Optimal Control
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Lecture - 40
Solution of Infinite - Time LQR Problem and Stability Analysis

So, last class we have discussed the solution of finite time regulated problem, the basic steps of solution of finite time regulated problem is, first we have to solve what is called dynamic matrix quadratic equation. Once that you have to solve by what is called knowing the final time straight weighting matrix, you have to backward integration the matrix quadratic ((Refer Time: 00:53)) equation, which is A n into n plus one by two dynamic first differential equation and coupled. So, you have to solve it backward integration and stored the values of this information of the matrix matrices at different instant of time from t f to t 0, you store it. Then you implement the control law u of t again, once you implement the control u of t, then question comes, then what is our optimal cost value?

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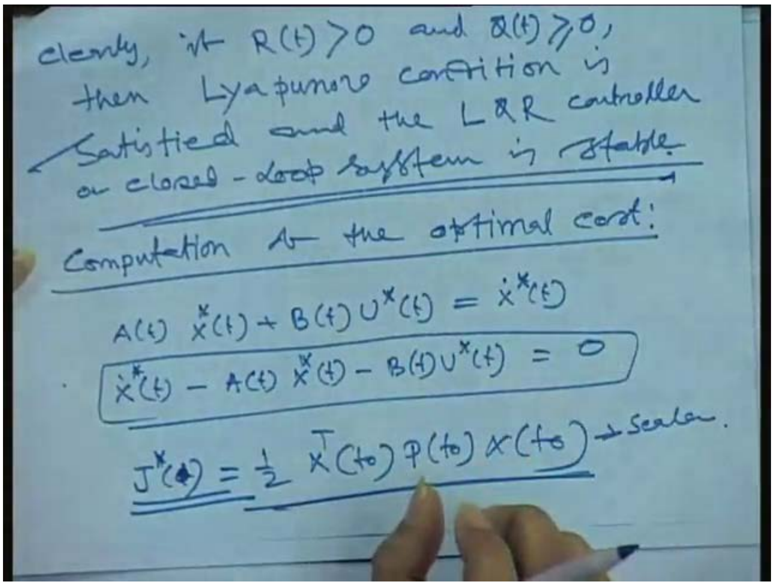

The Objective function below what is our objective value, that means we have A terminal cost plus integral of the quadratic terms, and to the objective function and that we have to compute from 0 to t is equal to t 0 to t f, so that we are discussing. Since we have obtained the optimal trajectory for control input which in turn we can get the optimal trajectory for the state trajectory. It must satisfy the solution of that our control in termed as state trajectory must satisfy the constance, what is that constance X dot is equal to b u plus b u, that constant you have to satisfy.

That means you could take this is that side this equation must satisfied and you can show that optimal objective function value is nothing but half X transpose t 0 P of t 0 X of t 0 X T 0 is the initial time of the state and P t 0. When you do the backward integration of the recartic matrix, recartic equation dynamic matrix equation from t is equal to t f to t is equal to t 0, then t 0 value what is the recartic equation. What is the value of P matrix, that you have to use it here that to get the optimal cost function value, so let us see how we achieved this one. So, today I will discuss that how we are achieved that optimal cost function is equal to j star is equal to half X transpose t 0 into P f t 0 into X of t 0, how you have achieved, so let us see this one.

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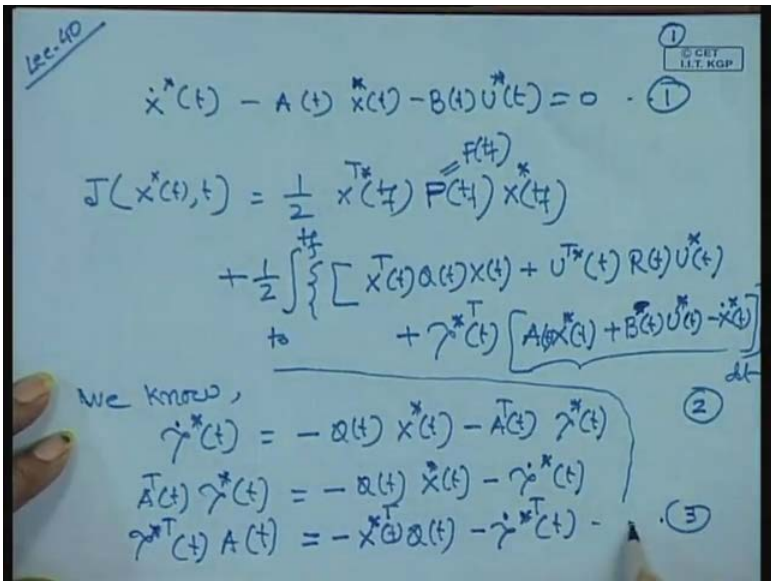

If you consider that one our equation that basic equation what we have considered X dot star of t minus A t X star of t X star of t minus b of t u of t is equal to 0. Let us call this is the equation u studies our optimal trajectory for controlling the documentary trajectory that we have found out by optimizing our performing index and correspondingly state trajectory we can get. So, this equation, the constant that exhausting is you must be satisfied by this optimal trajectory X star as well as the input optimal trajectory e star.

So, this is the basic information, so our performing index expression is like this way these half t is equal to half X transpose t f star along the optimal trajectory that A f of t which is nothing but P f t f. This is nothing but equal to terminal state weighting matrix this f of t which is nothing but A P of t into X t of t star.

This is the A terminal cost class integral of the what was it called quadratic term, this is equal to state and input vector is that one that 1, 0 to t 0 to t f. Then this one state weighting matrix X t u of t X of t plus U star of T plus r of t U star of T plus lambdas star of t into A X A function of t X star of t minus A plus b star b u of t v of t U star of T. This is b of t U star of T minus X dot star of t this quantity is X 0 0 multiplied by this objective function. Value will not change even if you multiply the optimal vector, it will not change, now let us say this one actually our objective function values.

This class, the term where you have added this, because along the trajectory these will lose this is 0. So, it does not, it will not change the objective function of this value of this function, now let us called this is the equation number two as we know from then derivation of the finite time regulated problem derived this one. We have seen that we know that lambda dot star of A is equal to minus t of t X star of t minus A transpose of t lambda star of T. If you take this this term in the left hand side this will be A transpose of t lambda star of t is equal to minus u of t X star of t minus lambda dot star of t.

This you can write it, that you can write it lambda dot if you just make it this the transpose of this both left hand and right hand side, you take the transpose left hand side and right hand side. If you take the transpose then you will come lambda star transpose of A into A of t minus X star of A transpose u of t actually q transpose of t since q is A symmetric matrix. Then I can write P of X next is this is equal to lambda dot star transpose of T, let us call that is equation number three. This is the equation number three, this is the whole equation number two and this is the equation number three, this one, so this is the one expression we have got it further.

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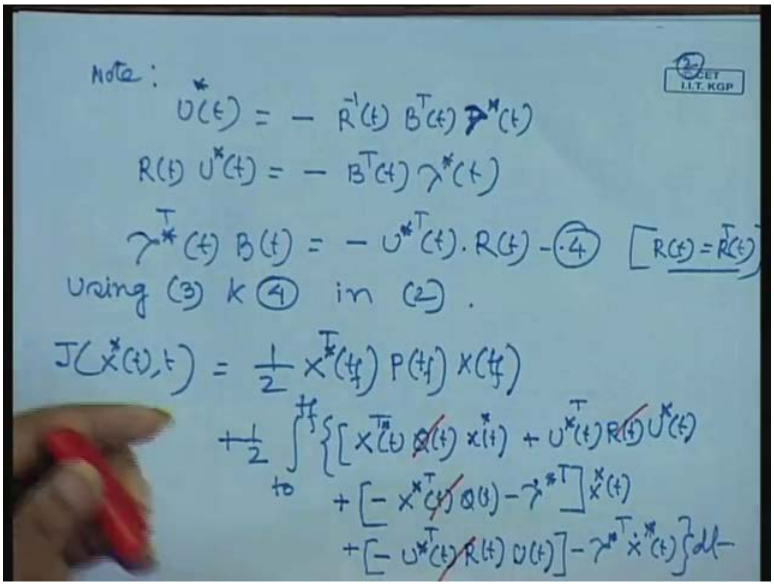

We can note it that u star note that our control input that U star of T optimal controlling minus are inverse b transpose P of t P of t. Actually, then we have A lambdas star of t when we have deducted this one U star of T, this c at the diversion you got it minus R lambdas transpose lambda star of T. So, this we can write it if you take this both side and multiplied by R, if you both side multiplied by R of t is U star of T equal to minus is B transpose of T lambda star of T. Take the transpose of both sides, then it will be A lambdas star transpose of t b of t is equal to minus u star t transpose of t into R of t transpose.

Since it is a symmetric matrix of transposes are R, so you write it since it is symmetric matrix are transposed is equal to R to this due to symmetric matrix. So, this is the equation number four using equation number three equation and four in equation number two. You just replace that one lambda star transpose of T into A t replaced by this expression.

Here, once again lambda star transpose B of t lambda star b of t, you expressed by this equation four using three and four in two we will get it that j star j X star of t of t is equal to half first term terminal cost will be as it is. There is no change in here P of t f X of t f then half t 0 to t f, then this will be A first term is X transpose X star transpose u of t. Then X of t transpose plus u transpose t r of t u transpose U star of T that is the first two terms as it is. Now, I replace lambda star T transpose of T star by A t into this expression this expression again, so if you replace this one by that one, you will get it plus and replacing that one by minus X transpose of t.

Then, your Q of A minus lambdas dot star of t the whole thing multiplied by see lambda star transpose t transpose A t and replacing by this one that is what you are getting then multiplied by X star of t. Then this lambda star transpose B t and replacing by plus lambda star B t replacing by that quantity minus v star lambda star B. I am using minus u star transpose of A r of t into u star that is u star is there, this u star, so u star that I am replacing by this one and what is that this one this two times lambda star transpose X dot star.

So, it is left with minus then 1 minus lambda star transpose X dot star of t the whole thing d t, now see this one this you push it in the bracket then this and these cancelled this and these cancelled. We can write it this one this is nothing but an integration of left is integration of minus lambda dot star transpose X star of t minus lambda star transpose X dot transpose, these two things combinely, we can write it into this form.

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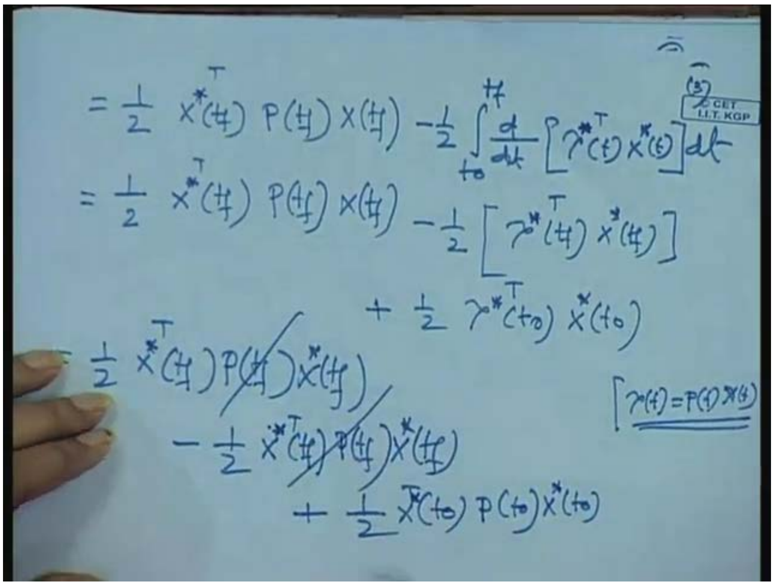

Rewrite this equation that half X star transpose t of f, P t of f, X t of f, then minus because both are minus term here minus term is there you see. So, I am taking minus half, then this I can write it t 0 to t f d of d t lambda star transpose of t X star of t d t. So, these two term, combinely these two terms combinely write it as form, so if you take integrate this one, I will get it first term will be as it is because it is constant depending on the value of X t f P t P f this constant minus. Then what is the limit of that one i will get it this X transpose but there is A one term this is this lambda star this X x dot so i will get it this.

So, if you do this one what you we get it that one lambda star t f transpose X t f star that lower limit, if you take lower limit if you take minus is there minus plus half X star transpose of t 0 X of star t 0. So, this quantity lower limit it is a upper limit, then lower limit I have written then lower limit I have written, now see this one we know lambda t f is nothing but A, our P X t f that they are related to the lambda t is related to P t into X t.

This one which I have used t f that I take that one equal to that transpose I will get this X star t f P t f X t f that minus half lambda star transpose those first will come X star this is the star X star transpose t f into P t P f is equal to t f into X t f star. So, this term is this one plus half that will be X transpose t of 0 star, then P t of 0, then u what is call that X t of 0 star, so this these cancelled.

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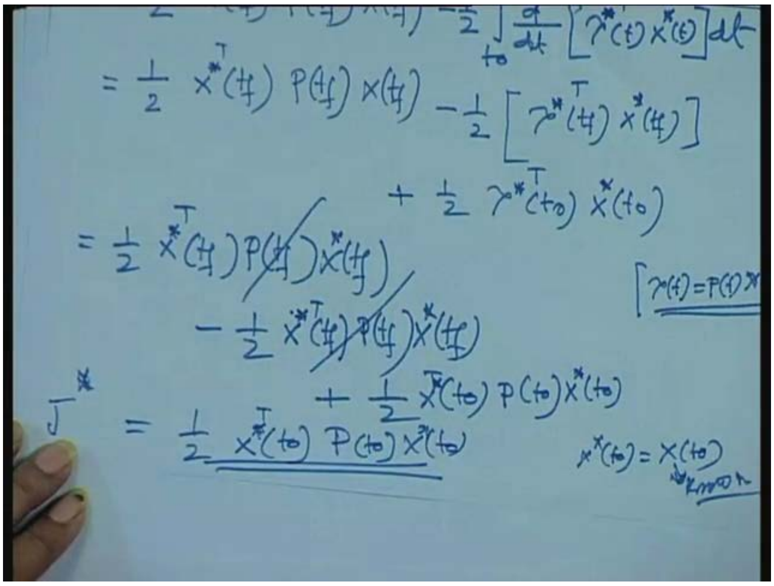

So, our optimal value of j star that our optimal j star will get it this half X transpose of this t 0 into P t 0 plus X t 0 star, now you see initial value of this system is known to us. So, this is nothing but A X star t 0 is nothing but A X of t 0 is known initially known below known and this P 0 we have computed the solution of matrix dynamic matrix backward integration. Backward in time, we have integrated and we have lambda with t of 0 this will be 0 this value is also known. So, you can we can find out what is the optimal cost of this functional or objective function corresponding to the what is called finite time regulated problems what is optimal cost this one, this is the important relation that is the half X transpose t 0 P of t 0 X.

Now, next week we consider what call finite time regulated problem we have consider is. Next time we will consider that is what is called infinite time linear quadratic regulator for linear time in variance systems, so next we will consider the infinite time regulated problem.

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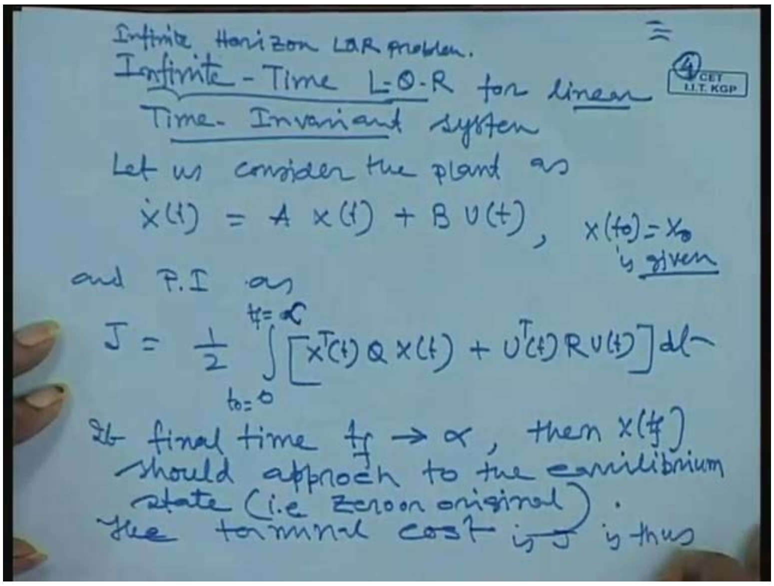

Infinite time linear quadratic regulators for linear time in variance systems time in variant systems. This is linear quadratic regulated problem, infinite time linear regulated problem also called that that horizon infinite, horizon finite time regulator problem horizon infinite horizon LQR problem infinite horizon problem. Previously, we have considered finite horizon linear quadratic problem, now infinite and L Q R problem also called infinite horizon L Q R problems.

Then, what is the problem is this different from this one previous case finite time regulated, if you recollect the our finite time f finite that means that time t is equal to t f our state from initial states t of 0 will drive. You will go from initial to final state which is close to the horizon or desired position of this one by using A what is called suitable control effort.

We have derived the state near to the as small as possible the near to the desired values that, so here we consider an infinite time regulator t f times to 0 when you are talking about t final t times 0 finite times 0 by using the control law. What we will see our state will go to the equilibrium position equilibrium point and equilibrium point is nothing but a linear system case is an origin. So, at t times to t f is equal to 0, so if you consider that terminal cost if you see that terminal cost terminal cost is not coming into the picture because at t is equal to t f when t f times to infinity. This state will go to the equilibrium point which is nothing but a origin of the coordinate systems.

So, that terminal cost is not coming to the picture into part of this the integration of the quadratic term associated the performance index that minute will come from t 0 to t 0 t f is equal to t f where t f times. So, that will be the our corresponding perform index, so let us identify that our problem that one let us consider the plant as X dot of t is equal to A of X t last b and we are considered in the problem we have considered linear time in A is not A function in A time now if it is A function in time still we can write it this one joint form.

So, A f t B replaced by A and B since it is A linear time in and you are initial state is given X t of 0 is given, so the corresponding performing index as j is equal to half 0 to infinity t 0 is equal to 0 to infinity t f is equal to infinity because t f. Now, infinite time is infinity then X transpose of t q X of A plus u transpose of t r of t u of t whole multiplied by d now you look at the terminal cost is not coming to the picture because when our t f the final time is t f is infinity it indicates the with the control effort. Then our state will go to the equilibrium point and equilibrium point in our system, real system is real point is origin. So, the terminal cost X t f will be 0, so terminal cost term terminal cost term will not come into the picture and our t f.

So, you have to solve this optimization problem, if you recast this problem in it will be like this way a problem is designed the control U. This performance is minimized subject to the condition that this dynamic equation system is satisfied that is our problem, so if that is what I have mentioned it, that I will write here for your convenience if the final time t f tensed infinity. Then X t f should approach to the equilibrium state, I get 0 or origin of the coordinate systems or in the all the states will write to the 0. So, naturally this this terminal will not come in this problem the terminal cost.

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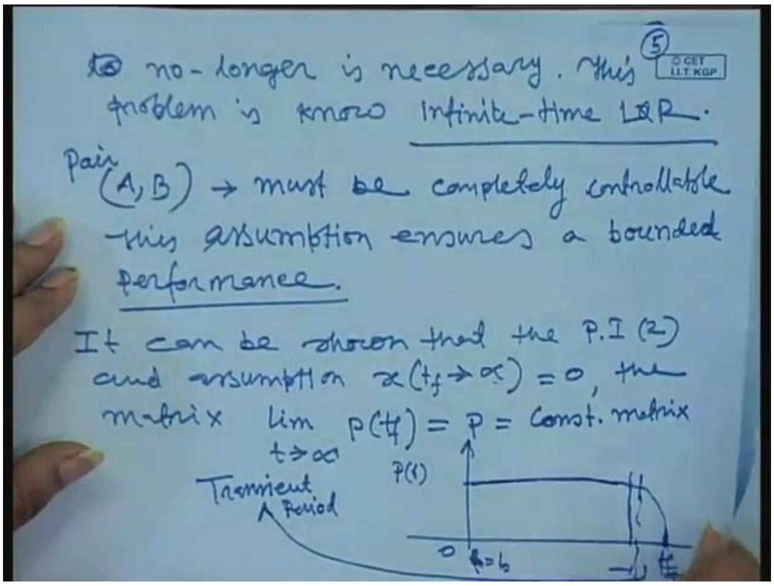

The terminal cost in j is thus no longer necessary, so this problem is known as this problem is known as infinite time help your problem or infinite time infinite time horizontal horizon problems. So, infinite time problem, so our previous formulation for infinite time regulator problem previous formulation for finite time regulated problem is still valid. There is an additional assumption is made that the system A and B must be controllable or at least it should be A stabilize able, so this is the addition of condition is paired.

That means in this case from pet to our finite time regulated A and B must be controllable then our performer index will be A bounded performance index this one. If it is A and B are controllable completely controllable or stabilizer stabilize able the state by due to using the control law. That can stabilize the state, so our assumption from the previous one, the finite time regulator that A B P r P r A B must be completely controllable.

So, this condition this assumption ensures a bounded performance bounded performance because time t is equal to 0 this performance will be bounded that means this or when you find out the objective function or that is our optimum control. If you use in this expression t is equal to time t is equal to 0 usual to infinity then this expression value will be bounded if this assumption is considered. That means the state trajectories of this one X of t will be stable one, so this is our basic assumption keeping this thing in mind these two things it can be shown that performing index to the various systems. This is the system equation, this is our system equation, let us call equation number one this is the performing index.

This is the index number two, so if you see this solution of this one that means it can be shown that the performing index to and the assumption X t is equal to t f tense infinity is equal to 0. The matrix limit t tends to infinity P f t f will be A P which is a constant matrix, what does it mean that if the assumption is consider A B and this t f is tensed to infinity. This value A is to the equivalent to the point of this system, that means origin then it can be shown that our solution of P is constant when t is tends to infinity solution of this one constant. If you recollect the infinite regulated problem that our solution is see it, let us call t f is very large origin and it has saved t f in finite case, so you have seen it, if you see this values are measure of the P is like this way.

So, this is A P of t expression this is the t f and this is equal to t is equal to t 0, then this period from here to here this period is the transient response of this our solution of our matrix P dot P dot is the matrix. So, this is the transient period this from here to here is A transient period transient period that means during this period the value of P is changing.

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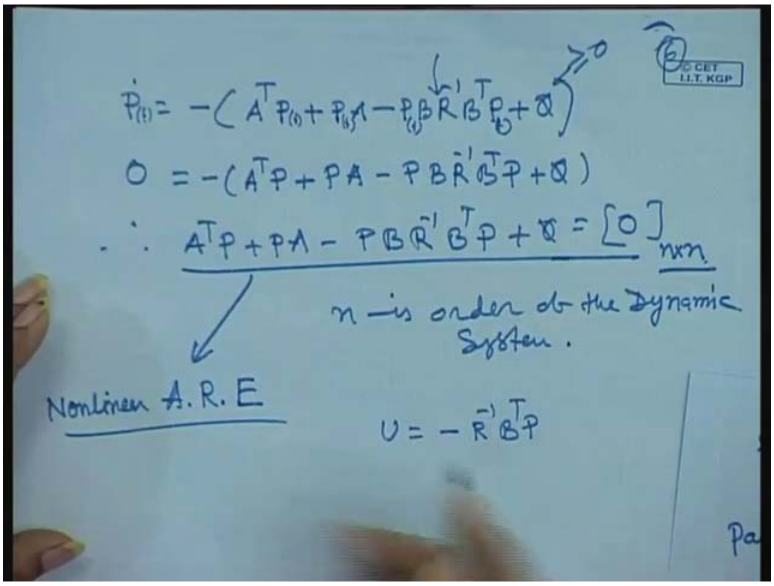

Let us call up to this after that the value of P is constant value of P is constant even for finite time regulator. Similarly, now if you put it in the limited in t f is now increasing in approach to the infinity. So, the solution will still will get same nature of just for sum period of time is A transient period is got P of t will be changing and then after that it is a constant.

So, if it is then our the finite time regulator expression if you see P dot is equal, then our P dot expression is this P dot is equal to in finite time regulator problem is A transpose because our system is time invariant system means it is not A function of time. These are equal to P plus P of t P of A plus P B are inverse B transpose P plus Q is that one since P is constant. You say these values when t is t tends to infinity that actually this is equal to t if you write it our P of function of t more specifically function of t if you write it. If you consider the our state weighting matrix Q is constant which is which is also positive semi definite matrix was it in semi definite matrix and our input weighting matrix are is also what is called constant.

Assume that it is A positive symmetric matrix, so since t f is infinity the most of the time after the after that transient period is over this is constant. So, during this period from t 0 to you see from t 0 to sufficient long time this one P is constant, so this term will be 0, then I can replace that one is A transpose if P f t is constant P then P f t is constant A, then P B are inverse B transpose P plus Q.

They are proved, A transpose P plus P A P b are inverse b transpose b plus q is equal to null matrix the dimension is n into n, n is the n is the n is the order of the order the dynamic systems. This is called this whole equation is called algebraic recartic equation, now linear more specifically linear algebraic recartic equation because there is no dynamic equation is involved and P dot is not involved for infinite time regulator problem.

Now, question if you must know how to solve that one if you can solve this one, then our problem becomes is solved that means we know our that U is equal to minus R inverse B transpose B P is constant. Most of the from t 0 to suppose in the long term this is constant only short period of time these value of P is changing by the time, this is the transient and form here to here is a steady state value of P steady is steady value of P.

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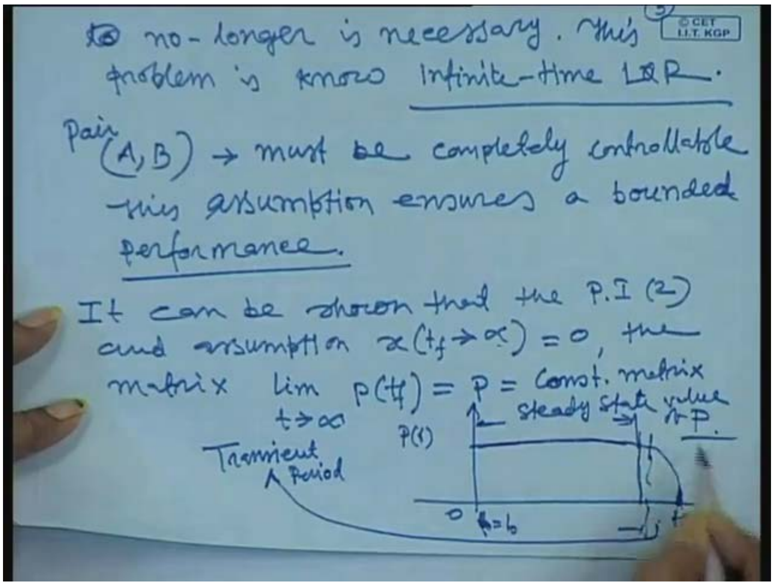

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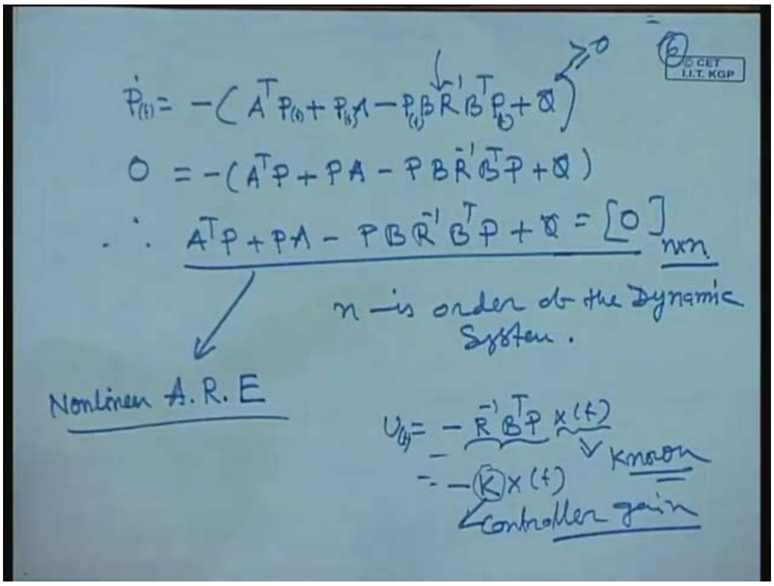

So, that u of t is nothing but A R inverse B transpose P into X of t and this X is known we are assuming that states are accessible from the system is known if is not known. We have to by some means we have to estimate or you have to get the information of this states which will call estimation or observer of we one has to introduce to observer that estimate the state of this system. So, this and this this is nothing but A if you use k R inverse B transpose to B is k, then it is A this k is this k is call controller gain.

One can obtain the controller gain is nothing but R inverse B transpose P and this P is constant and P is what the solution of this non linear algebraic recartic equation. So, our first step is solution of non linear algebraic recartic equation, this is the non linear algebraic recartic equation, you solve it you will get P once you get P. then you know what is k R inverse B transpose P is k, once you know k know what is U and then just because you solve this one or in real time what will I, you will get the information of X multiplied by k.

You have solve d this equation regarding offline and you know the information of A, so multiplied by X, so it is the state feedback controller k is designed based on minimizing the what is call our objective function. This is called infinite time regulated problems that due to some initial state disturbance this controller will drive instead to a linear position are equilibrium point.

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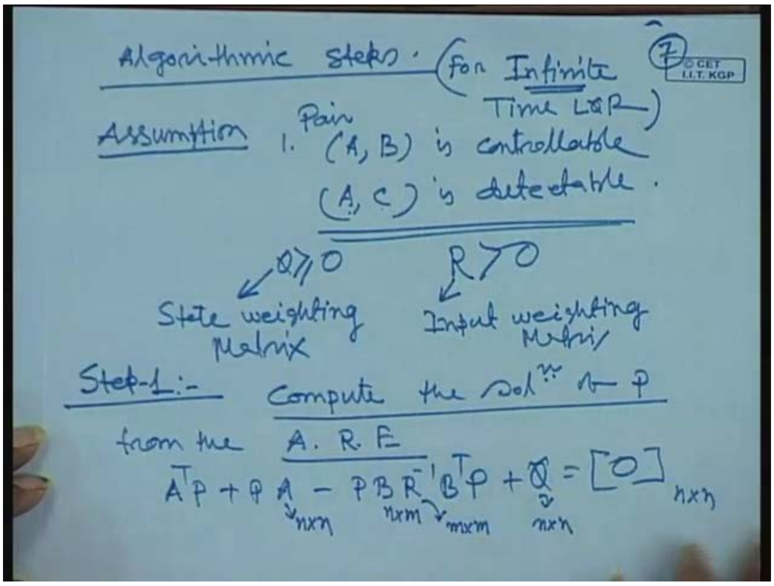

So, if you see this one that that what is our algorithmic steps algorithmic steps for finite time infinite time help your problem or horizon L Q R problems infinite time horizontal control problems infinite time horizon control problems. You can said in infinite time L Q R problem solution for this one, so first thing we have made the assumption we have to check it the assumption the pair A and b first assumption is pair A and b is pair. This A is controllable A and C is detectable, so what is controllable you might have been read it in a first course of linear control theory. With the system is controllable by using and control effort we will be able to drive the state at time t is equal to 0 to A deserved state at P is equal to t f as desired state we will be able to drive it.

If the system is controllable or at least it is stablizable and this A and c must be a deductable this this one, so this is the first assumption if this assumption is valid then and also we have made the assumption re correct that our state will get. This is the Q is the state weighting matrix state weighting matrix associate in the performed index that must be positive semi definite and in the input weighting matrix input weighting matrix must be positive semi definite matrix. So, these things is our first step with this assumption if the system is not controllable then will not proceed further that means we are the system must be controllable or at least stabilizable.

Step one, which compute the open loop system may be unstable, but it must be controllable or at least stabilizable, so compute the solution of P from the algebraic recartic equation. So, A transpose P plus P A minus P b are inverse b transpose P plus q is equal to null matrixes is dimension is and if X is the number of states which number of states is the small n or of the system is n. Then A is the dimension n cross n v is the dimension depends on the number of inputs u if u is n that dimension of b n cross m that dimension of b n cross m and the dimension of A is n cross n or at the system.

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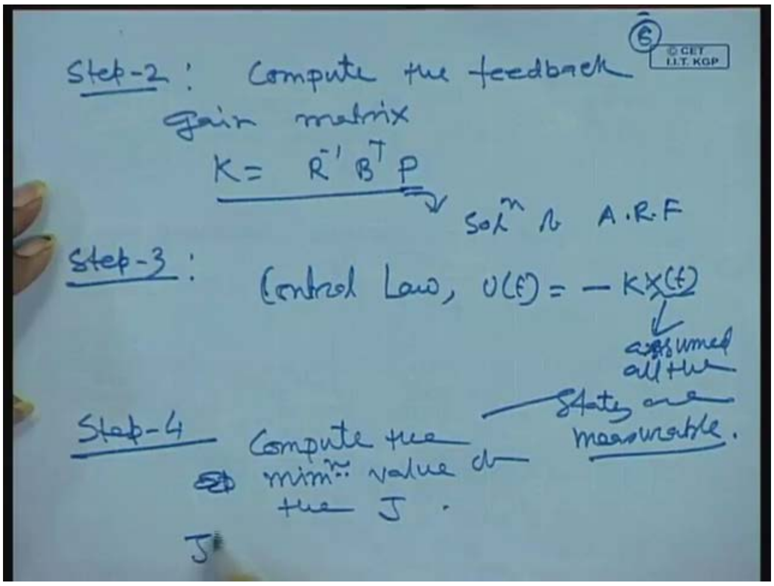

Then, this immediately this dimension is n cross n the dimension of r is n cross n it depends on the number of inputs and one has to solve this one. We will discuss later that how to solve the algebraic recatic equation which is non-linear algebraic recartic equation. What are the methods are available to solve such type of algebraic recartic equation so once you compute this one from this step you compute that one.

The step two, once I compute the solution of algebraic recartic equation solution of solution the our algebraic recartic equation, then step two compute the controller gain compute the feedback gain matrix k is equal to R inverse B transpose P. You know this is known by solution of this is nothing but a solution of algebraic recartic equation as it is known. Once you know this one, you can compute take four step three, you can compute that controller u of t is equal to minus A X of t assume all the states are measurable. Then once you know this one once you know this one then immediately you can find out that, what is called our cost objective function value that compute the optimal or compute the minimum value of the J that means J star.

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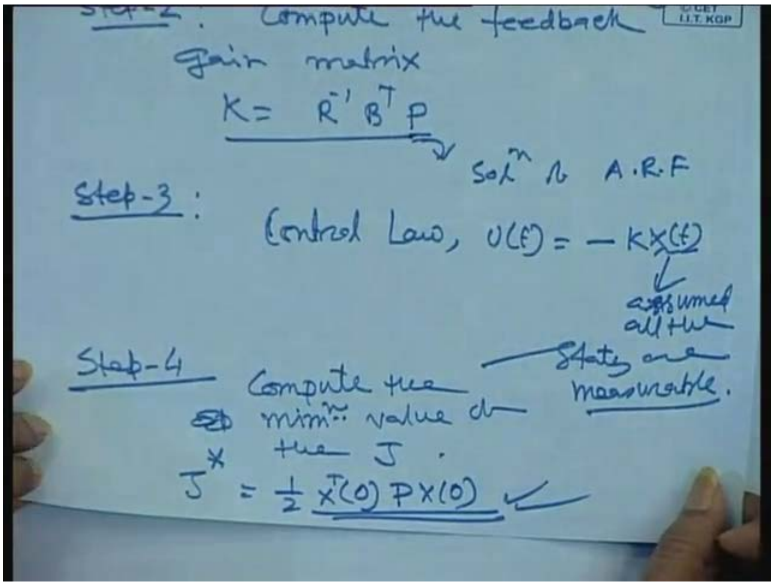

I can compute j star is equal to half X transpose of t 0 m t 0 is equal to 0, then this solution of the recartic equation P then X of 0. So, this you can compute this is the steps of how to solve the, what is called finite time regulator problems, this is the solutions. So, if you see this stability analysis close loop system it is exactly same that what you are considered in the finite time regulator problems.

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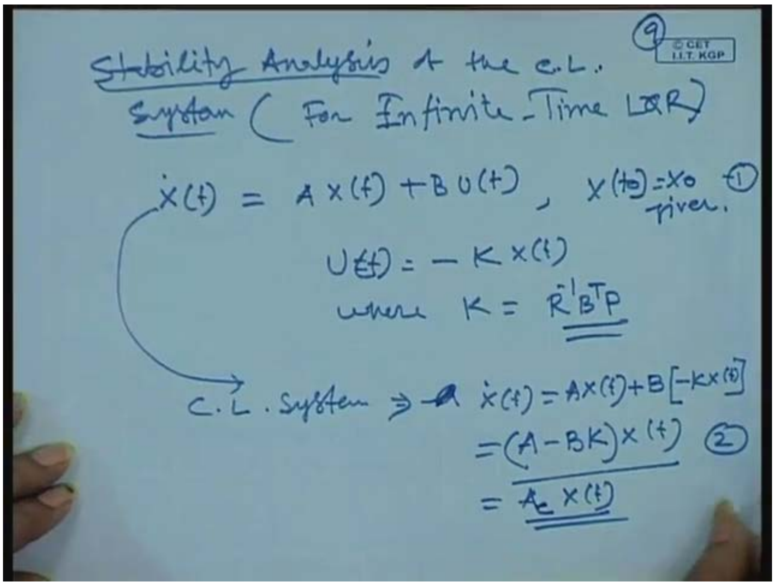

So, stability analysis of the close loop C L close loop system bracket for infinite time L Q R infinite time regulator problem that is in short it is called as regulator problem, so this ability. So, let us ask c destiny of the problem our system description is A is X of t A is the system is linear time in variance system, so B of t. Our system can state is given initial state is given if you say our control u of t is equal to minus X of t where is A is you have R inverse B transpose P. This way you have generated P, so what is the close loop system close loop system.

If you just write it use the value of k this expression u k X is the expression, then it is a close loop system X dot close loop system X dot is equal to A X of t plus b what u minus A X of t is so this is if you simplify. Similar way this one A minus B k whole X of t, now let us call this is the equation number one and this is the equation number two. So, this our close loop in short we have denoted by A suffix c close loop of X t you are denoted by this now look at this what is the what is the how will you check it whether the system is stable or not one is way of checking it.

Since you have calculated the A, put the value of k here find put the value of k here diagonal value of this one. Analytically, how to check, the still is I told you exactly same way what we have described for finite time regulator based on the function of the technique can check it the stability of the system.

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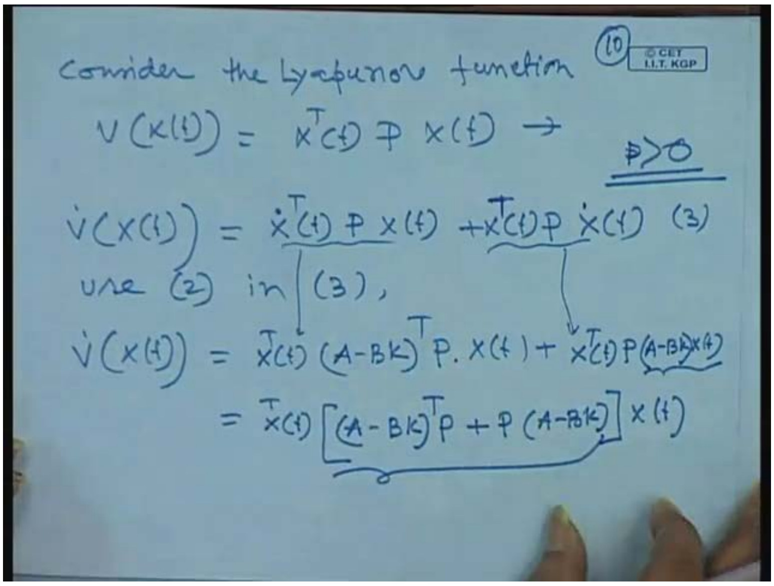

Now, consider the simplest way of function is like this way P of X t is equal to X transpose of P X of t agree and this is the quadratic form this is energy function always what is called the value A is greater than or equal to 0. When X is 0 will be 1 when the states of 0 will be A 0 and this value will be always greater than 0 is equal to 0 because P is positive definite matrix P is positive definite matrix. So, we have selected the energy function like this way now v dot, so if energy if due to some initial condition the energy contain in the system is that one. Now, I will see whether this energy is decreasing with respect to time or not that how to be v dot we have to find out with time as time is increasing whether the energy is decreasing or not.

That means negative definite, so what is v dot, so differentiate with these two time, then you will get X dot t of A P X dot of t plus X transpose of t then P is constant. We are not differentiating this is the constant in earlier case P was A finite time regulator P is the function we got it three terms, but here is two terms. So, put the value of X dot from equation two use equation two in equation three in three, then v dot X of A is equal to A minus A this is our equation two X dot is equal to this so X dot transpose to it will come to as A minus B k whole transpose.

So, I am writing X transpose A minus b k whole transpose then P multiplied by t this is the first term I have written here. Then next is this term I am writing here this term I have written here this term here and this term is transpose of P and then what you have this one A minus B k into X of t this is X of t. So, this is this is two terms I have written here X transpose P and this our transpose P, I have written, so if you just simplify this one X transpose of A is nothing but A minus B k whole transpose P plus P A minus B k this into X of A. So, the v dot v dot is the algebraic recartic function this provided this is a negative finite matrix again this is negative definite matrix and P is the algebraic recartic equation solution. So, these matrixes if you can prove it that this is the negative definite matrix, then our stability analysis proved that for infinite regulated problems.

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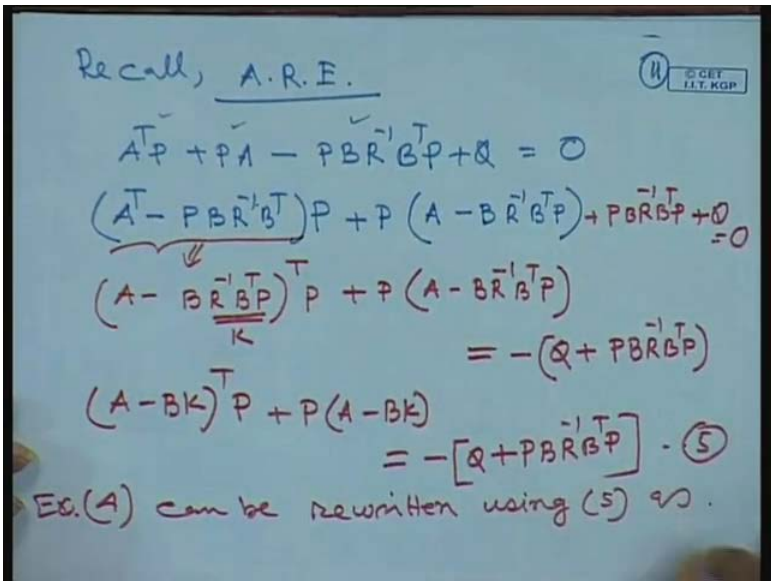

Let us call this equation is four, so recall our recall that algebraic recartic equations, so you recall the algebraic recartic equation that our A transpose P plus P A minus P B r inverse b transpose P plus q is equal to 0. Now, I will just say what am doing it here that P b t are inverse i am taking P common from right side post and b transpose. Then whole multiplied by P, so it is A transpose P this term is coming P B r in the transpose of P it is getting. Then I am getting P, then you are writing is that one is A then P B not P are inverse then b transpose P, then you see A transpose P B r in the transpose of this also gone P A.

This also gone, but there is an additional at the term P minus P B r inverse B transpose, I will just add another term of the term P B r inverse B transpose P plus this term, so this our equation just remains same. Now, you can write is this one you see A minus what you can write it for this one A minus if you write it transpose of that one. So, A minus b are inverse b transpose of P whole thing transpose into P so first term you see as it is.

So, if you take the transpose P transpose means symmetric matrix transpose on P B R inverse B transpose B this this one term I am getting. So, this is same as that one plus P then A minus B are inverse B transpose P and this equal to take this is to that side minus q then minus will be plus because minus am taking common. So, P B R inverse b transpose P, so if you see these quantity is nothing but A, so it is nothing but A minus B k the close group system transpose P plus P A minus b k is equal to minus q plus P b r inverse b transpose P.

Now, see the left hand side I can easily replace that is what called equation five is using before this equation using equation five and four where this term is same as this one. I can replace by the right hand side equation four equation four write here not here equation four equation four can be rewritten using equation five, that if equation four this equation four.

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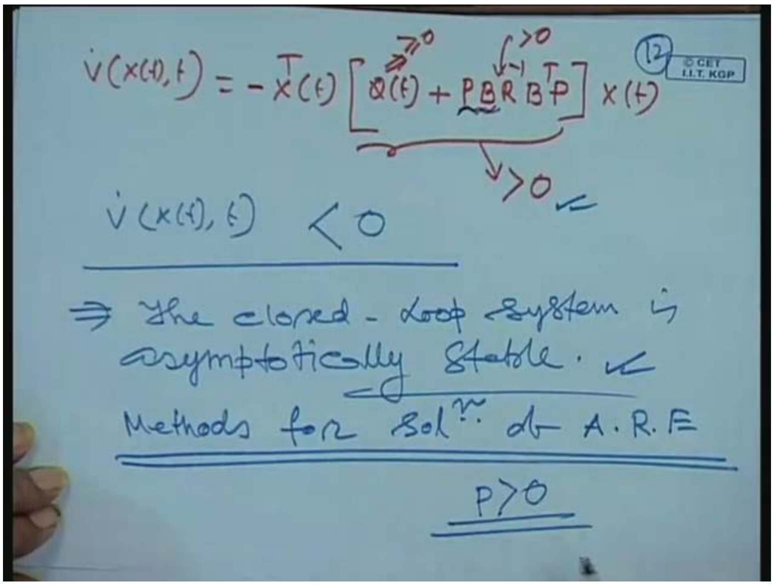

So, v dot equation four is v dot so v dot X of A of t. This one X transpose of A this one then am writing minus so minus time is taken out so q of t plus, then u of P B R inverse b transpose P again plus X of tense. So, this equation this equation I replaced this equation left and side replaced by that one in equation four drive, so I got it that one. Now, this change in this function will be must be negative when you have to become the system is stable. So, the negative sign, so this matrix must b this matrix must be B positive definite, I can in the similar minor what we have shown in the finite time regulated program same thing we can use here.

So, since our assumption Q is Q is our greater than equal to 0 Q is greater or equal to 0 R is positive definite matrix say if r is positive definite matrix say if r is the inverse is also inverse definite matrix is proceeded with the some matrix. P multiplied by some post multiplied by same matrix transpose, so this is the matrix you say this is the in A. This is the matrix, so r inverse multiplied by P multiplied by some matrix combination of B and B and post multiplied by transpose of that one. So, if it is so then we can also say the combined matrix is also A positive definite, since r is the positive definite matrix.

So, the resultant of this one since it is positive definite matrix since it is positive symmetric matrix resultant will be positive definite matrix, so this includes that v dot of A is less than 0 because this is positive negative sign. So, this implies that the closed loop system is asymptotically stable in the sense at time t is equal to 0. The states will come back to what is called it will be in position even though there is initial condition their initial disturbance is given to the system at t is equal to t 0.

So, the system is asymptotically stable next question comes in mind that how to solve that what is called algebraic recartic equation. There are different methods are there one some methods are there what is how to solve the algebraic recartic equation some methods are used for numerical solutions itetrative methods to solve the algebraic recartic equation. So, next class we will discuss the methods for solution of methods for solution of algebraic equation, so this again this is the method how matrix is algebraic recartic is solved.

Also, we will discuss the problem is given that whatever we are considered finite am regulated instead of t. Now, t is equal to sorry X t 0 is equal to t f is equal to t f, now t f is equal tense to infinity and correspondingly our algebraic recartic matrix equation. Then will be converted into an algebraic recartic equation and we have to solve the algebraic recartic equation. If you get the solution of P is positive definite to solve this algebraic recartic equation in such A way so that P must be A different matrix, then correspondingly one solution will get it that controller gain that controller gain will stabilize the system. The state will bring down to the control effort which will come to equilibrium state again; we will stop it here now.