Optimal Control

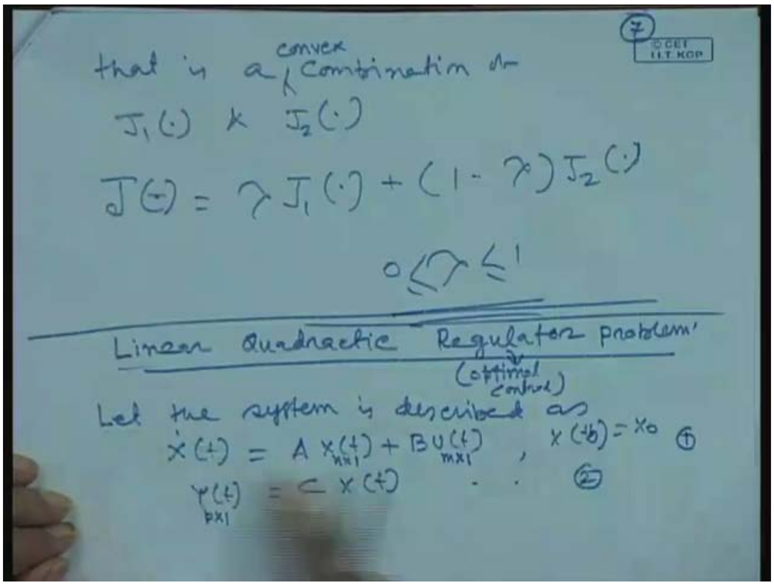
Prof. Dr. Goshaidas Ray
Department of Electrical Engineering
Indian Institution of Technology, Kharagpur

Lecture – 38

Performance Indices and Linear Quadratic Regular Problem (Contd.)

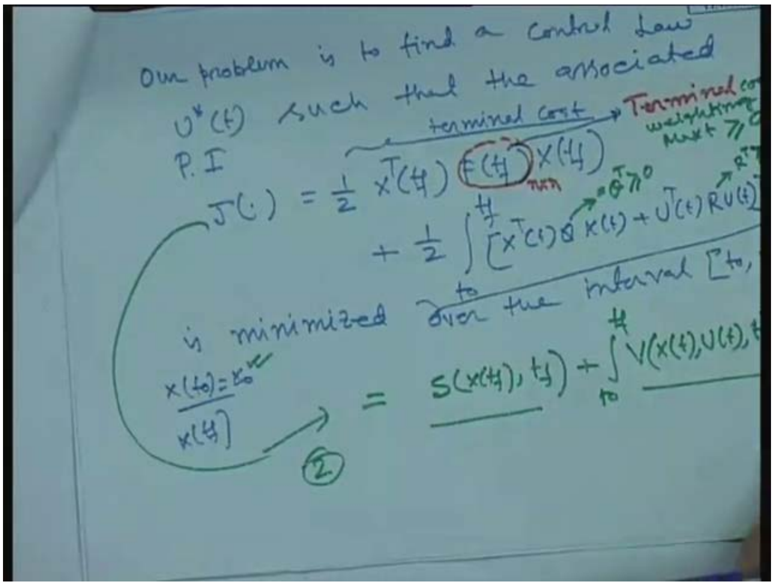
So, last class we have discussing about the linear quadratic regulator problem, let us recall this problem.

(Refer Slide Time: 00:21)



We have a dynamic system which is described by x dot is equal to A x plus B U and x is the state vector whose dimension is n cross 1, use the input vector whose dimension is m cross 1 and e y is the output vector. So, our problem is to find a controller U of t such that this performance index what we have described this one. The performance index is minimized in other words, our aim is to find a controller that will drive the state x t 0 t is equal to t 0 that will drive the state at t is equal to t 0 to a finite time t is equal to t f near the optimal near the origin or near the 0 state. So, for that one, what should be the choice of U so that it will satisfy the performance index, not only this one, it must satisfy our given dynamic equation one and two. This is our problem and we have seen this problem what we have considered the performance index, if U see this one performance that interpretation of this waiting matrices.

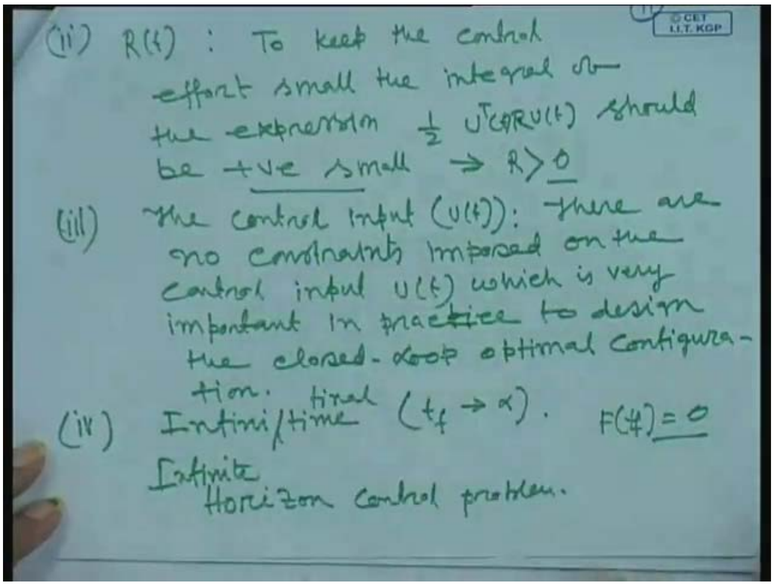
(Refer Slide Time: 00:47)



The state waiting q is the state waiting matrix, because this is associated with the state R is the input waiting matrix this is associated with that input. We have a terminal cost that may time t is equal to 0 t is equal to t f at finite time our state initial state will go to the near the origin not exactly origin, but near to the origin near to the origin at finite time t is equal to t f. So, this is the state terminal waiting matrix this one and if you do not consider that in the performance index that input term that control effect term.

Then, if you minimize this one, we can say that we do not have any control with the input effort, so input output may be extremely high, which you cannot directly apply it to the systems. So, in other words if you do not consider the state waiting matrix of that one quadratic performance index this part, then we do not have any control on the state. This is why we are taking the linear combination of state waiting matrix and input waiting matrix control. We are minimizing these two efforts that mean control efforts as well as the state waiting matrices effort this one.

(Refer Slide Time: 03:23)

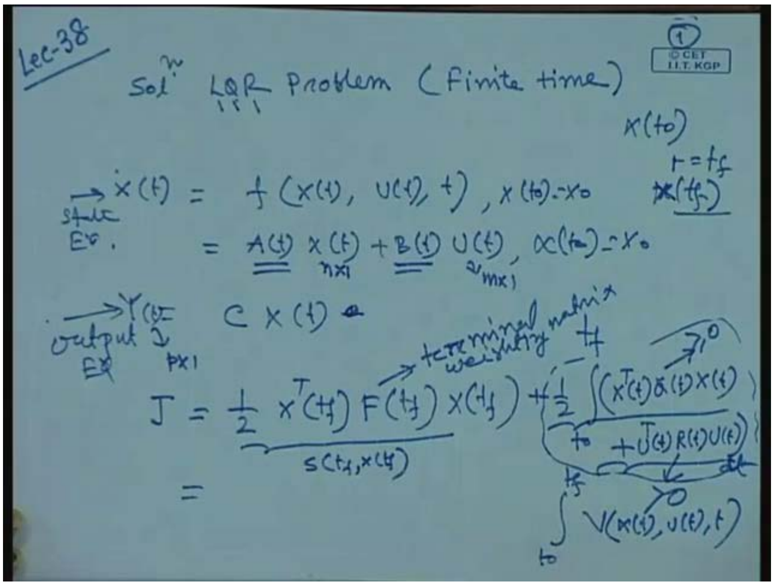


So, this is our basic state and the choice of q R what q should be a positive semi definite and R should be what is called positive definite matrix and in our linear quadratic regulatory problem, we have not considered what we will derive. Now, we have not considered there is there is no constraint on the U as well as x that x and U are free that no constraining impose on x and U, where you will see. Then, when t tends to infinity, instead of finite time t tends to infinity the state waiting matrix that is x t f will be assign to 0 because it has no physical significance of t is equals to t f.

That is function that is state waiting matrix value is assign to 0, because it has no physical interpretation there because all the states will come to 0 asymptotically as t tends to infinity. So, this type of problem when t f tends to infinity this is called this infinity, this is called the infinite horizon problems that also we will discuss later. So, let us see how we are going to derive what is called the LQR problem for finite time regulated problems.

Regulated problem means our initial state are in equilibrium position if there is a initial disturbance in the state is there. Naturally, this state will deviate from the equilibrium point and it should come back to equilibrium point as quickly as possible with the application of control effort. We have to find out control effort so that the performance index what we have considered that is minimized.

(Refer Slide Time: 04:59)



So, next is we are considering the solution of LQR problem, linear quadratic regulated problem for a finite time interval problem bracket finite time. That means once again we define our aim is at driving the state x is t is equal to t 0 to a final state t is equal to t f. That means x t f finite x t f near about the origin or 0 that is our problem for that one what should be the choice of our control input. So, it satisfies our performance index including the terminal cost that performance index, we have considered. So, let us call our problem is like this way x dot is equal to in general, it is A x of t U of t and this and x t 0 is equal to x of 0.

Since, we have described the system our dynamic system is described into step is form that is x dot is equal to a of t x t plus b of t U t and x t of 0 is equal to 0. We have assumed that our system matrices a and b are function obtained time varying parameter matrices if it is time in variant that a of t is constant all the elements of the matrix, so a of t is constant. It does not change with time, so our output equation this is the state equation state equation.

This is the output equation this is the output equation and output equation c into x t plus or you write is simply c of x t. so, this dimension n cross 1 that m U dimension number of inputs is m cross 1. Then, number of outputs you can p cross 1 though in your regulated problem this output will not into picture. We are assuming that will show you later we are assuming all that is all that state variable information x t are accessible to us or measurable to us or in the sense the information of state variable is known to us. So, then our problem is to find a U such that it will satisfy this performance index, so this is the terminal cost which is f t this is the quadratic performing state, but time at t is equal to t f plus half t 0 to t f.

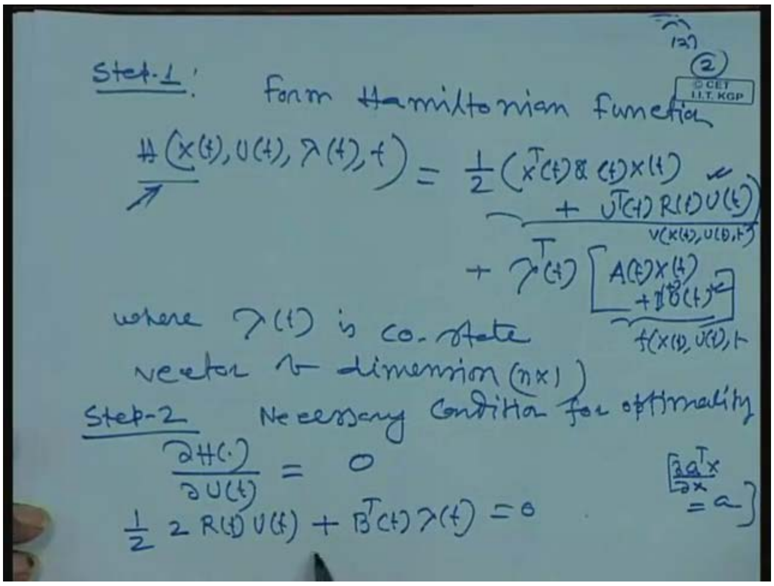
Then, x of t t U of t x of t plus this is inside the bracket plus U transpose of t R of t plus U of t bracket close d of t. So, I told you earlier that why half is taken into account that in the performance index that this half, when you find out the differentiation of Hamiltonian function with respect to U lambda or x. Then, you will see this half and from there it will come to two and this two and a half will be cancelled, otherwise if you do not consider that that two numerical number, two will come throughout this expression. To avoid that one, we just multiply by half, so at what point the optimal value of the function will get it.

That will not change the as you have seen in the static optimation problem that if you multiply a performance index by a constant or divided by a constant that what is called optimal point at which the function will be optimal. That point does not change it is we have seen it, so this you can say as far our notation, this is the final what you can call terminal weighting matrix. This is the state waiting matrix which is greater than equal to 0 means positive definite, this is the control letting matrix that value is positive definite matrix that we have to discuss.

Earlier, all that must be from physical consideration, also that must be positive definite matrix, it cannot be positive semi definite, because when it is positive semi definite control effort applied to the system will be 0. Then, this is why it is a positive definite matrix, so let us from our that is if you just compare with this one that is nothing but our s f s t f and x of t f function. This whole thing is nothing but this half, you push it in inside, then half x transpose U x U transpose r, this whole thing is nothing but a v, the integral point is v.

So, we can consider that whole thing that whole from here to here that part is v which is a function of x t U t and d t including half that is v as per our usual notation. So, this part is the integral part of that one, so this whole thing I can write it t 0 to f, this whole thing is will be replaced by that one, so let us see what is our Hamiltonian function we know.

(Refer Slide Time: 10:59)



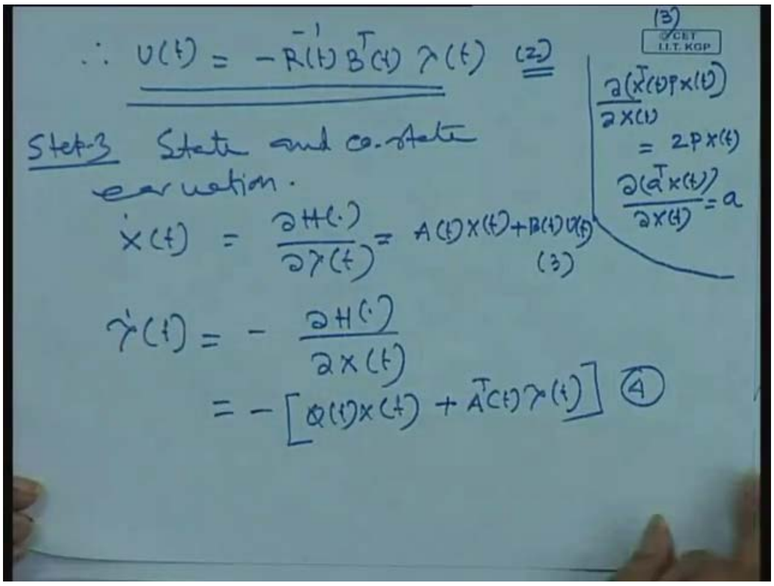
At this moment, how to generate or form the Hamiltonian function form Hamiltonian function you know U H h is a function of x t U t lambda t and t and which is nothing but a our original system equation x dot is equal to A x plus B U. So, I will write that our integral part of this integral part of the cost function that is half then your x transpose t q of t x t plus U transpose R of t U of t that part. That means first is what integral part of the performance index v of x v of x is nothing but a v dot x function of what is called U, and t is nothing but it is plus that in a what is called leg range multiplier of this multiplied by our x of t x t plus b of U t b function of t this one.

This is our Hamiltonian function, which is free from x dot that we have discussed earlier that one or if you see more carefully, it is nothing but our f of x t U. This whole thing is v of this if you see this is nothing but a v x of t U of t and t this one, so where lambda with t is co state vector of dimension that lambda is dimension n cross 1. So, this is the first step you form that linear quadratic linear quadratic regulatory problem is the system description dynamic system description is given in state space form and corresponding performance index is given. That terminal cost plus the integral of the quadratic form of in state and input matrix that is input matrix vector.

So, from this information I can find out the Hamiltonian matrix next step is step two necessary condition for optimality. So, what is the necessary condition del H del U of tis equal to 0, so what is del H of del, you see del H del H with respect to U. So, the U term is involved here and U term is involved here and we know that if you have A x transpose p x if you are differentiate with respect to x vector. Then, results is twice p x, so I will just apply that one that U transpose R t U t I am differention with respect to U t that result will be twice R t U t to this twice and half will be cancelled. So, it will be half as it is twice R of t U of t, I am getting from this part, again there is a function of U of t here,

So, it is a lambda transpose B U of t and we know that partial differentiation on a transpose x with respect to x. So, I am applying here, if you apply is here that one, then it will come plus b transpose t of lambda t is equal to 0, because this term you see this I can write it b lambda transpose B U t. I can write lambda transpose b, I can write B transpose lambda whole transpose and I made it into this form a transpose, then x means U.

(Refer Slide Time: 15:40)



So, from there, we are getting from there we are getting U of t our control input is equal to minus R inverse because R inverse b transpose of t lambda of t. Let us call our equation that Hamiltonian, this is equation number one given, this is equation number one and you give it this is equation number two. So, in order to get this one, what we have used it, we have used this two things differentiation of this with respect to quadratic function with respect to x of t. If you do this one, the result is twice p x of t another, we did it a transpose x of t this with respect to x of t is equal to a.

So, this two things we have used in order to get equation number two, so once you get it this one other to necessary conditions are this one. If you see the step three, the state and co state equation state and co state equation are given as x dot of tis nothing but a del H del lambda of t. That means H is a scalar function I am differentiating with respective lambda vector so that see this one the lambda is associated with this one, only the second part.

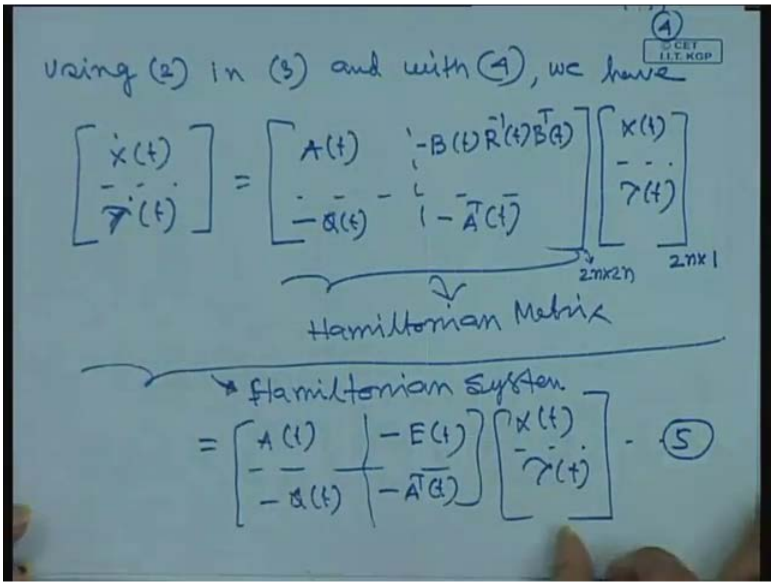
So, if you differentiate this one with respect to lambda, so I can write it this is nothing but a A x plus B U, if you differentiating with respective lambda, I am differentiating with respective lambda. So, I can write it this is nothing but A whole transpose into lambda that because this is a scalar quantity this and this multiplying is scalar quantity. So, I can write it A x plus B U whole transpose into lambda, it is same because it is a H is a scalar quantity, this part is a scalar quantity. So, if you differentiate this with respective lambda I am getting this expression is A x a t x of t plus b t U of t.

This thing is a scalar quantity what I take the transpose of that one multiply by lambda of t, so this will be same as this values, then I am differentiating with lambda t and that results is this one that using that property, I mean that expression. So, I can write it, now next equation post state equation lambda dot t is equal to minus del H del x dot x of t not dot x of t. Now, I am differentiating this with respective x, so x is involved here, x is involved here, so from there I will set first one, I will carry with if you differentiate with respective x, you will get twice q of t x t 2 cancel q of x t, I will get.

So, this will get it minus q of t x t the first part, then you see the this one lambda transpose A x t. So, this I can write lambda transpose A x, I can write it a transpose lambda whole transpose, because this two I am writing A transpose lambda of t whole transpose is same as that part. So, I am differentiating with respective x, so it will be coming plus a transpose of t lambda of t.

So, this let us call this equation number three and this is equation number four, so using equation number two means that one this this expression you use in in equation number three and with four. That means using equation number two, you o t replace U of t by this expression and then augment that equation with the co state equation.

(Refer Slide Time: 20:20)



So, I am writing using two in equation three and with four using it four we have, so x dot of t that means if you write equation number three in equation number three, you are using the expression of U of t. Then, augment that U of t with not be that, then augment that equation three change equation three with four, so I am just doing this lambda dot of t is equal to this a of t. Then, B of U t B is your B as it is U of it is what R inverse b transpose b transpose and lambda of t and that I am writing x of t and this is lambda of t. So, the lambda t can be multiplied by A x actually A x a into x b into U b as it is U I am writing minus R inverse b transpose lambda.

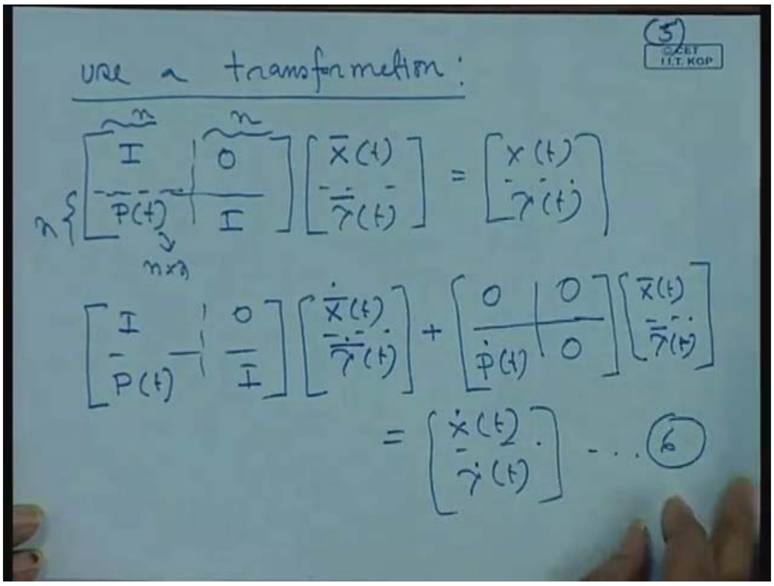
So, this is U of t see, this I am writing q of t minus a minus a transpose of t, so this dimension is since co state vector and state vector dimension are same each is n. So, this dimension n cross 1, now this is called is Hamiltonian matrix and this whole thing is called Hamiltonian system and this matrix dimension is 2 n cross 2 n. Now, look at this expression, since our given system a t b of t q of t R of t is the designer choice.

If you know the designer what is the designer has taken into consideration to design the LQR problem, then this whole matrix is known to us that means Hamiltonian matrix is known to us. We can find out the it is Eigen values, only difficulty is their the parameter changing with time, so naturally the Eigen values will also will change with time. Suppose, if you consider a of t B of t R of t and q of t is fixed, does not change with time, then this is a constant matrix and Eigen values. You can find out and this Eigen values is a closed loop system Eigen values, if x dot is there lambda dot of t is there.

We have a in addition to this, there are n Eigen values are there which is closed loop Eigen values and another n Eigen values are there which is associated with the co state vectors. So, let us see what is this represent what is the Eigen values of this one, what my main steps is here without calculating anything. If you designer has selected R of t q of t and system matrix. I can find out what is the closed loop system matrix Eigen values. So, this we can write it more compact rather simple form.

I am replacing by another variable e of t plus minus q of t minus k transpose of t, this equal to x of t dot lambda of t. So, let us call this equation number five, now I want to show you here with the similarity transformation. This Eigen values are nothing but Eigen values of closed loop system of that lambda dot co state vectors Eigen values of this system matrix of co state vectors.

(Refer Slide Time: 24:47)

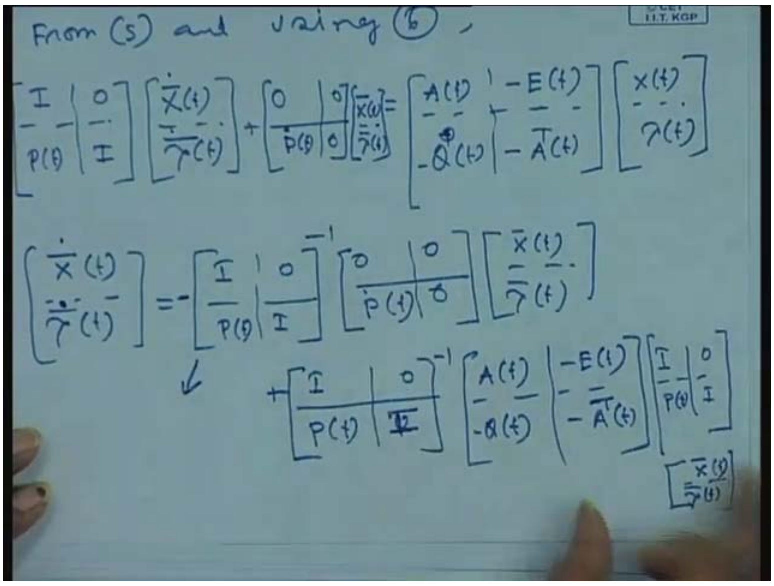


So we use a transformation and this transformation I have used it 0 i p of t and p of t is a matrix whose dimension is n cross n. That means this dimension is n this dimension is n, this dimension is n naturally all matrix dimension. You know this is invertible by look this one this is invertible because diagonal matrix blocks are unity and it is a non zero that all columns are linearly independent columns are there, so their inversion is exists.

So, I am using the trance function x is transformed to x bar lambda is transformed to lambda bar of t and that is equal to x of t and lambda of t.

So, if you differentiate this thing both side what we can write it first is this this p of t, then lambda bar dot of t lambda bar, sorry x bar dot of t this plus another term is differentiation of this block with respect to 0. Then, 0, then p dot of t, so this is that one, so this into that U x bar of t and lambda bar of t is equal to x dot of t write inside x dot of t lambda dot of t. So, I know what is the lambda x bar dot of t lambda bar U dot of t, so I will put it this expression in this side and then we make some manipulation and simplification we will get it. Now, this is the equation number, we are given up to equation number five, so let us call this is a equation number six, so from six.

(Refer Slide Time: 27:19)



So, from five and using six from five and using six, what we can get it, so what i told you that in place of lambda x dot lambda dot, I will replace by this equation using five x dot lambda dot, I will use that one. so, right hand side equation is x dot lambda dot is replaced by a of t dot partition e of t minus q of t, sorry not transpose or you can write transpose because it is a symmetric matrix minus a transpose of t. So, this is equal to x of t lambda of t, this is the right hand side, I have written left hand side will be as it is as you have seen it i 0 p of t i.

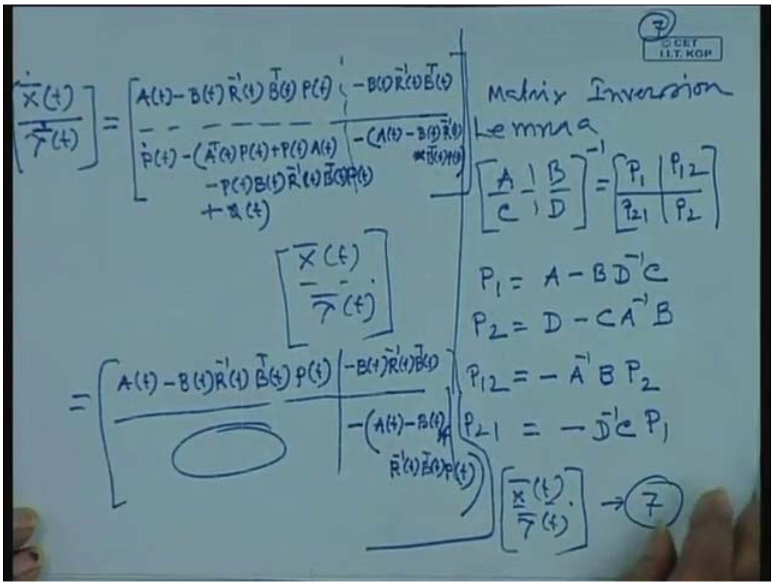
Then, lambda bar dot of t, sorry x bar dot of t lambda bar of t plus 0 p dot of t and 0, this multiplied by x bar of t lambda bar of t and this I have written. So, after simplification this then you bring it that side, right hand side, this part you bring it that side and x and lambda t you replace by transformation matrix.

How they are related x of t lambda of t related to with x bar of t lambda bar of t that we know with this expression. If you recollect this is the expression we have considered for transformation, this is the transformation. So, replace x of t lambda of t by this one in the right hand side of this part and ultimately if you do it that, what you will get it that one just after simplification lambda dot x bar of t lambda bar dot, this is dot of t is equal to i 0 p of t i whole inverse agree whole inverse. Then, this multiplied by 0 0p dot of t 0, then x bar of t lambda bar of t, this is minus preceded, because of this term.

I have taken right hand side, then this inverse I have taken both side, I have multiplied by inverse of that one. So, this is this part will be what see this will be inverse of that i 0 p of t i, this is i whole inverse, then a of t minus e of t minus q of t minus A transpose A transpose of t multiplied by this x and lambda t is replaced by x bar. So, if you replace by this one, this will get it you i 0 i p of t multiplied by x bar of t and lambda bar of t, this is the expression we got it x bar of t lambda bar of t.

So, after just this and this you club together and if you take this common then this multiplied by this matrix multiplied by this matrix and resultant, you will get a matrix of 2 by 2 before that I have to know inversion of this one. I have to get it because I know the inversion is exists because this matrix has a n linearly twice n linearly independent vectors. From this structure, you can say twice n linearly independent vectors are there. So, this inversion is exists, now using what is called matrix inversion lemma one can get these values are like this way matrix inversion lemma.

(Refer Slide Time: 32:07)



What I am just writing it here matrix inversion lemma, so if you have a matrix a suppose we have a matrix A B C D and their partition like this way. It is assumed that that A and D inverse is x is H to a partition in such a way that this as a square matrix and their inversion exists. So, we have to find out this inversion of that one so that is the inversion that is let us call this is p 1 that is p 2. Let us call p 1 2 p 2 1. So, what I have done, we have a matrix A B, let us call a matrix we have partitioned the matrix into four block in such a way that a matrix this block and this block which will be a square matrix and inversion is exists.

Then, I can write p 1 is equal to a minus a this one see sequence a minus then B D inverse D inverse C P 2 P 2 is this correspondent D, D minus C, D minus C. Then, A inverse B A inverse, then P 1 2, this one is equal to minus A inverse B A inverse minus A inverse B then P 2. When I have taken the this P 1 2, you see P 1 2, I starts with A minus A inverse B, then p 2 this one P 2 already I calculated then p 2 1, tell me this one minus P 2 1 is this one minus P 2 1, I am finding out minus D inverse, then C, then P 1.

So, this is the matrix inversion lemma, so if you put the matrix inversion lemma here, this this places and simplify this one. Ultimately, I will get it final expression, I am writing x lambda bar dot of t this what I will get it i just write it final expression, please see the final expression, what final expression this. You use the matrix inversion lemma use the matrix inversion lemma, then multiply this one multiply this and this this three matrices multiply, and then this, and this you add. Then, finally you will get it this matrix a of t minus b R inverse b transpose of t p of t this is the first block you will get it and second block you will get it minus b transpose R inverse b b transpose b inverse b of t R inverse b transpose b of t.

Then, this you will get it p dot of t minus a transpose t p of t plus p of t plus a of t, please remember that p is a symmetric matrix here we have just considered this p is is a symmetric matrix, the transformation what we have considered here. This p is a symmetric matrix and say p is p is equal to p transpose of t, so this plus this is inside the bracket this 1 plus p b R inverse of t b transpose of t into p of t plus q of t. So, this is the 2 1 block then the 2 2 block, what we will get it minus 2 2 block, I am writing minus a of t minus b of t R inverse of t into this is continuation b of t R inverse of t into b transpose of t p of t.

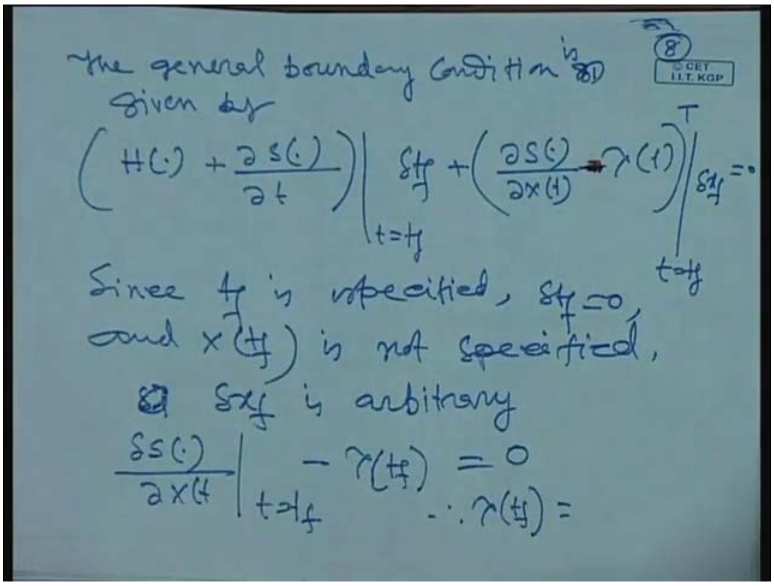
Now, you see these block a one block and a 2 block a 2 block is same as a 1 1 block preceded with a minus sign only and other two it a 1 2 block 1 2 block 2 2 block that whole bracket multiplied by what x bar of t lambda bar of t. Now, this is the equation after transformation, we got it this one, now I am telling you this is you see I you select p of t in such a way. So, that p dot of t minus of whole expression p A transpose p of t p of t a t al these things will be 0. So, you find out the p of t value in such a way so that this is assigned to a 0, it will be 0 or you can say it I will assign this quantity is assigned to a 0.

For that, you find out what is the solution of p that means this quantity, I will assign to a null matrix of dimension n cross 1. This whole thing I will assign for that one you solve it p such that this equation is satisfied, so I can write it that a of t minus b of t R inverse of t B transpose of t p of t this this is 0. This is B of t R inverse of t b transpose of t 1 and this one is minus of a of t minus b of t cross this is the cross multiplication R inverse of t b of t and p of t p of. This is just same as that one only preceded with minus, so this whole thing I am writing this whole bracket after this because I have assigned this is 0.

In other words assigned this one find a solution of what is called matrix equation, so that this expression H be satisfied find as solution for p for which this will be equal to 0. So, that whole thing multiplied by x bar of t that lambda bar of t, now you see the this is the block diagonal form the Eigen values of this one this whole matrix Eigen values is nothing but a Eigen values of this one and the block 2 2 block Eigen values. Whatever is the Eigen values of this block the Eigen values of this block will be same with opposite sign, because it is preceded with a minus sign. It will be opposite sign, so this we can conclude this one that Eigen values of total state enclosed state vector Eigen values.

This one is nothing but Eigen values of this block and union of Eigen values of this block and these Eigen values are same only difference is the Eigen values of this one, with preceded with a minus sign means opposite sign of that Eigen values will be here. So, let us call this is equation number seven, this is a equation number seven, now we can see this one. So, we got it what we made it conclusion that one that if you have a co state vector, if you do the transformation of this one, which we have seen the augmented cos state. The co state vector can be represented into a upper block form the Eigen values of x star is nothing but a Eigen values of this one Eigen values of lambda dot is nothing but a lambda.

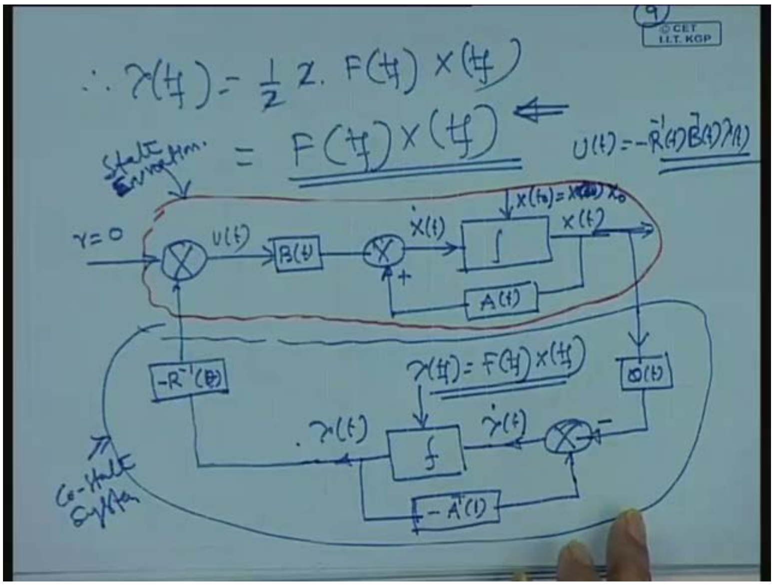
(Refer Slide Time: 41:38)



Eigen values of that one with precede with minus sign that is why sign in, so our general boundary condition, if you see boundary condition is given by H. We will refer our the terminal cost all these thin boundary condition t is equal to t f delta t f plus del s of del x t minus lambda of t this is minus lambda of t this is minus lambda of t a t this whole transpose t is equal to t f and delta x f is equal to 0. Since, our what is called t f is specified, since t f is specified, because we have considered our problem that our aim is driving the our initial state x of t 0 to a final state x of t f t is equal to t f near to the origin.

So, t f is specified, therefore delta t f is 0 and our x t f near to the origin means x t f is not specified near to the origin. Therefore, delta x f delta x f is arbitrary, so then what is our boundary condition, we are getting it here just see this one. So, you just put it here x delta s of this differentiate this with respect to x t put t is equal to t f minus delta t is equal to t f this equal to 0. Therefore, we can write x t f is equal to if you differentiate this one, if you see with x our if you go back to this one, if you differentiate this thing that s term with respect to x and put t is equal to t f this you are differentiating.

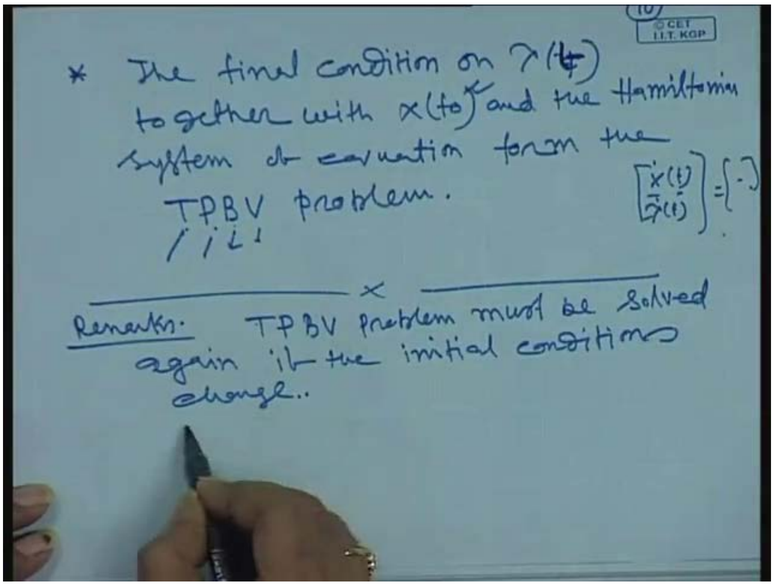
(Refer Slide Time: 44:31)



So, what you will get it this one twice f x t and put t is equal to t f, then it will come like this way, therefore lambda t f is equal to half twice f t f into x t f. So, this, this cancel, so f t f into x t f, so if you do the block diagram representation of this one, now we can find out our controller if you see our control/controller is minus R inverse B transpose of lambda t solution of lambda t. If you know it, then lambda t, then I can find out the controller, let us draw our block diagram of this representation our reference input. We have considered 0, because due to the initial condition what is this we are showing it this is b of t. So, this is our star and this is our integrated and this is x dot of t the output is x of t, then this is coming is from here this state a of t and t this is A.

If you just relate this one x dot is equal to A x A x plus b that must be U of t, then this is this from there what is lambda, you see lambda expression this is U of t. Then, this is coming to here this is lambda dot of t lambda dot of t and you see there are lambda dot of t. What we got it here minus q x, if you see this one lambda dot of t here lambda dot of t minus q x x t so x t is coming. This is sign is either you put it minus here or you put it minus here so this then then a transpose lambda t. We have to solve I told you that what we have to do this, now this in order to solve this one we have to solve that final condition, so note the final condition.

(Refer Slide Time: 50:03)



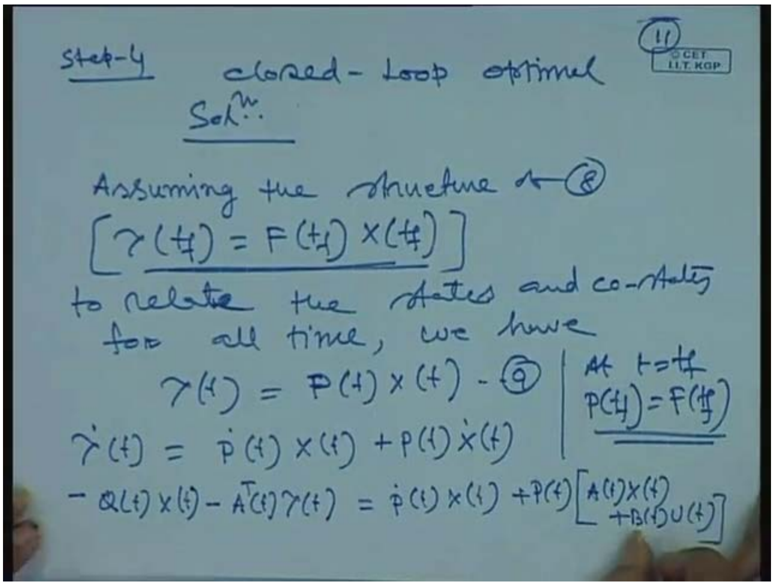
Together with final condition on lambda t f on t final condition on lambda t together with x of t zero and the Hamiltonian matrix system what is Hamiltonian system x dot of t and lambda dot of t. That expressions ag augmented system is the Hamiltonian system Hamiltonian system of equation form that means final condition on this together with x t zero initial condition of the state and Hamiltonian form the 2.2 point boundary value problem.

So, we have to now solve this equation knowing the lambda t and final condition on t f and together with the initial condition x t 0 and Hamiltonian system and Hamiltonian system is what x dot of this lambda dot of t from this augmented system. What is the expression we got it with this season, we have to solve this, so it is called two point boundary value problems. Now, look carefully see this one, now so how will you solve that that one, how you can solve that one that x t of that things because x lambda t f if the final condition of this one is known to you that lambda t f. Then, we can solve this one in backward integration form, now see this one lambda t f t is equal to t f final condition t f that if you know the lambda t f value lambda t f value.

Then, I can find out lambda t f minus what is called previous state of that one that what is called if it is the step size is capital T f minus t, I can find out by solution of differential equation in backward process. This whole thing is dependent on x 0 x t 0 if x to t 0 is changed, then it is also change that solution of this one also change it. Now, how do you will solve this one, next question is how you will solve this problems, there is a one next solution of this one is a. If you see carefully, this one this is a open loop solution in the sense it is dependent on the solution of this one dependent of x t 0 x t 0 means x 0 of that one.

So, now how to form it that remarks, you can write it remarks that t p two point boundary value problem must be solved again if the initial condition change if this initial condition is changed. We have to again solve this one, now what is this one, how you will get it this one x t f, let us call the x t f is known, where we want to keep our final state agreed because I told you x t is equal to t is equal to t f it is specified.

(Refer Slide Time: 55:06)



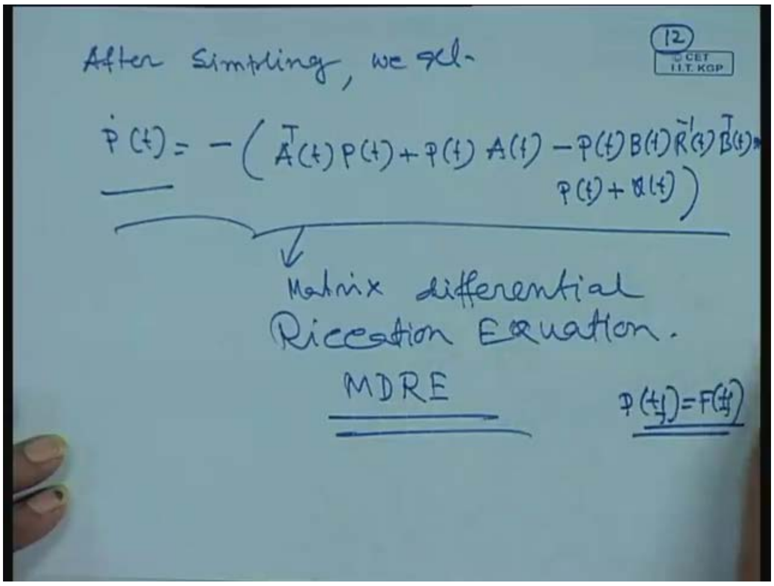
That means we are looking for a controller U that will drive the initial state is equal t 0 to a final time at t is equal to t f, where t is equal to t f the state x is equal to t f will be near to the origin. In final timing that is our problem, now question is this problem you have to solve in backward integration, if you know the x t f backward you to integrate and solve this one. That solution depends on the x t 0, so this is if you need to change.

Now, we are looking for a closed form solution step four the closed loop optimal solution so how you do this one we know how lambda is related at time t is equal to t f how lambda t f is related to x t f. So, assuming the structure of eight, we have what is the equation number, we have given equation number, let us call this is equation number eight, this is the equation number eight. This is equation number eight assuming the structure eight lambda t f is equal to bracket, you can write it that which one is x t f into x t f agreed to relate the state and co state the states and co state for all time for all time.

We have lambda t is equal to f of p of t into x of t, they are because they are at t is equal to t f is f of t. So, you are telling that for all the time t is equal to if it is related to this one, then at t is equal to t f, what should be the p t f same as f t f at t is equal to t f, p t f is same as f t f we know this relationship. We got it from the boundary condition of that one agreed so that is the boundary condition and that x t f will be near to the our origin mean equilibrium point.

So, if you consider the equation number nine, then differentiating this one both side, we will get p dot of t x of t plus p of t x dot of t put the value of x dot of t in the expression. Then, you will get lambda dot of t that expression, what is lambda dot of t it is nothing but a minus what is lambda dot of t. We got it minus here lambda dot of t you see lambda from equation three, possibly you are going four lambda dot of is nothing but a q of t into x of t minus a of t lambda of t. So, I will put it this values is there, so it is q of t x t q of t x t minus a transpose of t lambda t, this equal to p dot p dot of t x t plus p of t and what is x dot a of t x t our system dynamic b of t U of t, so after simplifying this one I can write it p of t.

(Refer Slide Time: 58:57)



After simplifying, we can write p dot of t is equal to minus a transpose of t p of t plus p of t into a of t minus p of t b R inverse B transpose of p of t this product p of t plus q of t is that one. After simplifying, we get this after simplifying we get that expression, now you see the solution of this one is not dependent on the initial condition of that one.

The solution of this one is not dependent on the initial condition, whereas when we have found out that this one lambda t f this one, when you will solve this equation in the backward integration the solution of this U is dependent on the x t of zero this one. So, this equation this whole equation is known as matrix differential Riccati equation or in short m d R e matrix and it is a non linear differential equation that our problem is solve this non linear equation differential equation. So, we must know our terminal conditions we know p t f is equal to f t f terminal condition is given. From there, we have to get the solution, we will discuss the solution of what is called matrix differential equation in details next class. So, we will stop it here now.