Optimal Control
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Optical Control
Lecture - 39
Solution and Stability Analysis of Finite-time L Q R Problem: Numerical Example

Last class, we have derived the expression for solution of L Q R problems let us describe this briefly what is L Q R problems, so our problems is to design a controller.

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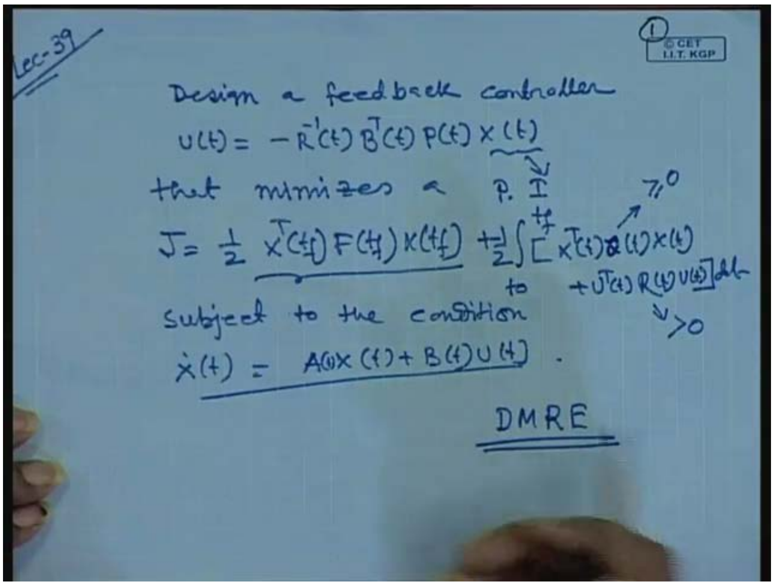

To design a feedback controller, feedback controller or controller which is described by U of t is equal to minus R inverse of t, B transpose of t P of t into X of t, we assume the states are accessible measurable. For feedback purpose this controller will minimize, that minimizes a performance index performance index. What is this performance index half X transpose t f, F t f, X t f then this is called is terminal cost and also t 0 to t f half then X transpose t U of t X to F of t plus U transpose R of t U of t whole bracket d t. So, our problem is to find a design a controller U such that this performance index is minimized subject to the condition subject to the equality condition, subject to the condition X dot of t is equal to A of X t, a function of X t plus B U of t.

This is our problem and we have derived the expression to find out the controller U of t and Q of t this is Q of t which we assume is a positive semi definite mean and R of t weight age with the input matrix input. This Q is weighted with the state vector and R is weighted with the input vector and that is positive definite matrix and we have derived the expression to find out the expression for P of t for a finite timing interval.

Now, we have seen to solve this controller we need to solve what is called dynamic matrix Riccati equations, dynamic matrix Riccati equation that we need to and this is nothing but a set of dynamic equation. Set of dynamic equation which is coupled to each other and in general it is a non linear differential equation, so let us write what is called the algorithmic steps that how one can solve the finite time regulated problems.

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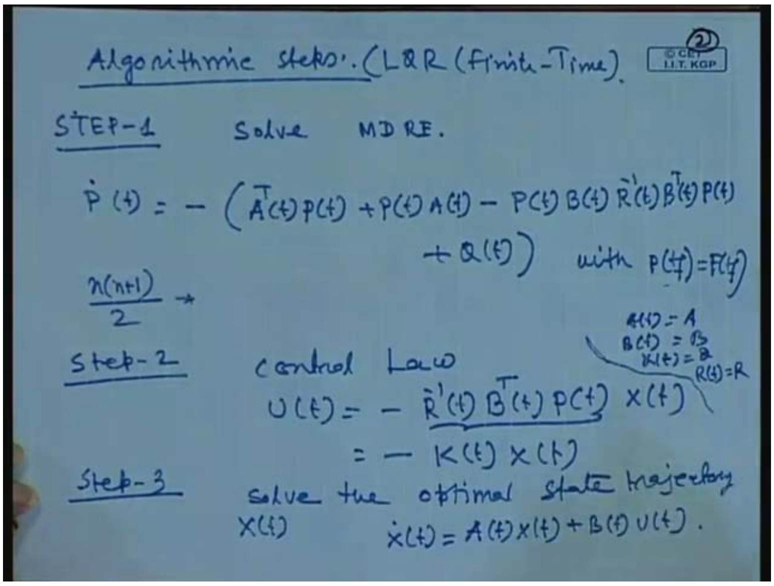

The algorithmic steps for L Q R finite time that is finite time L Q R problem, finite time L Q R problem, so first step 1 solve what is called the dynamic matrix Riccati equations. Step 1 solve matrix dynamic Riccati equations, what is that matrix Riccati equation if you recollect this one P dot is equal to minus A transpose of t P of t plus P of t A of t minus P B P of t B of t R inverse B transpose of t B transpose of t then P of t plus Q of t.

This equation we have to solve it since we have considered P is a symmetric matrix 1 can easily see from this expression that, since P is symmetric matrix one can see from this expression. Then we need to solve n in to n plus 1 by 2 differential equation first we are differential equation n in to n plus 2 by 2, since P is a symmetric matrix. So, this differential equation are dynamic in what is coupled in nature to agree and not only these they are coupled and in general it is non linear.

So, this set of different non linear differential equation we have to solve it, that is the first step this with the boundary condition that P of t f is equal to F of t f and F of t f is nothing but A, the terminal cost waiting function. This is the cost is what is cost is integral cost functional, this whole part integral cost functional, so with the knowledge of this boundary condition we have to solve the P dot of that is expression. This expression to find out P of t and that we have to do what is called backward integration in time we have to do backward international in time this one.

So, next step once you solve this one next we can easily implement what is our controller U of t is equal to minus R inverse of t then B transpose t P of t and X of t. This if you can write it, this whole thing is K of t and X of t it is just like a state feedback you can say that information of state it feedback and we are generating the controller and that k confide by controller is varying with time. This is a second step we can implement this one, and as I told you this solution we have to do backward integration you have to store all the information of Riccati matrix P.

Then in second step offline, this is an offline competition you have to do it then store it in a matrix and then you compete online. Then step 3 is then solve the optimal state strategic X star of t, by using this equation X dot of t is equal to A of t, X of t plus B of t, U of t, this U of t we can compete from step 2. So, this way you have to solve it and, now question is suppose the matrix A of t, B of t is time invariant that means this parameter does not change with respect to time.

Also waiting matrix Q state waiting matrix and also input waiting matrix does not change with time then you can write in place of A of t in place of A of t you can write A, in place of B of t you can write B, in place of Q of t you can write Q and in place of what is called R of t you can write R in this expression. But, still it is this expression is valid for this one, now one may interest one may be interested to find out what is called the optimal cost.

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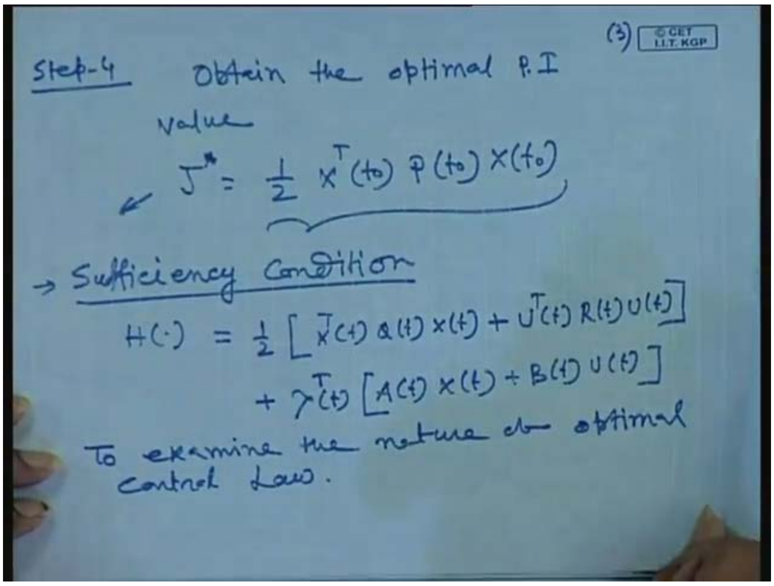

So, if you want to obtain the optimal cost obtain the time obtain the optimal performance index value then J is just star is optimal below is half X transpose t 0, P of t 0 and X of t 0 if you know the initial condition of state. Since, we have solved the Riccati equation matrix, Riccati equation and we have solved this in backward in time then store the value of P of 0 at P at time t is equal to 0. Then we can compute the optimal cost of the performance index over the interval 0 to t, what is the performance index, the performance index is that one the value of J.

You can compute over the time t is equal to t f what is the expression for 0 to t f, this expression you can find out with this expression. So, we know the al algorithm steps of this, now we will see this is the sufficiency condition to get the nature of optimality sufficiency condition. Sufficiency condition for the optimality to check the optimality means whether the function, their objective function is optimized in the sense maximized or minimized to check that one with this sufficient condition we have to consider, we check the sufficient condition.

Let us if you recollect that our Hamiltonian matrixes, this is formed from the system matrix knowledge and also performance index state and waiting state, waiting matrices and for the input waiting matrix expressions. So, this is the half X transpose of t, Q of t, X of t plus U transpose of t, R of t U of t this is the first part which is formed from performance index plus lambda of t transpose A of X t, X of t plus B of t, U of t. You recollect it may recollect that what is called Hamiltonian matrices form a function which is free from it is the X dot derivative.

So, this Hamiltonian functions to examine this, the nature of optimal controller to examine the nature of optimal controller. What does that mean to nature the optimal controller means whatever control law we obtained whether the performance index what we have considered earlier this one is minimized or maximized. To know that one, we have to check the sufficient sufficiency condition, so what is that condition without deriving in details because we have all ready derived earlier in some few situations.

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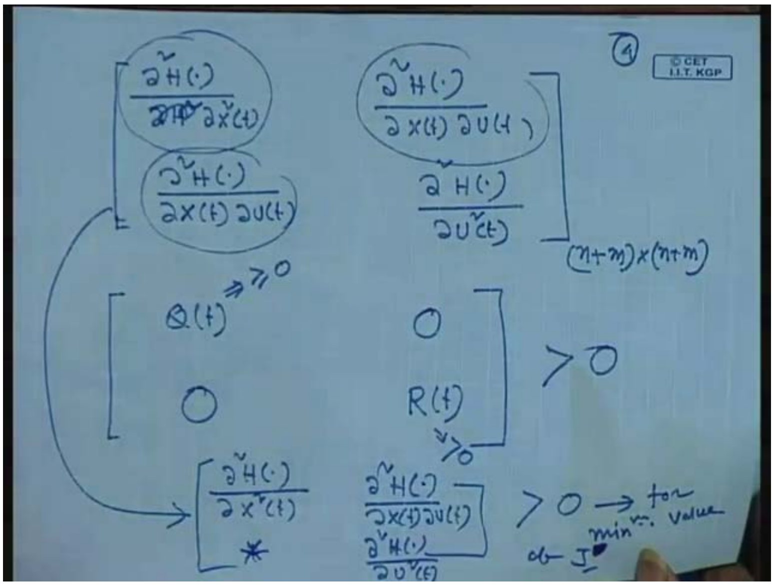

So, we have to form what is called this matrix H of this del U of t then this will be symmetric matrix as we have already seen del X of t del U of t plus del square H del U square of t. S, that matrix dimension, if you see this is X dimension is n, U dimension is m, so this dimension will B n plus m that is the symmetric matrix. So, this is called a matrix which is a symmetric matrix and that matrix we have to check whether it is a, it will give you a sufficiency condition n matrix. What is a sufficiency condition will give you whether it is maximum or minimum if this matrix is positive definite then we will get what is the minimum value of the function if this matrix is a negative, definite matrix.

Then we will get a what is called positive value what is called maximum value of the function, let us see this one we know since we know the expression for H we can easily compute this term, this term and this term and this term each is a matrix of proper dimensions. So, if you compute this one, now you see this, this I will get it Q of t, this I will get 0 because I am differentiating with respect to U and then to respect of X, this is also 0 and this is we will get R of t and that this is R of t. In order to become, in order to become the function that is objective function is maximum or minimum, it depends on the nature of this matrix, so this nature of the matrix if you see this one.

This matrix we have considered if you consider this is a positive semi definite this is a positive definite matrix, this matrix is always is positive definite matrix. So, if this matrix is positive definite matrix and which is obvious this and we have selected the Q state waiting matrix and what is called input waiting matrix based on some physical considerations. So, this will be greater than 0, so the objective function value that will get it that is will be equal to what is that it will give you the minimum value of the function obviously.

So, this implies that this must be, so this implies, this implies that matrix, that this matrix must be greater than equal to 0 that del square H, this del square of t, del square H del X of t del U of t. This is the symmetric of that one the star I can write it then del square H del U square of t must be positive definite for minimum value of the functional. Minimum value of the of j star of j write j, so we are this is the necessary condition, so if you select Q is positive definite and R is positive semi definite, R is positive definite this ensures that we will get minimum value of the functional.

So, let us take an one example and illustrate that, and illustrate and if you see this expression, if you see this expression it is enough more stress you can give it in order to become a positive definite this is enough R must be a positive definite this is enough. So, let us solve this problem by using what is called a numerical example, see how to solve the finite time regulated problem by solving a by considering a numerical examples.

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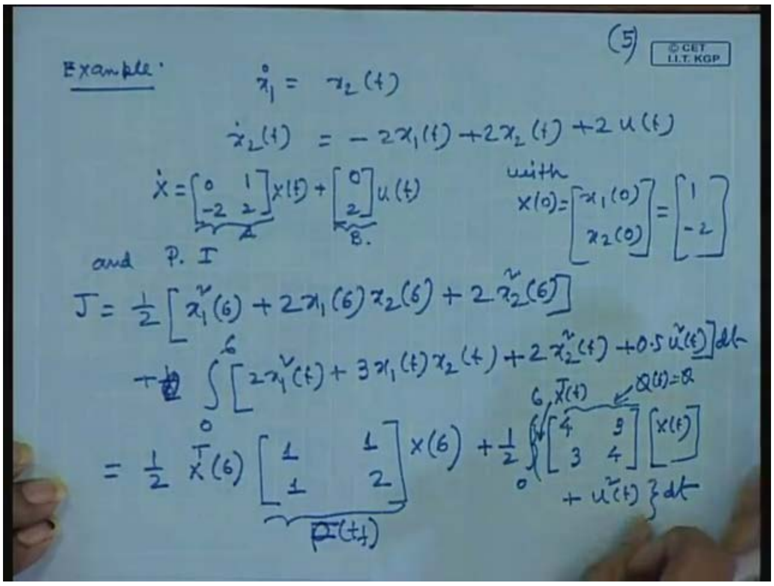

So, we consider our example is this X 1 dot is equal to X 2 and then X 2 dot is equal to minus twice X 1 of t plus 2 X 2 of t plus 2 U of t with initial condition their initial condition X of 0 is equal to X 1 of 0 plus X 2 of 0 which is given to you 1 minus 2. So, our problem is to find out the controller U such that the performance index and the performance index, this performance index is minimized. This per find the controller U such that this performance index is minimized subject to the condition of the dynamic equation, so this is our is given is X 1 of square of 6 plus twice X 1 of 6, X 2 of 6 plus twice X 2 square of 6.

This is, this is the terminal cost the sixth indicates at time t is equal to 6, the terminal time and initial time is t is equal to 0 is given condition plus half that integration 0 to 6 then twice X 1 square of t plus 3 X 1 of t into X 2 of t. Let us call this half is not given, we have to formulate this problem keeping half, so let us call it is not half is not given, so this is equal to then your 0.5 that is 2 X square then 2 X 2 square of t plus 0.5 U square of t whole d t this is how the performance index. So, if you consider this is in our extended format that means half you have to keep it here and multiplied by 2 this will be 4, 6, 4, 1.

So, we can just write it in short compact form X transverse X has a two component X 1 and X 2, so this we can write it if you see this one we can write it 1, 1, 1 and this is 2 is X of 6. This is we have written in quadratic form X transverse P, X form plus the I told you multiplied by half that is divided by 2 multiplied by 2. So, half 0 to 6 this will be 4, so the 4 then this will be 6, 3 is here 3 is here that we have expressed earlier how to convert a polynomial quadratic polynomial into a matrix and vector form. This is 4, 4 is here multiplied by X of t, here is I missed this one, here is X transpose of t that is this 1 plus this is multiplied by 2 multiplied by 2 divided by 2 that means it is a U square of t whole d t.

So, immediately you can find out, you see from this expression if you write it in the this two equation in compact form I can write X dot is equal to 0, 1 minus 2 plus 2 into X of t plus 0, 2 U of t. So, this is our if you see this is nothing but A, our A matrix this is nothing but A, our B matrix and this quantity is nothing but A, our F of t F this what is given and this matrix is nothing but A our Q of t which is equal to Q because it is not a function of time. This R is nothing but A equal to 1, here which is in the matrix that means which is in the control vector is 1, so I can write it now is like this way, so you can write this is into this form.

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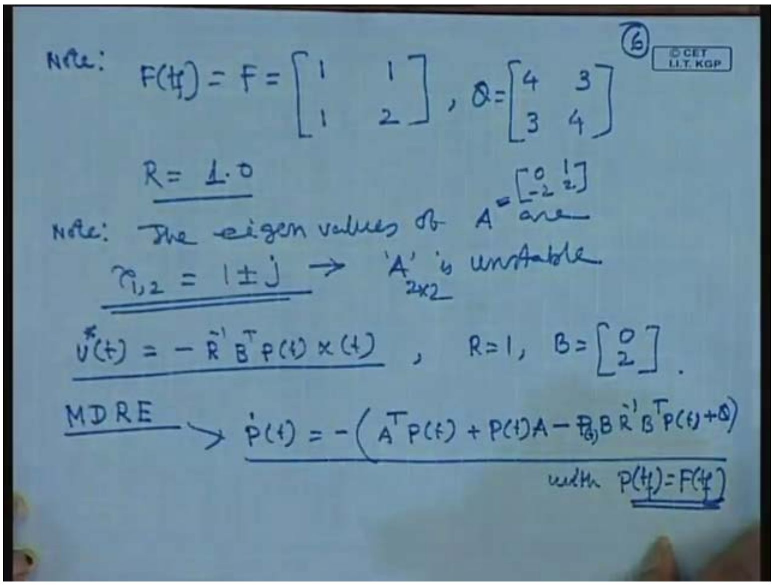

Note F of t is nothing but A means F which is nothing but A 1, 1, 1, 2 the same from the last equation then our Q is got it 4, 3, 3, 4 and our R is equal to 1 because it is a scalar input. If you look that this expression, if you look this expression the matrix open loop system when there is no control effort is applied to the system that open loop system, that a matrix is unstable. If you want to find out the note Eigen values of the matrix we will find the Eigen values, the Eigen values of A are lambda 1, lambda 2 is equal to 1 plus minus j.

So, this is the unstable open loop system, open loop Eigen values are this then matrix A is matrix A dimension 2 by 2 is unstable and this matrix is what if it is 1, 1 minus 2, 2 this matrix A, matrix is this 1. So, it is unstable natural we have to use and controller, so that the closed loop system response is stable one and not only this that we have to restricted that our, that is our terminal cost is given. So, within a time t is equal to 0 with a initial condition this the final time at t is equal to t f is equal to 6 the state should come near about that value near to 0 this one.

Now, let us solve this problems how you will solve this problems this, so U star of t that our optimal controller of this is nothing but A R inverse B transpose P of t X of t this is our controller and we know R we know B. The state you can find out by simulating the system or from the system you can get the information of X t, now what is this how to find out P the solving the matrix dynamic Riccati equation, dynamic matrix Riccati equation.

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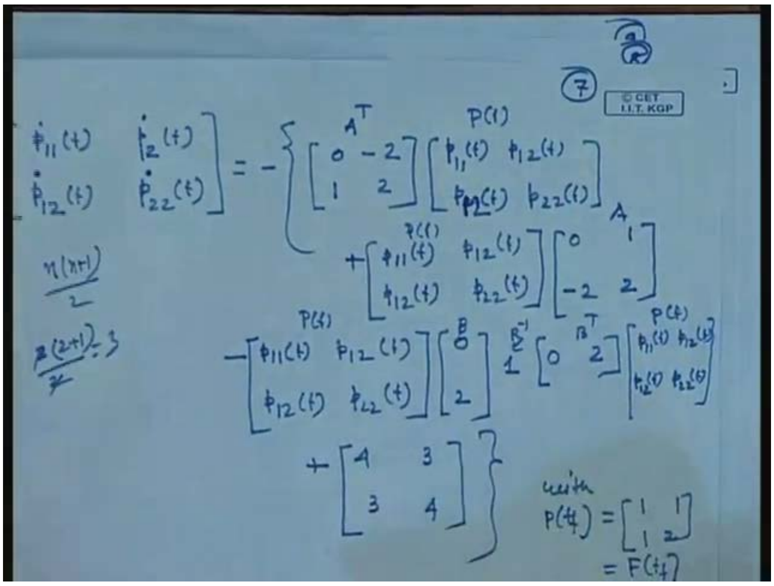

You just write it dynamic matrix Riccati equation, so this equation what is that equation if you see this equation or are you noted our R is equal to 1, B is equal to 0, 2 is and our Riccati equation P dot of t is equal to minus a transpose P of t because our A is constant matrix. So, we omit that symbol A of t plus P of A minus P, B R inverse B transpose P of t this is function of t, P is the function of t that P of t plus Q agreed this is the matrix Riccati equation dynamic matrix Riccati equation. This you have to solve with the knowledge, with the knowledge P of t f is equal to f of t f, so this value we know it from here f of t is the this one, so let us see how to solve this equation.

So, our P of t since p is an symmetric matrix I can write the each component of p like this way p 1 1, p 1 1 dot p 1 2 dot of t, since it is symmetric I can p 2 1, I can write same as p 1 2 of t dot. Then p 2 2 dot of t this left hand side is equal to minus, then you write it that one A transpose a is our 0 1 minus 2 2 the transpose will be one 0 1 minus 2 2, this is nothing but A our A transpose then p 1 1 of t p 1 2 of t p 2 1 of t p 2 2 of t. Since, p 1 and p 2, p symmetric matrix I will write is p 1 of t this is a transpose p of t, then p of t p 1 1 of t plus p 1 of t plus p 1 of t, p 2 2 of t into A.

So, A is if you recollect our system matrix minus 2, 1, 0, 1 minus 2, 2 then minus p, p 1 1 of t, p 1 2 of t, p 1 2 of t, p 2 2 of t, p B is 0 2, if you see our B is this is our B 0 2 then R inverse R is 1, R inverse is that. So, this is A, A this is p of t, this is p of t, this is B this is this one is R inverse then R inverse B transpose 0 2 which is B transpose. Then p and p is 1, p 1 1 of t, p 1 2, p 1 2 t, p 1 2 t, p 2 2 of t plus Q, our Q is if you see this is p of t our Q is if you recollect that our Q is 4, 4, 3, 3 symmetric matrix 4, 4, 3, 3 this and whole bracket.

Now, you just write you say this is a symmetric matrix of this one, as I mentioned earlier if the matrix dimension is n then how many differential equation we have to solve which are coupled to each other n into n plus 1 by 2 that you have. Now, our n is in the present case is 2, so 2 into 2 plus 1 by 2 that we have to solve 3 equations, p 1 1 dot one expression we get from the left hand side and right hand side p 1 2 another expression, p 2 2 dot another expression.

So, three equation we have to solve, so this with p t f is equal to if you see 1,1, 1, 2 which is nothing but A F t f value p t f value is same as F t f value. So, with this knowledge we have to solve it, now we write this after solving this one because you just multiply. You multiply the right hand side matrix, all these things and equate the position the left hand side 1 1 position and right hand side 1 1 position, if you do this one then you will get it.

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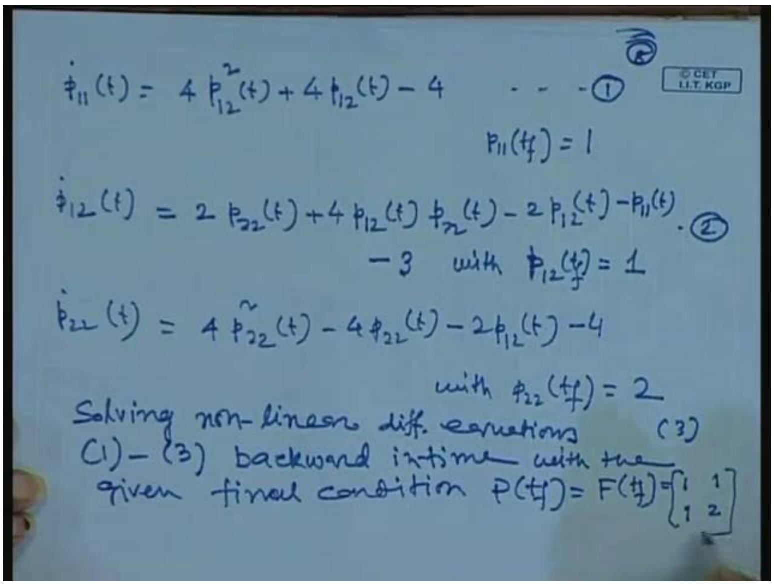

You will get p 1 1 dot of t is equal to 4 p 1 2 square of t plus 4 p 1 2 of t minus 4 and we know that our initial condition for this one, we know initial means not initial final terminal condition of p. We know that I can write it p 1 1 t f is equal to 1, then p 1 dot of t we can write, but this I am getting just I told you are getting by equating left hand side and right hand side.

By simplifying this whole right hand side expression into a matrix form equivalent way, matrix from this 2 by 2 matrix from then you will get p 1 2 dot is equal to twice p 2 2 of t, plus 4 p 1 2 of t plus p 2 2 of t minus twice p 1 2 of t minus p 1 1 of t. That value and minus 3 with p 1 2 of t is equal to 1, then p 2 2 dot of t you will get it 4 p 2 2 square of t minus 4 p 2 2 of t minus twice p 1 2 of t minus 4 with p 2 2 of t f this is t f t f is equal to 2. So, this equation you see there are three first order of differential equation, but they are coupled to each other and we have to solve, for solving this one we must know the boundary condition.

We know the final terminal condition of the matrix p or each element of this one p 1, each element of p we know this 1, so we have to do backward integration and not only this set of equation, if you see these are the non linear differential equation dynamic differential equation. So, let us call this is equation number one this is equation number 2 and this is equation number 3, so solving non linear differential equations 1 to 3 backward in time.

So, in time with the given with the given with the given final condition means p t f is equal to F t f is equal to in our case is 1 and our final condition is what 1, 1, 1, 2. So, one can solve this thing either two ways, either analytically which is very tough to solve this one or one can solve this one by using numerical techniques. So, I leave this problem as an exercise solve this problem what is called numerical method to find out the trajectory of p 1.

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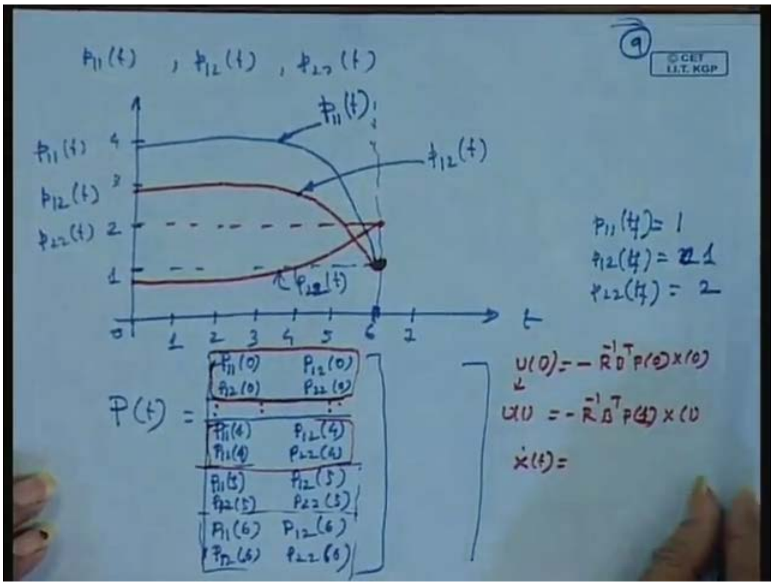

Find out the trajectory of p 1 1 of t, p 1 of p 1 of t and p 2 2 of t, so I leave it this exercise this one to find the trajectory of this one starting from t f to t 0 means t 0, means t 0 is equal to 0 value. This trajectory if you solve this problems by numerical methods the nature of this response what you may get it from this one, let us call this is the time and this is I am plotting the p 1 1 of t, p 1 2 of t. Then p 2 2 of t I am plotting this one, now see the value of p 1 1 of t f value is equal to 1, as you know from the terminal cost this is p 1 1, p, p t f, p t f is equal to f t, F is equal to this one.

So, p 1 1 t f is equal to 1, p 1 2 t f is equal to 1, p 1 p 2 2 t f is equal to 2, so our p 1 1 t f is equal to 2, p 2 2 t f is equal to, sorry this is one this is 2. So, let us call this is if it is A, this is one this is 2, this is 3, this is 4 and let us call this is 1, two time 1, 2, 3, 4, 5 and 6, 7 and this way, now you see this 1, 2 our final condition on p 1 1 is where it starts from here. Let us call the solution of this one if you to do the backward in time integration at this solution of this differential equation if you do by numerical method. That means by using what is called your rung out a method for solution of differential equation and if you use this terminal condition let us call this you are getting like this the solution.

This is corresponding to p 1 1 t and p 1 to t, p 1 to p 1 to t also is starts from here p 1 to also starts from here let us call this solution is something like this and p 2 2 starts from 2 here and let us call this solution is something like this. This is I am showing the nature the solution is not the solution corresponding to the problem what we are now discussing this is not the solution. So, this is your solution for p 1 2 and this is the solution of p 2 2, p 2 2 of t, so if you solve this set of equation that what we have considered. Now, here in backward integration backward in time then trajectory of p 1 1 you will get like this way and trajectory of p 1 2, you will get like this way the trajectory of p 2 2.

You will like this way is not that I am showing I have shown the nature of this one, this is not corresponding to the problem what we have considered. I have given you the exercise that you solve it the p 1 1, p 1 2, p 2 2 dot using the final terminal condition, using this final terminal condition as well as use the numerical method. Say the rung out a fourth order method or simplest way we want to solve if the sample step size is very small you can use what is called the Euler’s method also for solution of these 3 differential equations. So, once you get the solution you store how you store this one let us call I am storing the p of t how you are storing it and p is having how many elements, there are 4 elements are there for this particular example.

So, you start the 4 elements of this one that is, that is you see at time you stored it this one p 1 1, 6, p 1 1, p 1 2, 6, p 2 p 1 2, 6 and p 2 2, 6 with the knowledge of this is given with the knowledge of this one, with the knowledge of this one. Then you find out p 1 1, 5, p 1 2, 5, p 2, p 1 2, 5 and p 2 2, 5. So, this next you store it p 1 1, 4, p 1 2, 4, p 1 2, 4, p 2 2, 4 and so on and last what you will get it p 1 1, 0, p 1 2, 0 and p 1 2, 0, p 2 2, 0 in this way you just stored it. So, this competition you have to do offline competition and then you stored it in a memory whenever you need it.

Let us call I want U of 0 when to complete minus R inverse B transpose p of 0 into X of 0, so you retrieve this information p of 0, information from this stored memory from this one and then get it this one. Now, how to find out U of 1 is again R inverse B transpose p of 1 into X of 1, so p of 1 is retrieved, this information from the stored memory. So, this you this is p of 1 stored and then X of 1, how you will get it by simulating that one set of this or from the system you are getting the information of X 1 of t, sorry X 1 of t.

Then in this way you just solve this problem to get it that one agreed so in general that this is the procedure, how to solve the finite time regulated problems. Next will come what is called stability analysis of finite time regulated problems, stability analysis, so next is stability analysis.

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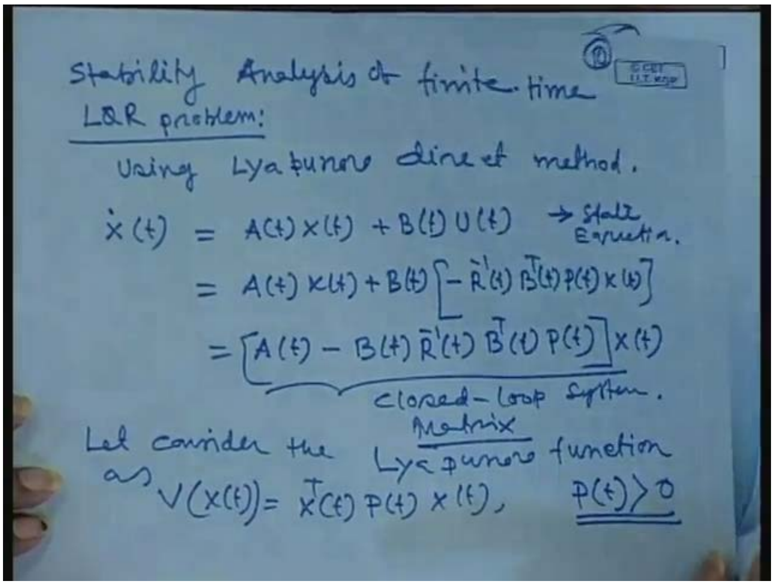

Next is stability analysis of finite time L Q R problems we will be using Lyapunov direct method, this analysis is very straight forward if you see. Let us consider our system equation X dot is equal to A of t X of t plus B of t U of t this is our system state equation, state equation and if you use the value of U of t. Then you will get it is nothing but A X of t B of t and U information is minus R inverse of t R inverse B transpose p of t into X of t.

So, this and this if you club together you will get A of t minus B of t R inverse of t B transpose of t p of t into X of t, now see this is the our if you say this is our closed loop, systems dynamic closed loop system matrix. Now, I want to study the stability of this systems, let us consider, let us consider the Lyapunov function as V of X of t is equal to X transpose p of t X of t. Say p is what is p you have considered is a positive definite matrix p of t that is all this is called Lyapunov function, we have considered for any this function you say when p is greater than 0.

This function value is always positive for any value of t all these things, now this nothing but energy function in Lyapunov function this is one of the big. This energy function value is either positive or 0, it cannot be negative this one, so with this one if you if the system is stable that if the system is stable the energy should decrease with time. That means V dot must be negative value and suppose the system is excited to the initial condition this whether the system will be stable or not. You can say that if the energy contained in the system due to the initial conditions what if the energy that energy should decrease with time that means V dot must be negative.

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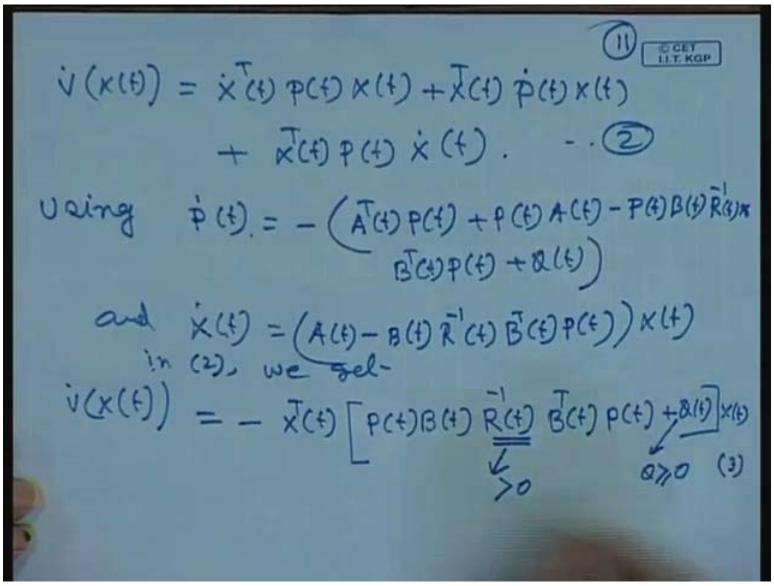

So, this X of X dot of t we can write it, now if you differentiate with respect to time t then we will get three terms associated with this one X transpose of t, p of t X t plus X transpose t p dot of t X t plus X transpose of t p of t X dot of t. We know the expression X dot of t expression of the closed loop system, let us call this equation number 1, we know X dot expression is that one we replace wherever when we will get the expression for X dot you replace by equation 1.

So, using p dot expression is that a transpose of this p of t, let us call this is equation number 2 plus p of t A of t minus p B, R inverse of t B. Then star B transpose p of t plus Q of t use this expression p dot expression and X dot of t is equal to X dot of t is equal to you got it. That one X dot expression what we got it just now we have seen it a of here X dot expression this or you write it X dot expression A of t is equal to B of t, R inverse of t plus B transpose of t, p of t into X of t. So, using this expression and this expression in 2, in this expression we will get finally after simplification we will get finally that one you see using p dot X dot in equation 2.

Using p dot and X in equation 2 we get using p dot X dot expression in 2 we get that 1 minus X transpose of t. Then p of t B of t R inverse of t B transpose of t and p of t plus Q of t whole into X of t, this must be a negative definite. In order to become negative definite because V dot if it is a system is stable then V dot must be a negative definite. So, in order to become a negative definite, this must be a positive definite because it is preceded with the minus sign, this must be a positive definite. So, Q is if you see the Q is greater than equal to 0 positive definite and this R is, R is positive definite this implies R inverse also will be a positive definite matrix.

So, clearly from the expression of 3, clearly from the expression of the 3 that whatever the logic we are put it Q is a positive definite matrix, positive semi definite matrix R is positive definite. So, R inverse also positive definite matrix, it is preceded with a matrix and its transpose post multiplied by its transpose and pre multiplied by that matrix, so this matrix also will be a positive definite.

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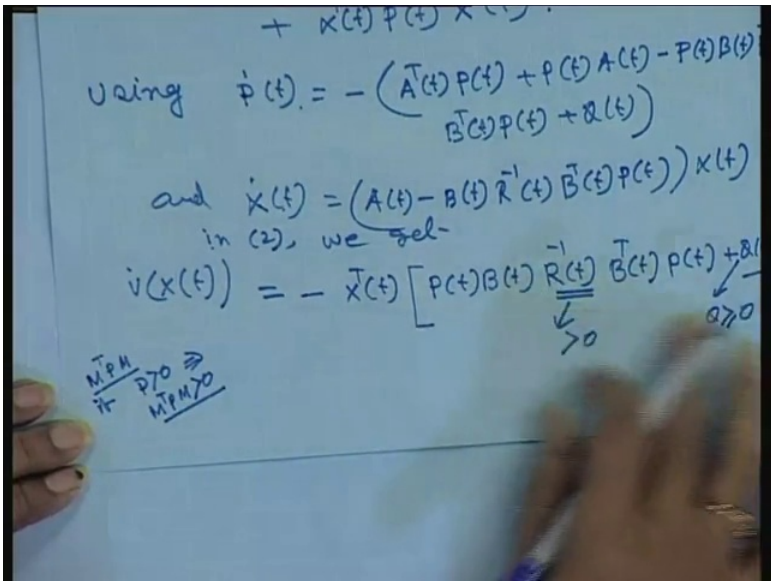

That means we can say M transpose P, M this matrix if p is greater than 0 this implies that M transpose P of M also greater than 0 means positive definite this we have used it here.

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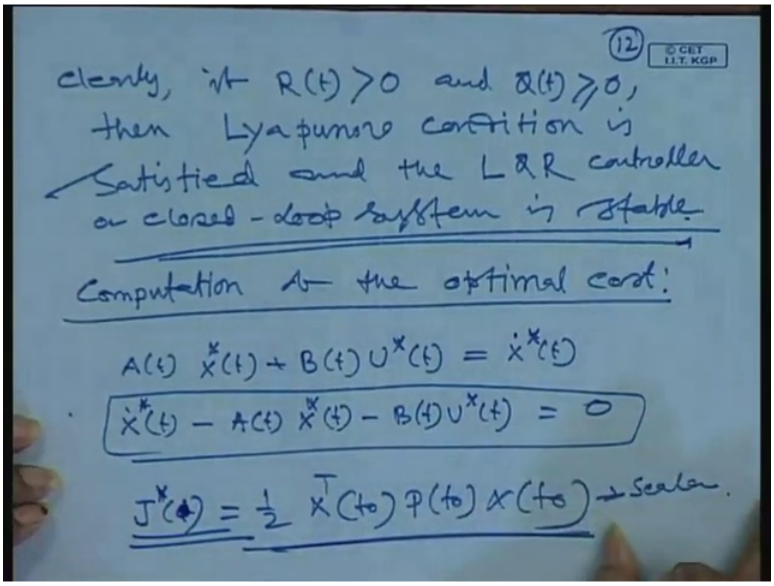

So, clearly one can say clearly if R is greater than 0 and Q of t greater than equal to 0 then Lyapunov condition, Lyapunov condition satisfied and the L Q R controller or the closed loop system is stable, so this so the stability of the. Now, this is the stability of the, now let us call if you are interested to find out what is called the as I told you if you use what is called our finite time regulated problems that the way we solved it.

Here, it is obvious that we have to check it what is called the sufficiency condition that means whether the controller will give you the minimum value of the functional or maximum value of the functional. That you have to check it and that one can check with this matrix this is called the Hessian matrix, Hessian matrix that hessian matrix must be a positive definite for functional value to be a minimum. We have seen that it is enough to check let us call if you select the R of t and Q of t is R of t positive definite and Q of t is positive semi definite that it ensure that function value what a function value.

You will get it minimum and stability analysis also shows that this R of t greater than if it is there clearly the closed loop system or L Q R controller design stabilize our original systems because if you see the our original system is unstable. Now, with this controller we can able to stabilize the systems, so next question comes, so as I mentioned in the algorithm if you see this algorithm step 4 of this algorithm the optimal value of the cost function is that one. Say how to prove that one this is next we will consider computation of the optimal cost, since we have got the what is called that our controller U of t that must be satisfied.

So, our what is called subject to that our statement of the problem is like this way design a controller U such that the performance index what is we consider it will be minimized that is the our problem. Find a controller such that the performance index is minimized and this must satisfy this out its own dynamic equation that means it must satisfy the constants. So, naturally the trajectory what we will get it corresponding to the optimal controller that must be A of t, X star of t plus B of t U star of t must satisfy that equation.

That means this implies X dot of star of t minus A of t X star of t minus B of t U star of t U star of t must be equal to 0, since the knowledge of the optimal controller U the optimal trajectory we got it. That must satisfy this, our own dynamic equations agreed this is our condition based on this one, based on this one we will derive the optimal value of the function. That we will discuss next class that how to derive to get the value of optimal value of the function that I have considered that J optimal value of the functional J.

This is equal to half X transpose of t 0 and p of t 0 X of t 0 that portion we will derive next class, now look this if you know the initial state of this one which we know the our statement of our problem which you know from the statement of the problem. If you know p of t 0, I know p of t f, t is equal to t f final value then p of t 0 we can find out by from the solution of matrix Riccati differential equation backward in time and stored it p of 0. Then this product of this one is a scalar and that will give you the optimum value of the functional, so we will stop it now.