n minus this one. We told you we have seen it that Hamiltonians matrix, there are two sets of Eigen values they are symmetric about the axis or the complex main. So, that this let us call this set of Eigen values are thus stable Eigen values of H M Hamiltonian matrix. These are the since this is the stable since it is the minus that will be unstable because this stable means the Eigen values that real term real term with a negative sign.

If it is a stable one and the negative sign and there is a negative of positive sign, so the Eigen values the real term real values of the Eigen values are with a positive sign that means un stable Eigen values of H M Hamiltonian matrix. So, this positive real parts this is the negative real parts this is the negative parts of the Eigen values real parts of the Eigen values negative real parts. This will be negative real parts of Eigen values of this n Eigen values are there.

So, this we have just defined this one, then if you that will just see this one that Eigen values are symmetric about the imaginary axis of the symmetry to show this one. We have already discussed this one earlier at this stage we just briefly discuss how we are making comments like this way there Eigen values of Hamiltonian matrix are symmetry matrix are about the real axis imaginary axis. So, do the similarity transformations, so let us call x of T lambda of T this cost augment vector of this one we have used this transmission like this way.

So, let us call this is the equation number it just this, this, this equation let us call equation number one again this is the equation number one from equation one or you can say equation one it can be written equation one can be rewritten as rewritten as that x dot. So, it is i is a constant matrix P i, then you x bar dot of T again than lambda bar of T lambda x dot lambda dot is equal to this dot lambda dot bar dot this equal. We are writing we have a Hamiltonians matrix H M, then we are writing for this one i, then 0 P i this into h bar of T lambda bar of t. So, this you can write it for this one that what is H M, we know the H M expression.

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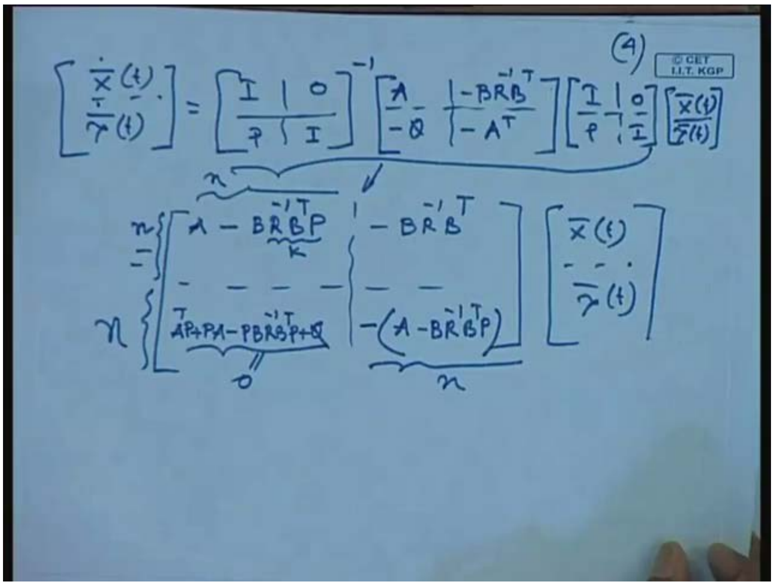

Lambda bar of T is equal to i 0 P i full inverse than our Hamiltonians matrix if you write the expression for the Hamiltonians matrix this is nothing but A B inverse B transpose that is minus q minus a transpose multiplied. Then, I just add simplified that one x bar top lambda bar of T, so if you use the matrix top of this one, we discuss earlier and if we do it after simplification will get the final results in this way a minus B are inverse B transpose P. This is nothing but our k, if you see this equal to minus B are inverse B transpose that equal to that side only A transpose P plus P a minus P B are inverse B transpose P plus u.

That will get minus of that matrix a minus B are inverse B transpose P this equal to x bar T lambda bar T. So, if you just multiply from here to here you will get this matrix. Now, see this one this is the algebraic equation this quantity is equal to 0 for this one again and this is nothing but structure and this is nothing but closed group system matrix. This closed group system matrix is stable it is enough if you consider U is semi definite positive matrix than this is the stable one.

If it is a stable one and this is specified with a minus sign, so this will be a unstable this matrix will loosen up again unstable. So, this matrix dimension was n rows this is n columns similarly, this is in n columns and this is the n rows, so n again values of this stable and this block will again are unstable. So, whole system will again after transformation we know the Eigen values does not change.

Only Eigen vector which transformation again vector will change, so since it is a block diagonal matrix the Eigen values of the whole matrix is nothing but this block which is the stable Eigen values. Eigen values of this block which is the un stable Eigen values because of that minus sign is presided with that matrix. So, that we have shown it earlier also here now see that this is the second step, if you just see the first step is the controllability test if the system is controllable go to the second step.

In the second step you just form Hamiltonians matrix and other things what we have discussed just to show Hamiltonians matrix Eigen values there are two values there are n Eigen values to the left top of the spring. In the sense the Eigen values with the negative real parts and remaining Eigen values are positive real parts there symmetric about the imaginary axis. So, once you find matrix, then find out the step three step three to find out the Eigen vector of the Hamiltonian matrix corresponding to the stable Eigen values. I repeat once again find out the Eigen values of the Hamiltonian matrix if it is 2 and 2 cross n to find out the Eigen vector corresponding to the stable Eigen values of H M that is our next step.

So, let that V i, U i its dimension is V i dimension is n row 1 column, this is the one column this is the n row that means this is the dimension twice into one. Let the Eigen vector or generalized Eigen vector of the matrix H M corresponding to negative real parts corresponding to in another words corresponding to mu i and what is mu i the stable Eigen values. What is stable Eigen values the Eigen values with negative real parts

Eigen values with negative real parts means this is the corresponding to the stable Eigen values Eigen value of H M. So, you find out the Eigen vector of that one corresponding to mu i.

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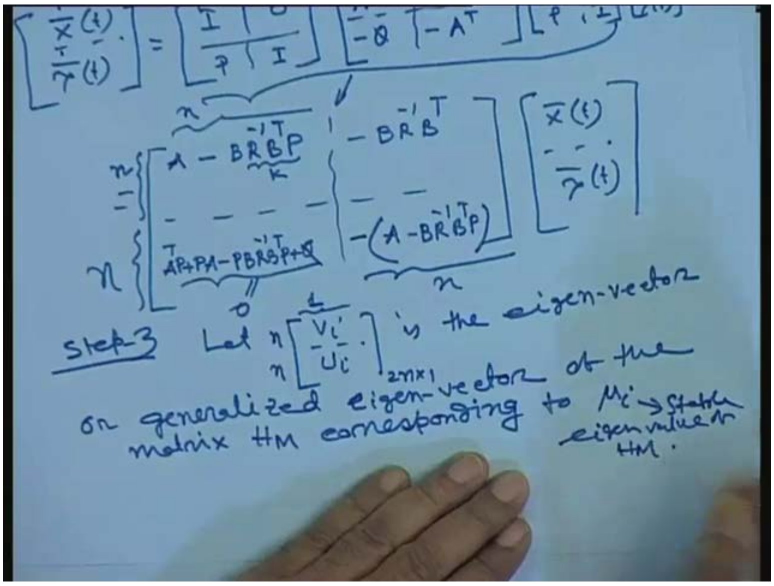

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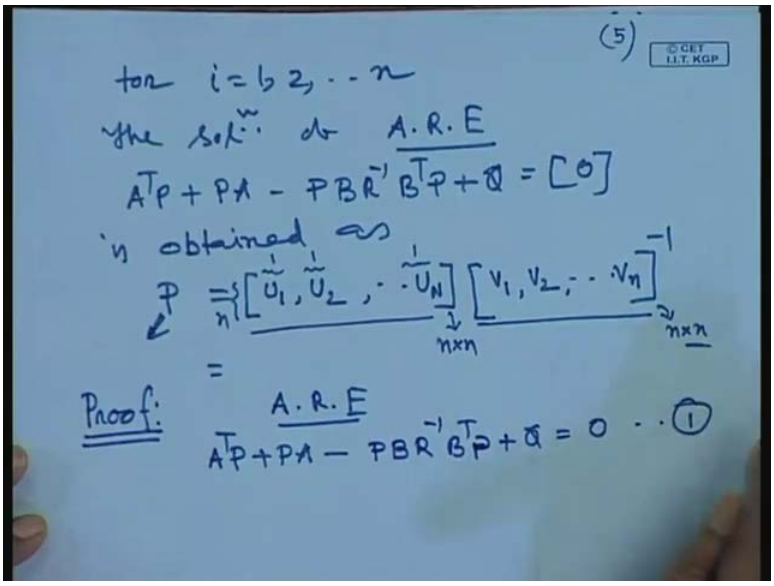

So, if you find out this mu one and we can find out that we can know the award for i is equal to one two dot dot n you see this one i how many stable Eigen values are there? We have a n stable Eigen values are there n unstable Eigen values are there we are we are re concentrated where attention to find out again vector corresponding to n stable

Eigen values of H M. So, for i is equal to 1, 2 n, so the solution of the two will give you letter the solution of algebraic that equation, then one we can find out that means a transpose P A minus P B are inverse B transpose P plus Q is equal to 0.

The solution of this one is obtained as P is equal to that you form A U 1 U two dot dot U n than this matrix B 1 B 2 dot B n whole inverse. Now, see this one which one that three four, now see this one, this is Eigen vector correspondent to Eigen value mu i which is again stable values and when you are getting a first n rows of this column vector, you partition as a V i remaining n power rows of this vector you partition as a v i. So, since we have what is called a n stable Eigen values the first partition and first n rows am writing B 1 B 2 B 3 dot B n.

Remaining last verse of the Eigen vector corresponding to the Eigen values I am writing U 1 U 2 U 3 U dot dot n do the solution of this equation so this is the stable Eigen vectors of H M corresponding to mu one again last n rows of this Eigen vectors is e one the last n rows of the Eigen values corresponding U is equal to mu 2 is the U 2 and so on. This is the first Eigen rows of the corresponding to stable Eigen values mu 1, this is stable Eigen values mu 2 first n rows of the Eigen vector in this way. So, if you compute this that will give you the solution of the equation, so this dimension if you see how many rows are there n rows this is column one in this way we have A n columns are there.

So, each dimension is if you these n cross n this dimension is n cross n and this inversion is because this is an Eigen vector of the composes that one B 1 all these things. So, let us see this proof of this one, so these are the Eigen vectors what you can write it for this one if you see that one. This is what is called this is the closed group system Eigen values preclusive system matrix a minus B a minus B R inverse B transpose P is the closed group system matrix. So, this way you can solve it this one, so let us prove that how we are getting this expression, so proof, so algebraic Riccarti equation we derive A transpose P plus P a minus P B R inverse B transpose P plus q is equal to 0, let us call this equation number one.

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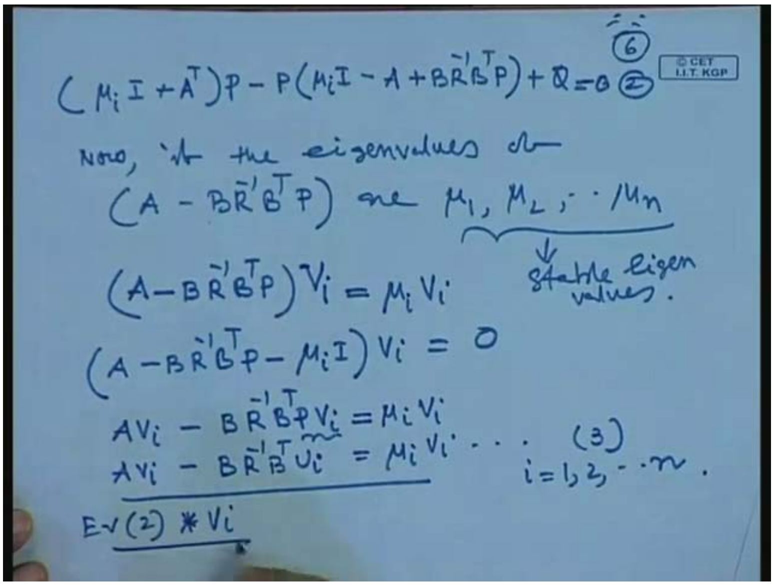

We are writing into different form look mu i is the Eigen values of the matrix cause exponent to stable Eigen values mu i this is added to the a transpose multiplied by P a transpose this is more when the term is added it is subtracted this way mu ii again. Then, A minus plus B are inverse B transpose P plus Q, let us call this equation number two, now see mu i P minus P mu i again that if you multiply by you is a scalar quantity again this and this cancel. So, a transpose P minus and that is minus plus P a minus plus this P B P B are inverse B transpose P plus q so it is same as equation number one.

So, next is the Eigen values, let us know if the Eigen values of a minus B are inverse B transpose P R mu 1 mu 2 dot mu n and that corresponds to stable Eigen values. If the Eigen values of this matrix are mu 1 mu 2 is again stabilized values, then we can write by definition of Eigen value and Eigen vector. The definition A B are inverse B transpose P this into that vector that vector is B i that is called the corresponding to the vector this is the Eigen values corresponding to the vector are V i is equal to 1 to n. Then, B 1 B 2 is the Eigen called Eigen vector corresponding to this matrix.

So, this plus result to that mew into B i since this Eigen values and Eigen vector, this is the matrix and collusive symmetric matrix and that Eigen values are mu 1 mu 2 which are the stable Eigen values vacancy. So, by definition that Eigen values and Eigen vectors you can write it closed group systematic in to corresponding Eigen vector if it is v i must be equal to mu i into B i. So, this I can write it after bring it in this side B are inverse B transpose P this minus you bring it mu ii full B i is equal to 0 push it B i in the center.

So, it will be B i minus P B are inverse B transpose P B i again minus mu i mu i v i if i take it right hand side it will be mu i v i. So, this is nothing but A v i minus P v are inverse B transpose, let us call this matrix this will be multiplied by the vector. So, it will be a vector which by e y is equal to mu i v I, so let us call this equation is equal to equation number three and this is valid for n number of equations at A. So, this is the one equation three, now if you multiply by this equation two by v i post multiplied by v i both sides. Then, what it will get it this equal to that is this equal to 0 null matrix from equation we can see the adjust equal to null matrix, so here I missed it, it is the null matrix here.

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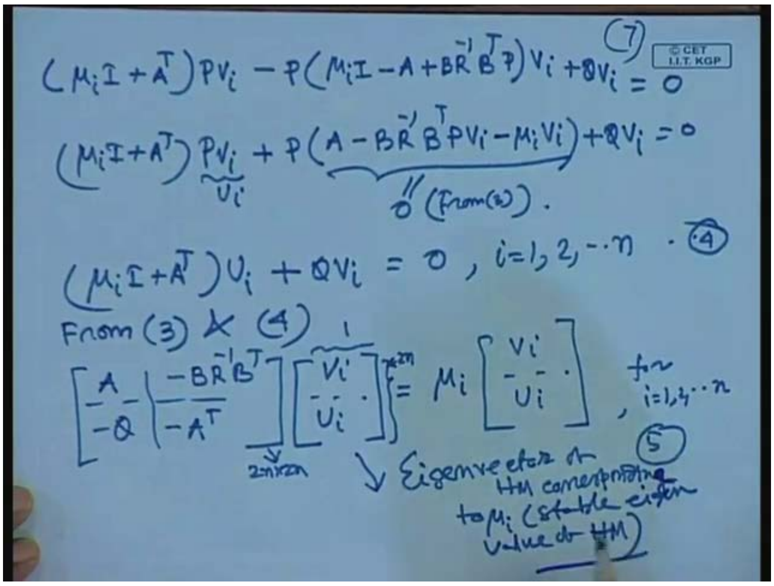

So, I multiplied by equation two equation two multiplied by B i then what we get it, let us see mu ii plus a transpose P B i minus P i that is c equation to v i that means i. Then, minus A plus B are inverse B transpose P multiplied by v i plus q v i is equal to null matrix. Now, look at this expression this expression, you see this expression what you can write it A V i if you multiply by a v i you see this one A V i are inverse B transpose v i minus mu i v i with 0. If you take this one, this side it will be 0, so is you see these are then a v i are a inverse v i, then mu i, but it is minus with this one this quantity is 0, this quantity is 0.

So, I can write it now mu i plus A transpose P v i again, then I am taking the minus sign common is equal to plus plus a minus B are inverse B transpose P B i again minus this is the minus sign. If I take it common this is plus, so it will be minus mu i V i plus 2 lyapunov i is equal to 0, so this quantity from equation 0 from three. So, you got at this one this is nothing but A U i, so you got it mu i plus A transpose U i plus q B i is equal to 0 and i is equal to 1, 2 dot n.

So, let us call this equation number four, now you just write from equation three and four from three and this from equation four you can write from three and four not v i. Now, what V i is called is the Eigen vector of the Eigen matrix corresponding to a Eigen value mu i. This is the Eigen vector and mu i is the what is mu i is the nothing but the stable Eigen values stable Eigen values of Hamiltonian matrix, so from three and four we can write it a minus B are inverse B transpose minus U minus A transpose. This is the Hamiltonian matrix and this equal to B i U i than is equal to U i and B i U i and that is for i is equal to 1, 2 dot dot n.

So, let us call equation number five and these are the what this is if you see the Eigen vector this dimension is what if you see this dimension is this matrix dimension to cause to n and this dimension one could cross twice m this is the twice m. So, one row one column twice m, so these are the Eigen vectors corresponding to Hamiltonian matrix again this what is told you earlier.

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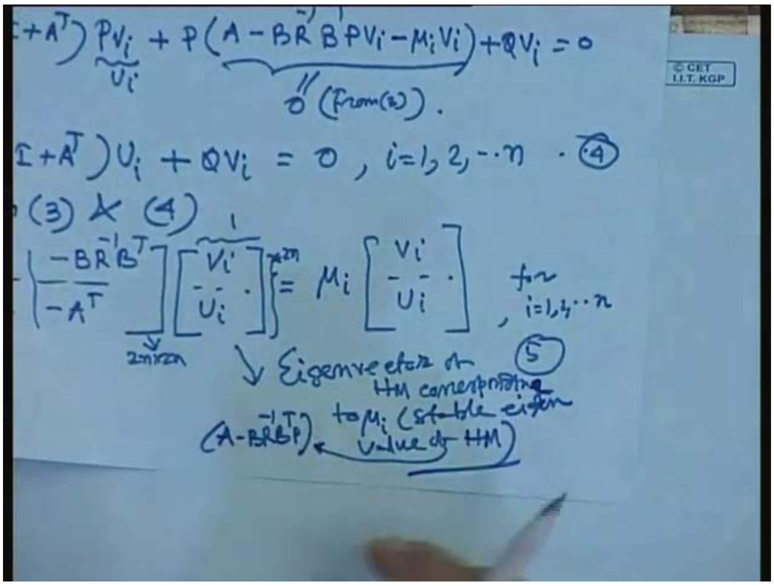

Also, you find out the Eigen vectors of the Hamiltonian matrix is called corresponding to the stable Eigen values of H M means Hamiltonian matrix. Then, you can find out of the what is called solution of the algebraic equation and this is you can write it Eigen vector of H M corresponding to by definition of values Eigen vector a of x is equal to lambda of x. That can be done corresponding in to mu i which is stable Eigen value Eigen value of H M this is or you can take Eigen values of a minus B are inverse B transpose P, this is the stable Eigen values of this matrix also, now this I can write it.

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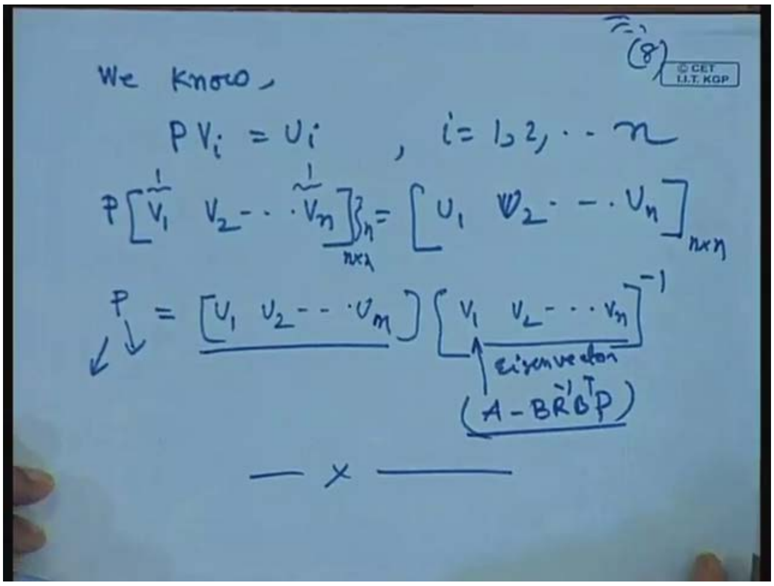

So, we know or we have defined you see P v is equal top U i is equal to 1, 2 dot n, so I can write it B one B two dot B n is equal to U 1 U 2 dot U n augmenting the is equal to 1, 2 n this matrices are like this way. So, this is nothing but this dimension is n cross n because this is the row is n rows this is the column one this is the column one, so you have this is also n cross 1. Therefore, P is equal to U 1, U 2 dot U n divided by that is inverse of post multiplied by this one B 1, B 2 dot B n whole inverse mind it that B one B two are again vectors of the matrix Eigen vectors of the matrix that is a B R inverse B transpose.

A closed system were ever again it is again vectors of this one this are the Eigen vector Eigen vector of this matrix Eigen vector of this matrix is correspondent to the Eigen values mu i corresponding to the Eigen value mu i. Since this vectors are it can be proved that V 1 V 2 V 3 dot V n is the set of Eigen vectors corresponding the set of Eigen values mu 1 mu 2 dot u n that is n again values are there.

They are linearly independent, so the inversion exists for this one again, so this way one can get the solution of this one, so if you summarize this one then what is the step to solve the algebraic equation. First step is check whether the system is controllable or not, the system is controllable, then preside second step if the second step, send the message that this we cannot design a control based on l Q R. That means solution of equation does not exist or we are not be able to stabilize this systems seventh step is here that you form a Hamiltonian matrix. Once you form a Hamiltonian matrix than you can find out you can immediately find out because the Hamiltonian matrix is known to you completely because you know A, you know B you know Q and you know R.

So, Hamiltonians matrix is known to you completely then you can find out Eigen values of the matrix consider only the stable values of the matrix and find out the corresponding Eigen vector of the Hamiltonian matrix Eigen correspondent. An Eigen matrix corresponding to the stable Eigen values again once you got the Eigen vectors of the Hamiltonians matrix correspondent to the stable Eigen values mu 1 mu 2 dot mu n. Then, first n rows of this vector you picked up that is we denoted by v i and remaining n rows and last n rows are U denoted by U i then you form a matrix like this way.

Take the inverse then this will be the solution of the A equation that is the proof of that one, but this is what method what we are considered the Eigen value to an Eigenvector method. This method is not numerically much sound in the sense when finding out the Eigen values and the Eigen vectors and the Eigen values are the repeated values. We have to look for what is generalized again vectors and competition of generalized vectors may create what is called numerically some problems cannot be reliable it is not reliable much to when will compute the real values by this method by this ordinary method.

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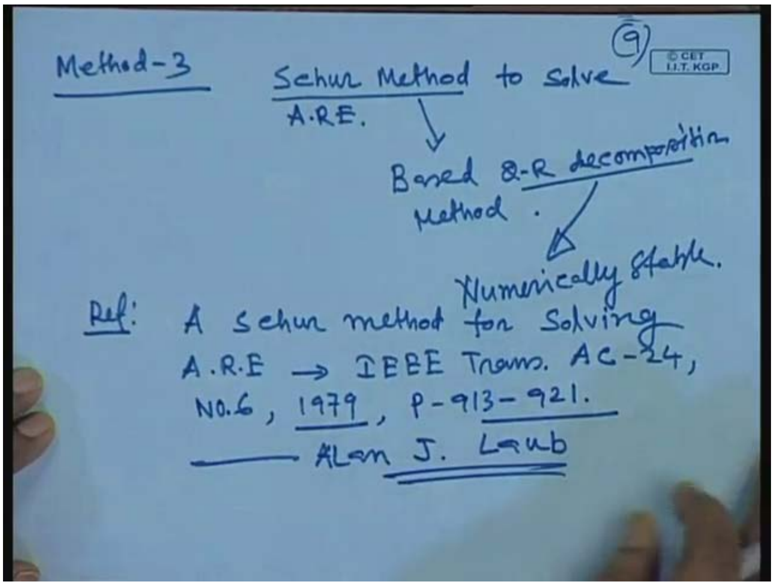

So, the next method is stabled method three is more numerically stabled method while we computing the Eigen values of the Hamiltonians matrix corresponding to the values of H M like it is called Sehun method to solve A R E algebraic equation. This full method is based on Q R decomposition method decomposition method again and that is numerically that Q R D decomposition method is numerically stable. More reliable when you will compute the Eigen values of Eigen vectors of the symmetric matrix when the Eigen repeated notes are there.

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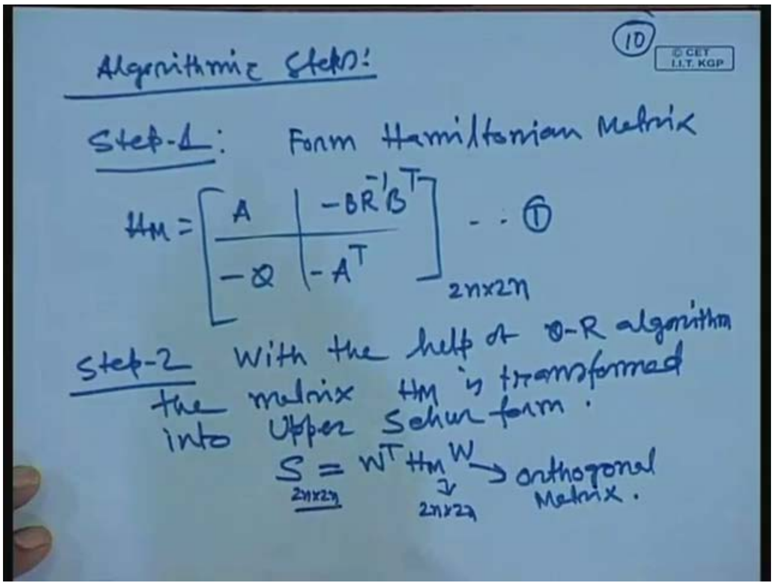

So, one can see the reference of this one this you can sure method for solving algebraic equation that is the I triple e to transition A C is to see volume 24 number 6 and 1971. Even though if it is a very old paper that it is a basic things is given here only and page 119 to 192 authors is Allan J Lamb. So, will just briefly discuss the solution of this equation algebraic based on what is called the sure compliment by a sure method.

So, step ever the mistake steps What will do will just find the Eigen vectors of matrix H M corresponding to stable Eigen values mu 1 mu 2 dot mu n that is the method only once. You know the Eigen values of this one we know how to find out the solution equation P meant the first n rows we picked up and will form be one B 2 B 3 B n and this is nothing but a Eigen value Eigen vector corresponding to close looks group system. Eigen values again and the remaining Eigen values that of this Hamiltonians matrix Eigen vectors that Eigen vectors are the 2 n cross 1 again dimension the first n rows which denoted by g 1 and the remaining n rows that means last n rows is denoted by U 1.

So, from that information find out the solution of P and this method Sehun method is giving the competition of Eigen vector corresponding to again vectors of H M corresponding to Eigen values of mu 1 mu 2 like that mu n. So, first step if from Hamiltonians matrix H M is equal to a B R inverse B transpose minus U minus A transpose, this is the equation number one which dimension is n cross twice n into twice n.

This step two with the help of with the help of Q R algorithm the matrix H M is transformed into upper Sehun form what is upper Sehun form first x is equal to w transpose H M of w and w is the H M matrix orthogonal matrix. An orthogonal matrix means if we take this matrix it is nothing but a transpose of the matrix, this is algorithm matrix transpose is equal to the matrix about the algorithm matrix. So, this is the algorithm matrix a structure of A if you look it this expression this dimension is H M dimension twice n and cross twice n dimension is twice n and into twice n because this n dimension is equal to twice n again.

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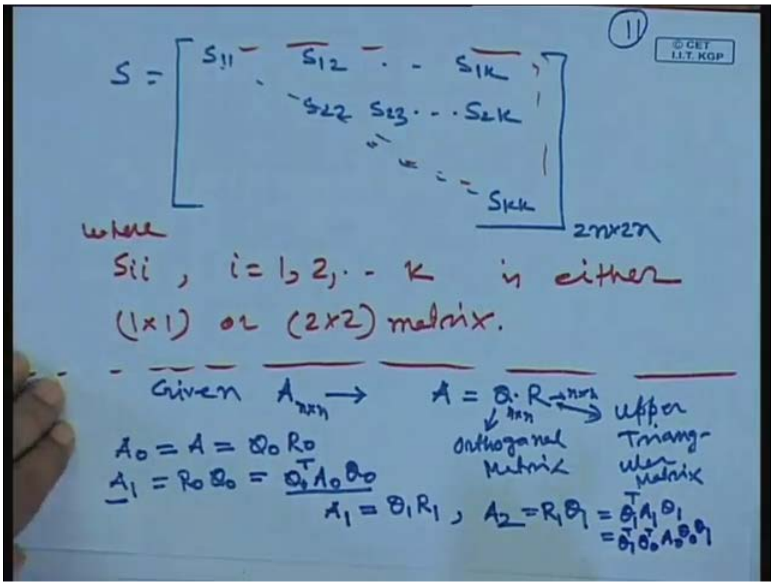

So, the structure of s the structure of s you will get it is 1 2 dot dot s 1 k s 1 2 s 3 3 dot s 2 a and this way if you just go on s k k. So, this four matrix dimension is called twice n just like here this dimension is twice n and into twice n this algorithm matrix. Now, what will do you do, the equation on H M such that the structure of this matrix when you multiply by w the structure of P matrix will be this form.

The structure this will be a structure again from this one and look at this expression at each block s 1 1 s 2 2 s 2 3 3 4 s a k. This block will be either one by one block or two by two block this will be s ii were s ii is equal to 1 to dot k is either one cross one means scalar or 2 cross 2 matrix. Again, this will be the this matrix, so this is the sequence of the we can make it on a H M matrix such that this matrix will be converted in to this form either one by one block or 2 by 2 block.

If the 2 by 2 block is coming in there, then this Eigen values of that one s you see Eigen values of say Eigen values of H M because you were doing the humiliation transformation w transpose nothing but w this one you can think of it like this. So, a Eigen values are remaining same, so this Eigen values are there either it will be 1, 1 this work or it will be 2 by 2 block. This mean Eigen values will be either complex or real this if it is a 2 by 2 blocks are there, so know see how it is done here this is just to get some idea that what we are doing in this equation.

Suppose, given our problem is converting to A decompose is equal to A into Q R again, so let us call the matrix a is decomposing to Q into R Q is orthogonal matrix and R is the upper triangular matrix upper triangular matrix again. Now, this is equal to this is the matrices can be decomposed into this q into R form this can be done by using its sequence of orthogonal transmission. In other words, one can do is sequence of transformation which will decompose in a it is into this even to R, let us call we know how we assume that this technique we know to decompose a matrix into R.

So, if it is then how you get Sehun form to this from a is equal to I am writing is the given matrix is say written assigned with a 0 which is equal to A 0, R 0. Now, I have divided A 1, if you reverse the water of this one because both are the dimension of n plus n this each is n plus n this is also n plus n. So, if we reverse this order the dimension this matrices will defer from A 0, so this matrix and show considering is A 1 is a R 0 k 0. So, what is R 0 if you from here take the inverse both side two, so it will be a q 0 inverse means transpose is orthogonal matrix.

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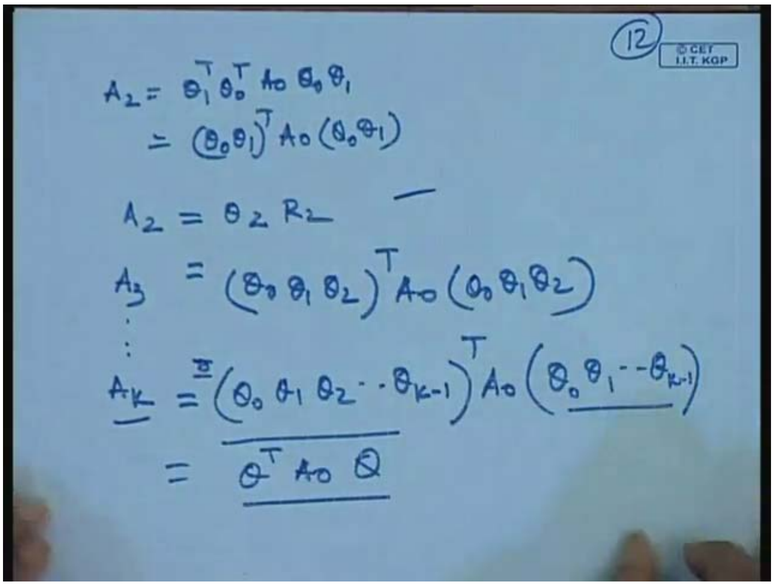

This is orthogonal matrix, so this will be this one into A 0 into k 0 again, so what is R 1 is nothing but A R 0 e 0 and whatever the decomposition is done reversibly write it R 0 k 0 A 1. Similarly, I can write it now A 2 that that A 1 is just like if you decompose this matrix is equal to decompose to one R 1 decompose in the which is denoted by A 2. It is equal to R 1 Q 1, so what is our R 1, R 1 is nothing but A Q 1 transpose A 1 into you writing you transpose A one multiplied by Q 1 again so what is a one just now you got it A 1 is this one, so it is a Q 1 transpose Q 0 transpose A 0, Q 0, Q 0, Q 1 again.

Ultimately, if you see if you do the k th iteration a k, what you can write a k that q 0 q 1 q 2 dot q minus 1 whole transpose a 0 U 0 q 1 q k minus 1 in this again. So, ultimately it is nothing but U transpose q 0 and q that two orthogonal matrices, it is multiplication system is that is again something like this way. If you do this sequence like this way ultimately I will get A is from the Sehun form into Sehun form is this form, just like into this form into this structure. This is all of that means either A 1 1's 1 1 may be 2 1 by 2 by 2 block or 1 by 1 block, this may be 1 by 1 block or 2 by 2 block, but it cannot be more than 2 by 2 block. It is made by 1 by 1 block or 2 by 2 blocks.

So, ultimately if you proceed like this way you will get it a k into Sehun form, so I just leave this in the next exercise please do this one again this. You just proceed it that one and next class I will just complete the algorithm how to solve that that algebraic equation using the Sehun complement is Sehun form. Sehun form is nothing but a numerically you are finding out the Eigen vectors of a of Hamiltonians matrix into a stable Eigen values in this that is all only again, so will stop it here.