Optimal Control

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Lecture - 36
Hamiltonian Formulation for Solution of Optimal Control Problem and Numerical Example (Contd.)

So, if you recall our last lecture that given a dynamic system X dot is equal to f of f function of xt ut dt t and subject to performing index or cost function is like this way. So, first term of the cos function is a terminal cos function and second part of this one is integral cos function.

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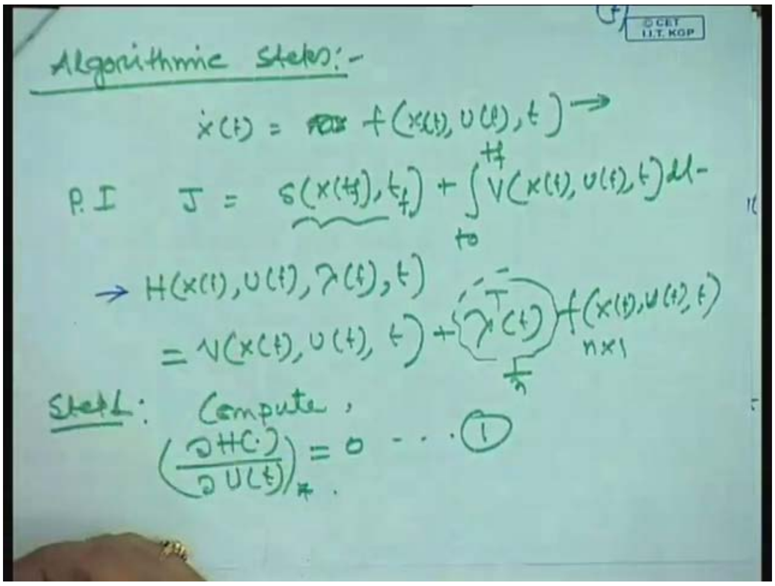

Then how we have solved this problem? Then first we have generated or formed a hamiltonian matrix, this is a hamiltonian matrix with the knowledge of integrant of the integral cost function this one plus that lambda transpose of t of f of t, this is the function of f of t. Behind this idea is this is the constraint optimization problem is converted into a unconstraint optimization problem, using what is called Lagrange multiplier. And Lagrange function is now expressed in hamiltonian function and ultimately we got some necessary condition and boundary condition to solve this problem. So, our first step during this process, our first step is del H, del U assigned to a 0, that is the one necessary condition.

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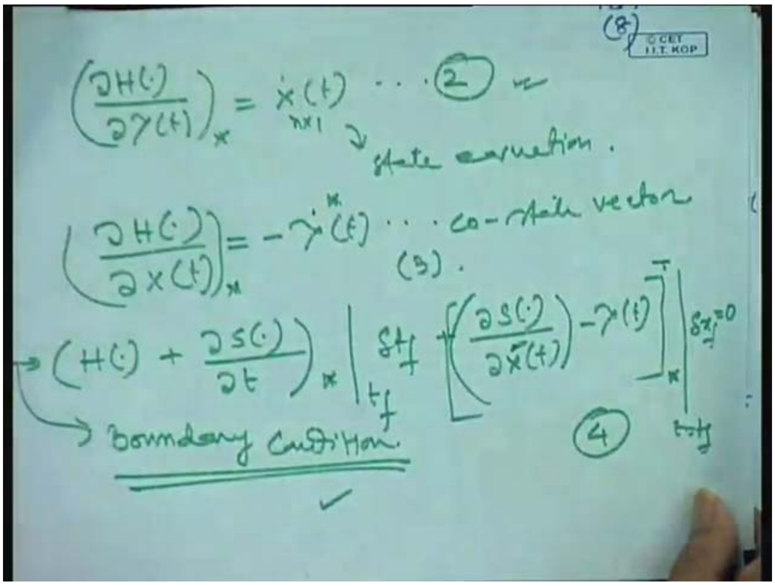

Then you have a del H, del lambda is equal to X dot, which is called the state equation. This is another necessary condition and del H, del X partial differentiate of H with respect to X is equal to minus lambda dot star is a co state vector dimension of this one. And dimension of X and lambda are same and their expression is exactly same, I mean in place of lambda you replace by X and that X dot is replaced by lambda dot with preceded minus sign and this is the boundary condition, you got it?

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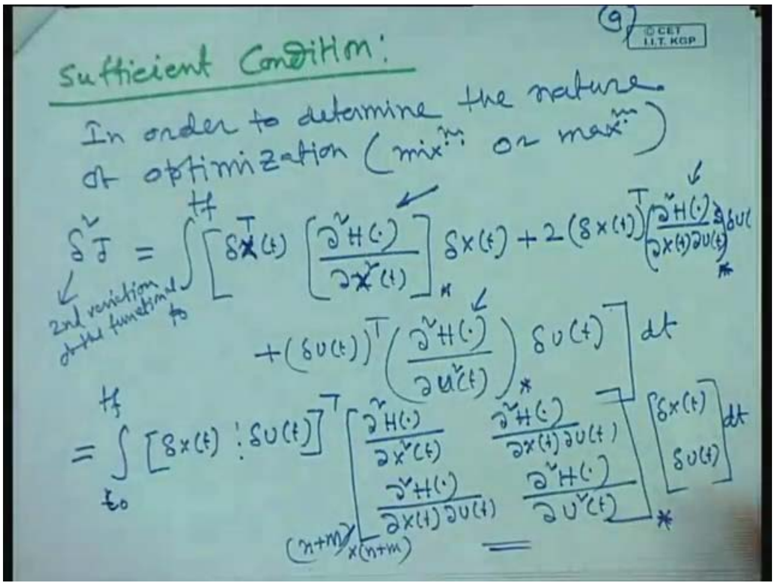

So, one has to solve this equation necessary condition along with the boundary condition to get the optimal solution of our control input, which in turn it will give you the optimal trajectory of the our step. Hence it will minimize or maximize the what is called performing index, that we have to check it with considering the sufficient condition. We have established the sufficient condition for the functional should be a cos function should be a minimize or maximize.

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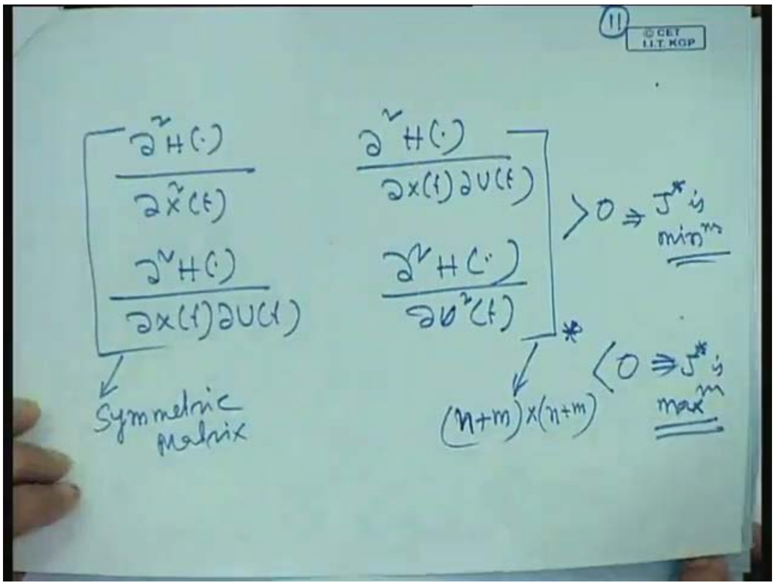

This is the matrix, we have to form this matrix which if it is greater than 0, that J which is the our performing index is minimum. If it is less than 0, that means less than 0 means it is a negative definite matrix and this matrix has a dimension n plus m into n plus m. If it is a negative definite then J is we will get it maximum that means, performing index value is maximum. So, there are different situation is we have discussed first case is fixed final time and fixed final state. So, their first three equation then del H del U is equal to 0, del H del X is equal to minus lambda dot of X, then lambda dot of t, then del H del lambda is equal to X dot. This three equations to solve only changes is boundary conditions and since both are fixed, time is fixed, fixed final, time is fixed and final state is fixed. Then our delta tf boundary condition delta tf is 0, delta xf is 0. So, there will be there is no boundary condition for that one, agree?

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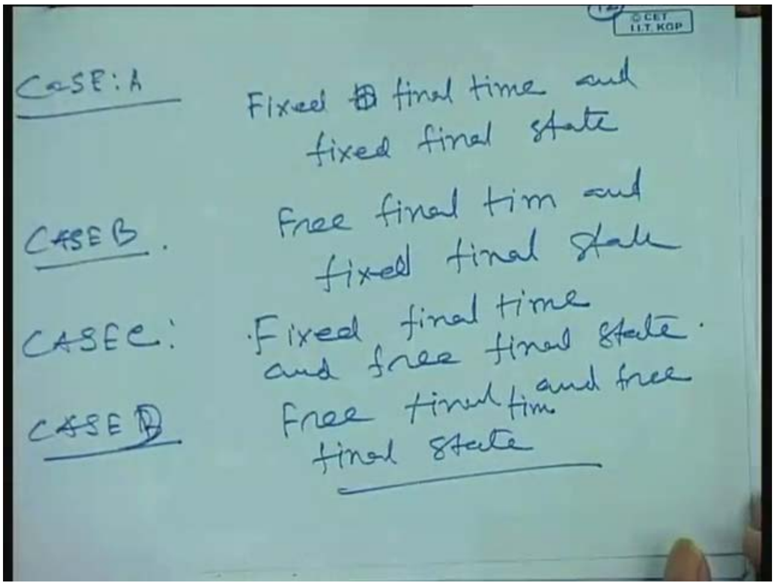

So, free final time, this is time free final time and fixed final state, that mean delta tf is free, agree? Delta tf is free, that means we have to make delta tf is free. This equal to boundary condition, this equal to 0, but delta xf is fixed. The final time is fix, the final time, final state is fixed, agree? So, this equal to 0 so only we have a case B, we have a only one condition is there, boundary condition.

Similarly, final time is fixed, final state is free, final state is free, so delta f is fixed, mean d 0. So, delta xf is free that mean this boundary condition this equal to assigned to 0 and we have to solve when both are free then this equal to 0. In addition to this there is another, this equal to this you have to make it and solve the problem for this optimization problems. So, let us now see how to solve such type of problems in practical problems using the hamiltonian principle formulation.

So, example our example is take and some let us call we have a resistance and we have one capacitance is there, which is connected in parallel with a inductor, agree? And that our problem is this is the current flowing through the circuit i of t which we denoted by u of t. Input to the circuit and that resistance is R and that value let us consider is 1. And that is the inductance, that value you are consider the value numerical value of l is equal to 1 Henry and C is 1 Farad, agree?

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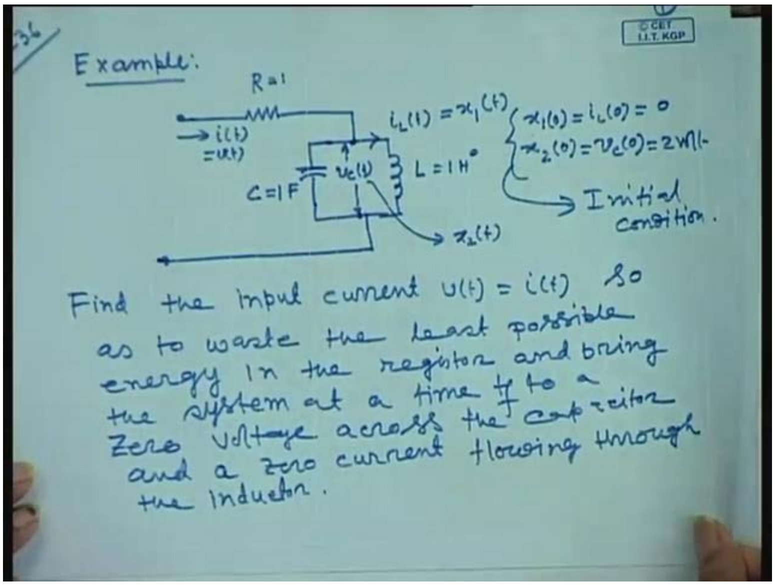

So, let us call current flowing through this one inductor is i L of t and which we denoted the current flowing through this denoted by a variable x 1 of t, agree? So, the voltage across the capacitor, voltage across the capacitor we take v c of t, agree? That we denoted by a variable, that we denoted by a variable x 2 of t means voltage across a capacitor.

So, what is our problem the initial state? Initial state x 2 of 0 is nothing but a voltage across the capacitor which is given to 2 volt and initial current flowing to the inductor is assumed to be this is nothing but a current flowing through the inductor is assumed to be 0. So, our problem is to find a control input u of t, in other words i of t such that after a finite time interval the voltage across the capacitor and the current flowing through the inductor will be 0.

So, our problem is find a control u of t, such that the loss in the circuit, loss power wasted in the circuit will be minimum. Not only this, the after final time interval t f, the voltage across the capacitor and current flowing through the inductor must be 0. So, we now these are the our if we can say these are the all initial condition for the circuit or for the system.

So, our problem is find the input current or input control signal u of t which is equal to i of t. So, as to waste the least possible energy in the resistor and not only this and bring the system state, bring the system at a time t f. At a finite time t f you can write a finite time t f to a 0 voltage across the capacitor, not only this a 0 current through flowing through the inductor.

So, now you see this is our control problem. What is that we have to optimize? We have to optimize the power. Loss in the resistor part is minimum, that is we have to and initial condition state of the initial condition of the inductor current flowing through the inductor is 0 and voltage across the capacitor is 2. We have to find a control signal i of t or u of t such that that the circuit get in a finite time. The voltage across the capacitor is 0, current flowing through the inductor is 0. Not only that, during this finite time the power wastage in the resistance part will be as minimum as possible, mean minimum.

So, this is our problem statement. Now in order to put this problem in the framework of what is called optimization of a dynamic optimization problem, then we have to write the dynamic equation of this system into x dot is equal to f of x which is a function of xt ut comma t. So, let us see how we can make it that one.

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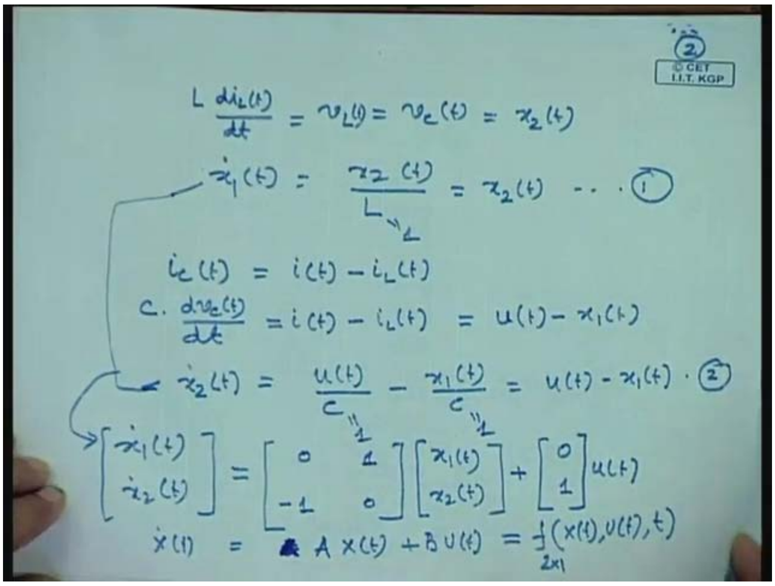

So, we know by basic what is called this voltage across this one, voltage across this one is the same as the voltage across the inductor, is same as voltage across the capacitor. So, I can write by the L di dt L d l current flowing through this one of t dt is equal to v L is equal to v c of t. And this v c we have considered voltage across the capacitor with a variable x 2 of t and current flowing through the inductor, current flowing through the inductor i L of t is a variable x 1 of t.

So, we can write x 1 dot of t is equal to x 2 of t divided by L, agree? Since, our case we have considered L is equal to 1 henry, C is equal to micro 1 farad. Then we can write this is nothing but a x 2 of t. So, this value is 1, so that is let us call equation number 1. Another equation we can write it if you see this one, current flowing through the capacitor is nothing but a by using the KCL law, Kirchhoff's current law at this point i L ift is equal to i L plus ic. So, ic current will be is equal to ic current flowing through the capacitor, you can write it here is current flowing through the capacitor ic of t, agree? Ic of t so ic of t I can write it total current minus current flowing through the inductor.

So, you know the current flowing capacitor i your c, dv voltage across the capacitor with respect to time is the current flowing through the capacitor is equal to i of t minus that iL of t and ift is u of t input and iL is x 1 of t. Now, this is nothing but A vc is x 2, so is it nothing but A x 2 dot of t is equal to u of t by c minus x 1 of t by c is equal to, since c is equal to 1 farad then it is u of t minus x 1 of t. So, that is equation number 2.

So, combining equation number 1 and 2, I can write x 1 dot of t, x 2 dot of t is equal to this is 0 1. Then it will be a minus 1 0, that is x 1 of t, x 2 of t then plus 1 0, then 1 just this equation. And this equation we club together this and this we club together and writing like this way, agree? So, this equal to this into u and this is the very well known structure of state space representation of a system. That is we can write it this is nothing but A x dot, this is x dot of t is equal to A is a matrix. This matrix is A or you write A x of t, A is a constant matrix plus B u of t which we can write, it is nothing but a function of x of t u of t comma t. So, it is a general form we are writing x dot is equal to function of this and this f is a two dimensional, because it has a two equations are there x 1 dot, x 2 dot.

So, this system dynamic equation, this is the system, this is described into a what is called a state space form. Or in other words this we can represent into a second order differential equation which can be converted into a two first order differential equation, which is written into this form. So, let us call this is equation number 3, agree? So, now what is our performing index? If you see what is our performing index according to our problem.

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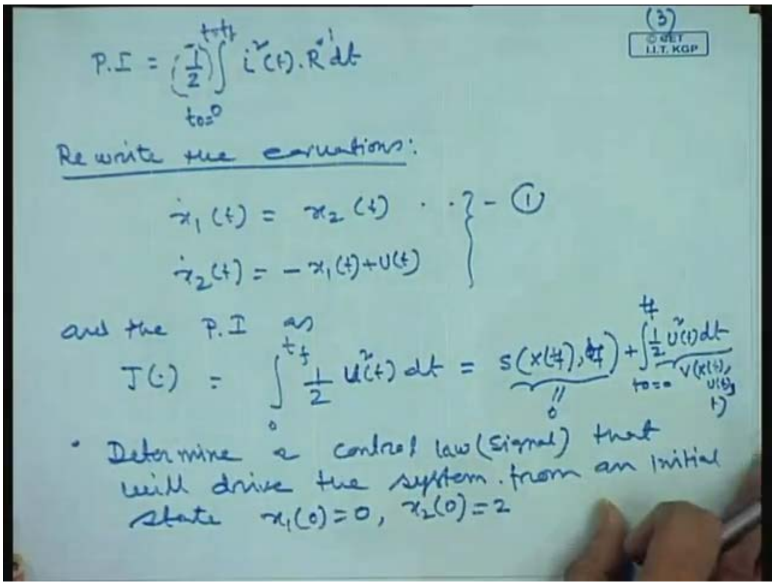

That our performing index is pi, performing index is you have to minimize the loss in the resistor part of the circuit, that means i square into R. So, I have A, that means i square into R of dt over a finite time interval t, 0 to t is equal to 0 and t is equal to tf and this is you have to minimize. But i we have already discuss in our static optimization problem, that if the performing index, if you add something constant term or you multiply by performing index by a constant term or divide by constant term, the optimal point does not change. But optimal value may change it, but optimal point at which the function has a optimum value that will not change.

So, we can make it this is the divided by two mean half. Why we made it? It is half, we will explain this thing later, because this portion, because when you do the what is called our derivation everything we have seen there will be a 2 will come and 2 and half will be cancelled. Otherwise, the 2 will be carried out throughout our formulation, that is why this half, but our optimal point at which this will be our optimal. Optimal that will not change it, but optimal value of function of the optimal value may change it.

So, let us see the how to solve this one. So, if you recollect our that numerical example we have considered. Let us consider equation number 1. So, rewrite the equation equations as part of our statement of the problem. So, we have the equation, if you see x 1 dot is equal to we got it x x 2 dot. Let us call this is 1, this set of equation are 1, x 2 dot of t is equal to minus x 1 of t plus u of t. This is equation number 1 and 2 combinely.

Now, I am rewriting in this equation number 1 and the performing index and the performing index that is what mean as J is equal to t 0 to tf half the half i square, i is our ut u square of t r is 1. This value R is 1, so dt, agree? Which this is, you see R value is what so it will be that one.

So, which we can write it S X of tf the boundary condition and this is tf, there is no boundary condition. So, this part practically it is 0 plus t zero to tf, t 0 is to tf t 0 is 0 or tf, then our half U square dt, agree? So, this quantity is now according to our problem it is nothing but a integrant of the integral cos function integrant. So, it is a function of xt ut and xt ut and comma t, so this part I can write it that one.

So, our problem is that if you see determine a control law or signal that will drive the system from an initial state, from an initial state x 1 of 0 is equal to 0 means current flowing through inductor is 0. From this condition current, voltage across the capacitor is a 2 volt, agree? So, this is initial condition of the system states. So, our job is derive a control law you such that initial condition will come to the final state x 1 tf and x 2 tf is 0 in a finite time and as well as it will minimize this performing index. So, that is our problem.

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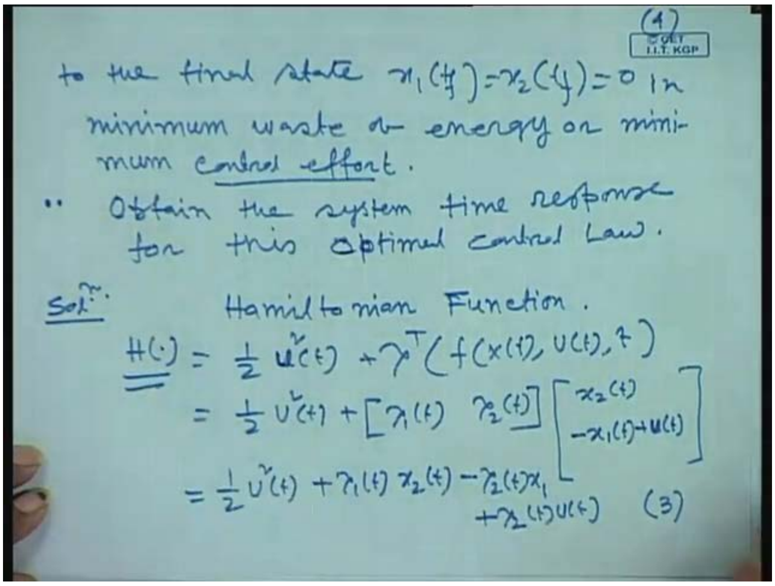

So, to the final state from a initial state to the final state x 1 tf is equal to x 2 tf is equal to 0 in minimum waste of energy or minimum control effort. What is the minimum control effort? Suppose, you have a resistance, that resistance what is if current is flowing through the resistance, what is this? Our energy involved i square R into t and in our case R is 1, so it is i square means U square. So, it is nothing but a multi input case is nothing but A U transpose, U indicate the control effort so that control effort you have to minimize for a fixed value of resistance. So, this is the control effort to minimize or the energy wasted in the circuit is minimized, this is this one.

Second point is obtain the system time response for this control law, for this optimal control law. So, this you see our problem is given the system given, the system minimize the performing index in such a way, minimize the performing index that is this by selecting U. My control effort you minimize this performance, and this performing index minimization means the minimum, what is called wasting. The loss in the resistor part will be minimized. So, you have framework, this we can write it. If you see this, this we can write it x dot of t is equal to f of x of t u of t and t. So, you have now made it in general framework, x dot is equal to this and our performing index is terminal condition. In our present case, terminal condition 0 plus that integral cos function, this is the integral cos function and this is the terminal cos, we have framework.

So, now you apply that whatever the method we have discussed earlier using the hamiltonian function. So, this now you are in a position to solve this problem using the hamiltonian function. So, if you see the solution and our objective is clear to bring the state from initial state to final state in a finite time interval, agree? As well as it should minimize the performing index that pi subject to that conditions. So, our solution is first you form hamiltonian matrix, hamiltonian equation or function.

So, H of this is equal to half U square of t plus lambda transpose f of x of t u of t of t, and this equal to you know half U square of t plus lambda. Lambda is A because the dimension of this state is 2 lambda is that lambda 1 of t lambda 2 of t multiplied by x dot of t is what you just see. When you put into a better form this then we can write it this is nothing but A x 2 of t minus x 1 of t plus U, U of t this is our f of x, agree? This so if you simplify this one, this into this multiply it is half U square of t plus lambda 1 into lambda 1 into x 2 of t plus lambda 2 into this one is minus lambda 2 into x 1 plus lambda 2 of t into u of t. So, this is our hamiltonian function. So, once you know the hamiltonian function, so this is our equation number 1. And let us call this is our equation number 2, that one equation number 2. So, and this is equation number 3, the hamiltonian function is equation number 3.

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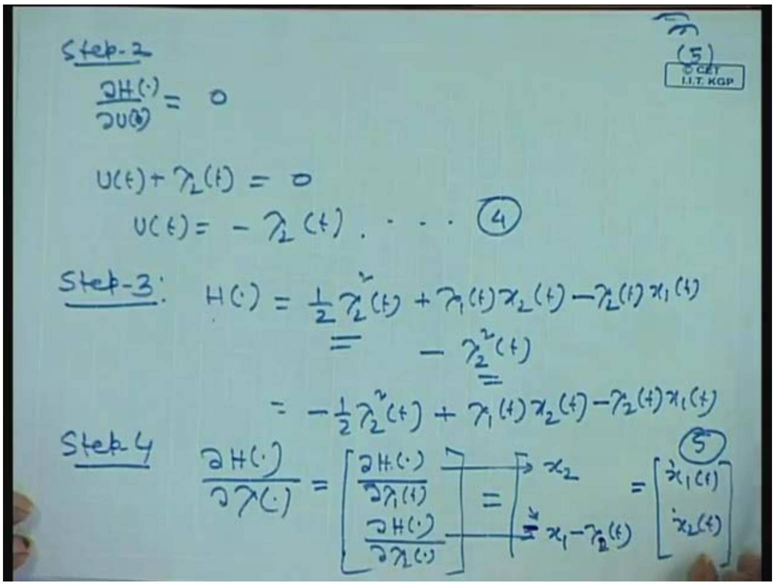

So, immediately we can find out what is called that del H, Del U, del U of t. So, this equal to 0, what is del H del U? U term is involved here and here. So, this is twice ut is equal to lambda t. So, if you see this one u of t lambda of t is equal to 0 u of t is equal to minus lambda 2 of t. So, once you do this one or you can write in step first step is 1 is formed. Hamiltonian function second step is your second step, step is formed del H del U.

Third step, that what is the algorithm. We have written according to that we are following third step using the value of this, H of this. We are writing use the value of u of t, replace u of t by lambda of t, then it will come half. This will be a half, replace u of t by minus lambda 2 of t. So, this will be lambda 2 square, lambda 2 t square, then plus lambda 1 of t x, 2 of t, just H. I am h expression what we got it?

We have written it and replace u of t by lambda minus lambda of t, only then your minus lambda 2 of t x 1 of t minus that is lambda 2 square of t. So, if you simplify this and this it will be A minus lambda 2 square of t, other terms as it is. So, I am writing other terms as it is lambda of t x 1 of t. So, this is let us call this is equation number 4, this is equation number 5 I am getting.

Now, what is we have to do it? We have to find it what is called del H, del lambda, del H, del x both the state and co state equation we can write it now. So, next is your step 4, del H, del lambda is equal to del lambda means what if you see this one is nothing but A that one del H del lambda 1, del H del lambda 2, that two things. And this I can write it del H del lambda 1, lambda 1 term is involved here is nothing but a first term. This term is nothing but A X 2 del H del lambda 1 is A X 2, this is the X 2 and this will be A and this is the differential with respect to lambda 2.

So, it will be a minus lambda 2, twice lambda 2, 2 cancel this is lambda 2 and this will be A X 2. So, this will be A minus X 1 minus lambda 1 lambda 2 of t. This one just this value I am writing, I will know lambda H by lambda dot is equal to x dot that expression. So, we can write it that is this we are differentiating with respect to lambda 2. So, this will be minus that is you see this is the minus sign, agree?

And so this equal to, what this equal to we know that will be a your x 1 dot of t, x 2 dot of t. Look, this expression del H del lambda is equal to x dot. What is x dot? X one dot is and x two dot. I am now writing the value of delta, del H is nothing but A del H lambda 1 whose value is x 2 del H del lambda, whose values is minus x 1. This is the minus, note this is the minus, minus x 1 minus lambda 2. So, from there I can write it that one equation that if you see I can write it x 1 dot is equal to…

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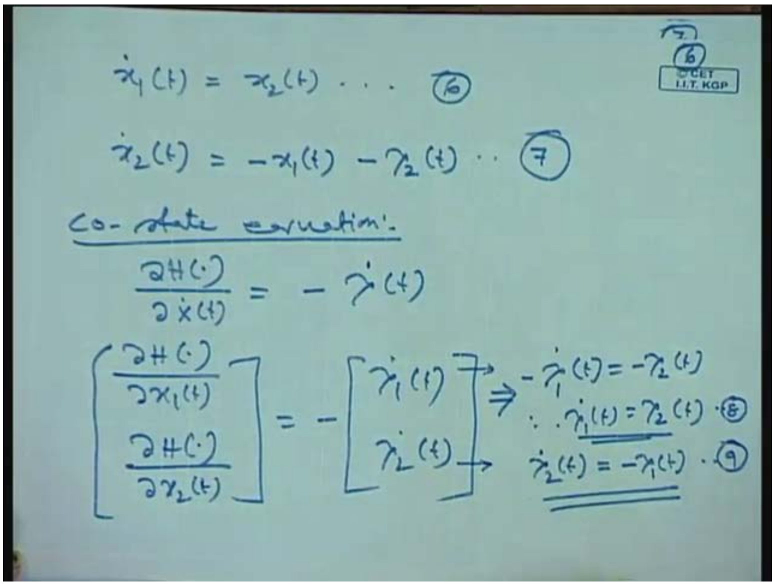

See, this one this equation x 1 dot is equal to x 2 of t, x 1 dot is equal to x 2 of t let us call this equation is 6. Then second equation you see, x 2 dot is equal to minus lambda x 1 of t minus lambda 2 of t. So, I can write it x 2 dot of t is equal to minus x 1 of t minus lambda 2 of t, that is equation number 7. So, this is the state equation that now co state equation, co state equation lambda h of dot x of x dot is equal to minus lambda dot of t.

So, this is nothing but A del h del x 1 of t del h del x 2 of t. This one is equal to minus lambda 1 dot lambda 2 dot. So, this now you differentiate this thing with respect to the del H, that is what we got it here. What is this expression that you differentiate this with respect to x 1 del h del x 1. So, x 1 term is there only it will be A lambda 2.

So, I can write it this equal, this is equal to your lambda 2, because you have differentiate with respect to lambda 1, lambda x 1. Then you are differentiate del h del x 1, you see del h del x 1. So, if you differentiate this one, it will be lambda 2 minus lambda 2. So, it is a you can write it minus dot, lambda 1 is equal to minus lambda 2 of t. Therefore, lambda 1 of t dot is equal to lambda 2 of t, this is so let us call this equation is 8.

Similarly, from this equation this is I am writing from this equation. This is from this equation differentiating h with respect to x 2, x 2 is involved here, only x 2. So, lambda 1 of t if we differentiate with respect to x t, so this is a lambda 2 dot of t is equal to minus lambda 1 of t. So, let us call this is equation number 9. Just this two equation co state equation we have written this is one equation and this is another equation we got it.

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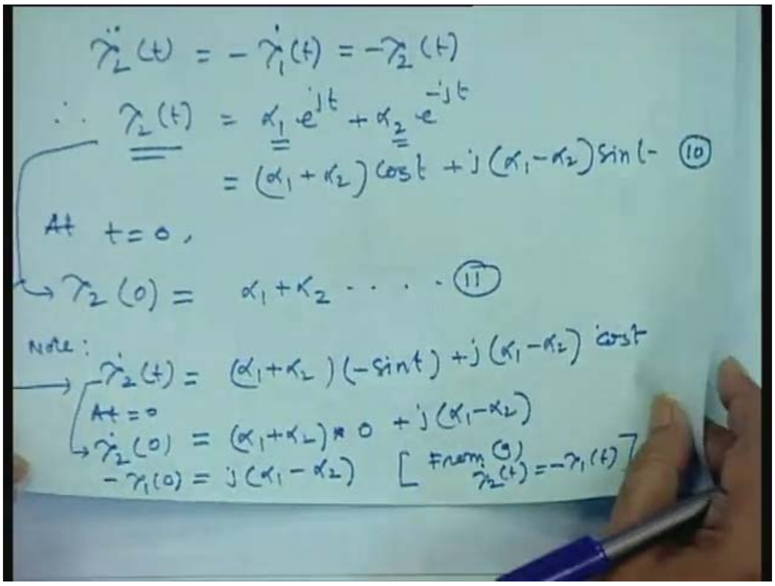

So, we go it co state equation. Now, we see from equation what we can do it from equation 8 and 9 if you differentiate this thing with respect to this t lambda 2 dot is equal to minus lambda 1 dot, agree? So, from 8 and 9 lambda 2 dot we are differentiate once again with respect to time t is equal to minus lambda 1 dot of t. Now, see lambda 1 dot of t is lambda 2 of t. So, I am writing is nothing but A minus lambda 2 of t. So, it is something like a x dot is equal to minus x. So, it is a homogeneous equation, the solution of this one we can get it like this way.

This is equal to alpha 1 e to the power of jt, alpha 2 e to the power of minus jt. This one and this if you take the hallers formula, all this things we will get alpha 1 plus alpha 2 cos t plus j alpha 1 minus alpha 2 sin t, that means e to the power jt is cos t plus j sin t. Here is cos t minus j sin t, then simplify you will get it this one. Let us call this equation is equation number 10, agree? So, this is the solution of x lambda t.

Now, you see note at time t is equal to 0 lambda. Here, lambda 2 0 is equal to alpha 1 plus alpha 2, sin t is 0, agree? This we got it let us call this is equation number 11, agree? Note lambda 2 dot is this again from here is equal to alpha 1 plus alpha 1, alpha 1 alpha 2 is the constant which we can solve the characteristic equation. If you solve it, you will get it that this roots and this is the associate with the that is a constant. This is the constant, real constant number may be complex. Also, this is the alpha 2 dot is equal to alpha 1 plus alpha 2. Differentiation of this one is minus sin t plus j alpha 1 minus alpha 2 cos t..

So, at time t is equal to 0 then we can write it lambda 2 dot of 0. This value we can write it lambda 2 dot of 0 is equal to alpha 1 plus alpha 2. This is 0 into 0 plus j alpha 1 minus alpha 2 this, agree? So, this we got it that lambda 2 is lambda dot is this and we know lambda dot is what, lambda dot t is equal to lambda dot t is equal to lambda 2 dot t is equal to minus lambda 1 t.

So, I can write it minus lambda 1 t is equal to j alpha 1 minus alpha 2, because this we know. Since, we know from 9, if you see from equation 9, from 9 that lambda 2 dot of t is equal to 2 dot of t is equal to minus lambda 1 of t. From that we got it lambda 1 0. So, this expression we got it. Now, from 10 and 12 let us call this is the equation number is 12.

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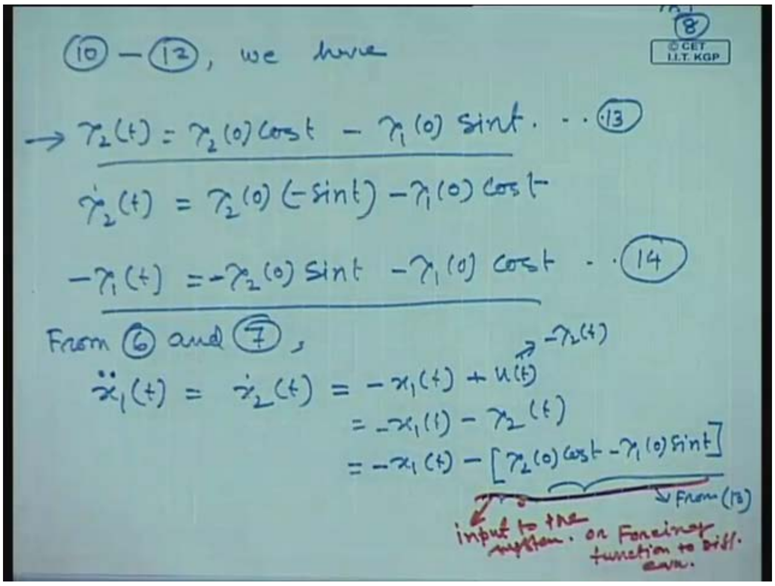

From 10 to 12, what we can write it from 10 to 12 we have say 10, 11 and 12. What we can get it, lambda 2 of t from 10 lambda 2 of t is equal to, now see this we got it. This value what is lambda 1 plus lambda 2, in this value is lambda 2 of 0. So, this equal to lambda 2 of 0 then it is cos t as it is the j lambda 1 j alpha 1 minus alpha 2 j alpha 1 lambda 1 minus lambda 1 of 0. So, this is cos t minus lambda 1 of 0 sin t. So, let us call this is equation number 13 and if you differentiate this one it is equal to lambda 2 of 0 minus sin t minus lambda 1 of 0 cos t.

So, this values we know minus lambda 1 of t is equal to lambda 2 of 0 minus sin t and minus lambda 1 of 0 cos t. So, let us call equation number 14, so I know the expression problem for lambda 2. I know the expression for lambda 1, agree? This is the final expression for lambda 1 and lambda 2. But still I do not know what is the lambda 2 and lambda 2 0. All those thing, if I know then our what is Lagrange. Multiplier description is known provided if I know lambda 1 of 0, lambda 2 of 0.

So, from equation 6 and 7, you see the from equation 6 and 7 I recollect this one from 6 and 7. What you can write it x double dot, if differentiate this thing with respect to once again with respect to time, x double dot is equal to x 2 dot and express the x 2 dot. This one replace x 2 dot by this 1, so I am writing. Now, from 6 and 7, x 1 dot, 6 is x 1 dot. So, I am differentiating with respect to time. Once again is equal to x 2 dot and x 2 dot is equal to you know minus x 1 of t plus u of t, and what is u of t minus lambda of t. So, it is u of t which is nothing but a this is minus lambda of t. So, this will be a minus x 1 of t minus lambda 2 of t, agree?

So, what is our minus x 1 of t? What is lambda 2 of t? Just now we have got the expression for lambda 2 of t, you see the expression for lambda 2 of t, we got it here, sorry. Here so I will write it this expression that expression is your minus lambda 2 of 0 cos t minus lambda 1 of 0 sin t so that I am writing, this I am writing from equation 12, agree? So, replace the lambda 2 value by this one from equation 13.

So, this if you look at this expression here that x 1 double dot is equal to minus x 1 of t some quantity is there. This whole thing I can consider, this whole thing I can consider as an input to the systems. That whole thing I can consider as if it is a input to the system, our differential equation the forcing function or it is a or forcing function to the differential equation, agree? So, now this solution you can easily find out because it is something like as double dot is equal to minus or you bring it this one x 1 double dot is equal to plus x 1 of t plus some forcing function so that solution, one can find out easily that one.

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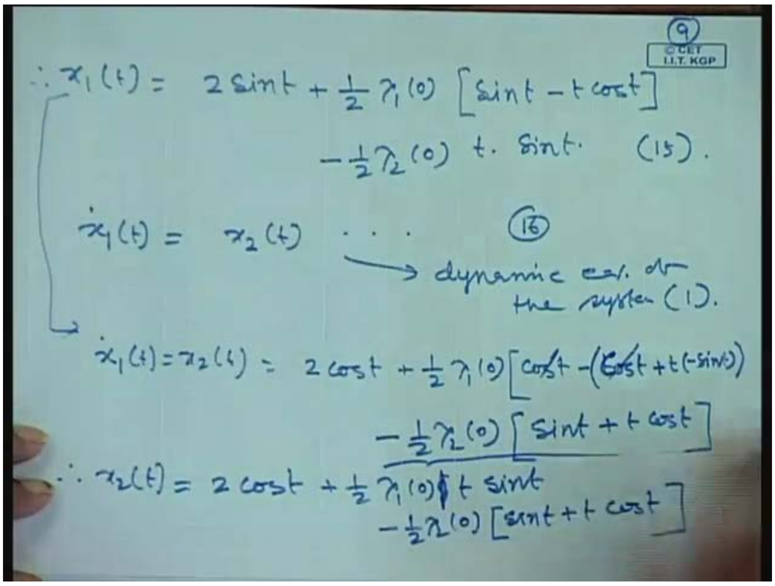

So, let us call the solution of x one of t. Therefore, x 1 of t is equal to twice sin t. So, I let this thing as an exercise to check the solution of that one is this that second order differential equation. What you got it lambda 1 of t sin of t minus t cos of t minus half lambda 2 of 0 t sin of t. So, you have done up to equation 14, this equation. Let us call this is the equation number 15. So, our control i is nothing but A U of t is nothing but A minus lambda 2 of t. So, lambda 2 of t if you know this one we can find out the control law.

Once you find out the control law, then we can find out the trajectory x of t. But here lambda 2 of 0, lambda 2 of lambda 1 of 0 is unknown. This is unknown, this is unknown So, yet to find out these values, then only lambda 2 is there. Once lambda 2 is there then we can find out the control law. So, our trajectory is this one. Still if you see that lambda 1 0, lambda 2 0 is unknown to us and not only this that this and another thing you see, x 2 this is the our x 1 t and we know from the dynamic equation x 1 dot of t is equal to x 2 of t, that is called from 16. This is from dynamic equation of the system equation of the system, see equation number 1.

So, x 1 dot if you differentiate this one, x 1 dot is equal to x 2 which equal to this right hand side. Differentiate this one twice cos t plus half lambda 1 of 0, this one will be what sin cos t minus t this one, you have to differentiate that thing with respect to this. So, first is differentiate this one. Let us call this will be then cos t then unchanged minus sin t minus sin t. Then this part minus half lambda 2 of 0 then sin t then plus t cos t, see just I have differentiate this one and that is nothing but a our x 2.

So, our expression therefore, our x 2 expression, x 2 expression is nothing but A twice cos t, twice cos t plus half lambda 1 0 then is you simplify that one. Then what will come? Just I am simplifying this things, then lambda 1 t, half lambda 1 0 t, t sin t minus half lambda 2 0 sin t, because this and this are cancelled. So, it will be A minus this is minus. This is minus minus minus plus, so it will be half half lambda 2 0 t sin t and this is as it is. This part will be this, this part will be as it is here reproduced sin t plus t cos t.

Now, this trajectory I mention is u of t is from the condition, we got minus lambda of lambda 2 of t and lambda 2 of t is the function of, if you see is the function of that our lambda 1 of 0, lambda 2 of 0, agree? Once I know lambda 1 0 lambda 2 0 then lambda t is known here, also you see x 1 t is a function. We need the information of lambda 1 of 0 and lambda 2 of 0, agree? Then we can find out x t trajectory, optimal trajectory x 2 trajectory which will control input as well as x 1 t, x 2 t, combinely it will minimize the our performing index that consider.

So, applying now what is our left with us only boundary condition. From the boundary condition you have to find out x 1 lambda 1 of 0 lambda 2 of 0 boundary condition and in the boundary condition our tf is free. What is fixed, that x t f is fixed, means both the state that current flowing through the inductor, current voltage across the capacitor must go to the 0.

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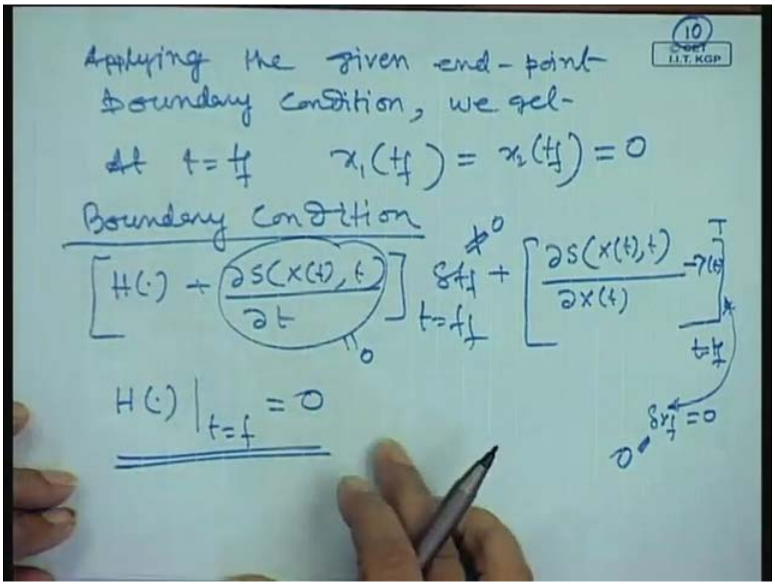

So, applying now applying the given end point boundary condition we get at t is equal to t f x 1 t f, current flowing through the inductor. This is x 2 t f, current voltage across the capacitor is 0. This is the we know the our boundary condition so if you apply the now boundary conditions this one. If you recollect this our boundary condition, boundary condition H del S X of t. This delta t, whole thing put t equal to t f, delta t f plus del S X t of t del x of t minus lambda t, whole t is equal to t f transpose. This transpose just see the boundary condition multiplied by this, multiplied by that is I am writing multiplied by U delta x f is equal to 0. Now, this delta x f if you see that delta x f is in our case is 0..

So, only this d, this is not equal to 0. So, our condition is delta dot and this since we do not have any terminal condition this part is 0, no terminal condition according to our problem statement. So this you find out t is equal to tf, this equal to 0. So, this is our boundary condition that finally, boils down to this equations. So, from equation 5, what is the equation 5? You just see that our basic hamiltonian equation, hamiltonian function. This equation from equation 5, I will write it t is equal to t f.

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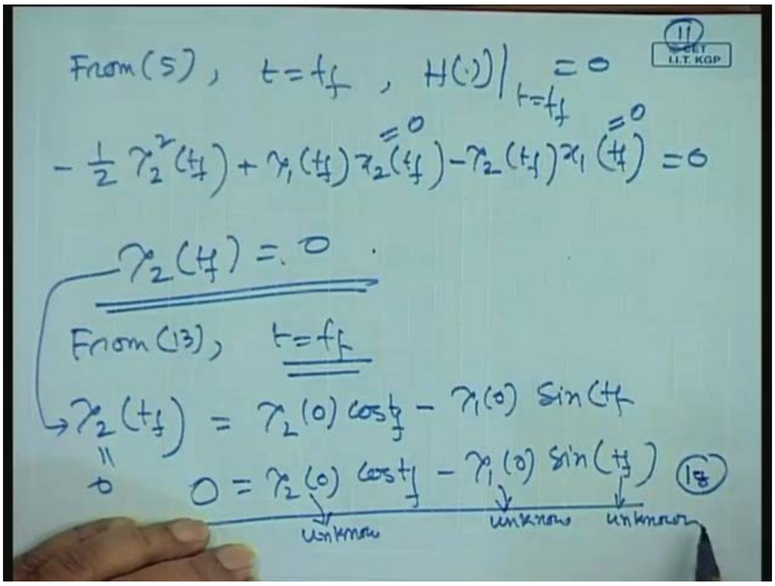

So, from equation 5 put t is equal to t f then your half minus lambda square of t f, agree? That is half lambda square of t f, this is what this equation you see half lambda square of t f, then lambda 1 t f x 2 t f minus lambda 2 t f, then x 1 t f is equal to 0. So, what I did it here, you see H of this put the value t is equal to t f is equal to 0. So, this equation is 0 because terminal what is called final state values is 0, this is 0. So, only this one so this one is lambda 2 t f is equal to 0.

So, from 13 so you again from 13, you see equation number 13, refer to equation number 13 that one lambda 2 of t is this one, agree? So, from equation 13 t is equal to t f. So, lambda 2 t f is equal to lambda 0 lambda 2 0, see this equation lambda t is equal to t f. This lambda 2 0 cos t minus lambda 1 0 sin t of t is equal to tf is tf this quantity is zero just we got it this is zero t zero so it is a we can say this is lambda 2 0 cos t f minus lambda 1 0 sin t f, this is equal to 0, that is agree?

So, let us call this is equation number, what is the equation number? We got last equation number 9, 16 then you give it this equation number is 17, agree? This x 2 expression that two expression what we got it x 2 twice cos of sin that is you give it 17, because this will refer 17, this we will give it 18, agree? Now, see this one, this is unknown, this is unknown and t f is unknown, agree? Three unknowns are there, we need three equation, which three equation are your. So, I can put it now that this 18.

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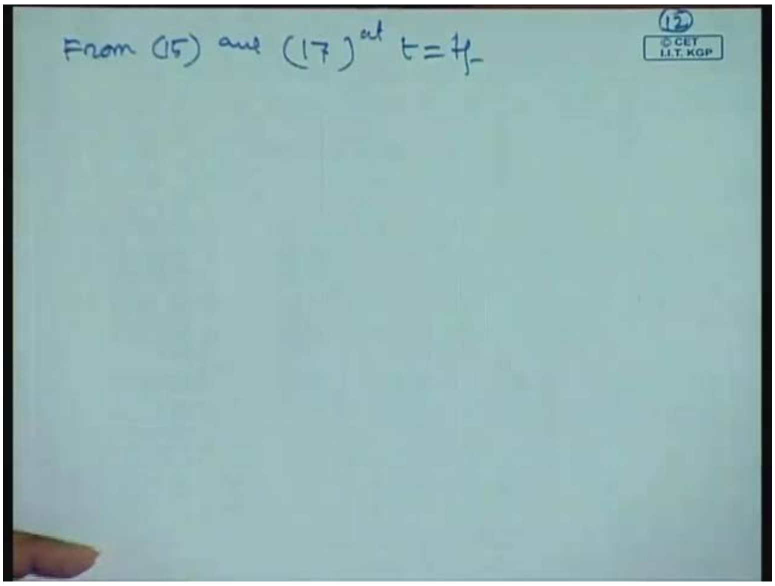

From 15 to 15 and 17 is equal to t f at t is equal to t f 15 is that one. You see 15 and 17 is that one, 15 and 17, 15 is this one, 17 is that one. In this expression t is equal to t f, you put it t is equal to t f. So, left hand side is 0, left hand side 0. So, you have a three unknowns are there and three equations are there. But that three equation are non-linear. So, you have to solve it to get the final value of x lambda of 0, lambda 1 of 0, lambda 2 of 0 and t f and that will give you the total picture of what is called, what is the optimal control law. As well as what is the optimal trajectory which will minimize the performance index of that one. So, a part of this one is left. So, I will continue next class again this one. So, I will stop it here now.