Optimal Control
Prof. G. D. Ray
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 4
Optimality Conditions for Function of Several Variables

So, last class we have discussed that what is the quadratic form of a function, that quadratic form of a function maybe positive definite, positive definite function, positive semi definite function, negative definite function and negative semi definite functions. Then we have discussed the, what we mean by that positive definite matrix, positive semi definite matrix, negative definite matrix and negative semi definite matrix.

Then, how to test a matrix is a positive definite matrix or negative definite matrix or positive semi definite matrix or negative semi definite matrix or function, quadratic function is positive definite function or not. That can be tested through Silvestre criteria and Silvestre criteria requirement is there, the matrix p must be symmetric. So, we have shown it if a function is expressed in quadratic form, if the function is in quadratic form that can be expressed, x transpose p x and that p always we can select, that from the quadratic function is symmetry, that we have shown it.

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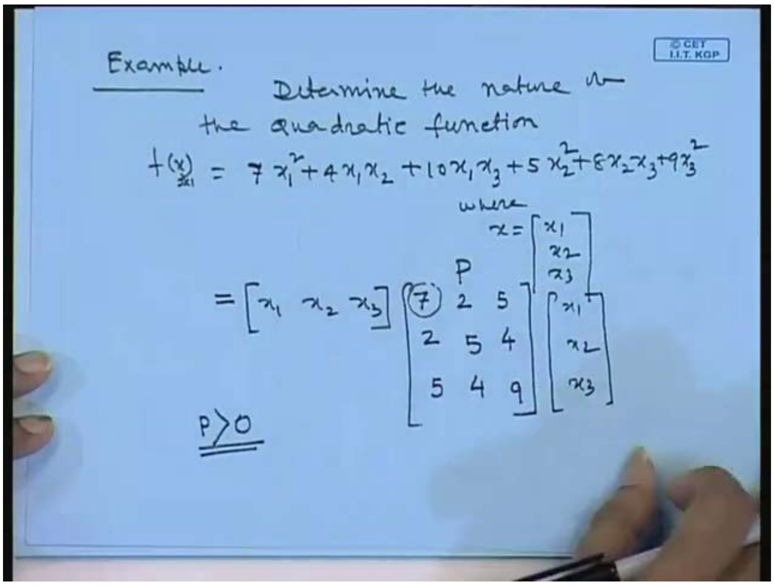

And we have, if you recollect that we have considered a function of, this function of x 1, x 2 and x 3, our job is to whether these, what is the nature of this quadratic function. Whether it is a positive definite function means, for all values of x, x 1, x 2, x 3 the function value is positive definite or not. Nature of the function it may be positive definite, maybe negative definite, maybe positive semi definite, maybe negative semi definite.

So, we have to check, we have to test that whether what is the nature of this function. With this one, we have formed x transpose p form and where, from this quadratic function we have written p is symmetric matrix and we have used, after that we have used the what is called Silvestre, Silvestre criteria for finding out the positive definite matrix.

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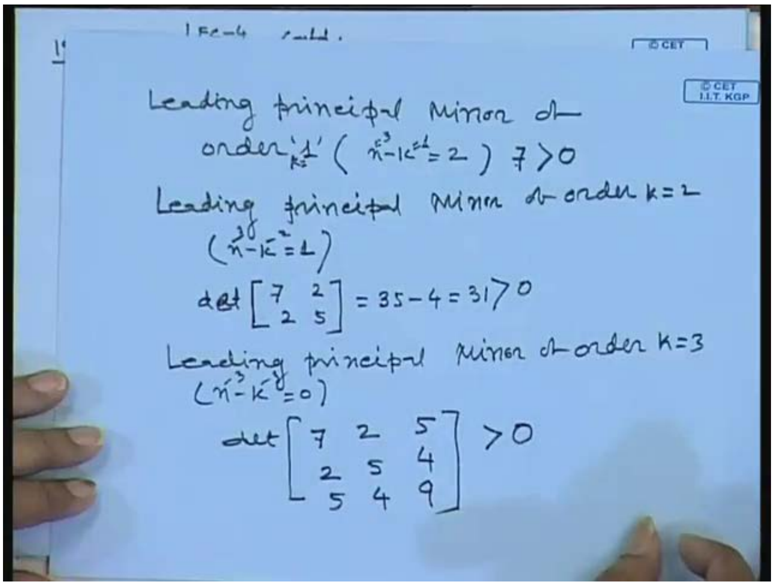

So, all leading principal, leading principal minors of order 1, order 2, order 3 and so on should be greater than 0, it that is. Now, next is suppose a matrix is, whether a matrix is negative definite matrix or not, how to test. The same idea, what we have considered for testing, for positive definite matrix same idea can be extended here.

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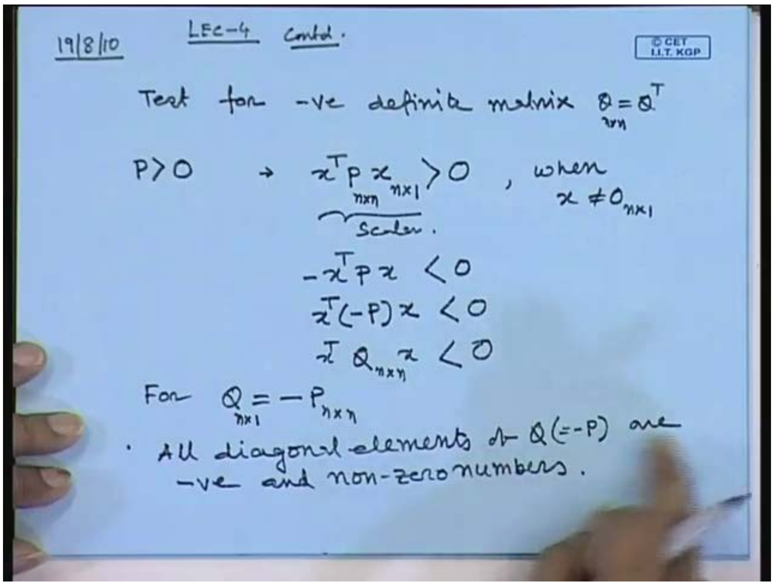

Let us take test for, test for negative definite matrix and when we are doing this test with using the Silvestre criteria, we assume that matrix is symmetric matrix. That means Q is equal to, let us call that Q matrix, whether it is the negative definite matrix, how to test it. So, and let us call this dimension is n cross n, if you recollect the positive definite matrix, positive definite matrix test that p is our earlier discussion, p is greater than 0, this indicates that p is a positive definite matrix.

In other words we can say, if we form a quadratic function with p and x, x is any vector of dimension, comfortable dimension with p. If you pre multiply it by x transpose post multiply by x and it will form a quadratic function. And this value, if p is a positive definite, this value which is a scalar quantity is always greater than 0, when x is not equal to null vector of dimension n cross 1, that is the by definition of positive definite matrix.

Now, if I multiply it by, since this is a scalar quantity, scalar if I multiply it by both sides by minus. Let us call 5 multiply it by minus 1 so I can write once I multiply it by this I can write p of x is less than 0. So, this is the important step see what we can do next, then minus sign I can just push inside with P, that is also less than 0. So, this quantity is less than 0, if P minus P is negative definite matrix, by the definition of quadratic functions. So, minus P I am considering as a let us call new matrix Q, whose dimension is n cross n because dimension of P and dimension of Q must be same difference between P and Q is we minus P is equal to Q.

So, now I have to check whether this, this will be less than 0, provided Q is negative definite matrix. Just now we have seen P is the positive if multiplied it by this one minus P, this matrix will be negative definite matrix, but how to test it that whether it is negative definite matrix or not. So, if you proceed in the same manner for positive definite matrix, we will see what are the changes are there so for our case Q we have considered it here minus P, whose dimension is n cross 1, this cross n. So, P I multiply it by minus that means, all the elements of P is multiplied by minus 1.

So, what we consider P is the positive definite all the diagonal elements are positive. Now, since it is multiplied it by minus 1 P so all the diagonal elements will be negative and non-zero number. So, our test for this one, all the diagonal elements of Q or Q is what this, are negative and non-zero numbers. So, this less than 0 means Q must be negative definite and Q is nothing but a P, P is the positive definite we have started from this one, we multiply it by minus 1.

So, all the diagonal elements previously was plus since it is multiplied by minus 1 so all the diagonal elements will be minus, that means all the diagonal elements from Q are negative and non-zero numbers. Next is, next is you see this one, the test for positive definite, next by using the Silvestre matrix is leading principal minors of order 2, order 1 we have seen the diagonal elements in case of negative must be negative, order 1. So, order 2 what will be there?

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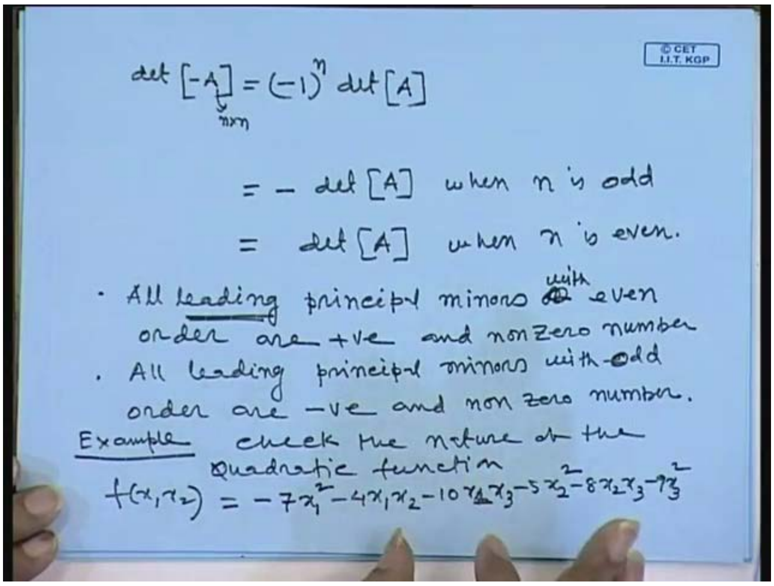

Before that I just tell you this things, you just, if the determinant of note this results I am using, determinant of minus A is equal to minus 1, whole to the power of n, where n is the dimension of this matrix A. I multiply it by minus 1 and finding out the determinant of minus 1 is equal to determinant of A. So, when n is equal to odd, when n is equal to odd, the determinant of the odd order or odd order matrix, determinant of odd order matrix with negative sign is equal to minus determinant of A. So, I can write it this equal to minus determinant of A, when n is odd. This equal to determinant of A, determinant minus A is equal to determinant A, when n is even. So, this results I will apply when I will check the test, when we will test the negative definiteness of this matrix Q.

So, you see Q is nothing but a minus P, that all the elements of p are now multiplied by minus. Now, you when you will find out the, what is called leading principal minors, there are two steps now. The all leading principal minors of, principal minors with even order, order are positive. All leading, please remember all leading principal minors with even order means, I know how to find out the leading principal minors. So, even order means the dimension of the matrix which will extract from the original matrix Q of either 2 by 2 or 4 by 4 or 6 by 6 or even dimension order of the matrix, we have to generate from the Q matrix.

So, the even order the principal leading principal minors determinate with even order are positive, not only positive it is non-zero numbers and non-zero. It cannot be 0 the determinant, non-zero number again, all leading principal minors with odd order are negative and non-zero numbers. So, determinant of odd numbers will be negative or it cannot be 0 number, non-negative numbers.

So, now this is the test of this one so alternative first order will be negative, second order, leading principal minors second order will be a positive, third order leading principal third order will be a negative. Fourth order leading principal minor fourth order will be a positive and so on, until unless you reach to the original system order that means, n cross n.

So, let us call the earlier example if I multiply it by, earlier example we have shown it that quadratic form is a, what is, what were the given is quadratic form is a positive definite quadratic form. That function is a positive definite function so I multiply it by both, I multiply it this function by minus 1. So, naturally this function will be a negative definite function. So, if you check this negative definiteness of this one, check the nature of the quadratic function and if you see the earlier example, I just multiplied it by the example by minus 1. And already example, we have seen that is the positive definite function that means, for any value of x this quadratic function value is greater than 0, when x is not equal to a null vector.

So, that quantity I now multiply it by minus 1 so that indicates that function value will be less than 0 for one. Now, how to test it, that function is the quadratic, the function which is given is a negative definite function, by the, what are the steps I mentioned you can check it with this one. So, I have just written, written this one x 1 7 x 1 square minus 4 x 1 x 2 minus 10 x 2 sorry, x 1 x 2 then minus 5 x 2 square minus 8 x 2 x 3 minus 9 x 3 square.

For this example, you check it that, that by using the Silvestre matrix, Silvestre criteria check what is the nature of this quadratic function, whether it I,s it should be negative definite function. Because, previously it was a positive definite function since I multiplied it by minus one, it should be negative definite. You check this thing whatever, I have discussed this one.

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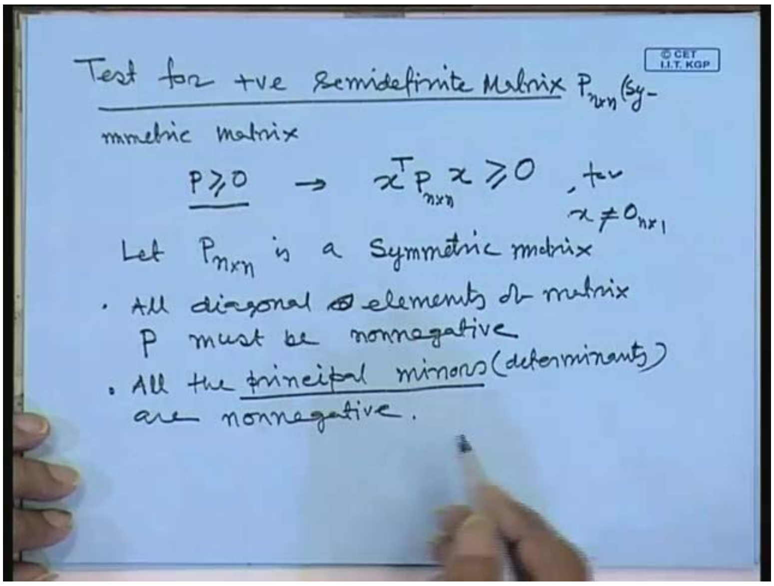

So, next is your what is called test for semi definite matrix using Silvestre criteria, test for, test for positive semi definite, semi definite matrix and that test you can do. So, semi definite matrix P, which is equal to n cross 1 and that matrix is symmetric matrix, I repeat once again that you can test the Silvestre criteria whether, the matrix is positive definite or semi definite, positive semi definite ,negative definite or negative semi definite.

You can test it if the matrix is positive definite, if it is not there I can actually easily convert into a positive what is called symmetric matrix because by definition of this one I can write that P is a, if it is a semi definite the symbol is this one. If it indicate P is greater than 0, means P is positive semi definite means, in other words you form a quadratic function with P and any vector x and its value, this value is always greater than equal to 0. It value maybe some value of x maybe 0 and other than x is equal to null vector and other values of x which is greater than 0, for x not equal to null vector.

So, this test how will you do it? The Silvestre criteria tells that let A is P is an n cross n is a symmetric matrix. Then what you have to do, you check first the matrix is given all the diagonal elements, it is a positive semi definite must be positive or some maybe 0 also, but it cannot be negative. So, it should be all the diagonal elements in other words, should be a non-negative numbers. So, all diagonal elements of matrix P, elements of matrix P must be non-negative. That once the matrix is given immediately I can say whether, it is nonnegative if it is a nonnegative then we can proceed further non-negative means it is maybe positive, some element maybe 0.

Next is all the, now I am not using the leading principal, only principal minors, all the principal minors means determinant are non-negative. This is non-negative, that means all the principal minors will be, the determinant will be a positive and some maybe 0, but it cannot be negative. Then we will call, this is a positive semi definite matrix so I must know, what is principal minors? How to find out the principal minors of a matrix Q, matrix P, which is a symmetric matrix.

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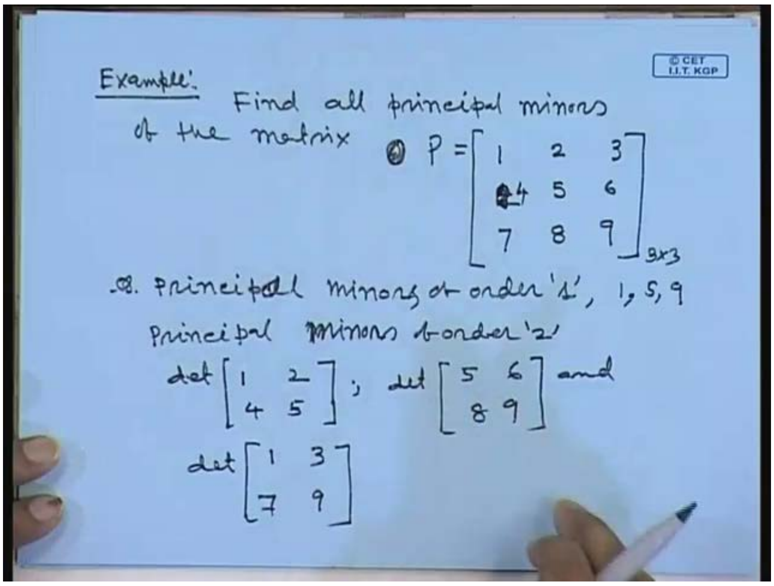

So, let us take x, let us explain this within example that how we can find out the principal minors of a matrix P, which is symmetric matrix. Find the, find all principal minors of the matrix let us call Q or some let us call P, P is now, how to find out P. Let us call that matrix is 1, 2, 3, 4, 5, 6 let us call this 2, 6, 7, 8, 9, but when we find out the principal minors, it is not necessary that how to find out the principal minors of this one, that not necessary P should be a symmetric matrix. But when you check it that, whether the matrix is positive semi definite or not even it is not a symmetric matrix, I can always convert into a symmetric matrix, that I told by definition of this one P plus P transpose by 2, if you multiply it by x transpose of x, the value of, will be remain same.

So, this let us call it is not a symmetric matrix, but our, my interest is to find out the all principal minors of this matrix. It is not a symmetric matrix, let us call this I am writing 4 so our aim is to find out the principal minors of this matrix. So, what I will do is first, find out principal minor of order 1. So, principal minor of order 1, you have to make all the one element that means, a matrix out of this from this matrix you pick up the, matrix of size 1 by 1. So, I have a 9 elements, so you have to select only that elements which are the diagonal elements also. This will come 1, 5 and 9 these are the principal minor of order one, you see I have 9 elements 1 by 1 matrix, out of this I am considering only 1, 5, 9 because these are the diagonal elements of the original matrix.

Similarly, principal minors of order minors of order 2. So, you have to pick up to a matrix of size 2 by 2 from the matrix of whose size is 3 by 3. So, 2 by 2 you have a different combination you see this is 2 by 2 and this and this also 2 by 2 this, this, this, this 2 by 2 and this, this, this also 2 by 2. But you have to pick it up those 2 by 2 matrix whose diagonal elements are the diagonal elements of the original matrix. In other words you can see that first row, first column you delete and what was the matrix is, what are the elements are left, that is the principal minor, what is called principal minor of order 2.

So, again second row second column if you it is, it is a 1, 9, 3, 7 and 1 and 9 is the diagonal elements, are same as the original matrix diagonal elements, part of this one. So, principal minor order is determinant of 1, 2, 4, 5 this is also one principal minors of order 2, that mean third row third column you delete it, that what is left is this one. Another principal minor of order 2 is determinant of first row, first column if you consider delete it is a 5, 8, 5, 6, 8, 9 determinant of this matrix.

Another is if you consider the, that is what we have considered first row first column, if you separate this now second row second column, second row with second column, if you do it this will be a determinant of 1, 9, 1, 9 then 3, 7, 3, 7. So, this is the determinant, there is no other choice is there, which size of the matrix is 2 by 2 and not only that these diagonal elements are the diagonal elements of this matrix. So, now is left is principal minor of order 3.

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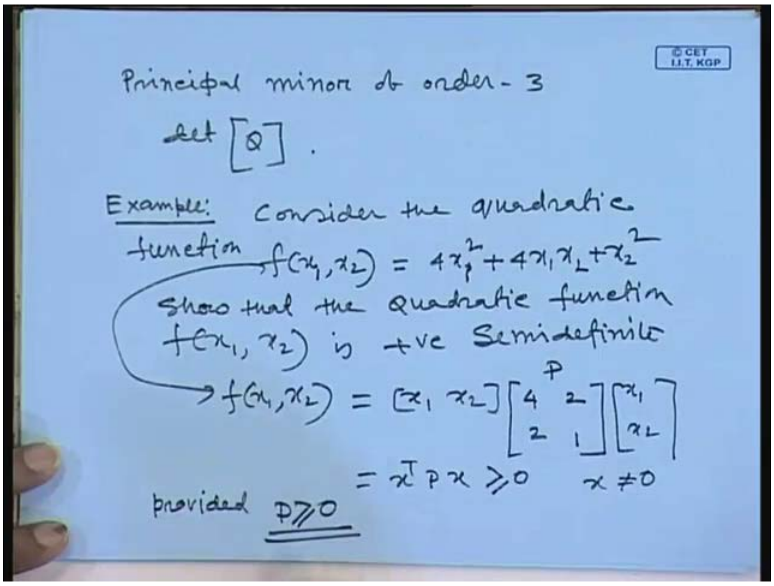

So, principal minor of order 3 is the matrix itself minor of order 3 is the determinant of Q itself this. So, I know how to find out the leading sorry, not leading principal minors of a matrix Q. Now, let us take an one example and check how to find out the, what is called that quadratic function example. So, consider the quadratic function and check function f of x 1 and x 2, I can write in short it is a function of x, x is a vector whose dimension, whose dimension is 2 cross 1 1 element is one variable is x 1, another variable is x 2. So, this is 4 x 1 square plus 4 x 1 x 2 plus x 2 square.

So, your problem is check the positive semi definite function, check the definiteness of this function or check this this quadratic function is positive semi definite. Show that, you can write show that the quadratic function f of x is positive semi definite that is our problem. So, this function if you see, this function I can always write into matrix and vector form. This matrix is positive semi definite means is nothing but a x, x 2 this is equal to I can write in matrix and vector form as I told you that x 1 and x 2. And this I can represent in different ways, but I prefer to represent this in symmetric matrix so that, I can test I can use the Silvestre criteria for checking this matrix is positive semi definite or not. So, I will write it 4, 1, 4 is there 4 x 1 x 2 here x 2 x 1.

So, I will divide into two parts 2, 2 so it is a symmetric matrix this P is a symmetric matrix. So, this will be positive what is positive semi definite matrix, in the sense the function below is always greater than equal to 0 for all values of x, when x is not equal to null vector. In other words, in other words I can write x transpose P of this will this quantity is greater than 0, when x is not equal to null means, this will be greater than 0, the function value provided P is a, provided P is positive semi definite matrix. So, we know if you use the Silvestre criteria for P, P, P is a symmetric matrix.

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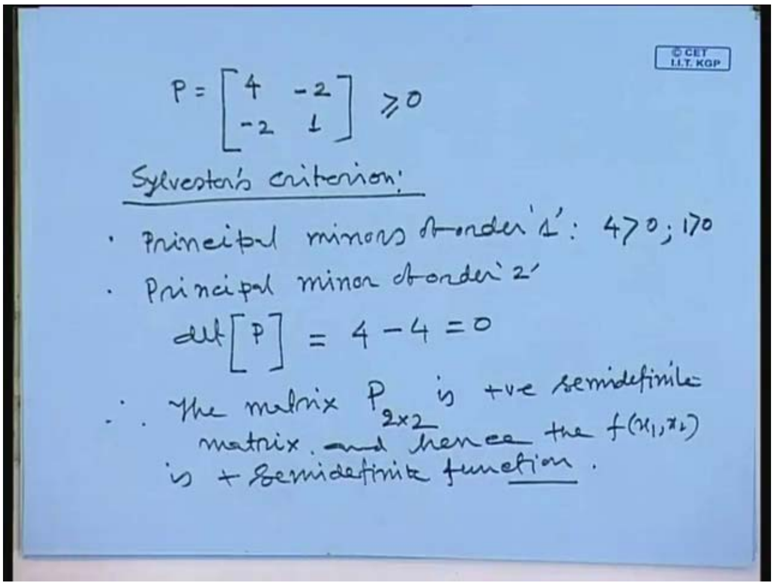

Then, you can say our P is like this way, what we got it here 4, 2 sorry, this is you see this quantity is minus, this quantity is minus. So, this is minus, this is minus so I mistake here, this is a minus quantity. So, this is minus because minus 4 minus 2, minus 2 here. So, I am writing P is this one, then matrix this one is 1, if you see this P.

So, I have to check it this matrix is positive semi definite matrix, according to the Silvestre criteria look at this one, positive semi definite matrix that function means P must be positive definite matrix. All the diagonal elements, all the diagonal elements will be greater than equal to 0. So, it is a positive, it may be 0, but here in this case one element, but it cannot be negative, non-negative. So, you test this, our satisfy first checking that principal minors of order 1 are 4, which is greater than 0 and 1 is greater than 0.

Next, I have to go for order 2 so principal minor of order 2, when it is a, we have to check semi definite, positive semi definite, negative semi definite we have to consider the principal minors. When only positive definite matrix or negative definite matrix we have to test it using Silvestre criteria, we have to consider leading principal minors. We know how to find out the leading principal minors.

So, this order is determinant of this this size is 2 by 2 determinant of our original matrix p and if you see the determinant of this p is 4 minus 4 is 0. So, one case is getting positive another is 0 so the matrix is positive semi definite matrix. Therefore, the matrix test the matrix P, whose dimension 2 cross 2 is positive semi definite matrix and hence, the function, given function is positive semi definite matrix. And hence, the function f of x is positive semi definite function. In other words, so any value of x you put it here, there is infinite number of x, you will see the function value will be either positive or some value of x it may be negative 0, but not negative, that is sure.

So, this is we know but if we ask to test for negative, what is called semi definite matrix then how will you do it? Test that means, if you multiply it by P matrix by, what is called minors then this matrix P will be a negative definite. And using the same logic that, what we have arrived with a from positive definite matrix to a negative definite matrix, same logic you can apply it here. Only the things, same thing in the sense your that first that, that elements of all leading all, all diagonal elements will be either negative or 0.

Then, the even the principal minors of even order, will be positive or some value will be 0 and negative that may, what is called the odd order of principal minors will be negative, what is called negative or 0, that is the same thing what we did it here you can do it this one. So, I am, I am leaving this an exercise to check it, this one, this function what we have considered this one, if you see this example what you have considered you multiply it by minus one.

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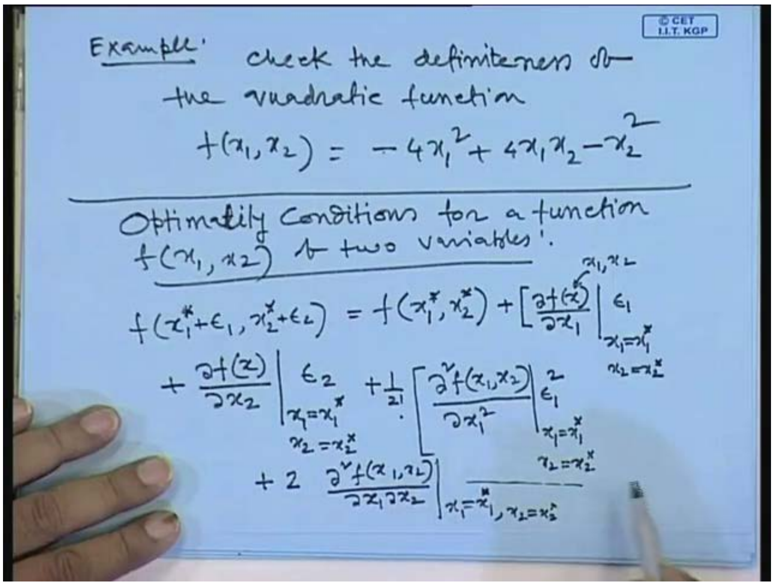

So, that this function will be negative definite function. So, check please do it this one, please so check the, check the what is it called definiteness of, check the definiteness of the quadratic functions, function f of x 1 x 2 is equal to minus 4 x 1 square plus 4 x 1 x 2 minus x 2 square. You see this, this function i multiply it by minus 1 so previously we have proved it, it is a positive semi definite. So, I am multiply it by minus so it will be negative definite so apply Silvestre theorem. Only the, what is called even order values and ordered are principal minor values, you check it all this things, you can easily prove it this Silvestre inequality criteria. So, I will leave this as an exercise for this one.

Now let us, that next is our optimality conditions, optimality conditions for a, for a function f of x 1 x 2 only two variable function that can be extended for n variable case function f of x of two variables. So, let us consider we have the function, this optimality condition can be obtained from the, by using the Taylor series expansion of the function f, which is a function of x 1 and x 2.

Let us call f 1 of the function f, which is a function of x 1 and x 2 is, we have the point here x 1 star and around this epsilon 1 x 2 star plus epsilon 2. Let us assume that x 1 star and x 2 star are the optimal point that means, when the value of x 1 is x star, x 2 is x star, x 2 is x 2 star, we get the function value, function value f of x which is the function of two variable case, optimal value of the function either minimum or maximum. Then around this we given a from x 1 star and x 2 star.

So, let us see the Taylor series expansion of this function. So, it will be a Taylor series expansion x 1 star comma x 2 star this, then this is a function of two variables. So, if you see this one I can write it this one, del f of x del x 1, del f of x del x 1 x is a you can say function of x 1 and x 2, into that you are doing the Taylor series expansion around x 1 star and x 2 star. So, write x 1 is equal to x 1 star x 2 is equal to x 2 star and what is this incremental is epsilon 1, plus another function is del f, just Taylor series expansion you did it. We are doing it now, this at x 2 is equal to x 1 is equal to x 1 star x 2 is equal to x 2 star, multiply it by the incremental delta epsilon 2.

This is the first order approximation then second order terms is what half factorial then what is this one, second derivative of f x 1 x 2 differentiate this with respect to x 1 square twice, with respect to x 1 twice. Then it is a epsilon, put this value you find out this value x 1 is equal to x 1 star and x 2 is equal to x 2 star because around this point you are doing. So, this is will be a x 1 epsilon 1 square half bracket then next term is your half is common. So, next term is twice del square f x 1 x 2 and differentiation of with respect to x 1, then with respect to x 2 this equal to at what point x 1 is equal to x 1 star and x 2 is equal to x 2 star, plus plus.

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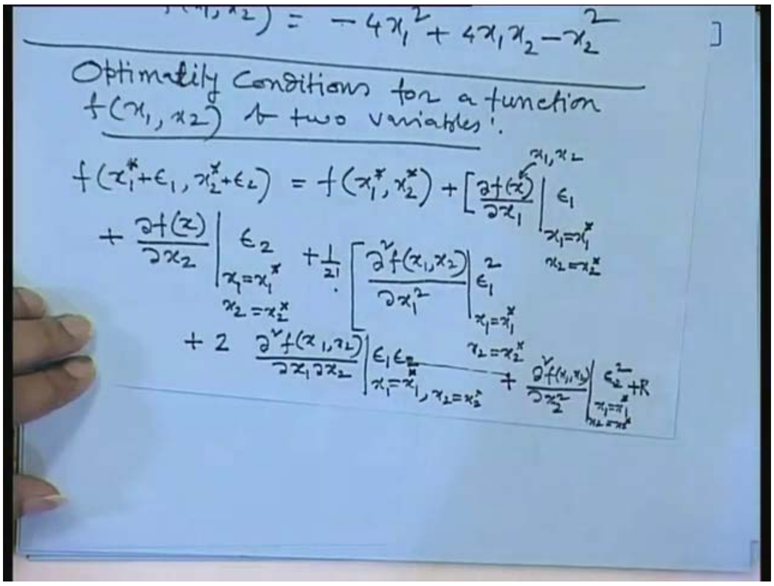

So, I am writing here plus, then del square f x 1 x 2 and this is, differentiate with respect to x 2 square, that you evaluate this value at x 1 is equal to x 1 star and x 2 is equal to x 2 star, multiply it by here I missed it epsilon 1 and epsilon 2. So, here you multiply it by epsilon 2 square plus plus some other terms.

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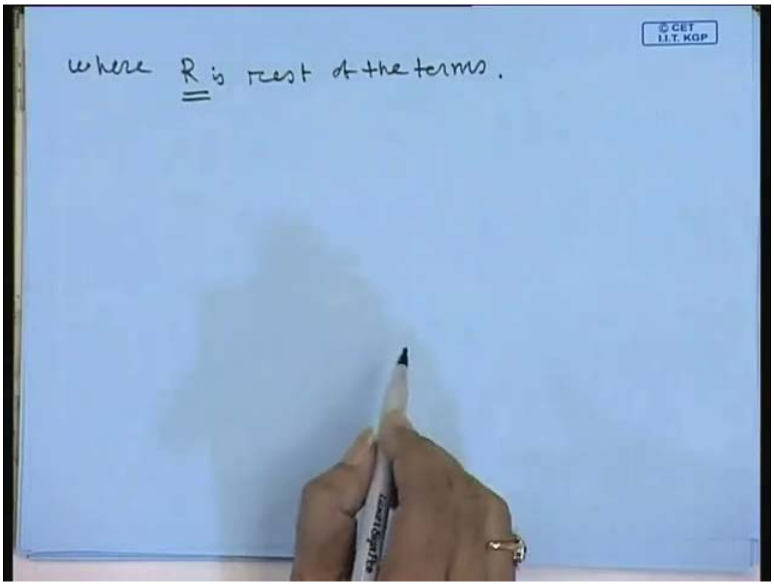

And let us call some other terms are, what is this, some other terms what we have considered is capital R, rest of this series terms, third order, fourth order, order of this what is called Taylor series expansion of this one is R. So, R is the where, you can write, where R is the, where R is the, is rest of the terms, this term is much smaller since that epsilon 1 and epsilon 2 the ((Refer Time: 41:38)) what we have given around the point x 1 star and x 2, that is very small. So, this contribution is small compared to the first term and second term. So, now if you investigate the terms, each individual terms, then we will see that, what we will get it that one.

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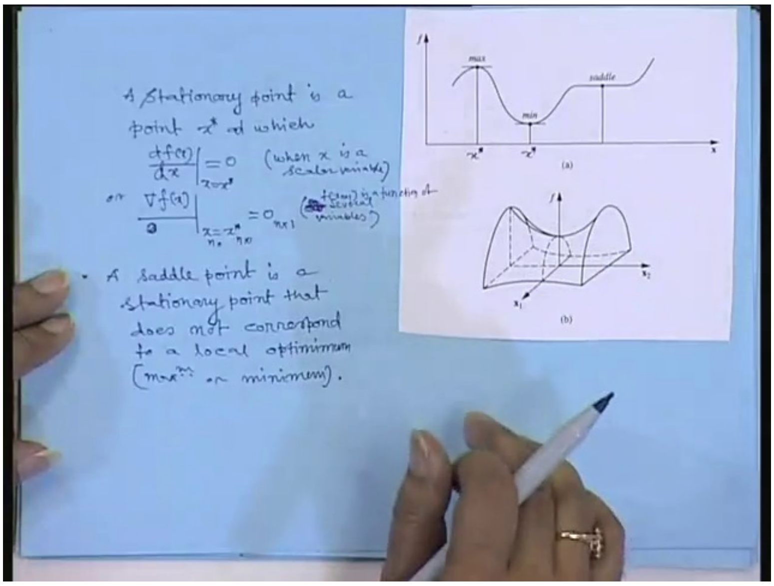

Now, see this one, let us call this one, this point, this slide I am keeping it here just to understand. Let us call f of x which is a scalar variable, one variable only that is single variable this. And we have f of function is like this way, then x is equal to x star, you see at this point, you got the maximum value of the function. For scalar case we know, the derivative at this point slope will be 0. So, we are getting the maximum value of this function here and this is the minimum value of function, at this point that slope is again 0.

So, the, for scalar case the derivative of the function, if the function value at that point is minimum or maximum or optimum the derivative of the function value is 0. And what is saddle point of this one, a saddle point is a point that does not carry any information about the local minimum or local maximum or optimum point. You see, this does not shows any local minimum or local optimum points. So, this point is called the saddle point.

So, our definition now, this is for simple scalar function, which is a function of only single variable. So, our definition for stationary point you know, whether the function will be maximum or minimum for scalar case. The derivative of this one must be 0, this one so a stationery point is a point, is a point x star, at which the derivative of the function is 0. But when this function is a scalar one, but in case of a function which is a function of multivariable case x 1 x 2 dot dot x n, then we have, we will show you the gradient of this function must be assigned to null vector. Then only, you will get at that point that maximum or minimum value of the function.

So, this is the, an, this, this figure shows that function is a, this function f is a function of x 1 and x 2 and the function value, we have got that in y direction, z directions and this is a function it is shown. So, you see it has a minimum value, maximum value at this, maximum value here and minimum value is here. So, let us see this one at this moment that, if you recollect we had a function is f which is a function of x 1 and x 2. And if we assume that x 1 star x 2 star are the optimum value, optimum value maybe maximum value or maybe minimum value of the function, at this point x 1 is x star there.

From there, we are given a, some ((Refer Time: 45:23)), then by Taylor series expansion we have written the function value at, at, at x is equal to x 1 star and x 2 is equal to x 2 star. And the first order derivative and second order derivative plus higher order derivative, whose sum of the rest of the series I have kept is R, this quantity R is very small provided, that of from the what is called x 1 star or x 2 star, we choose the optimum point of the function is small, then this is small compared to this terms, first order, second order terms all this things.

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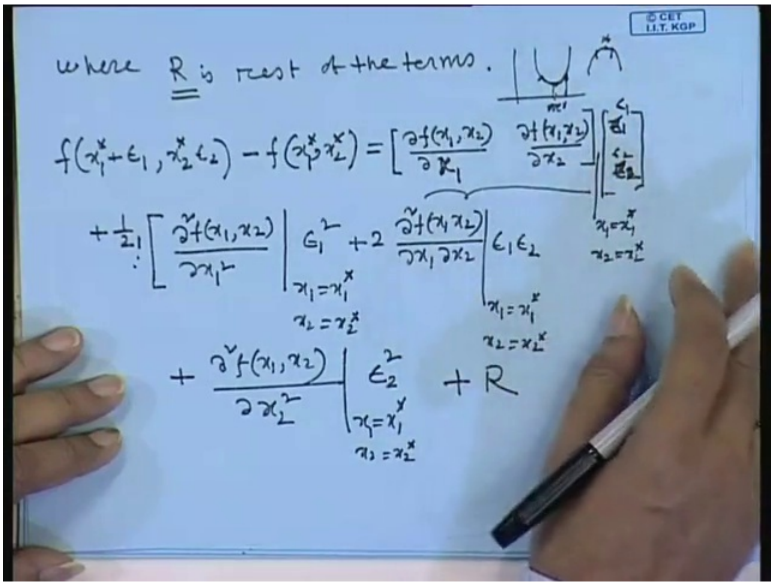

So, now let us see this things so we are writing the our original function x 1 star plus epsilon 1 x 2 star plus epsilon 2 star. And this f of 1 x I am keeping in the left term side, the function value at x is equal to x star and x 2 is equal to x star, this is equal to what is left in the right hand side? This part I have taken this side and what is left, first order derivative and second order derivative functions here. So, I am writing it this, you see carefully what I am writing into matrix and vector form only, is row vector x 1 star x 2 differentiate with respect to f, with respect to x 1 then del of this x 1 x 2. Then with respect to x 2 this multiplies it by x 1 and x 2. So, this is the first order part of derivative is coming.

Now, second part is half factorial this you see what I am writing, but this I have to evaluate, this I have to you have to evaluate I am writing x 1 is equal to x 1 star and x 2 is equal to x 2 star. So, this quantity is known to you, but this quantity, this maybe positive, this maybe sorry, this is epsilon 1, this is epsilon 2, see this expression. This is epsilon 1, this is epsilon 2, this is not x, this is epsilon 1 and this is epsilon 2, this. Now, this quantity when you put x 1 is equal to x star and x 2 is equal to x 2 star, this quantity you do not know it may be positive, maybe negative, maybe 0, all these things can be.

But what about epsilon ((Refer Time: 48:15))? Epsilon 1 epsilon 2 it can be positive side or it can be negative also. So, if you just multiply this into this, the resultant quantity you are not sure whether it will be plus or minus because once you assume that this is plus.

Let us call multiply it by epsilon, epsilon can be negative, can be positive because this it, it, it is the either side of this optimal point x y x 1 is equal to x star maybe either side.

So, this also if you assume, this is a negative and this epsilon 1 and epsilon 2 maybe positive or negative. So, you this whole product you cannot say confidently resultant will be a positive or negative. So, this things if you keep in mind I am now saying the, what is the second order derivative part, what we can write it. So, this is a delta square f x 1 x 2, delta x 1 square this, find out this value x 1 is equal to x 1 star and x 2 is equal to x 2 star multiplied by epsilon square. And this value you differentiate the, what is called the function of f with respect to x 1 twice that and put the value of x 1 is equal to x star x 2 is that multiplied by x 2 square.

Now, next is 2 then this x 1 x 2 then del of x 1 del of x 2 put this values x 1 is equal to x 1 star x 2 is equal to x 2 star. And multiplied by epsilon 1 and epsilon 2 plus del square f del x 1 into del x 2 multiply differentiate with respect to x 2 twice, put the value x 1 is equal to x 1 star and x 2 is equal to x 2 star multiply it by epsilon 2 square plus, the rest of the terms which we denote it by R. Now in this expression, you see this minus this these two quantities can be positive can be negative, you do not know because if it is a optimum point is, it is like this way if it is the x star. If it is this side and if it is this side that x plus epsilon this one, that means this is a positive quantity difference.

If it is this side, this will be positive side, positive then I can say, if it is positive that this is the optimum point of this one, if it is the like this way from this and this, then then this value will be a negative. Now I cannot say that one, with whether it will be positive or negative because due to this factors, this factor I told you if it is a, let us call this is a positive. Due to epsilon, it may be positive negative epsilon 1 can be positive, epsilon 1 can be negative.

Similarly, this term if it is a negative, it can be a positive negative, total resultant whether it will be positive or negative we cannot say. So, similar to that one, this I can write it, this is nothing but a gradient transpose. So, assign gradient of this vector f of x assign to 0. So, this part is vanished now so this ambiguity because we know at this point slope is 0, if it is case that gradient will be 0, assign this one.

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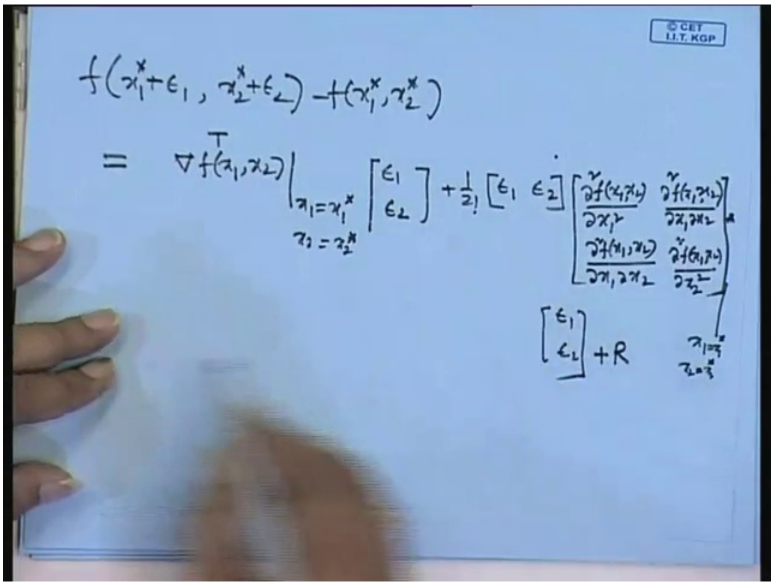

So, only this one is left and this I can write it, if you see this I can write it now into a that form x 1 star plus epsilon x 2 star plus epsilon, this minus f of x star x 2 star is equal to I can write it, this gradient of f of x 1 x 2 transpose x 1 is equal to x 2 transpose x 2 is equal to x 2 star into what I can write it that one, your epsilon 1 epsilon 2. See this one, this I am written is gradient transpose epsilon 1 epsilon 2 and what we can write it this one, you see something like this, this is the scalar quantity you can say a epsilon square, two b epsilon 1 epsilon 2, c epsilon 2 square.

So, it is quadratic form I can always write that one into this form, we have shown earlier that what we can write it, I can write always as epsilon 1 epsilon 2 transpose, this is, this form. Then I can write it, gradient that is x 1 x 2 del x 1 square del square f x 1 x 2 del x 1 del x 2 and del square f x 1 x 2 del x 1 del x 2. And this one is del square f x 1 x 2 and del x 2 square and that is, that one and this value you have to calculate x 1 is equal to x 1 star and x 2 is equal to x 2 star into this, into this is, into epsilon 1 epsilon 2. And what is the term, it is left plus R so if you consider this, this is a vector of epsilon.

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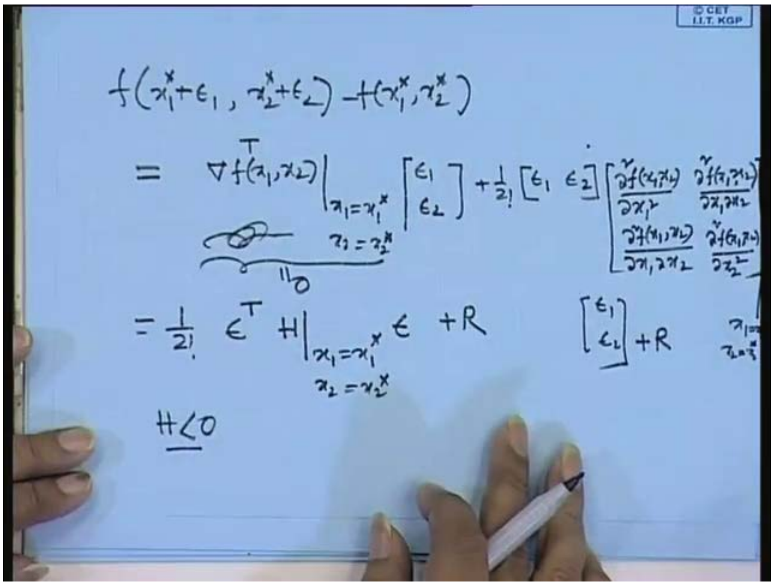

Ultimately, I am writing because you see this quantity will be positive or negative, I cannot take decision with this one because of it can be anything. So, I assign this quantity that, this whole quantity is assigned to a null vector. So, if it is so the right hand side now I can write it half epsilon transpose, epsilon transpose into this is the hessian matrix H, you calculate H, you calculate x 1 is equal to x 1 star x 2 is equal to x 2 star into epsilon plus R.

Now you see, I just mentioned it that R is sufficiently small compared to this previous terms because that epsilon 1 epsilon 2 because higher order derivatives will be negligible because I have epsilon 1 epsilon 2 are very small around the x 1 star x 2. So, if you assign this is 0, this term is not there only term is left, this plus this.

So, I can easily now tell that what is called this quantity will be positive provided H is a positive definite matrix, quadratic function. So, if it is a positive definite function then what does that mean? If you give a form x 1 star and x 2 star that quantity is positive means, we have obtained the, what is called local minimum. If this quantity is negative, when it will be negative?

H is negative definite matrix, H is negative definite matrix then this quantity will be negative. But that contribution is negligible, because of epsilon 1 epsilon 2 are very close to x 1 star x 2 star. So, this will be negative the function value difference is function value negative means, it has reached to the local maximum that difference is coming. So, today I will stop here and next class we just continue that what is our final conclusion.

Thank you.