Optimal Control
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Lecture - 34
Numerical Example and Solution of Optimal Control Problem using Calculus of Variation principle (Contd.)

So, last class we have solved a problem, that is we have to optimise a functional, we have to optimise a functional without any constant. That means unconstant optimisation problem we have seen how to solve using calculus of variation technique, then we have consider that application of calculus of variation to control problems if and we could not complete that whole exercise. So, let us recollect what we have discussed in last class. Consider a dynamic system described by x dot is equal to f which is a function of the variable state, and the control input to the system u t and time.

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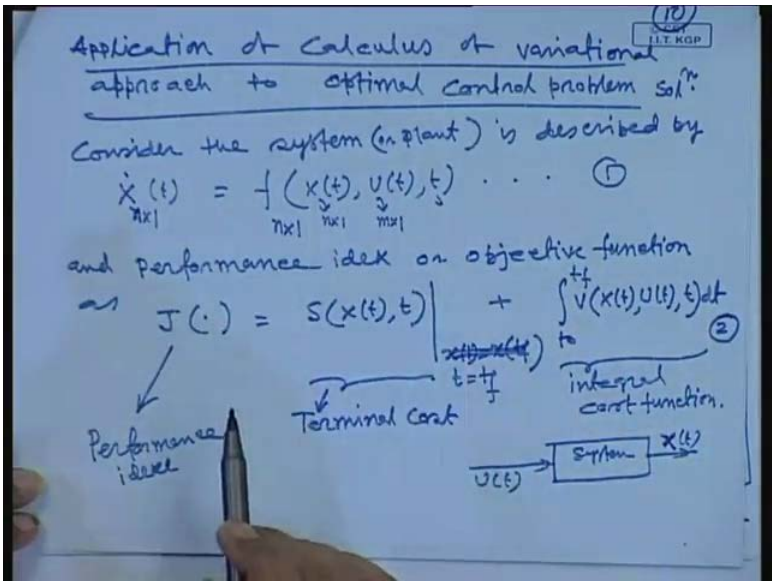

So, this may be a linear function, may be non-linear function which we did it, that a dynamic system which can be described by a anathora differential equation. That dynamic equation is converted into a first-order differential equation and they are coupled each other and that differential equation may be linear or non-linear. So, our problem is to optimise this performing index. What is this performing index? One term of the performing index is the functional integral of the functional and another is a fixed, whether it is terminal cost. That means x time t is equal to t f, this state is x t f is known and t is equal to t f, that terminal cost is given.

So, you have to optimise this performing index in order to find out u 2, u of t control input such that that performing index is minimised and it as well as it satisfies the dynamic equation. So the control input will dictate the response of what is called state. So, we have it performing index, this is index in omitted. So, in index, so this is the system input is that we have to design that input such that the performing index is minimised. In our case in the sense that this is the integral part of the performing index and it is the fixed part of the performing index which is the terminal t is equal to t f. This terminal cost is known to us.

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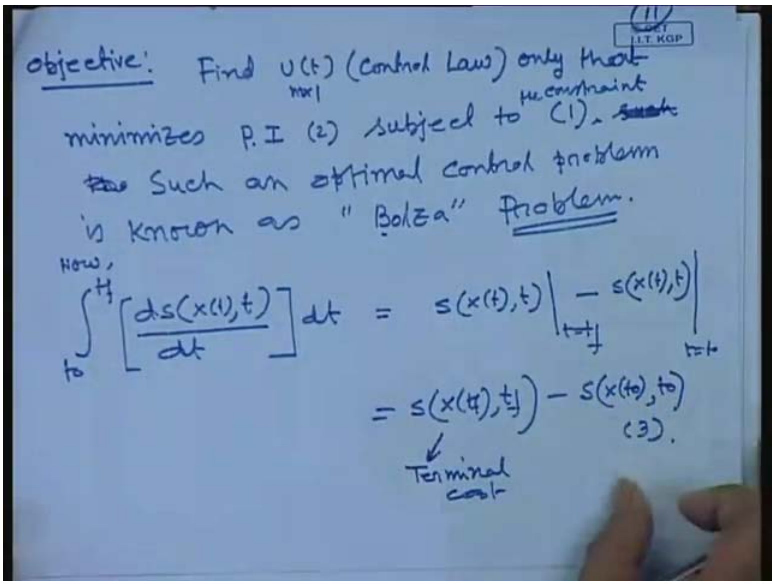

So, our objective is now that if you see u of t, you find out that u of t that minimises the performing index that's what we have considered subject to the constant 1, subject to the constant 1 that our constant is that dynamic equation. This dynamic equation is the constant. So, this is a what is called constant optimisation problem, previously we have discussed the minimisation of a functional without any constants. But here now we are considering the constants and this problem is also known known as Bolza problems. So, what we did it that terminal cost, what is the terminal cost that we want to like to push it in the integral part of this performing index.

So, this terminal cost that d of s dt we can write it in this. So, this terminal cost, this is the terminal cost will be expressed in terms of integral and that constant term where t is equal to 0 and the terminal cost is fixed known.

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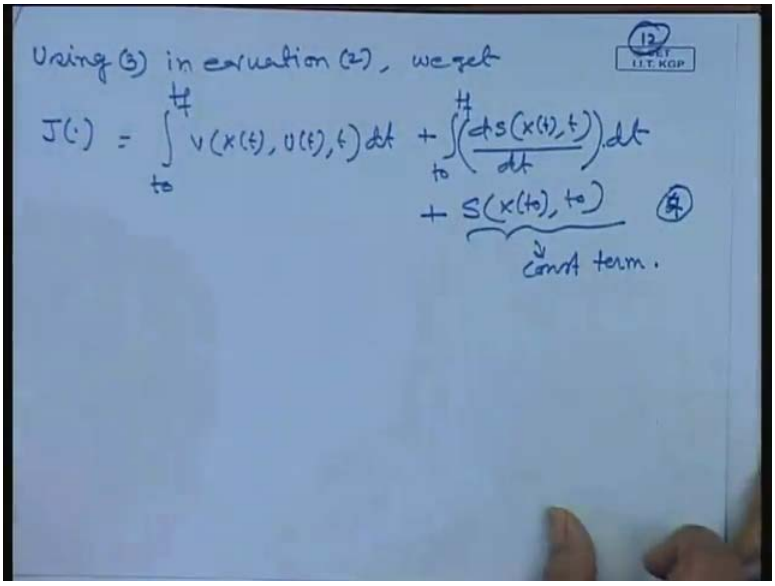

So, using this equation in our original performing index that we get it this one, just we put it the terminal cost expression in terms of this and this, that what we have seen in earlier slide. So, our problem minimisation of performing index, to minimisation of performing index, to that that one minimisation of performance to this one. That performing index to this is because we have replaced this thing in terms of integral part. And constant part is same as minimisation of the performing index 4, subject to the constant x dot is equal to f of x, that constant. So, up to this we have discussed last class.

So, now this is the constant term. So, minimisation of four is equivalent to minimisation of this integral part of that one because constant term, what is this minimisation of that one at what point. That what point or what trajectory of u t the function will be minimum with constant term also at that value of u star or trajectory, the function will be minimised.

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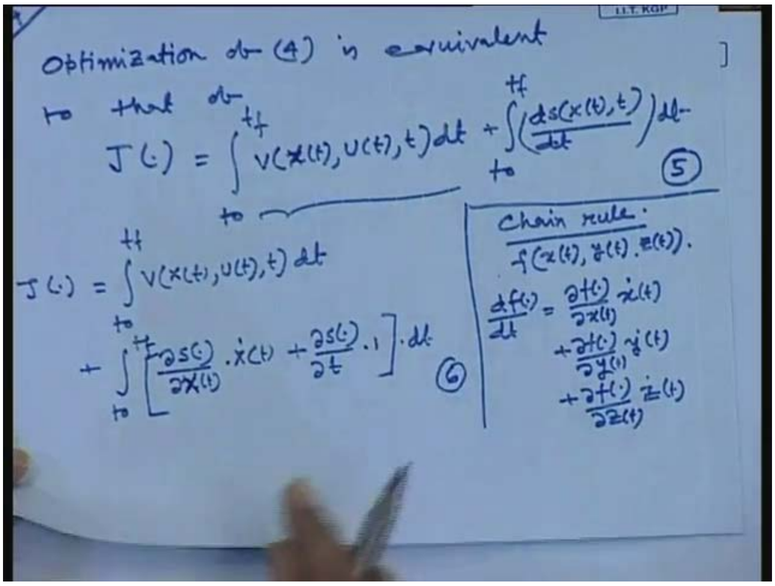

So, instead of equation 4, we can rewrite the equation 4 then we can write it optimisation of equation 4 is equivalent to that of J is equal to this indication of t 0. I am writing the same expression except that constant term x of t u of t dt plus t 0 to t f ds x of t, which is the function of x and t s terminal cost into dt. So, minimisation of that function is same as the minimisation of this objective function. So, before that further derivation we just recollect what is called chain rule in differentiation.

So, let us call we have a function f which is a function of x t, x is the function of x t, y also function of time parameter, time t and z of t, agree? We want to differentiate this with respect to time t, differentiation of this with t, this is by chain rule dl f dl x into x dot dx dt x dot of t plus dl f del y, y dot of p plus dl f dl z, z dot of t z is the function of time. So, if we differentiate this z with respect to time t z dot t, so this chain rule I will apply here to find out this one. So, say f is the function of xyz and x is the function time t y is the function of time t.

So, differentiation of x with respect to time t is nothing but a partial differentiation of f with respect to x, because it is a function of xyz. Keeping yz constant, you differentiate this one and then differentiation of x with respect to t. Differentiation of x with respect to y keeping x and z constant, then multiplied by y dot. Similarly, differentiation of x with respect to z, keeping x y constant, then multiply by z. So, this chain rule I apply it here because here s is the function of time x t and t, agree?

So, this I can write it then J is equal to J dot of this t 0 to t f, then v as it is we write as this term. We write as it is this term x of t, this is capital x of t u of t t dt and this we will write it by using chain rule. So, this will be a plus t 0 to tf. So, what we will write it? Differentiation of ds dt, differentiation with respect to time t because s is the function of s t and t. So, what we will write in that one that we will write dl f dl x which is equal to x of t into that x dot of t x dot of t. This is capital X plus dl f dl x of this with respect to dt and t, differentiate with respect to time t. So, that means it is one that whole thing into dt.

So, for this portion we have written that one. Now, see what we can simplify that one. This expression if you consider, this is equation number 6, again this performing index that means this equation 4, minimisation of equation 4 is equivalent to minimisation of equation 5 because in equation 4 there is a constant term is there, agree? So, optimise at what value of u you will get the maximum or minimum value of the objective function without considering, what is without considering the constant term. I will get a same stationary point u star of t to optimise this function, agree? So, equivalent it is equivalent to say the optimisation of objective function of 4 is equivalent to what is called optimisation of objective function 5 subject to constant, same constant x dot is equal to f x of t u of t comma t.

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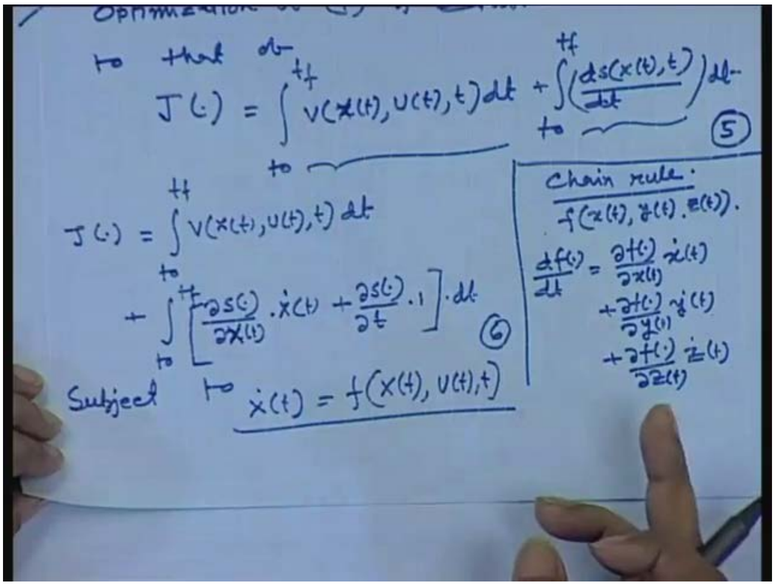

So, this equation, this by using chain rule we have written this subject to our same constant subject to x dot of t is equal to f x dot of t u of t this, this is the subject. So, this is the, what is called constant optimisation problem. We know very well how to in static optimisation problems also you have seen how to convert a constant optimisation problem to a unconstant optimisation problem. Then what we have discussed in calculus of variation, our first problem that if you have a functional what is called J, which is the integral part of V of x t comma x dot of t comma t dt. Then we know how to mini optimise this function.

So, first our job is to convert problem, what is called constant optimisation problem to a unconstant optimisation problem. Our problem is find u such that optimum statutory of u, such that this performing index or objective function is minimised subject to this constant. And that constant is a dynamic equation so that let us call this is equation number 2.

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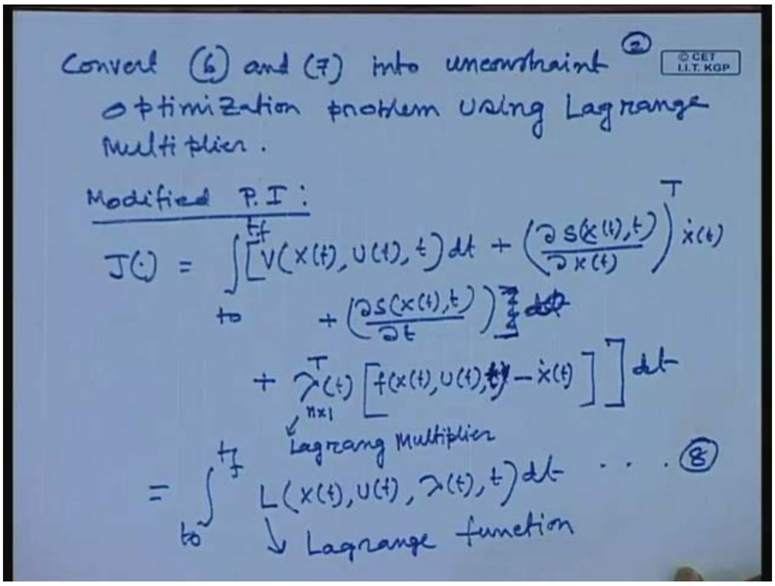

So, our job is to convert equation 6 and 7 into unconstant optimisation problem using Lagrange multiplier. So, that is what we have discussed earlier that where we can do it. Next is what is our modified performing index, when you use the concept of Lagrange and multiplier, what is the our modified performing index. So, our J is equal to t 0 to tf V x of t u of t t dt plus integration of del s, make a function of time xt. This del x of t whole transpose, see this one, that part I am writing, but here you see if you consider x is a, consider x is a vector of dimension n cross 1.Than you have to it will be a row vector, row vector multiplied by column vector. Then it is a scalar quantity, the whole thing is a scalar quantity, agree?

So, this you have to take transpose, ok? Transpose that one. So, that will be transposed into x dot of t plus dl x x of t t del t. This one and this whole is dt, just this equation, this and this I club together. I am written in that one, this plus because it is a unconstant optimisation problem, constant optimisation problem. Now, I have converted into unconstant optimisation problems again. So, Lagrange multiplier is used and that dimension is if the dimension of x is n cross n, that dimension will be n cross n and that must be a row vector, because the product of this must be a scalar one.

So, that is equal to f of x, the dynamic equation, the constant equation you can say this is t bracket, this is t bracket minus x dot of t and this part is 0. So, I multiply by constant what is called our Lagrange multiplier, this is the Lagrange multiplier, lambda a is Lagrange multiplier.

So, this is the equation number I can write the whole thing, if you write in this whole thing I can write that is this bracket. This whole thing you bracket its completed here and dt. Now, see this performing index of that one is same as before, because this part is 0 when it is optimise. The function of that is our or objective function, this part will be bigger. This must satisfy our constant, it must be 0 that one. So, this I have just written it so that is equivalent to t 0 to tf, t 0, this is tf t 0 to tf that I used a another function name is Lagrange function. So, that is a function of if you see x t u t then lambda t and t whole into dt, that whole this plus, this plus, this plus, this, that whole I am denoting it by l, this is this l is called Lagrange function.

So, let us call this equation number is 8. Now, you see it is now it is our original problem what we have considered. We have to optimise a functional without any constant. Now, it is becoming same problem. So, we can apply the same technique what we have discussed earlier to find the optimal value of the functional subject, there is no constant, agree? So, what is the necessary condition? What is the sufficient condition we can easily derive? So, for convenience I just will derive this one because l function now is different from V.

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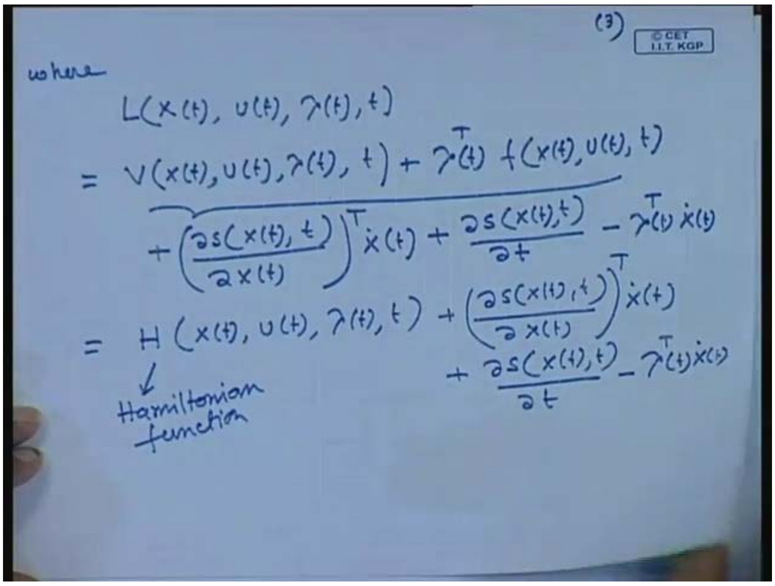

So, let us see this one where l which is a function of x of t u of t lambda of t and t which equal to v, the integral part of that equation 8 I am writing. So, that is equal to V function of x of t e of t lambda of t and t plus if you see this one plus lambda, this transposed than f of x of t u of t then t, ok?

So, what I did it here, this term integrate part this term and only this part omega I am not considering. Only this and this part I have written together, this plus the remaining term. So, what is the remaining term is 7 dl x dl x of t of t dl x of t, this whole transpose into x dot of t plus dl x which is a function of xt of t differentiation with x respect to times t minus lambda transposed x dot of t. So, this is the things, so this, this and this that means which is a V and that lambda transpose of x transposed t. This we denoted by a function x each which is a function of lambda x ut and lambda t and t, see t this one.

That means, if you just consider the integral part of the objective function, that plus the constant what is x dot is equal to fx f. That constant f right hand side of the constant multiplied by a vector, that term I have considered a function which is denoted by h and that h is called Hamiltonian function. We will see if you split up this Lagrange function into this form, that will be convenient when we apply to a our problems and that problem gets description when it is given into a straight paced form. It will be convenient when you want to express this thing into a Hamiltonian form and what is the leftover terms, this this, this. So, dl x xof t of this dlx xof t whole transposed x dot of t plus dlx x of t dlt minus lambda transpose x dot of t. So, this is the game. So, Lagrange equation is nothing but a Hamiltonian function plus sum of the differentiation ofterminal cost with respect to s transpose x dot plus differentiation of terminal cost with this dt. And then minus lambda transpose into x dot is that one is expressing this one.

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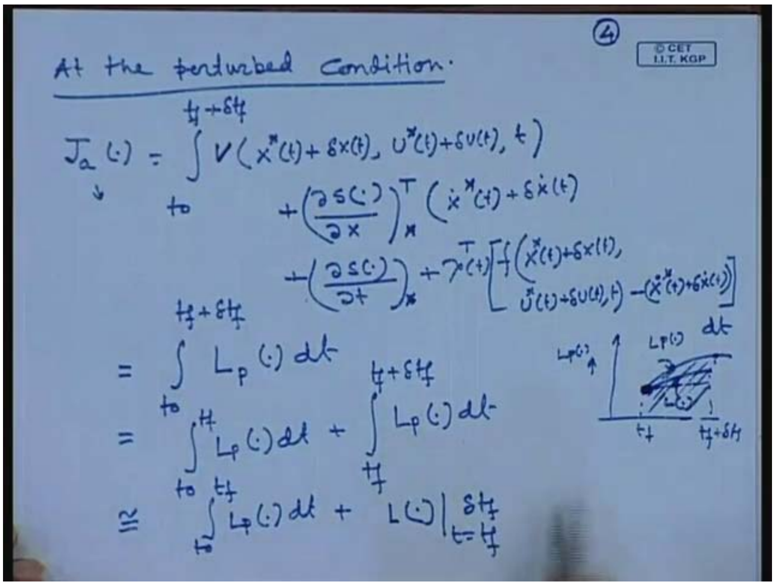

So, once you do this one than I can write. Now, this perturbed at perturbed condition what is the value of at perturbed condition, perturbed condition means you see this sequence, that J is defined this one when x is perturbed with a xt plus delta xt ut perturbed with a ut delta t and t is that one, t is equal to t. So, this trajectory what does it mean, our job is to if you see this one we have to look for optimal trajectory of u star, agree? Which in turn this performing index will be minimise subject to this constant. So, let us call whatever the u star is there u is the optima trajectory around this, u around this optimal trajectory, u there is another route of u, there is another trajectory, agree? And that trajectory if you consider that trajectory what is the and corresponding functional value of the objective function will be one corresponding to that part of trajectory, when u is perturbed by u t of delta u of t. Similarly, with the control action of this one, x will be also perturbed.

So, I am writing this one. Now, t 0 to tf and that time t, panel time is t it is perturbed with delta tf. Then V and if x star is the optimal trajectory and we are given the partition of delta xt. Similarly, which u star is this and perturbed with u star, t u star is the optimal trajectory and it is perturbed form the neighbourhood of u star delta u. So, which in turn x also will change from optimal trajectory xt to delta xt, and this t plus delta s of this delta x whole transposed that and x dot. Because if you see this one, this is the value of this one, we find out this value of that one differentiated value with respect to this along the trajectory of this one.

So, this star plus x dot star plus delta x dot of t, this is the that perturbed region perturbed trajectory of that one, then what is left? The dl x whole star is there, just that 1 plus this term plus lambda of t transpose than f x star of t delta x t plus u of t u star of t plus delta u of t, then t, ok? Minus x dot minus this f of x I have written this minus x dot is what is x dot x dot star of t plus delta x dot of t. So, this of indication dt, so what I did it, this suppose J is the value of the function value. Now, I perturbed u by u plus delta u, naturally x will be perturbed x plus delta u, agree? Then we are finding out here what is the new objective function when we perturbed, the trajectory u star to u star plus delta u and x star to x star plus delta u.

So, this and that I will integration t 0 to delta t because our final time also that is changed to tf to delta tf. Now, this is the objective function value, what is the incremental of incremental functional value. So, one can find out this is nothing but a, if you see this is nothing but a, I can write it t 0 to tf l p, means perturbed model, perturbed functional l p dot of t dt. This whole thing is perturbed p stands for perturbed, the whole thing is this.

So, this I can write it equal to this delta tf . Now, whole thing I just can write it this, that t 0 to tf l p dot dt plus tf to tf plus delta tf lp dt. So, it is a perturbed Lagrange function, this lp. So, this I can write it nearly equal to if you think of this, this is as it is tf lp dot this dt and this is nothing but a lp is a function scalar function and area under this curve from tf to tf plus delta f. If you just consider just like this way, it is the trajectory form, this is the tf, agree? This is the your tf plus delta tf and this is our lp dot function, this is lp, I am plotting this function.

Now, what is the area under this curve, tf to tf is nothing but this whole thing, agree? So, one can write this is nearly equal to find the ordinate of the non-linear function below at t is equal to tf. So, find l p this is lp, what is called if you see this is lp and there is a another curve is that one. What is that one, this is l dot, this is equal to tf, this and this is the t is equal to that is tf to tf plus delta, delta tf. This is a without perturbed Lagrange function without perturbed Lagringian function. So, area actually I have to find out the area from here to here with a perturbed model is same as I can write it is same as the area. Find out the function of the Lagringian function value at t us equal to tf, this function that is this ordinate you find out multiplied by delta tf.

So, this will be approximate because delta t is very close to tf. So, I can write the area under this curve is same nearly equal to area under the lp from from t j f to tf plus delta tf plus delta tf, again that you can write it. So, that is why I have written nearly equal to that one. So, if it is so then I can what is the variation of functional value, variation of functional value delta J a.

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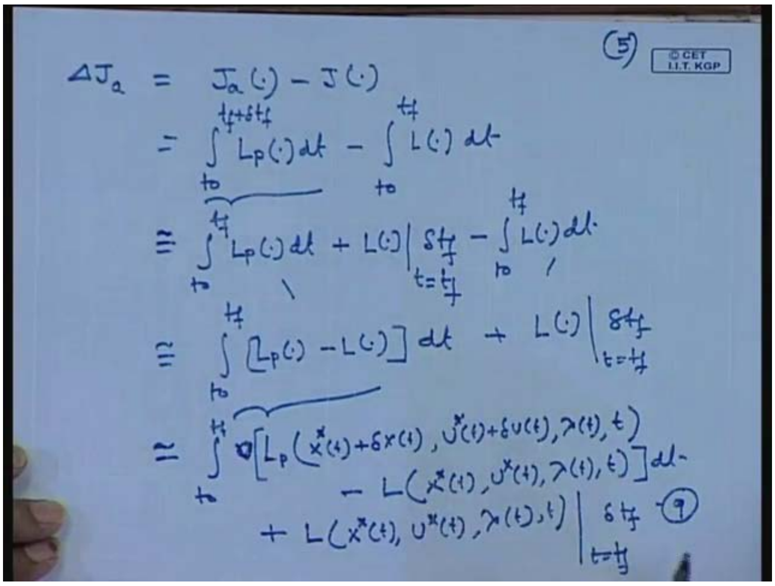

The variation of functional value J a dot minus J a is nothing but t 0 to tf plus delta tf, agree? l p perturbed function Lagringian function value, this one J minus t 0 to tf, that original Lagringian function without perturbation. This one this we have so that just now we have conclude this part, we can write it t 0 to tf lp dot dt and that is nearly equal to, I can write it nearly equal to l dot value. The find the value, the value of the Lagringian function t is equal to tf into delta tf minus t 0 to tf l dot delta t. So, you club this and this club together. So, this nearly equal to t 0 to tf lp dot minus l dot. This dt plus l dot t is equal to tf delta tf. So, this we got it that, that one. Now, one can write it this is now as before. We have discuss this one again, now what we can write it for this one?

This nearly equal to if you write more details, t 0 to tf lp is what V lp is that lp function I am writing. What is the function, lp is equal to x dot t plus delta x of t u star of t plus delta u of t is a function of this and lambda of t of t minus that one I am writing. That is minus l function of x star of t u star of t lambda of t of this whole bracket dt, that this part in details I have written the function of perturbed input and perturbed output. That we have written and then left remaining term is that one plus l x star of t u star of t lambda of t t bracket close, find the value t is equal to tf multiplied by tf delta tf.

So, let us call this equation number is we have given equation number up to 8. So, now this equation is equation number 9. Now, see this one, this part if you see the Taylor series expansion, then use the what is called chain rule. All this thing as we did in earlier we can simplify this two first part of the integration of that one we can simplify.

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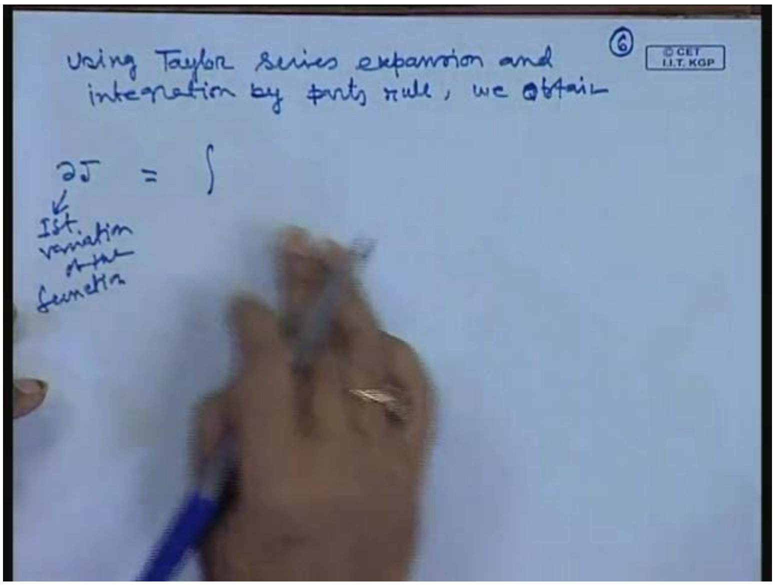

How, what is the final expression will come using, I am just writing it using Taylor series expansion and integration by parts by parts rule we obtain. I do not want to repeat this one because this we have already discussed when we have considered the functional J is equal to 0 to t 0 to tf V function of x t comma x dot t comma t dt. When we are deriving that of what is the necessary condition for the functional to be optimised, they are we have used that operation. Please refer that derivations than you will get it will miller's equations.

So, if you take the first variation of that one, if you take the first variation of this, if you do the Taylor series expansion and take the first variation of the functional then you will get del J is the first variation of the functional. First variation of the functional, that equal to if you do the Taylor series expansion and use the what is call integration by parts and simplify, then ultimately you will get the first variation.

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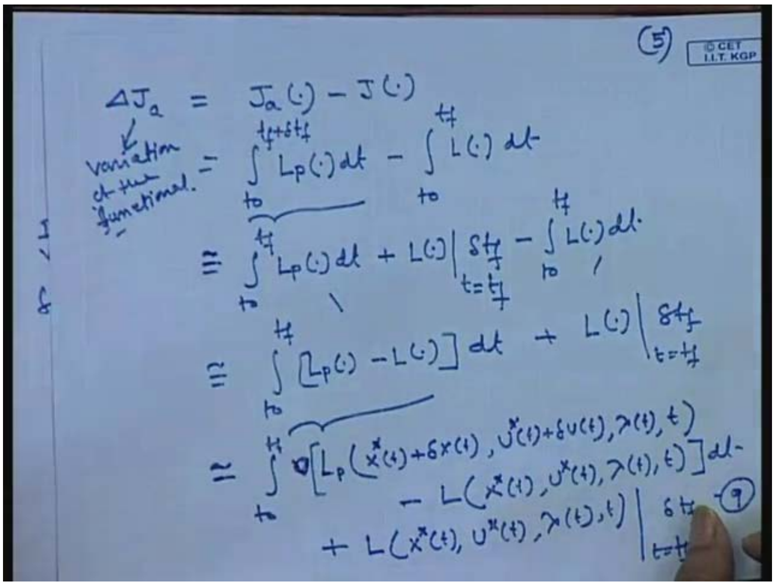

This is the this delta J is the variation of the functional variation, where you variation of the functional means when t 0 to tf plus delta tf and perturbed states and inputs are perturbed x and in turn x also perturbed. That is denoted by J minus without perturbed Lagrange integrant of the that Lagrangian function difference is first, that is variation of the functional. Out of these we split up into two parts, that first variation of the functional, second variation of functional is there, a third variation functional is there and so on. So, we are just concerned the first variation of functional for the necessary condition, for the functional to be optimised. So, let us see that one what is the functional variation after simplification.

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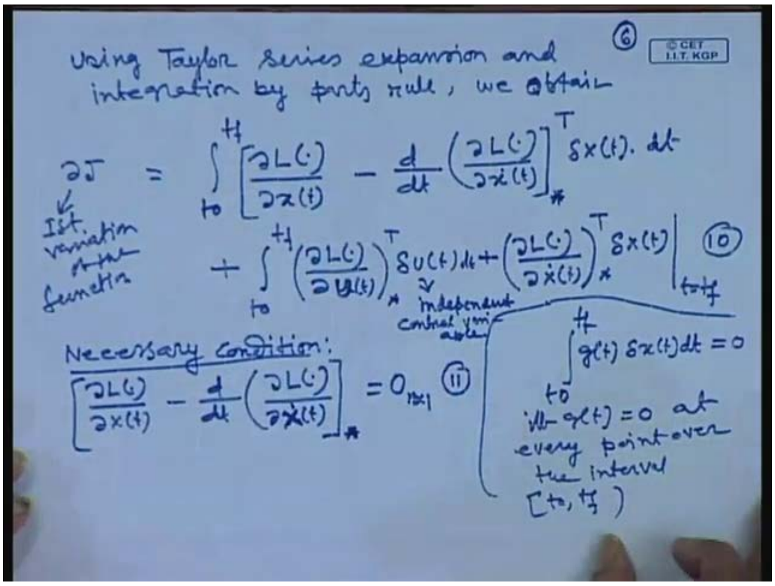

It will be t 0 to tf del l dot dl x of t minus d of dt del l dot del x dot of t. So, this you compete at optimal trajectory, optimal trajectory this whole transpose delta x t dt plus some of the terms are remaining terms are like this del l del u because you will it will come from the what you called Taylor series expansion in first order terms, ok? So, this star transpose del u of t. So, use a vector of dimension m inputs if you consider, use the vector of dimension m inputs in the beginning. If you see what have considered that one here, if you just no not this one here, if you say consider our original function x of t ut t x u is the number of inputs, that m cross 1 x is the m cross 1. So, the partial derivative of l with respect to u if it l is a scalar thing, u is a vector of m cross 1.

So, this will be a vector, so you have to transpose multiply by delta, then only you will get a scalar quantity that. So, this plus del x dot of t whole transpose star delta x of t, ok? That is e is equal to tf, that is we have derived this. This by a substitution Taylor series expansion and substitution you will get this expression. Then now, see you consider our that lambda is 0 to t 0 to tf. I am repeating this one once again here, you have g of t, then delta x of t dt is equal to 0.

So, g of t is the continuous function and it is differentiable each and every point in the interval this and delta x is the small change in that variable. Small element integrate with respect to time t, that value will be 0 provided if and only if and only if g of t is equal to

0 at every point over the interval at every point over the interval, over the interval t 0 to tf.

So, if you consider this equation number is 10, ok? 9 we have done it, if you consider this as equation number 10, then using I can write this quantity will be 0, provided that this quantity is 0. If delta x is not 0, this equal zero. So, our necessary condition just as before we did it that this del l del x of t minus d of dt del l dot, del function of this, del l x dot capital x dot of t whole, this is equal to 0. What is the dimension of this one l is a scalar, I am differentiating with respect to n to respect of x whose dimension is n cross 1. So, this dimension will be n cross 1. So, this star indicates that if you solve a differential equation, for that one you may need some other boundary condition. Let us say if solve this one, than whatever the trajectory you will get it that is the optimal trajectory. The star indicates the optimal trajectory of this one.

So, let us call this equation is equation number 11. So, this part is 0, now you see when you made an increment u star to delta u, this is not equal to 0. This is not equal to 0 and this is independent of this is the you can write independent control variable. So, this is not equal to this part, so this must be equal to 0 in order to make that or dt is there. Here if you see the last equation, there will be dt is here, dt here we missed this one. So, this equal to will be 0.

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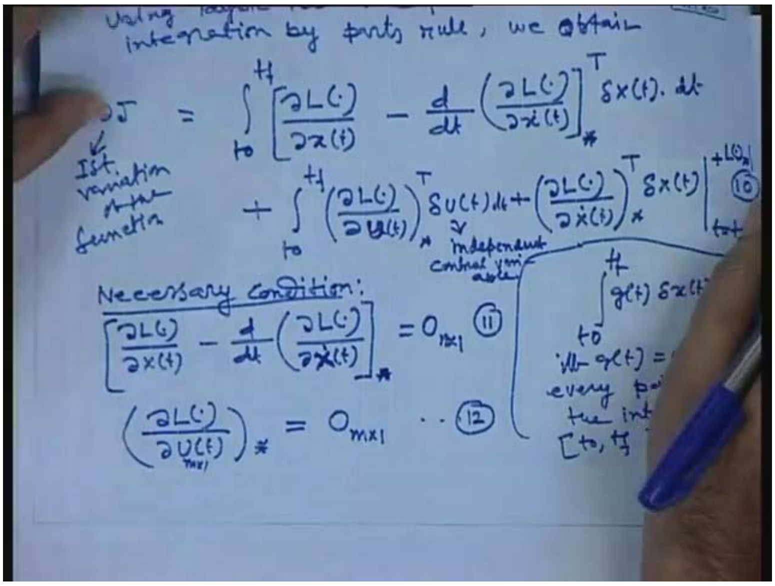

So, next condition is del l dot del u of t that compete around the trajectory. If we do it this will be a 0, whose dimension is m cross 1 because u is dimension is m cross 1, I am differentiating with respect to scalar quantity. So, that will be m cross 1, let us call this equation is equation number 12. So, we have a, you see when we have a two necessary condition this and this necessary condition. So, this is 0, this is 0, only the term is left with you with us is that one, another term is left here because I have just another term is left here plus, please correct it, plus dot star l dot star t is equal to tf. Find out the by tf into delta tf, this is that term.

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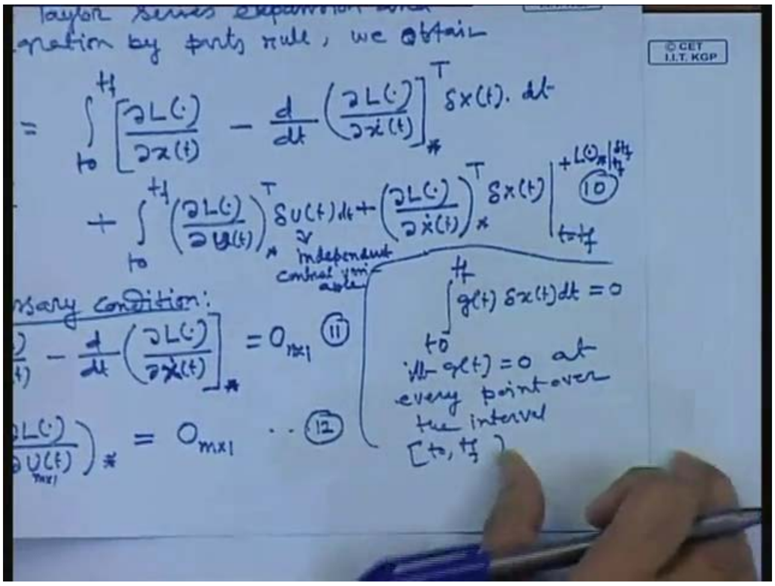

If you recollect that that one we did it that is the this term because this we have done in Taylor series expansion, that one Taylor series expansion of that one and that term is the this one, that term we missed it here. So, this so first term, second term and third term what we have got it due to the Taylor series expansion of this and this we got it up to this than what is the leftover term is there. So, our when you use the lemma this and this then we got it that what is call this equal to 0 and that equal to del l del l with respect to u to transpose. That will be 0, that two necessary condition. In addition to that still delta J, first variation of function is not 0.

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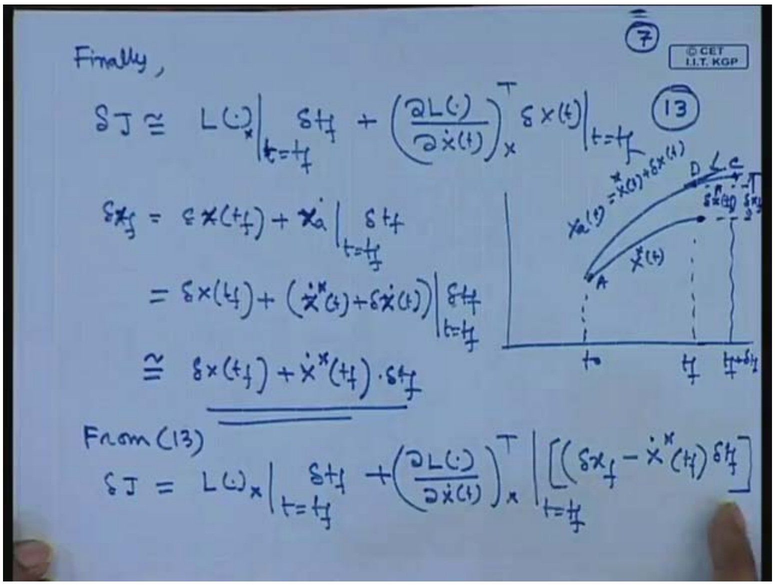

What is the finally leftover with delta J, finally leftover with delta J is equal, nearly equal to you can say that delta l dot star put l dot star. Or you can star you can give it here, t is equal to tf into delta tf plus del l dot del x dot whole star delta xt, t is equal to tf. That means x tf means x is equal to x x tf, t is equal to tf.

So, that value is I can write it. Now, this if you see this one our figure, let us go back to our figure, the optimal trajectory of this one. This is our A, this is our B, this is our T and this is our C. So, this is our let us call t 0 and this is our tf and this is our tf, tf plus delta tf again and this value is this value is, if you consider these value is and these value and that that we have consider delta x tf and these value we have consider delta xf as we discuss earlier.

So, you can write it delta xf, that means from this to this point that my coordinate of C and the this distance of that one is delta xf is equal to delta x tf plus x tf plus x a dot x. This is capital X, xa dot xa dot, this is our xa of t, this is our x star of t which is equal to x star of t plus delta x of t. I just find out the slope at this point. So, this is nothing but this slope is x dot, type t is equal to tf multiplied by delta tf. This I can write it that means this height plus this tangent at this multiplied at this height is equal to delta x.

So, this I can write it delta x tf plus what is this one, I can write it delta a dot of this delta x dot you can write it delta x dot is equal to x x dot star t plus delta x dot t, whole t is equal to tf delta tf. Now, see this is a small quantity very small quantity, this is also small quantity, the product of this is you can neglect it. So, we can write it the nearly equal to this, we can write it delta x tf is equal to x dot star tf into delta tf. This product is third term, you neglect this one. So, this we can write it, now equation what we got it equation 12 and then let us call this is equation number, that is equation number 13. You can see the equation number of 13 is that one.

Now, use this value xt f x delta xt is equal to tf delta x tf. I can use the value form here using in using you can write it from 13 using this expression. That means I will write x delta tf is equal to xf minus of that one. So, what you can write it from 13? Form delta J is equal to l dot this star t is equal to tf and delta tf plus this one delta l dot x dot of t whole transpose star t is equal to tf.

Now, I am writing x delta tf this is x delta tf value is delta xf minus x dot star tf into delta tf. This equal to that this is I am writing is that quantity in place of delta x tf I am writing is that quantity, you can see that one just like it. So, this equal to we got it, now you see what we can write it delta tf delta tf, if it comes out together than what you can write it. See growing that rotation if you symbolise this equation I can write it.

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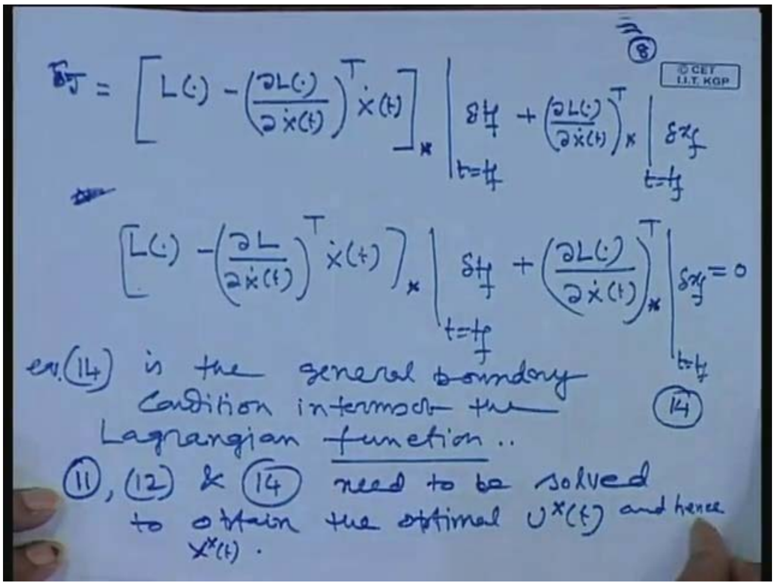

Or delta J is equal to l dot minus del dot del x dot of t, whole transpose x dot of t x dot of t that whole star, the whole star. Then put t is equal to tf, just I am writing that one. This term plus this term into this term I am writing t is equal to. So, first is star I am writing it here, you calculate the star than t is equal to tf. Then what is left, delta tf plus what is left? Only this term is left, this term, this and this term is left. So, that is I am writing is del l del x dot of t whole transpose star t is equal to tf is equal to delta xf, see this is delta xf delta xf.

Now, in order to become this is 0, that means what is the condition that delta J first variation. Our necessary condition is if you see is the optimic basic necessary condition, delta J must be 0. So, in order to make the necessary condition we got it two condition in addition to the two. The third condition is that l, this minus del l del x dot of t whole transpose x dot of t, this star evaluate t is equal to tf delta tf plus delta dot delta x dot of t whole transpose star t is equal to tf delta xf is equal to J, that means J is equal to 0.

So, our first variation in order to become first variation of this one, this part is 0, this is one necessary condition and that part is 0. This second condition and third part is what is call that we got it, that must be 0 and that depends on the our condition. Let us call our final time is fixed, tf is fixed, then tf is fixed means delta tf is 0, delta tf is 0. So, this is automatically 0, so only an x tf is free, that means delta xf is not equal to 0. It is in order to make it 0, this must be 0. So, in other words you can say in others way you can say if tf is free and delta xf delta xf is fixed, this is 0. So, this part will be 0 and tf is free when delta tf is not delta tf is not 0. So, this must be 0 in order to make this 0.

So, let us call this equation number is equation number, last equation we have given is equation number 13. If you see the 13, last so this is let us call equation number 14. So, equation 14, equation 14, equation 14 is the general boundary condition in terms of lagrange function. Now, if I summarise this one in order to optimise the our original problem, where the terminal cost is there and the integral part of term what is called performing index is there.

In order to minimise that one and subject to the constant x dot is equal to f of x comma u of t comma t. Then our necessary condition is first this, you have to solve it once you form the Lagrangian function, then you calculate that one or solve this one and also dl dl dl with respect to u is equal to 0. You solve this equation using the boundary condition of that one and this boundary condition if both the end point, final point that means time is free. The x of tf is also free, then you have to assume two boundary condition, you will get this equal to 0, this equal to 0, ok?

So, this indicates the equation number 11, equation number 11, 12 and 14 need to be solved. 14 to be solved to obtain the optimal solution to obtain the optimal u star of t and hence instead of the because u is an independent variable. That will derive the state in a optimal trajectories and hence x star of t. So, this is what we got it natural, I repeat once again what is the our problem was here. From the very beginning just see this one ok? Next class we will discuss the, what is called that using Hamiltanoin function, how we can solve that one problem.