Optimal Control
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Lecture - 41
Numerical Example and Methods for Solution of A.R.E

Last class, we have discussed what a derivation of infinite time linear quadratic regulated problem, and then corresponding optimal control if you use to the system. Then its stability analysis is also studied, so let us explain or illustrate that L Q R problem with a numerical example.

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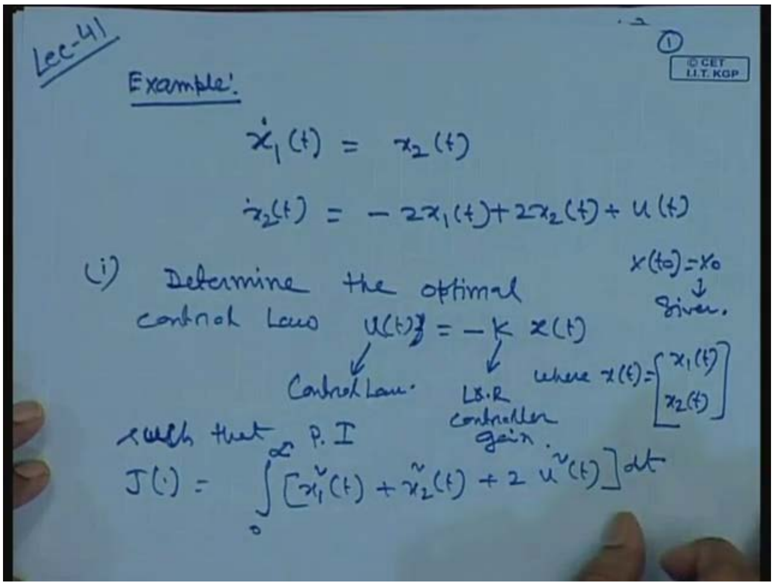

So, let us consider our dynamic system which is given is a second dynamic system is equal to x 2 of t and x 2 dot t of t is equal to minus twice x 1 of t plus 2 x 2 of t plus u of t. We have some initial condition is given this is given, so our problem is to determine a control law if you recollect our infinite time regulator problem. Our problem is to determine the controller u such that this performing index, quadratic performing index is minimized. The first problem first is you determine the optimal controller u of t small u of t, because is scalar input small u of t is equal to minus k into x of t where x of t is equal.

We have two states are there x 1 of t x 2 of t, and this is the controller gain LQR and is nothing but a state feedback what is the information about this state. You know from the system that we are gaining from linear combination of the states will give the control of the law, this is the control law. So, determine the optimum controller law of a u of t such that such that such that the performing index j is equal to 0 to infinity because it is infinite time regulated problem.

Problem is a x 1 square of t plus x 2 square of t plus twice u square of t whole d t is minimized given a controller u of t such that this performing index is minimized and this interpretation we have already discussed. Suppose, this is the input free system, this equilibrium position is the linear system in equilibrium position is origin is equilibrium position. Due to this initial disturbance or initial condition of the state the state x 1 x 2 will deviate from the equilibrium position and our this interpret this term interprets division from the equilibrium point means on the origin x 1 state is over.

The interval 0 to infinity that quantity have to minimize not only that addition to that control of four u square u transpose Q u transpose R u t which is e R is in this case twice u square t of n this. Also, you have to minimize, so this is the first problem we have to solve, first problem first part is that one.

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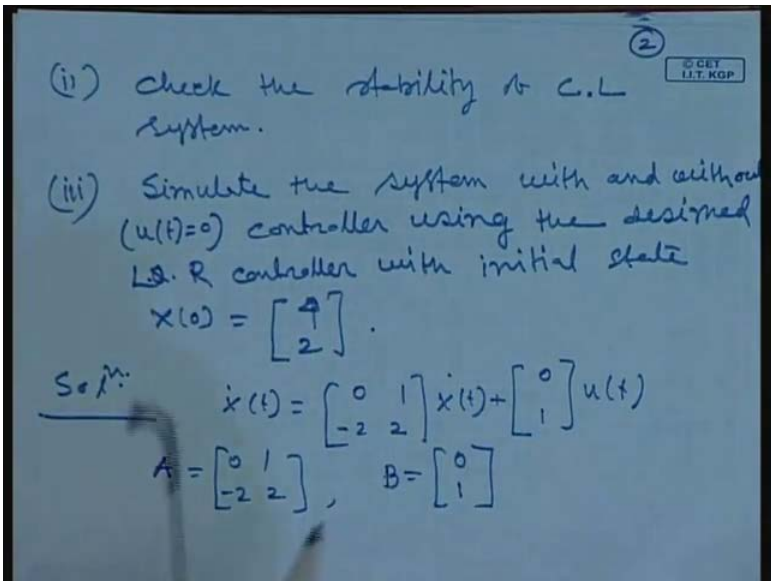

Second part this problem is check after deciding the controller check the stability of the close loop is the close loop close loop system this thing. Then third part of this problem is simulate the systems or get the response of the system simulate the system with and without controller with u t is equal to 0. This means without controller without when u P is equal to 0 without controller whether without controller without controller using the designed L Q R controller L Q R controller which initial state X of 0 is equal to 2, 4, 2. So, let us solve this problem and if you recollect our basic steps of definite problem is first step is to solve the algebraic recartic equation.

First, you have to solve the algebraic recartic equation what is the algebraic recartic equation solution we are writing solutions. So, first we write the algebraic recartic equation, now our algebraic recartic equation is before that you say you have to identify which one is A matrix, B matrix, Q matrix, R matrix. These are the four matrices information is required to solve the infinite time regulated problem, so we have to identify from the system module what is system matrix A input matrix B.

Stating matrix is Q and control matrix R that you have to identify it is usually can be seen from the system equation. We can write it that x dot is equal to x dot t is equal to 0, 1 if you write matrix in vector form is a minus two into x of a plus 0, 1 P of t. So, from this one immediately you can write A is equal to our 0, 1 minus to 2 and this is B is equal to v is equal to 0, 1. If you see this one, you just if you see this our Q and R matrix, if you see here our Q and R matrix this matrix we can easily write it this this perform index, we can we have to convert into standard form.

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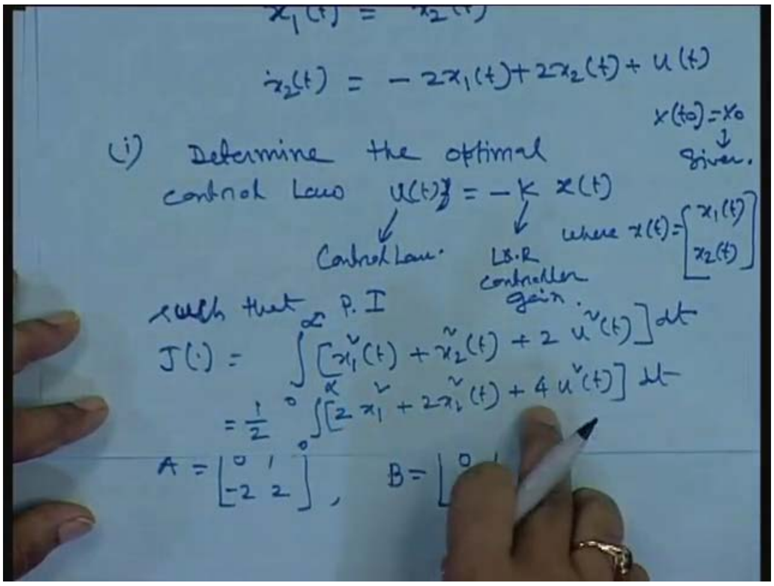

That means here itself you can write if you like, if you permit me, I can write it half because that e what is called the performing index that one it should be half into j to infinity. Since I divided by half multiplied by twice x 1 square plus twice x 2 square plus 4 u square of t whole d t, I made it half our standard formulation is half. There, this half is there created there in the derivation because when you do the perfect derivation of this L Q R problem. So, the derivation possess the two will come two and this two will be cancel for this one and based on this one you have derived all our expression, so that is why I just invert standard from I divided by half multiplied by 2.

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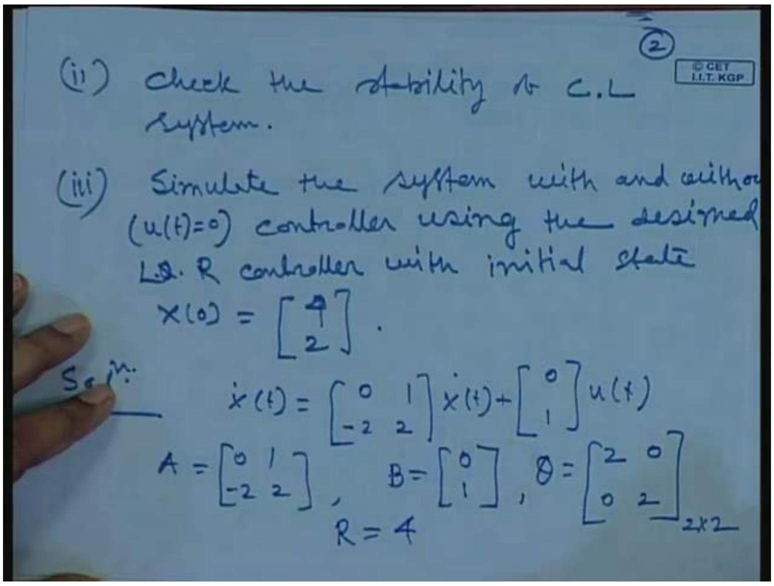

So, this is the standard of four from this equation even you can say that Q is what R is what from this, so our Q A if you say this is nothing but A 2, 0, 0, 2 and our R is equal to 0, 2 our this is 2 of 2 matrix. So, once you know A B Q R, this is the first step, I defined A B Q R from the statement of the problem. Once you identified this one, then you could write it next step is to check that what is what is stability condition, check the controllability condition.

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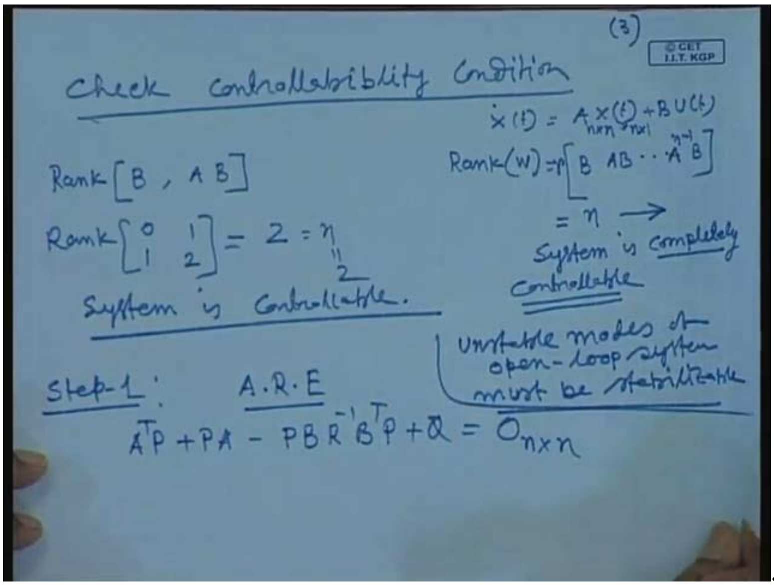

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You check the controllability condition whether you will be able whether to drive all the states into what is called desired location. If the open loop poles are on stable whether you will be able to drive that trans stable poles to the stable poles in the from half poles to the left half poles. So, whether you will be able to do it or not that you have to check it and that is done, suppose you have a system before that I am just telling you how to check this one. If you have a system described by x dot is equal to a x plus B u of t, then how to check controllability of this system means controllability of this system.

That means you will be able to drive the state from initial state to a deserved state to the finite time within a finite time by using appropriate control law whether it is possible or not. Before that, we can check it whether system is controllable or not, you can check it, there you can find out that rank of this matrix. Let us call the matrix is w is how you form the rank of the matrix B A B and dot A n minus 1 B, this you form it this is A n cross 1 n cross n matrix.

Then, the state of the system is n states so you form it matrix up to A and minus system to B, so find out the rank R small R is indicates of the rank of this matrix if the rank of the matrix is n the system is simplized. The system is completely controllable system is completely controllable the system is completely controllable that may all the states will be able to drive from any position to what is call any other this this at position. So, will check whether system is controllable or not at least it should be stabilize able, so if you check this this one, then we will find out that our rank of rank of B A B.

In our case, rank of B matrix is what 0, 1 is vector in this case or you can be matrix of dimension of 2 cross 1, this B, then A B if you multiplied by A into B that will become a say this a into B A into B. This will reflect the last column of that one agree that will reflect the last column of that one. So, it will become a that 1, 2, so that rank of this matrix clearly is equal to 2 and which is equal to n in our case it is one if you say number what are the system is 2. So, the system is controllable, so these are then we can proceed it if the system is not controllable or at least is not stabilize able. Then you have checked whether system is stabilize or not stabilize able means if you have opened the system is a un stabled mode.

That un stabled mode should be stabilize able in the sense you will able to drive the un stabled poles again values to the from the left top from the right top of the explain to the left top of the explain able to drive by using at the control effort e. If you satisfy that condition, then it will be to the system is stabilize able, but uncontrolled mode is not controllable. Then it is a not stabilize able, hence it indicates the you cannot apply the L Q R control problems, so unstable because in other sense you can un stable modes mean in values un stable modes of open loop system must be must be stabilize able.

So, this is the important thing, so first if you check the control ability test if it is completely controllable we can proceed first. Then our step is recollect one by one step we have to solve our algebraic recartic equation, since it is infinite time regulator, the time is 0 to infinity in the performer in the x indicate power this infinity. So, it is infinite time regulator problems, so our equation is transpose P plus P A minus P B R inverse B transpose P plus U is equal to 0 null matrix of the dimension n cross n we know A B R Q, so unknown is P.

You have to solve it by some means, we discuss in details what is the different methods of their to solve the algebraic recartic equation. By the time being, we just use since it is a small of dimension 2 by 2, 3 by 2, we can do algebraic solution of this one because we will get a set of non linear equations set of algebraic non linear. Then how to solve this one if this smaller dimension do it with a hand calculation, let us see I will put the value of that one A transpose A is what this, this is the A transpose you write it.

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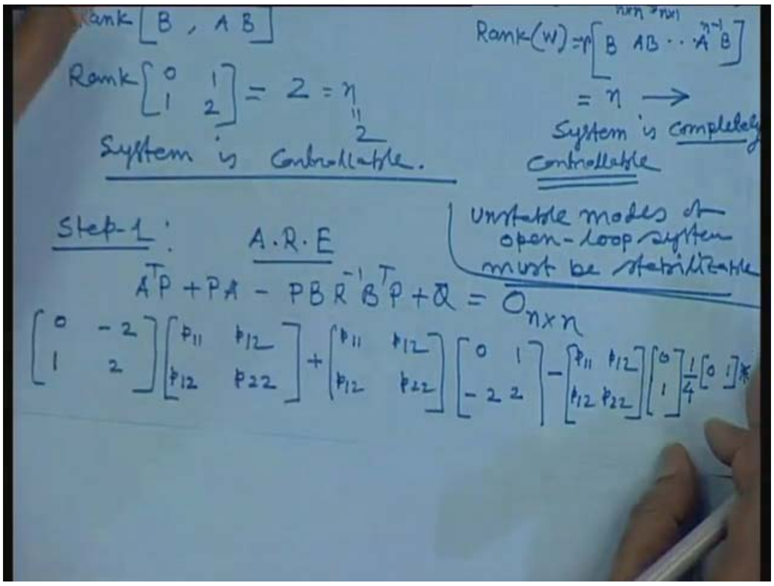

So, this will be a 0, 1 minus 2, 2 A transpose and then P is a symmetric matrix than positive definite solution is should get positive definite P 1 1, P 1 2 since it is a symmetric matrix P 1 to is equal to P 1 to P 2 two plus again this one P 1 1 P 1 2 P 1 2 P to two then multiplied by a 0 minus two one two this is that one P a all these things, then minus P, I am writing pone one P 1 2 P 1 2 P 2 two P B b is what 0 P B 0 one then R r inverse R is four the inverse one by four then B transpose B transpose is 0 one then multiplied by P.

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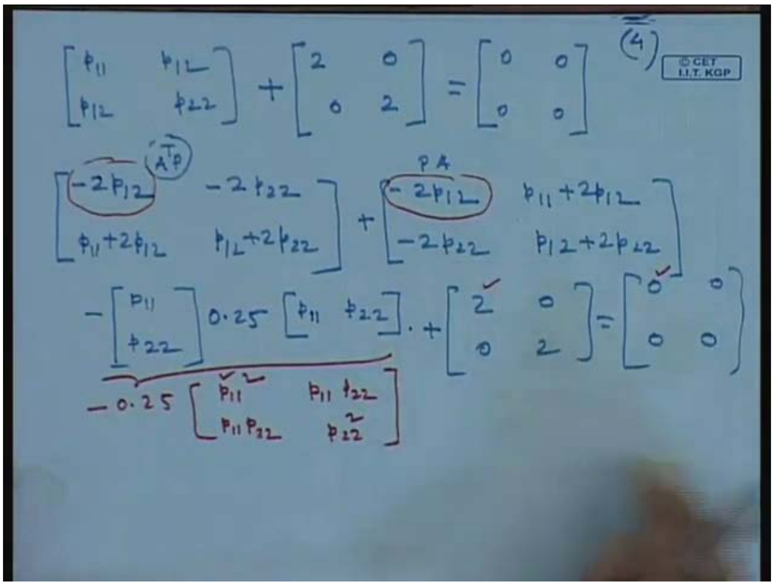

So, this cause I am writing here next page class multiplied by that is our P is P 1 1 P 1 2, P 1 2, P 2 2, then plus Q is a diagonal matrix each element is 2 if you see A. So, this right hand side A is equal to 0 0 0 0, now if you see you multiply by this into this A transpose B unknowns are three unknowns are there three equation. At least, we are getting three equations, so you can be able to solve, but when equate left hand side and right hand side you will get in general is non linear algebraic recartic equation. That we have to solve it, so that you multiply by this into this this into this is a minus two P 1 2 so i will write it left hand side minus P 1 2 is equal.

So, first I will write it minus this you say this into this into this this into this, so it will be a minus two P 1 2 then it will be next is this row is multiplied by this minus 2 P 2 2 minus 2 P 2 2 minus 2 P 2 2. Then this row multiplied by this column 1 twice P 1 2 P 1 1 plus twice P 1 2 then this row P 1 2, then this row multiplied by this P 1 2 P 1 2 plus twice P 2 2. So, this and this part this is a transpose P this one and this we have shown this value, now look at this one, we cannot solve one not necessary to solve P A.

So, P A is nothing but A transpose of this one, once you got the product of this one, take the transpose to get the P A to take the results of that one A transpose A transpose P take its results transpose then you will get P A. So, I will take the transpose of that one, so this will be a minus twice P 1 2 minus 2 P 2 two P 1 1 plus twice P 1 2 agree then P 1 2 plus twice P 2 2. So, this continues nothing but A transpose of this is nothing but A P A transpose of that quantity is nothing but A.

So, you can adding this composition by taking this you can transpose by this one and write it for the value of P into a that one, so next is if you see multiplied by P B are inverse B transpose P agree if you do this one. This second part of this one that P B if you multiplied by this you will get the last column of P because you see this 0 1. So, it will come last column of this and this into this this into this, so ultimately it will come from this last column of this one that will be P 1 1 P 1 the project P 2 2 that multiplied by R inverse. R inverse is 1 by 4, that means 0.25, now you see this one P B R inverse and P B if you know it if you take the transpose of that one B transpose P so this you need not to compute from the knowledge of this results take the transpose of P B.

Then, if you take the transpose of P B, I will get it P B transpose P, so this is P 1 1, P 2 2, then multiplied by what is this P B transpose then P B transpose we have done it. So, this part is over P B we have calculated R inverse is 1 by 4 means 5.5 and transpose of this one calculated this plus Q is 2, 0, 0, 2 is equal to 2, 0, 0, 0. Now, element wise left hand side, right hand side you equate this element means A 1 1 position, so you add this one plus this one plus after multiplying by this, you add it.

So, if you say that quantity this quantity is multiplying by this it will be a one fourth this is one fourth means minus 0.25 and this is nothing but P 1 1 square P 1 1. Then this will be a P 1 1, P 2 2, then P 2 2 square, I am writing this quantity only. So, just now like adding this A 1 can A 1 1 element 1 1 element this one is this one element then A 1 element is equal to that one.

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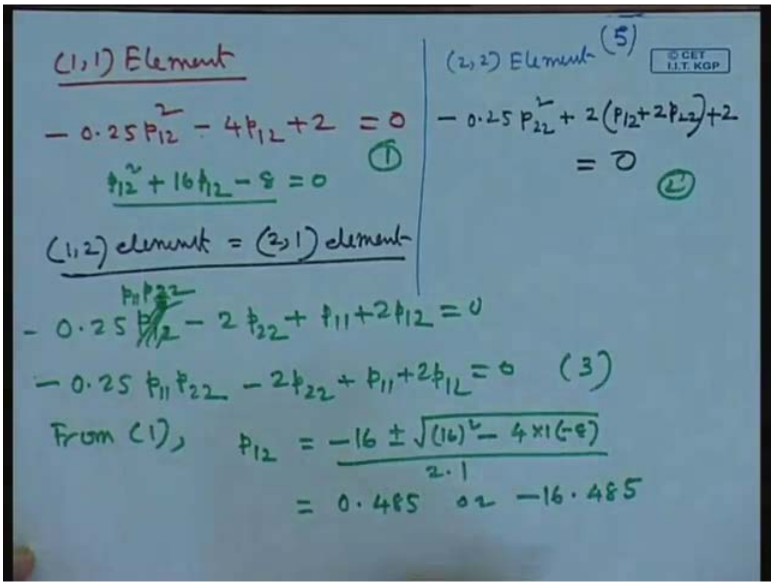

So, element wise if we equate now you will get it element wise, so I am writing 1 1 element left hand side and right hand side. If you equate I am getting minus 0.25 P 1 square minus four P 1 2 plus 2 is equal to 0, you say these and this 4 this is 2, 4, 2 and this is coming 0.25 P 1 square 0.25 P 1 square what is this? This is I have did mistakes here, you just here if you multiply by this and this that this and this if P 1 2 this row multiplied by this it is P 1 2. This row is multiplied by P 2 2, so it is not a P 1 1, P 1 2, so this also P 1 2, so this will become a P 1 2 will be multiplied by this is P 1 2.

Now, I am getting point five P 1 2 square plus four P 1 0 one element, then I write it 2, 2 element 2 2 is that one you see this plus this this is the 2, 2 element is this 1 plus 2, 2 element is 0 here right hand is 0. So, if you see that 2, 2 element minus 0.25 minus 2, 2 5

P 2 2 ,P 2 2 square, then you will get it twice see this one P 2 2 element P 2 2. Then this will come, now I am telling sorry that P 2 2 element that that means forget about not this one P 2 element is that that one.

So, this one this one and your case is that that one this is the P 2 element I am writing, so this is 0.25 P 2 2 square 0.225 P 2 square what I have written it then this twice here is this one P 1 2, P 2 2 then P 1 2, P 2 2 the twice of this 1 plus 2. So, I am writing this one twice of P 1 2 plus 2 P 2 2 plus two is equal to 0, this is another reason now I am writing either P 1 2 element is same as P 2 1 element because it is a symmetric matrix. Now, in this case you can see what is that quantity P 1 2 element is this one agree this one, then this one and this one this is 0, this one this is this one.

So, if you write it this one I can write it P 1 2 elements are that minus this is point minus 0.25 P 1 2 whole square, then you minus 2 P t 2 2 P 2 2 minus 2 P 2 2, then your plus P 1 1 plus twice 2, 2 is equal to 0. Just I am writing this quantity from two element 1, 2 element point P 2 2, P 1 2 your 1, 2 element this is P, this is I think P 1 2 element minus 0.25 P 1 1, P 2 2, P 1 1, P 2 2 this quantity. Then I am writing then am writing P 2 2, P 2 2 P 2 2, then I am writing P 1 2 P 1 1 plus twice P 1 2 plus P 1 2. So, I am writing to only this 1.25, P 1 1, P 2 2 minus twice P 2 2 plus P 1 1 plus twice P 1 2, so you have A 3 equation 1 2 and 3 equation. This three equation if you solve it, this three equation you will get the solution of P 1 1, P 1 2, P 2 2.

So, let us solve this three equation, then what we will get it is one equation let us from equation one from equation one what you can write it this is the quadratic equation. We can write it P 1 2 is equal to that is minus B plus minus if you just divided by multiplied by what is four both side. Then it will come this if you multiply by both side by 4 minus 4, then P 1 2 square minus 16 P 1 2 this is minus plus minus 8 is equal to 0 that quantity is that one this is multiplied by minus 4.

Then, this will plus minus, so it will be minus B plus minus root at this B square in 16 square minus four A C divided by 2 into A. So, we will get this it will be solve it you will get two values of the P 1 2 that is value a is 0.485 or I will get minus 16.485. So, out of this two whether we have to accept both the values and one of them we have to accept that we will say later stage.

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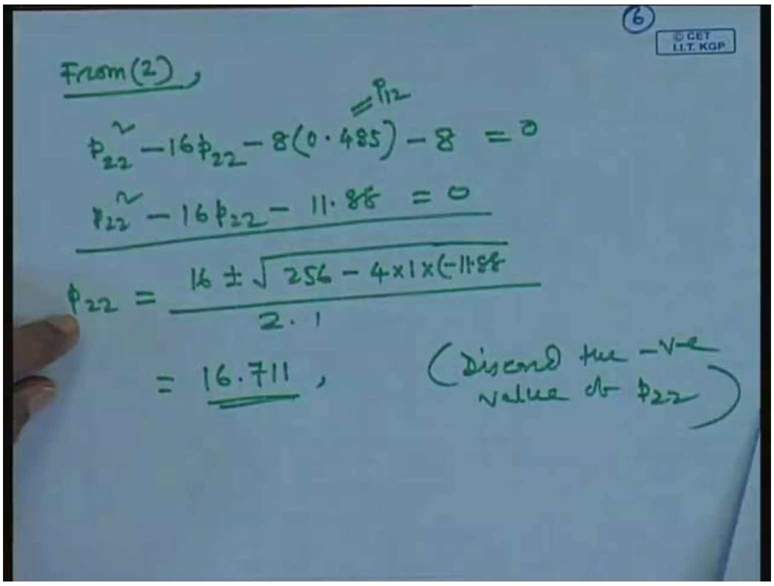

So, let us say once I know P 1 2 from equation two, so from equation two i know P 1 2, P 1 2 value P R if you put the P 1 2 value here you will get a quadratic equation in P 2 2 form agree quadratic equation form. So, if you just put the value of just that one P that expression, once again I multiplied by minus 4 both side both sides minus 4, so it will be coming if you multiplied minus four by both sides. It will be coming 2, 2 square minus 16, again 16, then it will be P 2 2, this is 2, 16, how it is coming 2 to 4 multiplied by 4 means it is a 16 minus 16 P Q 2 minus 8 and multiplied by 4 minus for this minus 2 is there. So, minus 8 P 1 2 minus 8 P 1 2 P 1 2 value is what if I consider 4, 8, 5 first this is the P 1 2 value and minus next is minus 8 is equal to 0.

This is minus 4 both side this is minus four this minus 0, so ultimately our equation is that one P 2 2 square minus 16 P 2 2 square minus 11.88 is equal to 0. This is the one, this solution of this is equal to 16 plus minus 2, 5, 6, 16 squares minus four into A into 11.88. So, there divided by twice A and that value will get two values of this one 16.111 value.

Another value will come what is this, is plus minus it will be what is call minus sign consider with a the minus more than 16 you will get. So, this next quantity will be a negative term, one can easily find out that negative term is what, so this negative term cannot be considered because whoever if you are Sylvester inequality test. If you see a matrix should be positive definite first condition is all diagonal elements must be positive, but here the other value are of P roots are in negative. So, we cannot discard the negative values P 2 2 discard the negative value of P 2 2 agree, so you are consider this one and this is the positive definite, this is the one from below to P 1 0.5 A 5.485.

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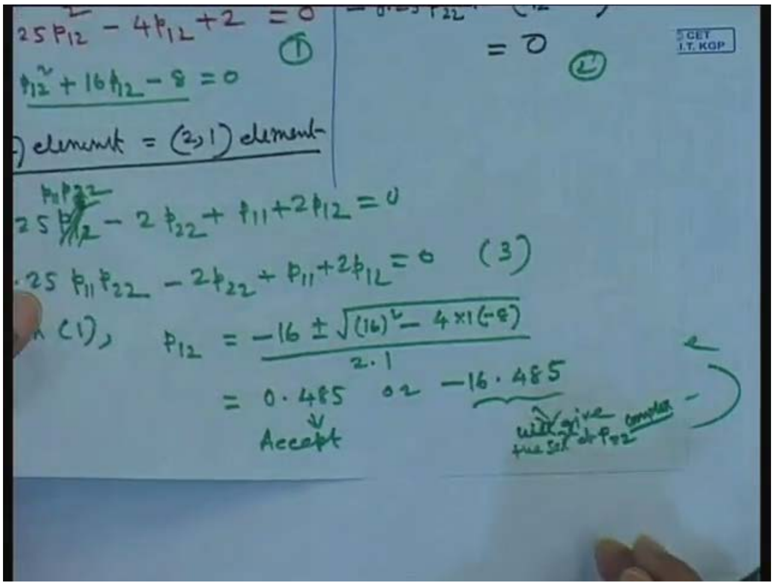

Suppose, if you consider the value of P 1 2 is that one, then you will get solution of P 2 t is complex, so will not consider that values. So, this will give you the will accept the value accept and this will give you this will give the solution, this will give the solution of P what we got P 2 2. This will give the solution of P 2 2 complex value, so will consider this is we will accept it P value is this one P 2 2 value once I know P 1 2 value. Once I know P 2 two value and P 1 2 value, immediately I can find out from equation three the P 1 1 this is known this is known P 1 2 is known and which value we have accepted that we know and we can get the solution of this P 1 1. So, from three using P 1 2 is equal to 0.485 and P 2 2 is equal to 16.711.

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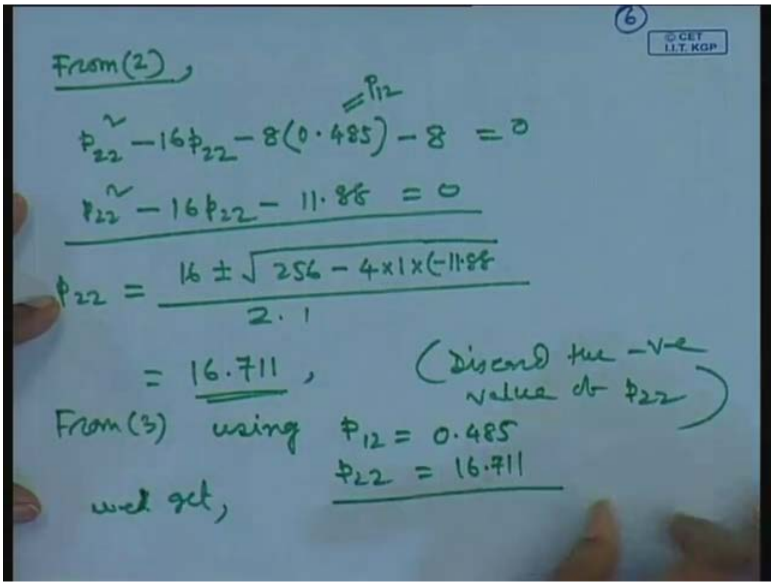

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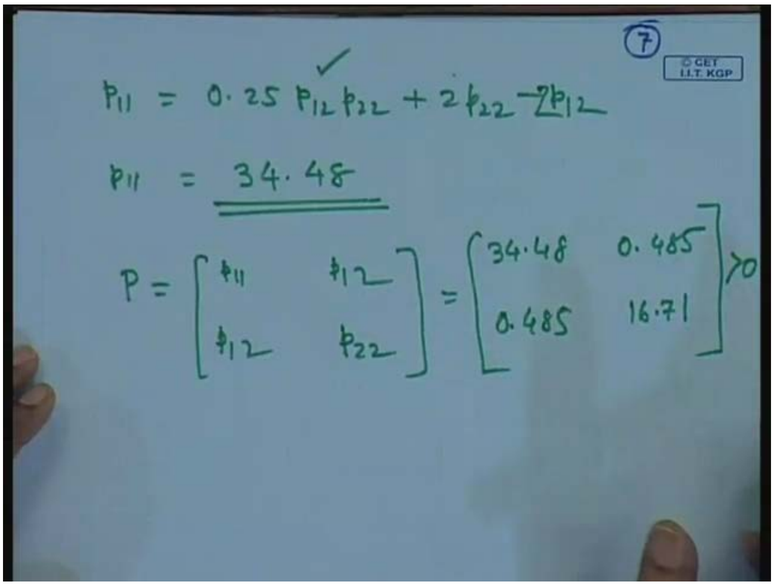

We get P 1 1 is equal to we get 0.25 P 1 2 minus P 2 2 P 2 2 plus P 2 2 minus P 2 1 that equation number three. So, this equation once again I multiplied by 2 that we will be A minus 4, so this is 4, 1 to this as it is keep does not matter this 1.25 P 1 1, P 2 2. If you take that side agree that P 1 2, P 2 2 values this P 2 2 values this P 1, P 1 2 values twice P 1 values. From this equation, you see one can write it this equation P 1 2 this if I take this 0.2511 P 2 2 which I have written in that one then it is a P 2 2, if you that side.

Then, plus P 2 2 then you twice P 1 2 just see this one P 1 twice P 1 2 equation to twice P 1 2 agree then P 1 1 so minus twice P 1 2 this is twice P 1 2. So, if you use this values of this P 1 1 value you are getting 34.8 P 1 2, from this equation I just find out this one is only unknown thing this is the unknown. So, I can get by using the values of P 1 2 P 2 2 with appropriate values P 1 2 P 1 2 P 2 2, now you can check it P should be a positive definite matrix P is P 1 1 P 1 2 P 1 2 P 2 2. So, if you put this values of P 1 2 34.48, then P 1 2 is minus 0.485, then 0.485 and then 16.71 and that matrix is positive definite matrix.

You can check it either finding out the n values of matrix or n will be positive or you can use the Sylvester test for testing the what is the matrix is similar what the matrix is positive definite matrix or negative matrix. So, our positive simulative definite matrix or negative simulative definite matrix can be checked by using the Sylvester inequality conditions.

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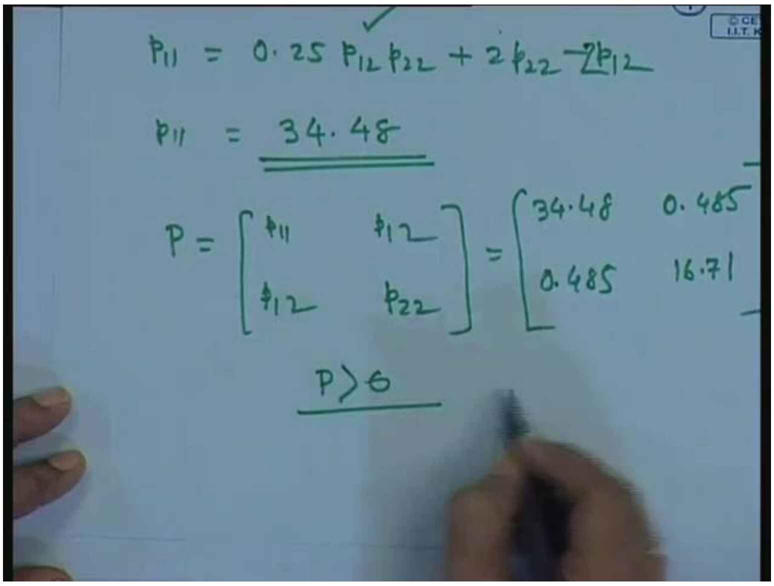

So, our case is P is greater than 0, so positive definite matrix, so this implies that our the controller the way we will design the controller will stabilize the system, so let us find how to find out the controller gain controller gain.

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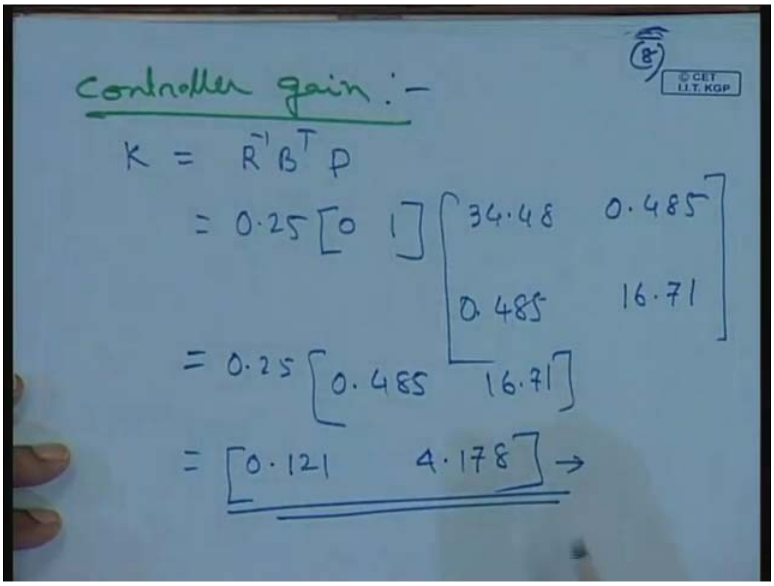

So, controller gain you know k is equal to R inverse B transpose P, you just use the values R inverse means point into five B is 0 1 and P is just now you got it is 34.48 and 0.4085 and that you got 16.71. If you multiplied by this last row will be reflected, so 0.25 into last row 0.25 into 0.85, 16.71, so multiplied by 4, but divided by this each element by 4.5. Then you will get it this is point 1, 2, 1, then 4.178, so this is the controller gain, so this is the controller gain we got.

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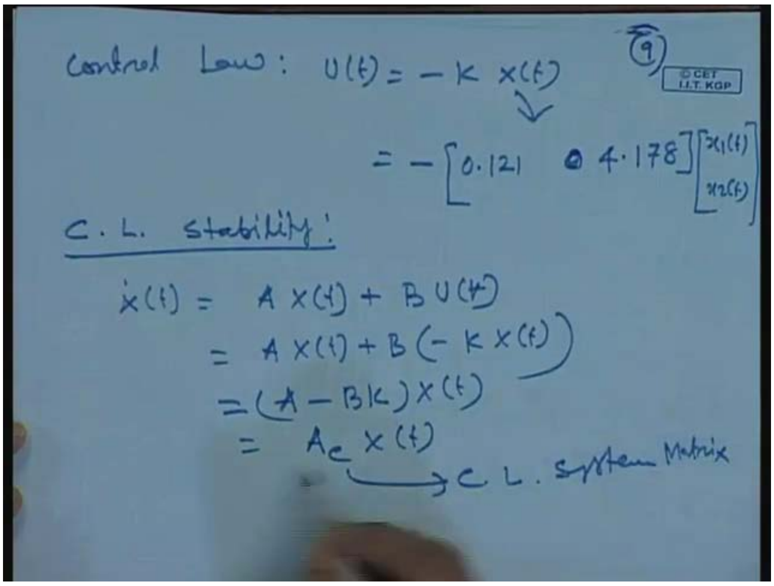

So, once you know the controller gain immediately I can find out the control law control law and which is nothing but U of k is equal to minus x of t. We assume that these are all accessible measurable by the users this one, so you know k immediately you can implement P of t. So, U of t is nothing but A minus 0.121 not 0 4 point, just now you got it 4.178, this into that our x is x 1 o t x 2 of t, so this is the control law and if this control law you applied to the system. Then it will stabilize the system not only stabilize it will drive the state to what is called equilibrium position from the initial state of the x 2 0 to the equilibrium position.

Again, in an optimal manner by minimizing that performing index, so our check the close loops system stability. So, our x dot is equal to is equal to a of x t plus B u of t u of t and if you put the value of u of t is equal minus k x of k and our closed loop system will become you take the x of t common. Then B of a is x of t and this is nothing but a suffix a x of t and this is the our close loop system matrix now system is stable or not. You just find out the Eigen values of x matrix the Eigen values of this matrix with a negative real, then system is stable, so find out the what is call Eigen values of this matrix the closed loop system.

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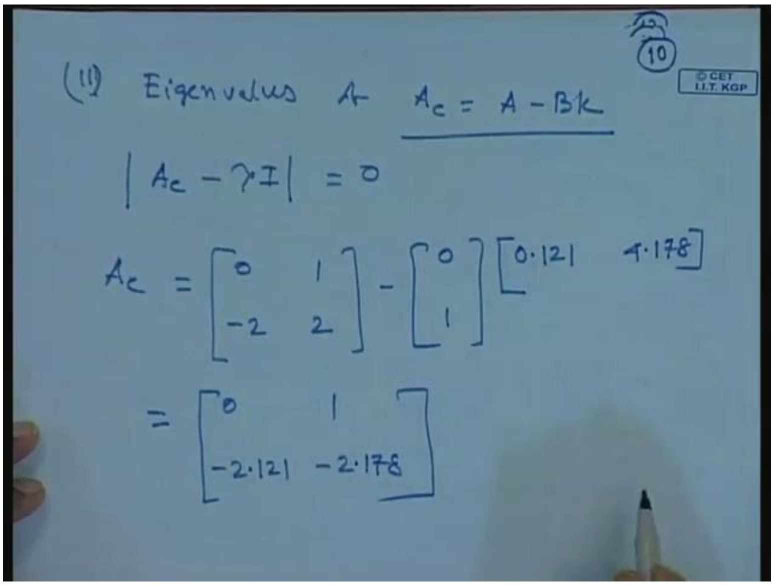

The part two is if you see the part two of the problem or the check the stability of the close loop system part two of the problem is check the stability if the closed loop system. So, you find out the Eigen values of Eigen values of A C matrix A C means minus B k matrix if all the Eigen values are of this one with negative power the system is stable. So, find it is just you know how to find out the Eigen values the determinate of A C minus lambda I of this equal to 0 and you will get because A C is known lambda is unknown.

So, you will get a polynomial of order in general n where n is the dimension of the system matrix or dimension of the state variables. So, this will be the order polynomial you will get in terms of lambda which order will be n same as n is the order of the system matrix. So, you just find out first is A C is our A n is what 0, 1 minus 2, 2 and B is 0, 1 and your k, we got it 0.121, 4.178. Now, if you simplify with this this one ultimately you will get this matrix is 0 1 minus 2.121 minus 2.178, this is the close loop system, so the Eigen values of this one you how will you find out.

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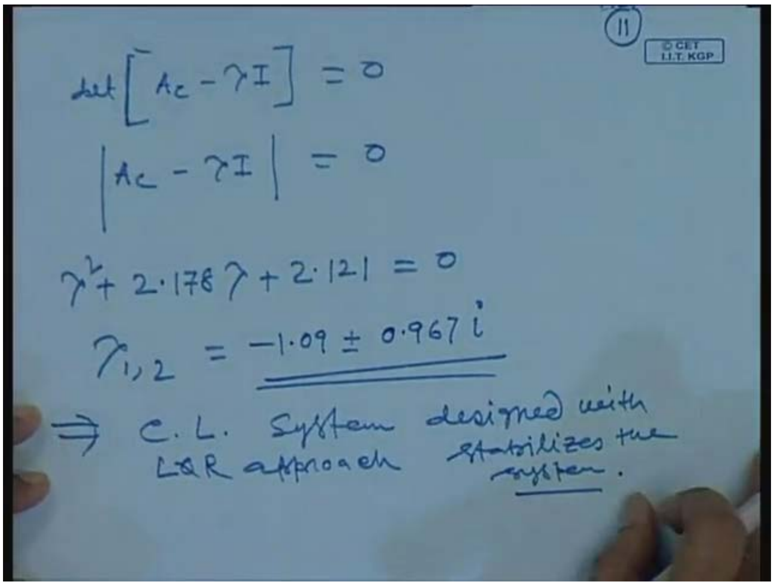

So, determinate of or determinate of what is called matrix A C minus lambda I is equal to 0, so if you do this one, so I will live it exercise to this one is nothing but A C symbol is lambda I of this equal to 0. If you do this one you will get it the determinate of lambda square is equal to plus twice 0.1, 7.8 lambda plus 2.121 is equal to 0. That Eigen values of this two will give this quadratic equation minus B plus minus will go this square you see if you do it you will get it minus 1.01 plus minus 0.967.

So, this real part of this one is equate, so this system is stable that means with that A, with that application of this controller the system response will be stable means. It will bring back to the state from initial state to an equilibrium state, so our conclusion this implies that our the close loop system designed with L Q R approach stabilize the systems he value of close loop system designed with L Q R approach stabilizes. So, this is our conclusion, now third part of the third of this problem is if you see it is nothing but that is what is called find that system response finds the system response. With the application of that that what is called optimal control law and optimal control law is designed based on the L Q R.

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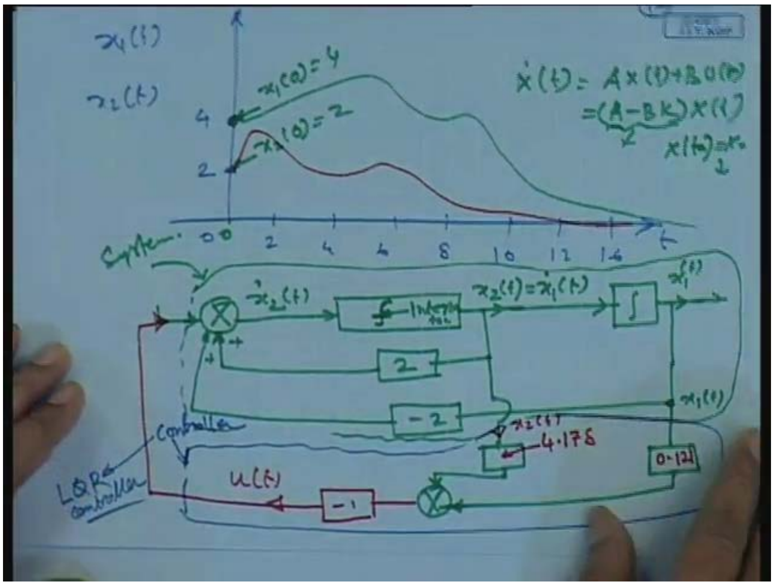

So, you are as to stimulate the system take the response t verses you find out x 1, then x 2 of A, this is our initial state. If you see it is x t if it is A 2, this is 4, 0 and it is a 2, 2, 4, 6, 8, 10, 12 and 16, something like this. Then our initial state is starting from that our x 1 0 is this this one which is initial state for x 1 of 0 is equal to 4, this is the initial state x 2 of 0 is equal to state by using this what is called control law. You find out the response of the system, let us call response of the system just say you have to solve it, how to solve it is equal to x dot is equal to a x plus a x of t plus B u of t agree or equal you can write a x minus B k of x t. So, it is a solution of autonomous that autonomous system is a close loop system, you know the value of given x 2 x t 0 is equal to 0 is given.

So, you can solve this one, so it is called solution may be like this way ultimately come to the 0 and this is the solution of other state solution may be like this way it is coming to 0 like this. So, you find out the response of this system and you know this matrix then you can solve it, so this is I live so this live it in exercise to solve this one. Now, let us say with close loop control system then how it looks like an block diagram form, so if you see our system is this is our f integration this is the integration and this our x dot 2 x dot of t. Output of this is the integration agree output of this one is x 2 of t which is nothing but a x 1 dot of t as per the description of the system dynamic and that goes to the integrator agree this goes to the instigator.

The output of the integrator is the integrator, so output of this one is x 1 of a and this output is going to the minus and this is the summer and x 2 is multiplied by 2 gain 2, you can say x one is gain 2. Here, you can realize this this one the amplifier, this is the U plus and this is state information is minus 2 and then it is gone. So, this our this portion x dot is equal to x, so you have B, so you have this, so x and x 2 information must be available to the controller. So, our controller this is one so this is our x one x 1 of t and this value is what 0.1211 and this value is your this value is 4.178, this our x 2 f t.

These two signals multiplied and then it is a minus k x negative and this multiplied by f B u and what is our B u and this is u is coming only the you see that in dynamics of that one u is coming only in the second state the x two dot expression. So, this directly I can write it this is nothing but our U of t, so if you see this portion from here to here this our plant or system plant or system and our controller is that part.

This is the controller that is the controller and this controller is what type of controller that controller is L Q R controller. So, one can look at this expression one can change the controller gain by selecting the state weighting matrix Q and R. This is the only the tuning parameter designer have that I tune the controller parameters that k a values by tuning the Q and R. So, we long our response is not satisfied, we can always tune Q and R to get the desired response of this one, so this is the infinite time L Q R problem solution this is the thing. Let us now go back to what we call the how to solve what is called the Algebraic recartic equation, how to solve the algebraic recartic equation by different techniques or different methods.

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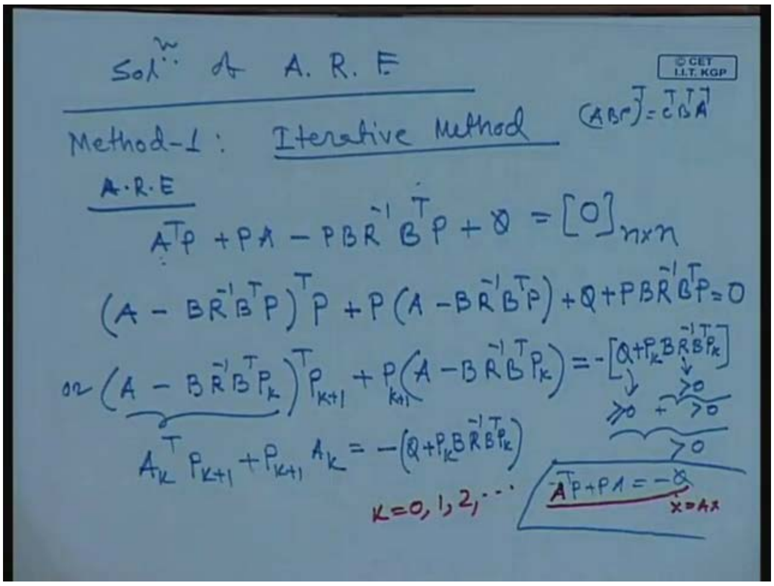

Solution of A R E algebraic recartic equation solution, so first method is a simplest method, but it takes much time to get the converse value of P. So, method one is the iterative method iterative, so A R E solution equation a transpose P plus P a minus P B R inverse B transpose P plus Q is equal to null matrix of dimension n cross n where n is the order of the system matrix or the number of states in the systems this. So, this can we can rewrite it in this form, look carefully how I am writing this one A B A minus B R inverse B transpose P whole transpose P then plus P A minus B are inverse B transpose P is plus Q plus P B R inverse B transpose P is equal to null matrix.

Now, we would see this manipulation who has done also earlier when you have discussed finite time regulator problem what I did it here this term is this and this term is have written a transpose you have to see a transpose P this term is there. Then this will be transpose, then multiplied by P, so it will be P, then it will come B R inverse B transpose into P. So, I am getting that one you know this A B C whole transpose is equal to C transpose B transpose A transpose.

That part I have used it here, so this is the that equation agree then this equation then P A this is like a P A minus P R inverse B transpose P. So, this is one term I have taken more, so I have added P B R inverse B transpose P, so this this cancel ultimately this remain same. So, this I am writing into this one a minus B R inverse B transpose, let us call this is I am writing P suffix k th iteration the value of P transpose P k plus 1 then P A minus B R inverse B transpose.

This I am writing k plus 1 k plus 1 is iteration this is k is equal to if I take this that side is equal to Q, then P k B R inverse B transpose P k what are the k indicate the k th iteration was the value of P. Let us call starting iteration is 0 2 equation P of 0 and this is means next t 0 equation what is the value of P that is k plus 1. In this way, I just inside the bracket it is the k th iteration outside the bracket is P value is k plus 1 iteration, in this way now you see this matrix Q is positive definite matrix, but it is semi linear R is positive definite matrix.

So, this product will be because it is multiplied by some matrix then positive definite matrix and transpose of that must be post multiplied by transpose of P multiplied matrix, so this matrix in turn the whole matrix is a becoming a positive definite matrix. So, positive definite matrix and positive semi definite matrix result will be positive definite matrix, so it is something like a standard reopen of function method function if you consider. Then you to find out stability of the autonomous system we know our condition is a transpose P plus P A is equal to minus Q you essence some positive definite matrix.

if A is stable the solution of the p must be positive definite, so that we are using here, so our expression now it is coming if you see I consider a k. This is the whole thing is a k a k transpose P k plus 1 plus P k plus 1 a k is equal to minus that 1 Q plus P k plus B R inverse B transpose P k. So, this place note down what I am written is here suppose e f a has A x dot is equal to x is there you want study the stability of this system by using open up function method. Then our condition is the solution if you do the solution of this one and with an accent value Q is equal to positive definite.

If you get solution of A is if you know a P if you get the solution of P is positive definite it indicates the matrix A is and stable matrix is all Eigen values are of A matrix is negative real parts. So, here also how to solve this iteration this one k is equal to 0 one two dot t when is k is equal to 0 means P of 0. So, you accent P of 0 P of 0 value was in positive definite matrix in such a way, so that this matrix is stable. So, you choose initial value of our recartic equation solution P in such a P 0 so that a minus B R inverse R transpose B is stable. Then this is I just mention this this is the positive definite matrix, so this is this is a stable matrix this is the positive definite matrix.

Then, the solution of this one i will get positive definite matrix whatever you got it this positive definite you use it here once again. Then you solve this equation and repeatedly you do and you say, finally that you will get the solution of P as k tends to infinity P is conversed constant a value. That we will discuss in detail next class, that conversion of solution of recartics manner, you solve it the conversion of this one solution and this is it turns to some constant value. So, here I will stop it.