Optimal control

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Lecture - 33
Numerical Example and Solution of Optimal Control Problem Using Calculus of Variation Principle

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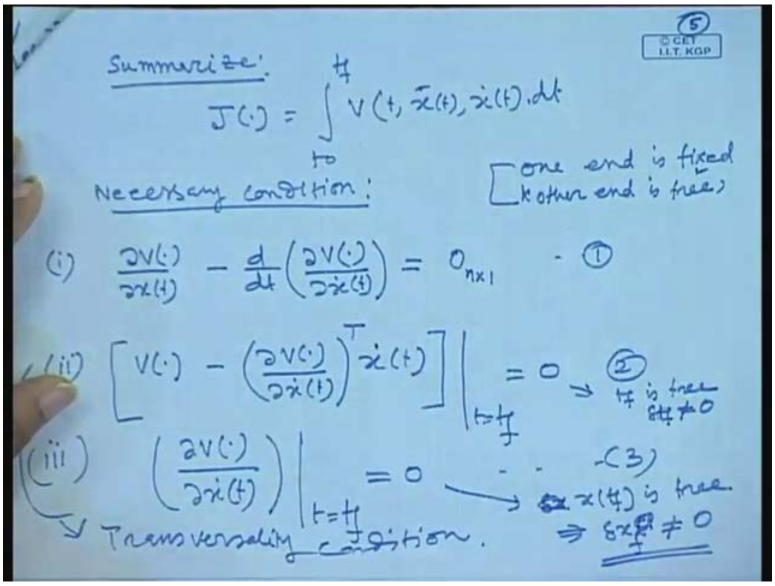

So, last class we have discussed that if you have a given functional J is equal to J dot is equal to t 0 to t f and V of t x, V is a functional of t and x t and x dot t. If you consider that one point is fixed and other point is free, then in order to have our job is to find a trajectory or full trajectory of x t, such that this functional is x has its extremal either minimum or maximum. So, that necessary condition we have derived it this one. So, the first condition is the Euler's Lagrange equation.

This equation is Euler's Lagrange equation. So, you have to solve this one. In order to solve this one there are two conditions, a boundary conditions are there. This condition when t f is free, when t f is free then you have to use this equation and x f is x t f is fixed, then out of this one only this is we have to use it when x t f is free. But t f is fixed, then you have to use in addition to the Euler's Lagrange, this equation when both are free.

That means x t f is free and x t f is free, then you have to solve Euler's equation in addition to the two boundary equation, boundary condition and this two boundary condition is called the transversality condition, agree, that we have seen it. We have considered the different cases if you recollect, we have considered the different cases, case A, B and C and correspondingly Euler's Lagrange leverage equation.

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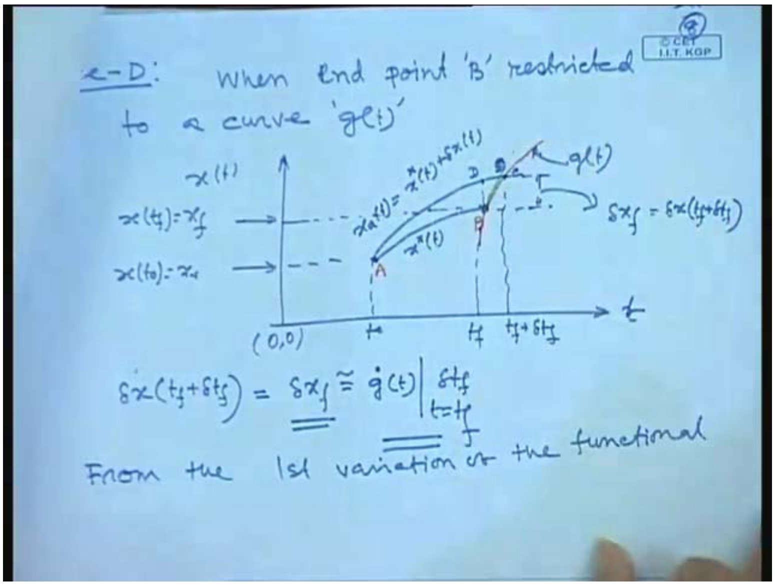

You have to solve it and the corresponding boundary condition, further we have considered. The next is we have considered the case 4 when the end point is restricted on a curve which is the function of t, agree? This and in this equation that you will get a what is called transversality condition.

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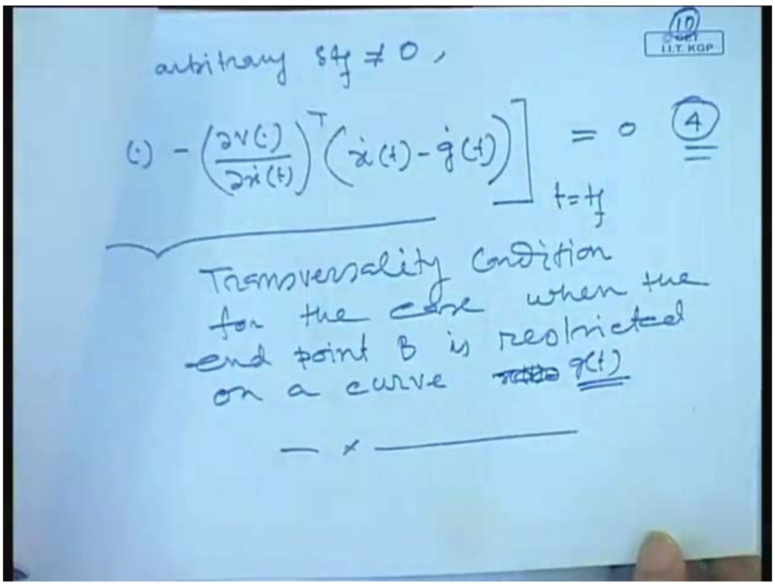

One is that x dot of t is state of x dot t, x dot of t minus g dot of t as t is equal to t f. This equation you have to solve. So, this is we have to now after solving this equation that will got the will get the trajectory whether this trajectory is what is called optimal. This trajectory will maximize or minimize this functional or not that we have to check it by using the, what is called sufficient condition. We will now discuss that sufficient condition for the functional J used to be optimized, what is the sufficient condition for the functional J will be optimized. We will either maximize or minimum value of the function.

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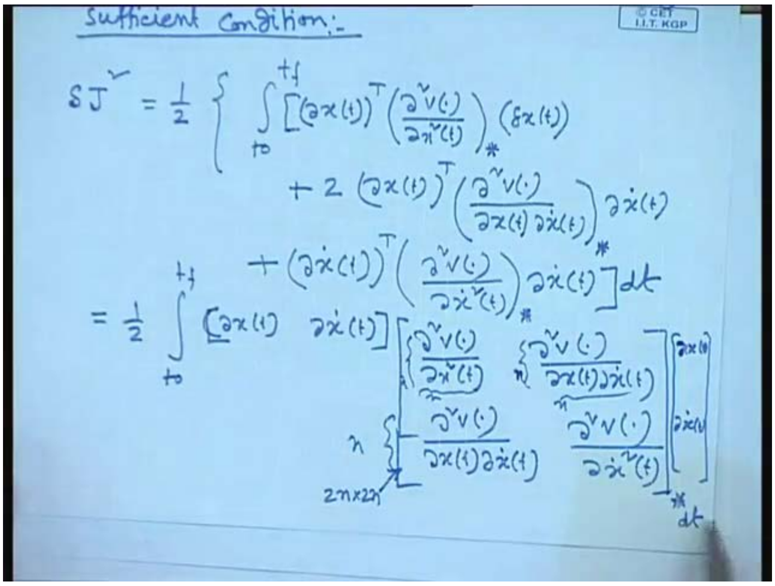

So, our sufficient condition, sufficient condition it is same as what we discussed in case of what is called the static optimation problem for multi variable optimization problems. The same condition we will consider here, also sufficient condition. The sufficient condition will give you whether the functional is maximum along the trajectory if you use in the functional that, whether it will be maximum or minimum value of the that will give you the, you will get it from sufficient condition.

So, if you see this our second variation of the second variation of the incremental what is called functional, that we have denoted by del J square. That is equal to if you see the Taylors series expansion of what we have disused earlier, that means that incremental functional value. The second we have we will consider up to second derivative of the Taylor series expansion and that second derivative we will consider. Now, as a what is called the second variation of the functional, that is equal to integration of this t 0 to t f, agree?

Then del x of t that whole transpose del square v dot dell x square of V, this is a matrix because we are finding the gradient V with respect to x. Again we are differentiating with respect to x. So, this is the matrix, so that matrix multiplied by delta x of t vector. So, this and second term, the second term is twice delta x of t whole transpose. Then delta square V, differentiate gradient of V with respect to x, then x, x dot or reverse order x dot. This you calculate along the trajectory of this gradient multiplied by delta x dot of t.

Then plus delta x dot of t delta x of dot of t transpose delta x dot of t transpose, this delta square V dot delta dot square of t, this compute at the this is the matrix compute at along the trajectory star means along the trajectory delta x dot of t, that this bracket and this bracket is complete, this and this complete and then differentiate with respect to t. So, this quantity inside the bracket of this one is a scalar quantity. So, we can write it in terms of quadratic form that you know quadratic form is nothing but a x transpose p x from we can write it.

So, how we are writing this one you see just see this one, this integration of t 0 to t f, then we have a delta x of t, then delta x dot of t. So, this is a matrix form, you write it vector in case then you write it this del square V dot del x square of t. Then del square V dot del x of t into del x dot of t, then del square V dot del x of t into del x dot of t del square V dot del dot square of t. Thus whole matrix, agree.

This you evaluate along the trajectory multiplied by delta, multiplied by delta x of t, multiplied by delta x dot of t. Now, see this one this is a matrix of dimension if x is a dimension of n variables, then this is the matrix of dimension n cross. This is n rows n column, this is also n rows n column and similarly, the whole matrix dimension will be twice n into twice n, if you see. Similarly, this one is a n rows n columns, this whole thing now in order to this whole thing differentiating with respect to d t with respect to d t.

This whole is a vector matrix multiplied by column vector, then differentiate with respect to this whole quantity is a scalar quantity same as this one. Now, in order to that we knows that sufficient condition of second variation of the functional must be if it is greater than 0. If it is greater than 0, then the functional value along the trajectory we got it should be a negative, sorry should be a minimum value of the functional. If it is a delta square V is less than 0, then functional value along the trajectory will be a maximum value of the functional. So, how to decide this one? You see now it is in quadratic form.

So, our this matrix, compute this matrix whose dimension is twice n by twice n along the trajectory. If this matrix is positive definite matrix, this matrix is positive definite matrix, that indicates that del square del J square means second variation of the functional will be positive value. That means the functional value along the trajectory is minimum.

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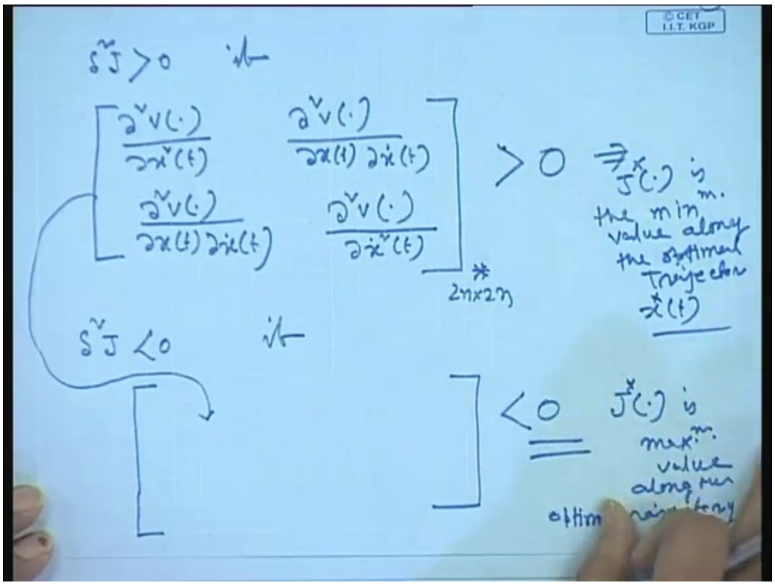

So, our condition is if delta square J is greater than 0. If the matrix delta square V delta x square of t delta square V delta x t and delta x dot of t delta square V delta x of t delta x dot of t, this order may be changed. Delta x dot and delta x, this value will be same. So, it does not matter for this one, we have discussed this thing in details while we have discussed the static optimation problems. So, this x dot square of t, compute this matrix along the trajectory and if it is a if you want to delta square is greater than 0, implies this must be greater than 0.

So, we will check this matrix whose dimension is twice n, twice n where x is the n variables of the what is called functional. If this matrix is positive implies that delta square J is greater than 0, that means the functional value, what we will get, agree? If you put this x star that is the optimal trajectory in the functional values, then we will get it minimum value of the functional. And if we can write delta square J is less than 0, if that same matrix that this matrix, this matrix is less than 0. This implies that the functional value J star of this is maximum value along the trajectory.

Similarly, this implies that J star of this is the minimum value along the optimal trajectory. You can write optimal trajectory means x star of t, this is x star of t you can write it. So, this is the sufficient condition for this one, agree? Now, let us solve one problem to get how one can solve such type of optimation problem, dynamic optimation problem using the calculus of variation principle.

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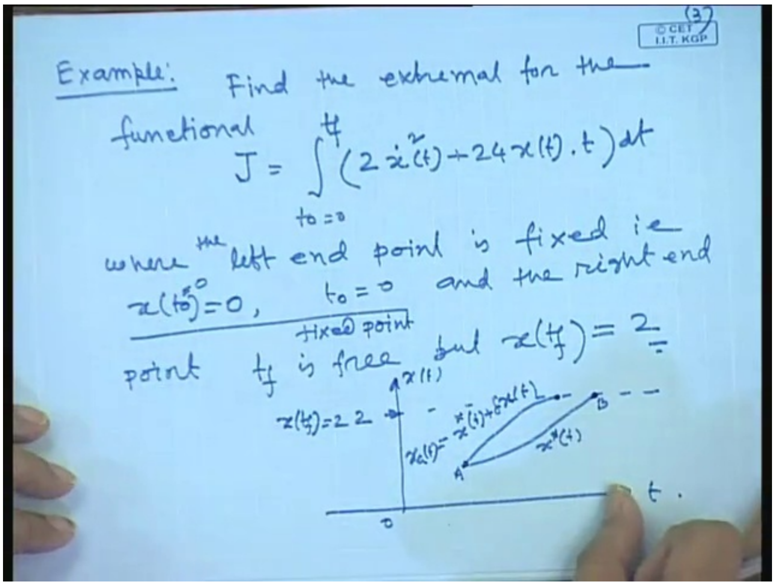

So, example take an example, so find our problem is find the extremal for the functional J is equal to t 0 to t 0 is equal to 0 to t f, agree? And this functional is given x dot square plus 24 x of t into t d t, agree? So, this functional value J is up to optimal extremal whether minimum or maximum, that we have to find out and corresponding what is the trajectory for this one.

So, where left where the left end point is fixed, end point is fixed. In other words x of t 0 is equal to 0, agree? x of t 0 is 0 and t 0 is equal to 0, here t 0 is 0 that means you can write t 0 is 0. So, this point is fixed, fixed point. You can assign this point as A, this point is fixed point and the right end, the right end point t f is free, but x t f value is equal to 2 is fixed x t f value. Whatever the t f value is that free, but x t f finally this trajectory should reach at time. Whatever the time is there at 2 you needs it may reach.

So, corresponding to that problem if you represent that one, it is something like this. So, if you consider x star of t and this is a x star of t near about delta near about the optimal trajectory is there is a another trajectory, which you called x star of t never root of the optimal trajectory is this.

So, this we are calling it a point and this point is B, if you say that that point is t f is free, agree? But x t f whatever the x t f is fixed, that is 2 x t f is equal to 2 and we have find out x star of t, bring optimal trajectory for this for the functional for which the J will be maximum, whether minimum or maximum that for that we have to check the sufficient condition. So, if you just follow our methods what you have first we have to consider our set solution.

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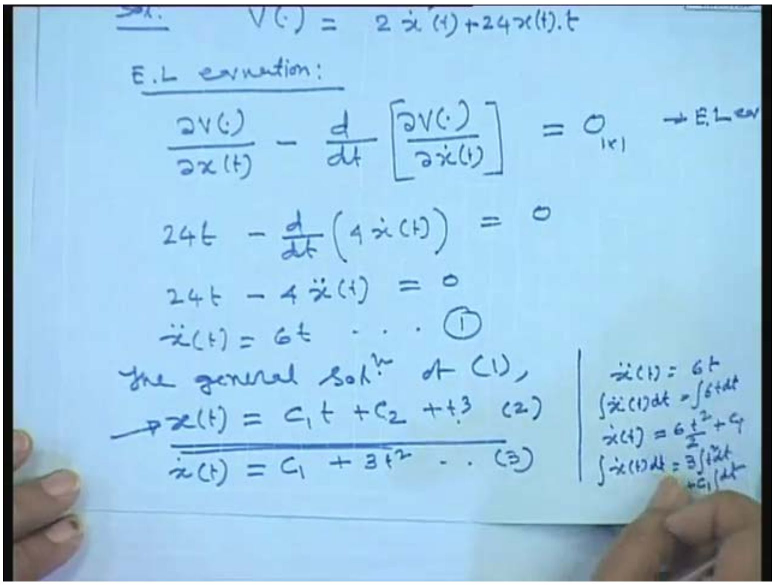

So, first you write the v function, v function is what in our case 2, x dot square of t. See this is our, we v is a function of x t x dot of t and t. So, the v function is what functional v is x 2, 24 x of t into t this way, so that the Euler's equation, if you recollect the Euler's equation del V del x of t minus differentiation of del V with respect to x dot del x dot of t functional differentiation of this one, agree? Then this is equal to 0 and since x is a in this dimension is 1.

So, this dimension will be 1 plus 1, agree? So, I will just put the value V and differentiate with respect to x, that is our problem. So, V is then if you differentiate with respect to this x, I will get 24 t, first part of this Euler's equation. This is called the Euler's leverage equation. Now, second part of this one you see we have to differentiate with respect to x dot, so that will be a your 4 x dot of t is equal to 0. Then 24 t is equal to 4 x double dot of is equal to 0, then your case is x double dot is equal to x double dot of t is equal to six t, which is equation number 1.

This is a simple differential equation, I will advise you to recap the solution of differential equation in presence of foreseen function, that you recollect this one since this is simple case. One can find out the solution of this one by using what is called double integration. If you do it first term, you integrate, you will get one constant term, second term you integrate you will get another constant term of this. So, you can this in this case it is very simple. So, I can write that general solution of equation 1, the general solution of 1 what you can write in general solution of this one x of t is equal to C 1 t plus C 2 plus t cube, for your convenience just write.

So, you x double dot of t is equal to 6 t, integrate both side, integrate both side x double dot t d t with respect to d t. Integrate 6 t d t, then you will get x dot of t is equal to 6 into t square by 2 plus C 1. Then again you integrate this one, once again if you again integrate that x dot of t, again integrate with respect to t. Then three integration of t square d t plus C 1 integration of what is called d t.

So, if you do this one, the solution is x x of t is equal to t square t cube by 3. It is cancelled 3 3 cancel the t cube we got it. Then you have a this C 1 into t this one and another integration constant you will get C 2. So, this the general solution of that one, let us called this is equation number 1. Now, let us try if you differentiate this one once again then what you will get it, that we will see C 1 into t, differentiate 1. This C 2 is constant, this is 0 then 3 t square, let us called this equation is 3, equation 3.

Now, this is the Euler’s equation, from Euler’s equation we get the what is called trajectory, optimal trajectory of this one, but still it is the task that how to find out C 1 and C 2. That is and that two constant we can find out from what is called the initial boundary condition using the boundary condition. So, let us see that how one can find out that C 1 and C 2 of equation 2 at time t is equal to t 0, that is 0.

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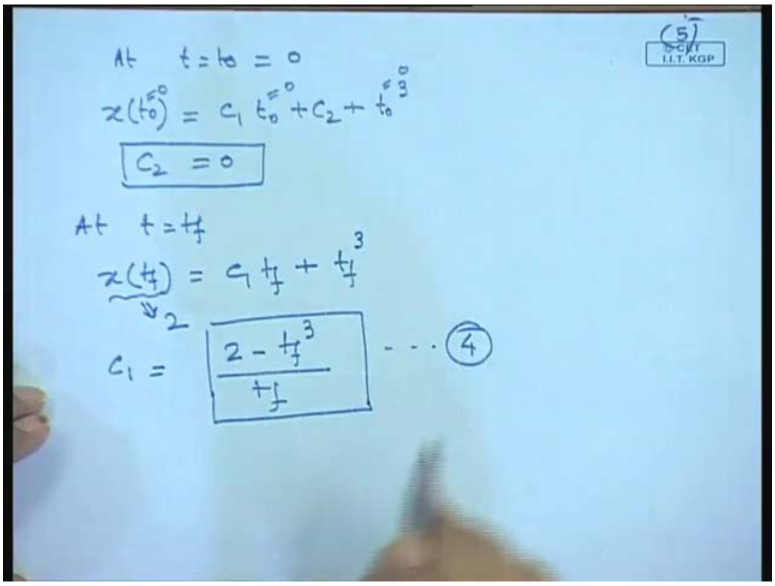

Let us see what is our trajectory t 0 value. So, this equal to C 1 into t 0 equal to t 0 plus C 2 into t 0 cube, t is equal to t 0. Then say t 0 is 0, this is 0, this is also 0. So, C 2 value is 0, the constant C 2 will be 0. Now, from the initial condition we got the condition of this, then next is at time t is equal to t f, t f is unknown. It is free then from equation 2, this is from 2 we can write it x t f is equal to C 1 t f plus C 2 value is 0. We need not to write it here t cube this.

So, C 1 that this value is give the problem is 2. Now, our C 1 value is what we can write it, C 1 value is your t f, C 1 value 2 minus 2, take it this side, t f you take it this side, then t f cube divided by t f. So, this is the value of t f, but that depends on the final time t f because t f is free for that. So, let us called this equation number 4, so now this is the condition, that means C 1 value you know, C 2 value you know. So, the description of our optimal trajectory is known provided if t f is known that to what to what time is more t f, that will reach to the our x t f is equal to 2. Now, condition is coming, transversality conditions.

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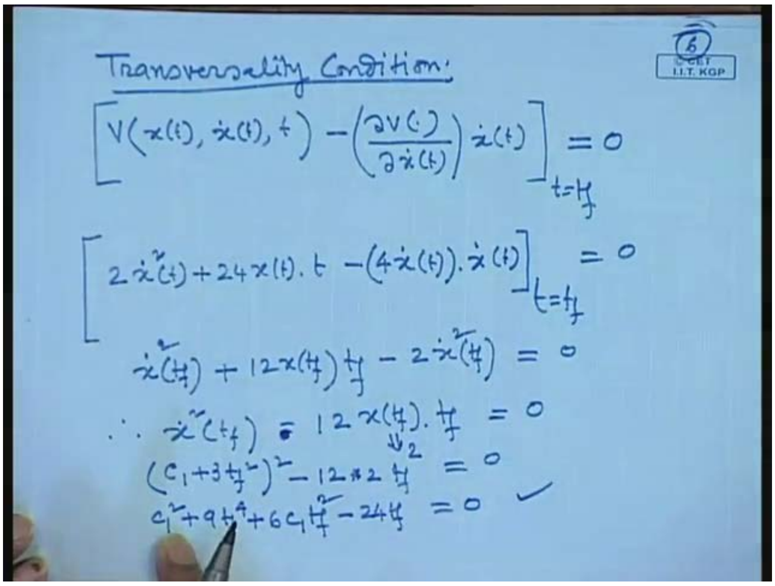

So, V, so our x t f if you recollect this, our terminal conditions that one x t f is fixed, that means delta t x t f is 0. So, this condition will not come into the picture, only this condition will come, Euler's equation and this condition will come into the picture because here x t f is fixed. Not free, but t f is free, the t f t f means delta t f is not equal to 0. This implies that this boundary condition must satisfy, that we have explained earlier. So, let us see this one if x t x dot of t of t minus and if you recollect this one, see I have not put it star. You can put it star because whatever solution you get it, that is only optimal trajectory star.

So, it not necessary put it star, that only if you get the solution of Euler's Leverage equation, you will get the optimal trajectory x x star of t or you can write x of t this one. So, I am just not writing, if you like you can put it star also there. So, this and del V, the transversality condition when t f is free, but x t f, x delta x t f is fixed, means delta x t f is 0. Then this is equal to x dot of t, whole this I should write it transpose if x is a vector, but it is a scalar. So, not necessary to write transpose for this one. So, this into x dot of t, but do not forget t compute this one as t is equal to t f. We have to put t is equal to t f in the transversality condition. See our last equation transversality is equal to t f. So, do not forget to write t is equal to t f when the t f is free, agree?

So, this now write this equation, the delta x is a V of x is equal to x dot square of t plus 24 x of t into t. Now, this you differentiate with respect to x dot, that means this is the v, you have differentiate with respect to x dot, means this is a 4 x dot bracket 4 x dot of t into that other term is 0 because I am differentiating with respect to x dot.

So, it is a x dot of t is equal to put t is equal to t f is equal to 0. Now, see this one there is a 4, there is 2, you divide by this into 2 then it will be x dot t is equal to t f. I am now putting the t values t is equal to t f plus 12 x t f into t f minus 2 x dot t is equal to t f square, this is square I missed it. So, this equal to 0. So, finally therefore x dot square t f is equal to 12 x dot, t f is equal to 12 x t f into t f, correct? So, this is I am writing x 2, x dot square 24 x t f, so 24 t f so 4 4 that is your x dot, 4 x dot t into x dot.

So, it is a twice of this one, agree? So, if you take it this, that side left hand side, so it is minus of this equal to 0, agree? So, this is minus, now I know this value of what is called t f x, x value x solution of this is here, solution of dot you see solution of x dot. I know that C 1 3, sorry C 1 x dot is x dot C is C 1 3 t square as t is equal to t f as t is equal to t f x dot t f C 1 3 t f square.

So, I will write x dot value is C 1 plus 3 t f square, whole square minus 12. So, x t f value is what say x t f value is 2, agree? x t f value is given 2, so this is 12 into 2 into t f is equal to 0. So, if you simplify that one C 1 square, C 1 square 9 t f plus 6 C 1 t f minus 20 t f is equal to 0. So, what we get C 1 square 9 t f 4 plus 6 C 1 t f, 6 C 1 t f square.

Here, t f square then 24 t f we got it, this is equal to 0. Now, C 1 expression already we have derived the C 1 expression, if you see in terms of t f. So, I will put the value of C 1 expression 2 minus t f cube by t f in this expression. So, if you put this it is in this expression, then what we will get it just see this one.

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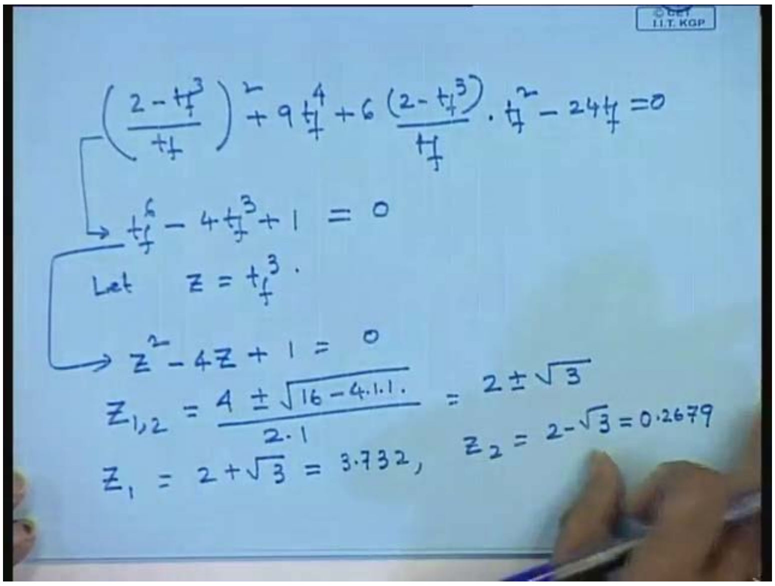

I am just putting this in this expression. So, C 1 is what 2 t f cube by t f whole square this is C 1, this value is C 1, this value is C 1 plus 9 t f, 4 plus 6 C 1 is what 2 minus t f cube, 2 minus t f cube divided by t f into t f square minus 24 t f. As it is minus 21 t f as it is f, look in that if you see this one, this is the equation of this one. I am just putting the value of C 1, hence where C 1 is 2 minus t f cube by t f. This values you and I putting in this expression and then I got it it. So, if you simplify this one if you simplify that one finally, you will get the expression.

So, I am skipping few steps for the there is a just is some academic exercise you have to do it to arrive in this expression. So, t x this t f 6, 4 t f cube plus 1 is equal to 0. Now, this is a equation which one can solve for t f. Let z is equal to t f cube, agree? Now, if it this one I can write it that this t f expression, that x square minus 4 z plus 1 is equal to 0.

So, z is equal to it has two roots quadratic form. In fact it is a you will get it how many roots is here for the time being z, I am getting it minus V is 4 plus minus root over b square, means 16 minus 4 a c divided by twice a. So, that will be 2 plus minus that is root 3. So, it is a 12 2 take out z has a roots of z 1 has a root of 2 plus root 3 1, which value is 3.732. Another is z 2 is equal to 2 minus root 3 and that value will be 0.2679 z t f, how t f is related z is related t f 1 cube.

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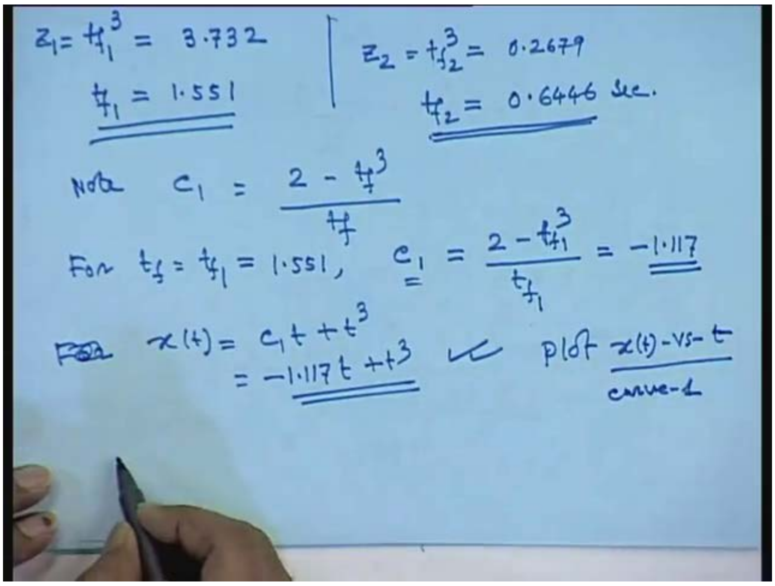

So, our t f 1 from x 1 if you see z 1 is equal to t f cube, t f 1 cube is equal to what we got it that is 3.732. So, t f 1 is equal to 1.551 this one and z 2, another case is z 2, z 2 is equal to t f 2 cube, which is equal to 0.2679. So, t f 2 this values is your 0.6446. So, there are two final time is there 1.5 times the state will reach x t f is equal to 2. Another time is 0.26, 0.6446 the our final state will reach to two units, agree?

So, once I know the t, I can find out the C 1 for t is equal to, note C 1 is equal to 2 minus t f cube by t f and for t is equal to, for t f is equal to t f 1. That is your 0.15, second one 5 1 second, C 1 value you will get it 2 minus t f 1 cube divided by t f, this is t f 1, ok? So, that value will come minus 1.117, this for another case for t f is equal to or you can say what is our trajectory, x t of this is our C 1 t plus t cube. And I know C 1 value, C 1 value one case is minus 1.117 t plus t cube.

So, this is the optimal trajectory for this one when at time t is equal to 1.5 ones 5 5, once again this x t f 1 will reach the state is very well is two units. This is one solution and if you plot it let us called you are you are advice to plot this x t verses that time, agree? And that trajectory at time that trajectory, let us call is some name. You give it curve 1, that trajectory is called curve 1. Now, next situation that is what we will say that is for t f is equal to our time is that for t f is equal to t f 2, that is our time is what.

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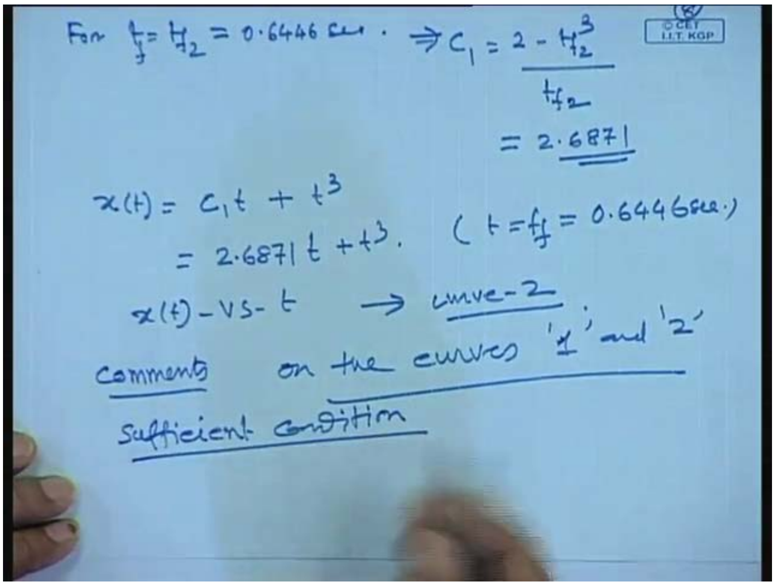

If you see the t f is equal to t f 2, that is what we have got it that point 0.6446 second, agree? Then C 1, this implies C 1 is equal to twice minus t f 2 cube divided by t f 2. So, this value will come 2.6871 constant. So, corresponding trajectory optimal trajectory that x t is equal to C 1 t plus t cube, so it is C 1 is 2.6871 t plus t cube, agree?

So, you are once again you are advice to plot t verses time and call this curve, that this optimal trajectory curve is curve 2. You can easily plot it this t verses t 0 to you know how long you have to plot it. This is valid for t is equal to t f, that value is your 0.6446 second, you plot it up to this. t is equal to zero to this and that one you plot earlier one t is equal to you have to plot this. This plot you do it t is equal to t f 1 which is 1.1551 second, agree?

So, there are two trajectories are there which will give you the optimal value or extremal value of the function, both are what is called optimal trajectory. One case you justify, you this one case will give you the maximum value of the functional. Other case you will give, it will give you the minimum value of the functional. Out of this two trajectories, one will give you the maximum value of the functional, other will give you the minimum value of the functional. Both the trajectory reaches final value at x t f is equal to unit is two units, but different times this one.

So, I just advice you make your comments, make your comments on the curve 1 and 2 which I asked you to plot it, and make your comments which one is giving the optimum.

In the sense maximum value of the functional which trajectory will give you the minimum value of the functional values. So, the weather minimum or maximum value functional will give, you can find out how that out our sufficient conditions, sufficient condition you use it. What is the sufficient condition if we recollect this one, that our matrix that Hessian matrix.

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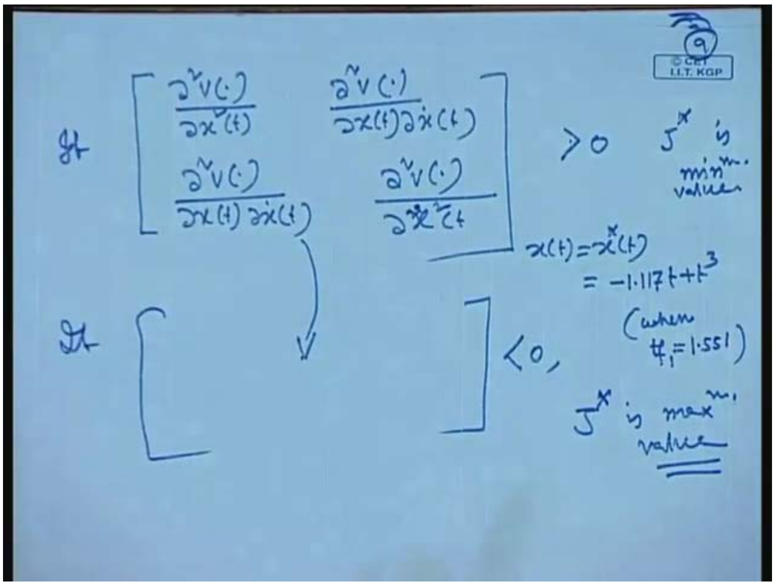

If you recollect in static optimation problem, we have discussed as a Hessian matrix. The Hessian matrix is del V dot del x square of t, del square V del x of t into del x dot of t del square V dot del x of t into del x dot of t del square V dot del square del x dot square of t. This value you find out the trajectory x is equal to x t is equal to x star of t, what is z star of t, one case we got it x star of t. If you see first case when t f is equal to 1.5 second, we got it minus 1.117 t plus t cube, when t f is equal to t f 1 is equal to 1.157.

So, you put this values because in this expression you differentiate with respect to x, gradient v, gradient you find with respect to x. Then again, once again differentiate and whenever you got it x you put this values of that x is equal to t f, agree? Then you x is equal to t f, you put it and find out the what is the that whether it is a positive definite matrix or not. If it is a positive definite matrix, if this is positive definite matrix then functional J star is minimum value. We will get if this is less than 0, then J star will give you the maximum value of the functional.

So, this way you can check it, so this we have shown it how to solve a problem, dynamic optimation problem by using the calculus of variation concept. Now, we will see slowly, we will got to application of what is called calculus of variation to control problems. The control problems means you have dynamic system, any system one can model into a mathematical model, either transformation model or state phase model or deferrable. Our problem is concentrated to describe the dynamic system in terms of differential equation, in terms of differential equation.

Let us call if the dynamic system, this system is described by another differential equation. One can convert by selecting the suitable variables into convert the differential equation which is order of n can be converted into a n first order differential equations. So, that is called is a state phase representation of a dynamic systems, whether the dynamic system may be linear or non-linear, but you can apply the what is called the calculus of variation principle to find out the optimal value of the functional or objective functional optimal value.

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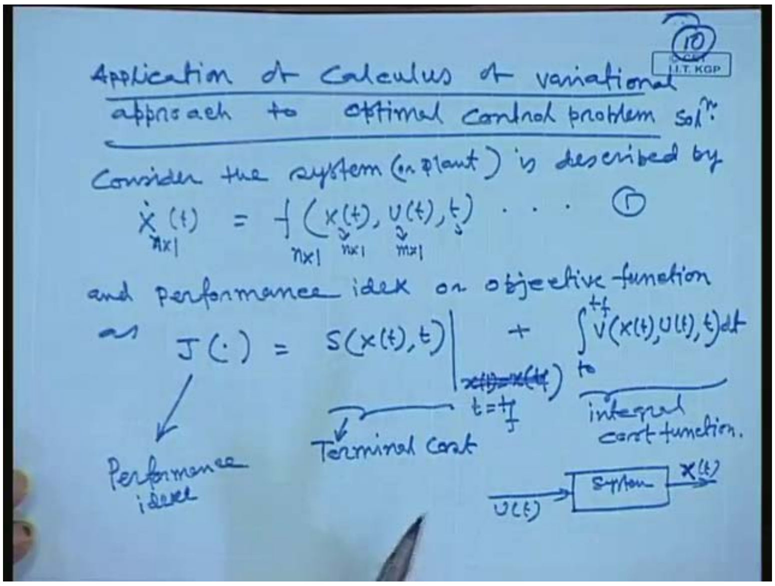

Let us see how to do this that one now application of you can say the application of calculus of variation approach to optimal control problem solution. So, that is our problem. Let us consider our system or plant, consider the system the dynamic system or plant is described by x dot capital X. Now, I am now using to represent if the variables are more than 1, then I am representing with the capital. So, let us called that variable is n cross, small n cross 1 n variable are there.

So, that is a function of x t u t and function of time. So, this function also we have a n function that they are this is a vector. So, let us called this is equation number 1 and if you see there that vector dimension is n cross and we assume that we have a m inputs are there to the system. So, and this is the time, so this is a description of what is called our dynamic system in terms of variables and how many variables are there, n variables. How many inputs are there, m inputs.

So, our problem is that you find a corresponding performance index or objective function, corresponding performance index or objective function is j is equal to s function of x t of t. And that x t is equal to t f x t f or you can write it here, better you write t is equal to t f. The terminal time the this function below this is a scalar because objective function is a scalar quantity.

This is also scalar, scalar one it can be quadratic form, but it is a scalar. It is a function of x t and t and then we have a functional t 0 to t f, V is a function of x t is function of u t and t that is called d t. This is the function of that one so and this we call this term first term is called terminal cost, terminal cost and this x is a functional of x, x of t and t it is differential with respect to time and t that is all. That function is a continuous function and this term is called the cost integral, integral cost function and that J is you performance index.

So, our objective is if you see our objective is to find a control signal in the system. Suppose, we have a system is there, that system and we apply the input u of t and the output is let us called the states are the output. So, we are excess in the state, our problem is to find a control input u of t such that this performing index, what is the performing index. That performance index is indicating that indicant is indicating 0 to t f and it is a function of x t u t and t u t is the controlling port. Once the control input is that is that regulates the state x of t, agree?

So, this plus that means as time t is equal to t f, what is this function value, that is called the terminal cost. So, that means we want to drive the state at final state at particular point, agree? That represents in terms of a quadratic form, agree? You cannot write the x t f only this quantity, but that quantity must be a positive quantity or 0. So, our problem is find control input which is, which will control the state of the system x t such that this performing index is minimized. Not only that, that control input and state because state will satisfy this equation. That means this you are find out the control input you in such a way this functional is minimized or maximized control input control problem is minimized.

Let us call and then subject to this constrain and that constrain is the equality constant. So, our problem is now becoming a what is called a constant optimation problem and that constant is equality constants are there.

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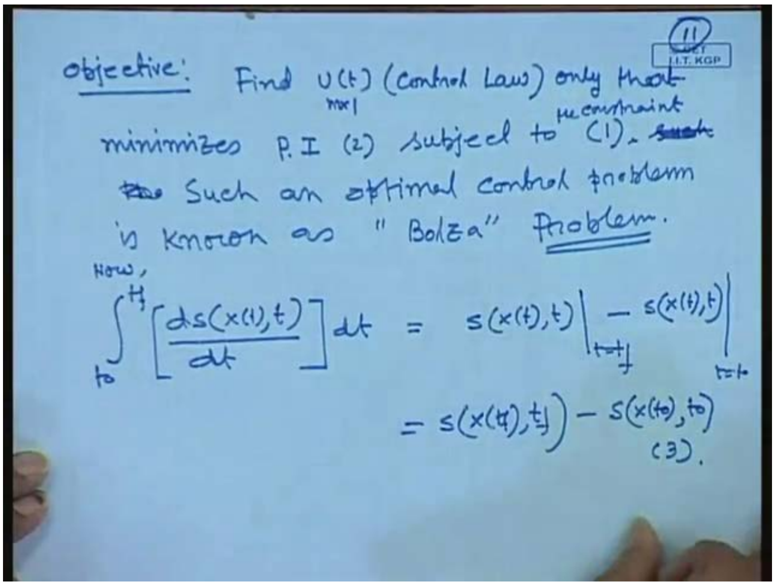

So, just write the statement of the problem objective, find U of t whose dimension is m cross 1, number of inputs is m. That means control law only that minimizes performance index, P I means performance index. Let us called, we just write this equation is equation number 2, agree? That performing index 2 subject to plant dynamics subject to equation 1 plant dynamics system dynamics such that, no our problem is find U t only. That minimize the performance index subject to the constrain 1, that means whatever the control input we generate, n x generates this much. Satisfy the equation number 1, that is such an this type of problem is called d, such an problem optimal control problem is known as Bolza problem.

So, this the our problem, now let us say if you look at this thing, what is difference from the earlier problem. We have a same thing here, earlier problem we have to what is called integrant V is there. We have to integrate 0 to t f and that functional we have to minimize, but in addition to that there is another term is there which we will call, what is call is the terminal cost and that can physically represent the final state of the system.

In terms of it quadratic form this and that quantity is must be a positive point that one. So, this not only this there is another constant is there, it must satisfy these equivalent constants and this equivalent constant is nothing but a what is called that dynamic equation of the system. This equation is the dynamic equation of the systems.

So, by some means if you can push it this term into the integrant part of this equation, that equation then our problem is similar to the earlier problem. Let us see that how one can do this one, so not because our job is how to push this tern in to integrant part of this functional, agree? Now, see this one we can write it now t zero to t f d s x t which is a function of time and d t we have just mentioned earlier that this is the continuous function, which is differential with respect to x t as also t.

So, this I can write it this into d t is nothing but a, that if you this one its nothing but a x of t comma t, t is equal to t f minus x. You can write it x t of t t is equal to t 0 which is nothing but a x x t f t f minus s x t 0 and t 0, agree? So, in place of this one, I can write this term and this term. So, this limit is an integrant limit of this one is same t 0 to t f. So, I can push it in equation number 2. So, if you push it let us call this is equation number 3, agree?

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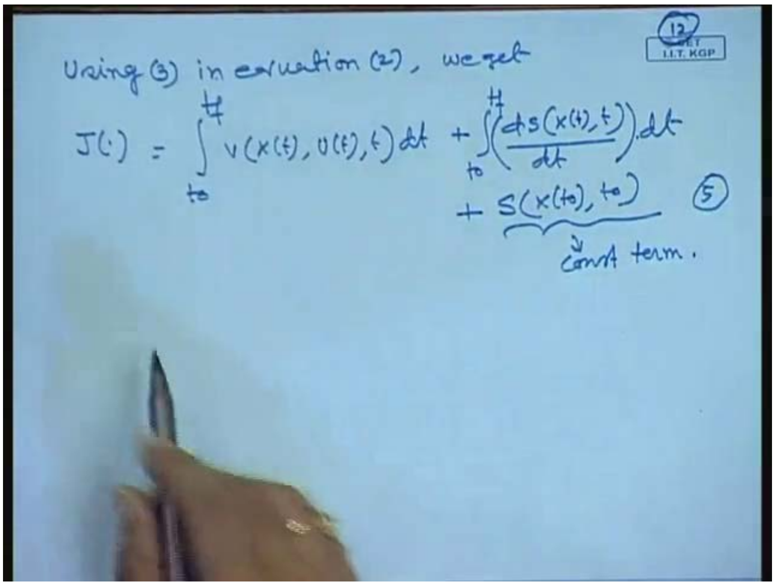

Using 3 in equation 2, what we get it just look at this expression that this is this minus, this one I am using this one. So, if you do this one J of dot is equal to integration, I am just skipping one step. This V of x of t u of t, then t d t, that one plus t 0 to t f d s x of t t d t whole bracket d t plus s of x t 0 and t 0 this and this is the constant term because you know that t is equal to t 0. What is that? Initial condition of this one you know, this one this is the constant term.

Now, our whole problem is now leaked, if you see our problem is now this one, minimize this one in place of minimizing this one, minimize this term. Let us call equation 4, agree? This is equation number 2, 3 and this 4. Minimize equation 4 is same as minimize equation number 2, minimize equation number 4 such that minimizing the you find that optimal value of the controllers such that this is minimized and as well as this equation is satisfied. Since this is a constant term that optimal trajectory will remain same, if you consider the minimization of this part only. So, instead of whole thing, minimization this part, minimization whatever the optimal trajectory you will get it, agree? For U transpose and x transpose that will minimize that as well as it will satisfy this one. So, we will stop it here. Next class we will continue this lecture in detail.