Optimal Control
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Lecture - 03
Optimality Conditions for Function of Several Variables

So, last class, we have discussed that, from the description of the problem, how one can formulate the multi objective optimation problem in mathematics. Then, we have discussed the, what is called, the primaries of vector calculus. The function is given, which is a multivariable function. How to find out the gradient of a vector and simultaneously, you have found out the gradient vector, if you once again differentiate with this respect to x, then we have seen it becomes a matrix.

Which may matrix is a symmetric matrix and its dimension is n cross n, where the n is the number of variables involved in that function. That matrix is a symmetric matrix, we have seen. With an example, we have seen, how to find out the gradient of a vector. Today we will just discuss how to find out that hessian matrix. That means, if you have to differentiate that function twice, second-order differentiation, you will do it you will get a hessian matrix. How to compute that matrix?

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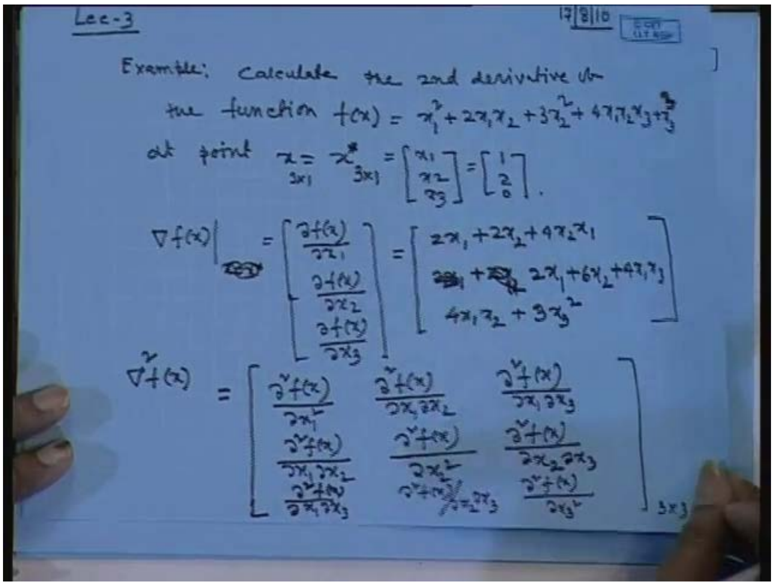

So, let us take one more example. That example, same example what we have considered in last class. For a to find out the gradient of a vector. So, the example, calculate the second derivative of the function f of x is equal to x 1 square plus twice x 1 x 2 plus 3 x 2 square plus 4 x 1 x 2 x 3 plus x 3 square. Calculate the same derivative of the function, at point x is equal to x star. Here, the dimension of x is 3 cross 1 that means, this has x 1 x 2 and x 3. And these values are given, 1, 2, 0. So, in order to find out the hessian matrix of this function, that means the gradient of this function, once again, if you differentiate, then this is called, the second derivative of this function for a multivariable function.

So, that can, first you have to find out the gradient of this function f of x. That last class we have found out, f of x value at x is equal to x star. First you, let us call you, find out the f of value f of x gradient of this one. Is nothing but, a del of x, with respect to x 1 then, del of x with respect to x 2, then we have to find out del of x with respect to x. Since, we have three variables are there, function is a, this function f of x is a function of x1 x2 and x3.

So, this if we differentiate this function with respect to x plus plus, we have seen it is 2x 1, recall this expression x 2 plus 4 x 2 x 1, then differentiate it f of x with respect to x 2, that will come 2 x 1 square plus 2 x 1 plus then 2 x this is x 1 with respect to x 2, this is a bit, sorry. This is 2 x 2 sorry. This is 2 x 1, then 6 x 2 then 4 x 1 x 3, and then is, next you differentiate f of x, with respect to x 3. Then is a function of these 2 terms, it is a function of f 3. So, it is 4 x 1 x 2 plus 2 this is the cube, not square. See, recall the last x function, then it is a 3. So, 3 x 3 square, So, this ,so this function you have to once again, you have to differentiate with respect to x. That means, vector this gradient of this vector once again. We have to differentiate with respect to x.

So, if you differentiate this with respect to x, that is generated by this symbol f of x. So, this one we have, it is like this way, so del of x del x 1 square, del square of x del x 1 del x 2 and del square f of x del x 1 del x 3. Similarly, this is del square f del x 1 and del x 2, this will be del square of f x x this is del x 2 square del square of f x, then del x 2 del x 3 and last of this one is del square f of x is equal to del x 1 of 3 del x 3, then del square f of x is del x 2 of del x 3 then del square of f is equal to del x 3 square. So, this is a 3 by 3 matrix and this is a symmetric matrix, because a 1, 2 position a 1, 2 position is a 2,1 a one position a 3, 1 a two position is a 3, 2 these are all identical, these out diagonal positions of this one. So, one is to calculate, since we know the...

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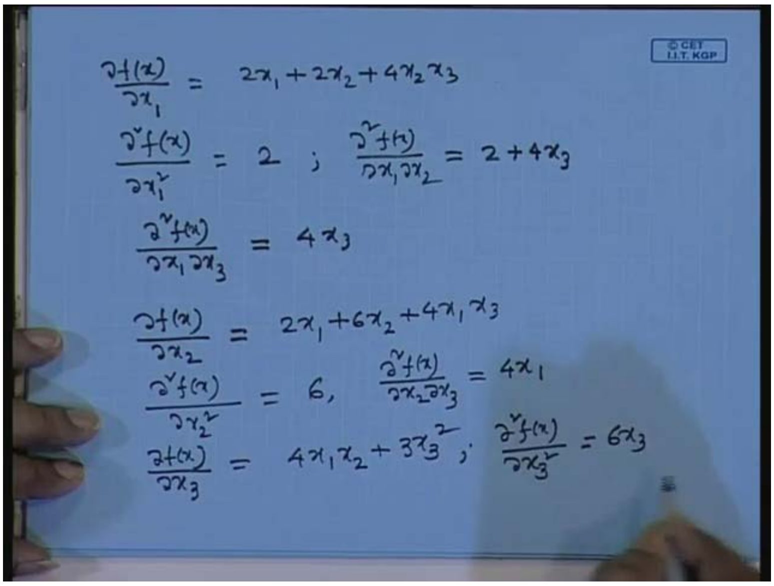

If you see this one, can calculate this one. We know, del f of x with respect to x 1 that value we know. This value is 2 x 1 square 2 x plus 2 x 2 plus 4 x 2 x 3. So, once again you differentiate with respect to x 1, that value will be coming 2 only. Similarly, you differentiate this, del x del x 1, already you have differentiate with differentiate with respect to x 2, then it will come 2, then what is this ? With respect to this is, 2 plus 4 it will come 4 x 3, then again you can differentiate del f x plus del x 1 del x 3, this one with respect to x 3 you differentiate, that will come to 4 x 2. Next is we know that, del of x in del x 2 expression, this is the expression. 2 x 1 plus 6 x 2 plus 4 x 1 and x 3.

So, you differentiate del f of x with respect to del x 2 square. So, differentiate with respect to x 2. So, that will come, if you 6. Then del f square of x del x 2 and del x 3. So, differentiate these 3 things with respect to x 3. So, that it will come 4 x 1. So, only left is differentiation of del f of x with respect to x 3. We know this expression is 4 x 1 into x 2 plus 3 x 3 square. Now, you differentiate once again this one, with respect to x 3. So, this will be del f of x del x 3 square will be equal to 6 x 3. So, put these values in the, that matrix, this one and find out this matrix value. So, you have already calculated these values, this all partial derivatives, secondary derivative, all these elements we have calculated earlier.

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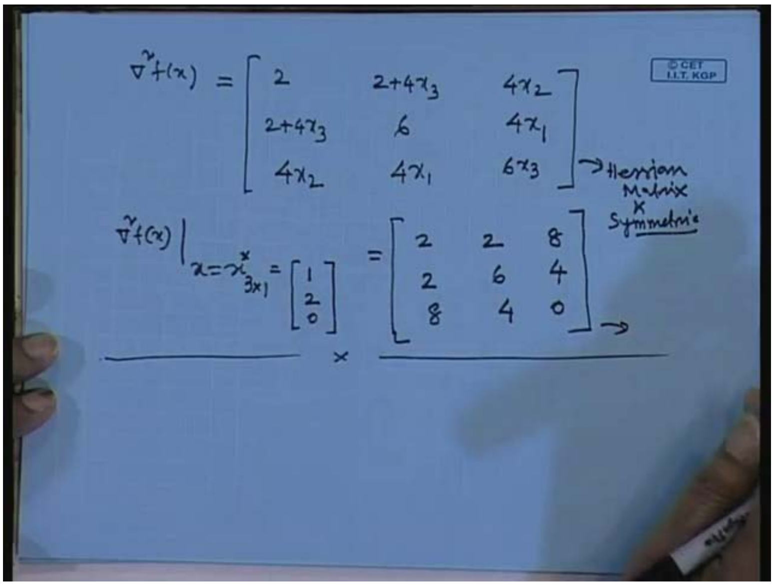

Now, if put you these values there, then our matrix which is called hessian matrix of del square of x is coming. Is 2 2 plus 4 x 3 plus 4 x 2 2 plus 4 x 3 plus 6 4 x 1, then 4 x 2 4 x 1 and 6 x 3. Now, you have to find out the hessian matrix at a point, that means we have to find out, the hessian matrix that matric point, x is equal to x star. That matrix dimension is 3 cross 1 and that is given its value is x 1 is 1 x 2 is 2. That is 0. 0 if you put in this expression, that is 2 as it is this, is x 3 is 0. So, this will be 2 then x 2 is 2. That will be 8 then x 3 will be zero. To 6 as it is. Then x 1 is your 4 this x 2 value is 2 8 that is x 1 is 1 4 then x 2 is 0 this one.

So, this our hessian matrix. This matrix is hessian matrix, that means if you differentiate gradient of a vector once again with respect to x, that matrix you will get a hessian matrix. Then that matrix for that particular example, when you use x, the value of at point x is equal to point 1 2 3 this is our iso-matrix. It is a symmetric matrix of this one. Symmetric you can write it.

So, next is before we go to this, our what is called, in order to find out the optimum value of this function. Which is a function. Function is a several values of x 1 x 2 dot dot x n. Let us call the function, f of x is a function, which is a function of x 1 x 2 dot dot x n. In order to find out optimal value of the function at a point, then we must have few more mathematically preliminaries.

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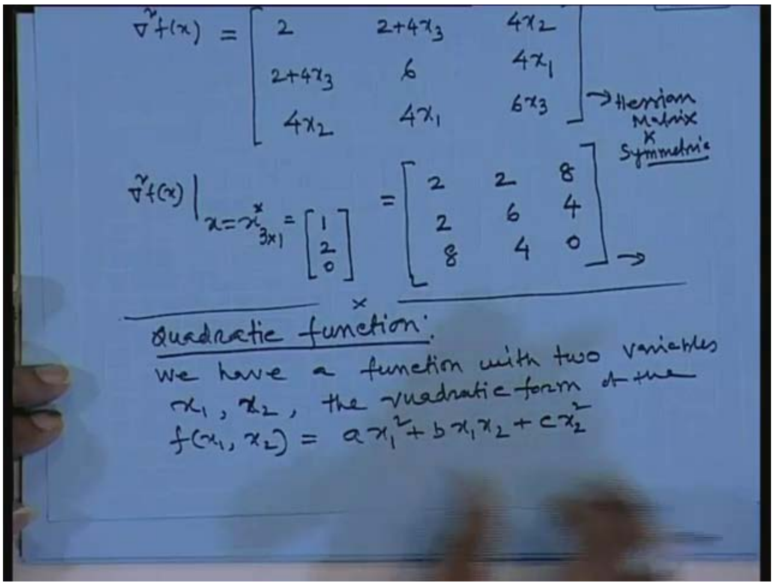

So, that is next is quadratic function. So, let us call that, we have a function with two. We have a function, we have a function with 2 variables x 1 and x 2. x 1 and x 2. The quadratic form, of the function is, of the function is a x 1 square b x 1 x 2 plus c x 2 square. It can be a, in addition to the constant term plus the linear terms. Let us call for the time being is this one. It is a quadratic form, quadratic function. Then this quadratic function, our problem is the, this is the quadratic form. So, this quadratic form, one can express into a matrix and vector form. So, let us call this is a equation number 1.

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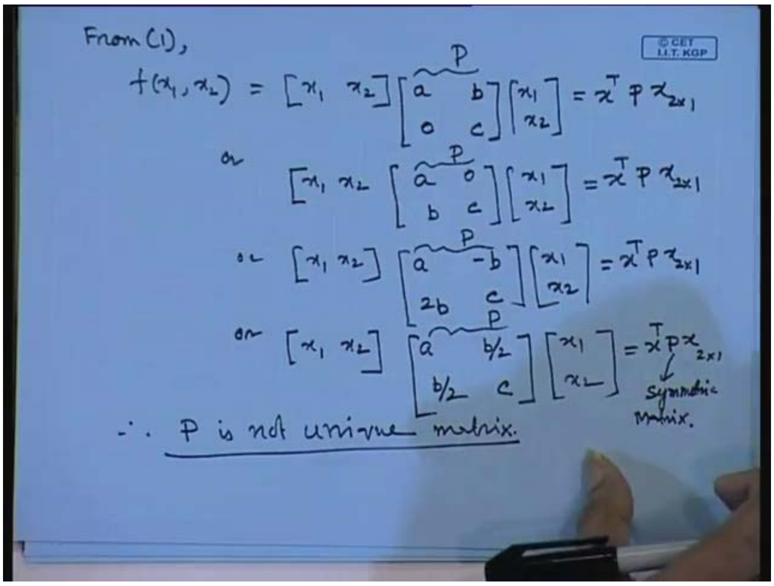

So, one can express the equation, one can express the equation number equation 1, as like this way f of x equivalently one can express like this way. This equal to x 1, x 2. We have considered for the two variable case, than we extend for n variable case. This we can write it like this way. You see, this what is a coefficient of x 1 square is a. That we will write in a 1 1 position of the matrix which are going to form.

So that is a then a 1 2 position is will be your our this matrix. a 1 2 position means, 1 2 position coefficient is b, that we can write it here. Then 0, then what is a coefficient of x 2 square? That we will write it, a 2 2 position. That means c into x 1 and x 2. So, if you just do the operation, matrix vector operation, then whatever the results we will get with the row operation with this one. We will get exactly what are the function is given. This is one choice or one can write it like this way. x 1 x 2 then a 0 b c that a 1 2 position or a 2 position I have just written it here, this one. This is x 1 and x 2 and also this if you do the operation you will get exactly what are the function is getting.

Now, one can write also like this way, there are infinite number of choices to put, that matrix, this matrix. So, I can write it here, because x 1 square coefficient is a. So, that will remain in a 1 1 position, x 2 coefficient x 2 square coefficient is c, that will remain a 2 2 position and this is x 1, x 1 x 2 coefficient and this is also one can write it x 2 1 position. That this x 1 coefficient x 1 and x 2 coefficient can be distributed with this ob diagonal things. Their sum must be equal to the coefficient of x 1and x 2.

So, I can write it also like this way, there is no problem if I write it, minus b, 2 b and c is like this way. So, you see if I write this expression, it will a 1 a x 1 square then cx 2 square, then I will write it here. Minus b x 1 x 2 again there is a 2 b x 1 x 2. So, if you simplify, this will come only b x 1 x 2. So, this is one the most of the cases it is selected the matrix b is or if one can write it in symmetric matrix form x 1 x 2. Now see how I am writing in symmetric matrix form. This one coefficient of x 1and x 2 is b, the half of this b by 2 I will write in 1 2 position and b by 2 I will write it in to 1 position.

So, it will be a x 1 square a x 1 square coefficient is a x 2 square coefficient is c. Then coefficient of x 1 is b. b I have divided them into two parts, to make the matrix symmetric. So, it will be a b by 2 and b by 2. So, you can easily realise that we have a infinite number of choices but, this choice has a. I mean most popular choice of this one, because we can conclude something from, if the matrix is symmetric matrix regarding the function. So, our conclusion is p is not, if you consider this matrix is P or this is also P this matrix this matrix is also P. This matrix is P.

P is not unique matrix to represent the function quadratic function into this form. What form, what form this I can write it, if you consider x 1 x 2 is a vector form, you see here I can write x transpose. This is vector, so it is row vector. So x transpose, I can write it that p into x, this is 2 cross 1 for this alone.

Similarly, I can write x transpose, but here P is different. This form here also, I can write x transpose. P x of t but, P is different but, all these cases it is same as equivalently same as function of x 1 and x 2. And in this case, P is symmetric matrix. In this case P is symmetric matrix is not unique. So, let us see that in general, we have seen that out of the four, this is out of the four P matrixes, which will give you exactly the same value of expression of at f of x 1 and x 2. Out of this one is considered the symmetric matrix choice.

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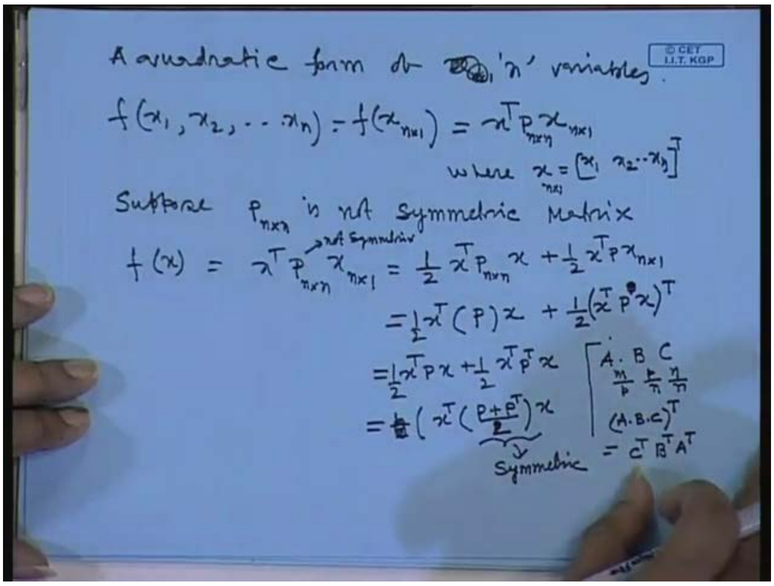

So, in general for n variables, function is a n variables f is a function of x 1 x 2 dot dot and x n. It is quadratic form of n variables n cross 1. Which is a n quadratic form bracket n variables, what we can write it f is a function of x 1 x 2 dot dot x n. Which in vector form is x of n, whose dimension is n plus 1 is I can write it this is equal to x transpose P x. Where x is equal to x 1 x 2 dot dot x n. Whole transpose means, it is a coulomb vector of dimension n cross n. x is a dimension n cross n.

Suppose, the P is not symmetric. Suppose P is not symmetric, suppose P which is n cross n, this is n cross 1, this dimension will be n cross n. Suppose P is not symmetric matrix, one can always suppose of this one, f of x. Just now, we have written x transpose P n cross n, into x whose dimension n. P is not symmetric. Look this expression this is a scalar quantity, the function is a scalar quantity this. So, I can always write, there is no harm, I can always write that x transpose p cross this x and half x transpose P x, because suppose this constant is 10. I can put it here 5 here 5. So, it is a 10 ultimately, it is correct

So, this quantity is scalar, for this further we can make it after some manipulation of this one. I can write it P into x plus, since this is a scalar quantity, again then I can take the transpose of that one. So, this x transpose, P x the transpose here. Since I have considered it is not a symmetric matrix, that will be at P transpose. Note how I have written this one, if A is a matrix of dimension m into p, B is a matrix of dimension proper dimension of p cross let us call n. C is a matrix with dimension n cross r.

So, we can multiply A B C, this you see, they are with proper dimension. If this is this, then this product if I result, resultant of this matrixes product matrixes after multiplication, if you take transpose. This is, we can write C transpose reverse order. We can write C transpose B transpose A transpose and this property I have used it here, to take the transpose of that one. Taking the transpose of that one or you can say just you can forget about this, I am now taking the transpose of that one.

So, this equal to, now I can write it x transpose P x plus half. This half is there, half then x transpose, you can say this is A, this is B, this is C. C transpose x transpose then B transpose P, transpose that. This transpose of a transpose, that means x. So, if you make this simplification of this one, after you take common, then you can write x transpose common. Then you write P of P transpose by 2. Sorry, 2 then you push it inside, then x so, this matrix is A P is A that is our non-symmetric matrix. Take transpose of this one and divided by 2. And this matrix we know, if A is a matrix, non-symmetric matrix, if add with a transpose, result is a symmetric matrix.

We are dividing by scalar, by quantity 2. So all elements of P plus P transpose will be divided by 2 only. So, this is a symmetric matrix. You see, this we have derived from this expression. So, x transpose P x if P is not symmetric, is not symmetric, we can write it this one x transpose P plus P transpose by 2 into x, where this matrix is symmetric. Let us, this matrix is symmetric. So, whatever the results we get it, the expression in terms of what is called f of function f, which is a function of x 1 x 2 is the same expression.

We will get it here also, that may x transpose P plus P transpose by 2 into x same. But, only the advantages we are getting, this matrix is now becoming new matrix, which is a symmetric matrix, but scalar this function value, scalar value expression in terms of x 1will be same expression as this one. So this is our conclusion of this one that, even if p is not symmetric I can make the x transpose P plus P transpose by 2 into x, is an expression whose values are same as this values in terms of x.

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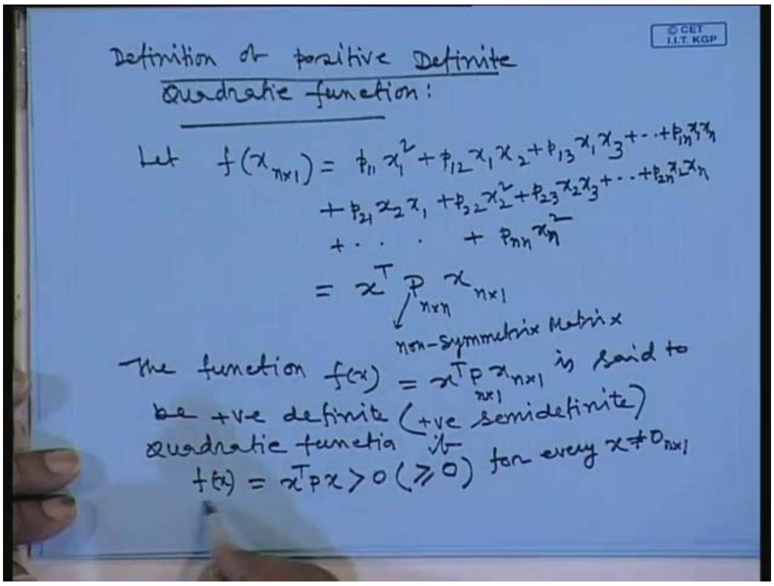

So, let us call now, we define some of the function, positive definite function. Definition of, definition of positive definite quadratic function. So, let us let us, we have a nth order quadratic function is there f of x is a function of n variables. Which is in general way I am writing, p 1 1 x1 square p 1 2 x 1 x 2 p 1 2 x 1 x 2 then p 1 3 x 1 x 3 and dot dot. We have n variables are there p 1 n x 1 x n plus p 2 1. Let us call this is x 2 x 1 p 2 2 x 2 square p 2 3 x 2 x 3 plus dot dot p 2 n x 2 x n. And in this, in this way you can consider the all these things. The term of this one will be p n n x 3 x n x n square, which we can write it into matrix and vector form.

This is matrix and vector. So, this dimension n cross n and this dimension is n cross 1, where P is, see clearly p 1 2 p 2 1 is not a symmetric matrix. We will get these two things are not same a 1 p 1 3 and p 3 1, third the equation will come is not a same quantity. So, you will get a non-symmetric matrix, non-symmetric matrix. So, our definition of positive definite matrix is that, first the function f of x, which is equal to x transpose P x n cross 1, this is said to be positive definite positive, definite, bracket, when you read the bracket, you will read all the, all in bracket terms or will kill positive semi-definite, definite quadratic function.

The function will be said to be a, this function f of x means, this function here, which you have written into matrix form. It is in vector form, is said to be positive definite quadratic function, if f of x is equal to x transpose P x is greater than zero or bracket term will said to be positive semi-definite when, it is greater than equal to zero, for every x and x is not equal to null vector. When x is equal to null vector, x 1 is zero x 2 is zero, this function will be zero.

So, this will be a, so I repeat once again, the function is said to be f of x, is said to be positive definite function if f of x, x transpose x that may f of x function value will be greater than zero for all values of x, except x is equal to, if it is not null vector or a zero. If it is a semi-definite, the function value maybe zero or greater than zero, for every value of x, which is not null vector. If it is x is null vector, null vector. So this is the definition of this one, then second definition of this one, that function is negative definite.

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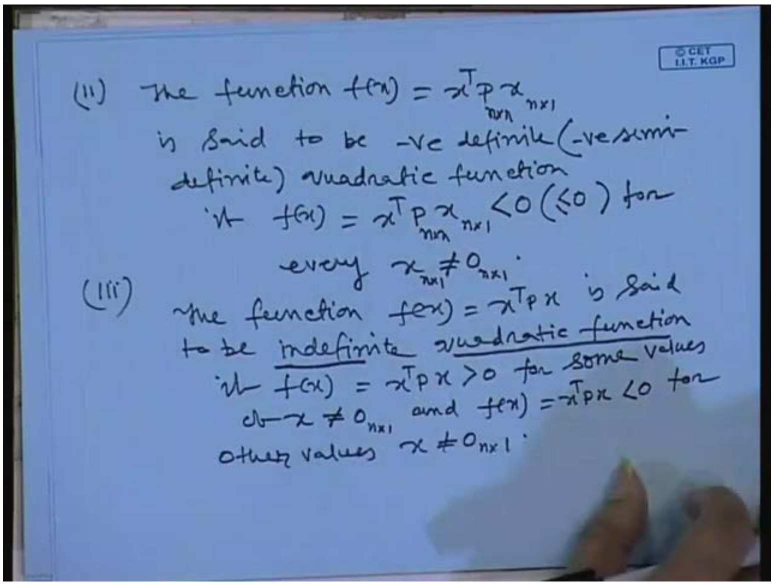

Similarly, the function f of x is equal to x transpose P x, which is a, this is said to be negative definite or negative semi-definite, definite quadratic function, if f of x is, f of x is a function of x 1 x 2, which can be expressed in terms of matrix and vector form, x transpose P x. This value will be, if it is negative definite, this value function value which is a scalar is always less than zero. If it is a semi-definite, this function value will be less than equal to zero, if function value maybe, negative or zero.

So, this is a negative of for every x whose dimension n cross 1, that may n cross 1 not equal to zero. Now, this function may be indefinite, that means a function is said to be indefinite, when this x function value can be greater than zero or less than zero or maybe zero. So, we cannot say anything, that for all values of x which is not equal to zero, the function value is either positive or negative or zero. It can be anything.

So, that is called, what is called function is indefinite, the function f of x which is equal to x transpose P x is said to be indefinite quadratic function, if f of x is x transpose P x is greater than zero, for some value, some values of x, which is not equal to zero. And f of x which is a, x transpose P x is this value is negative for other values of x, for other values of x, which not equal to zero. So, this is called indefinite quadratic functions. So now, so we can see next.

So you now, we know the, what is the positive definite quadratic function, negative definite quadratic function, positive definite, positive semi-definite quadratic function and negative semi-definite quadratic function. Along this things, in the same line, what is called definition of positive definite matrixes it is just linked with this only. Definition of positive definite matrix. So, let us call for a P which is a n cross n, matrix is positive definite matrix, if and only if, you multiply it by P matrix whether P is a symmetric matrix or non-symmetric matrix, whether it is a positive definite matrix, you multiply it by P matrix, with proper dimension. Pre multiplied by a row vector, post multiplied by a column vector of same.

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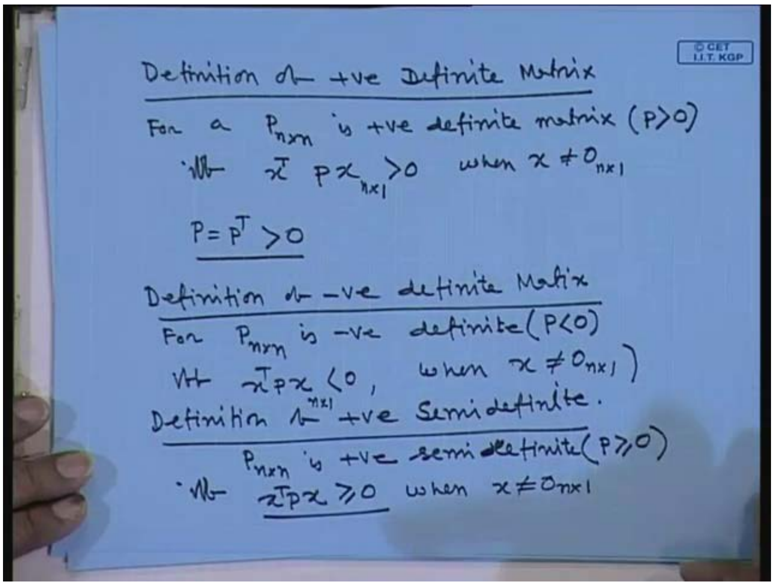

So, it is a column vector, post multiplied column vector and pre multiplied by same vector, which is row vector. So this, if this quantity is greater than zero, when x is not equal to this, then we will call the matrix is positive definite. But see, this is a, this has become a quadratic function. Now x is this, so what is this means, if P is positive definite matrix pre multiplied by x transpose, any vector x and post multiplied by x, if this quantity is greater than zero for all values of x, except x is equal to null vector. Then, we will call matrix is symmetric matrix.

So, next is positive definite, you can say this, if it P is equal to P transpose, that means P is a symmetric matrix. Then we will call that, this P is a symmetric matrix. That symmetric matrix is, if it is greater than zero and one thing is there, if P positive definite in and in short it is written like this way. P greater than 0 means P is a symmetric matrix, P positive definite in short is written, P greater than 0 means P is positive definite matrix.

So, if it is a P, P transpose greater than 0, it indicates that, P is a symmetric matrix but, this does not indicate P is a symmetric matrix. P maybe non-symmetric matrix also. Next, is the definition of positive semi-definite matrix or positive semi-definite or negative definite matrix or you can say it is a negative definite matrix. ((Refer Time: 39:20)) matrix definition is same the for a function P, whose dimension is n cross n, is negative definite matrix.

That means, in short it is written P less than zero is like I will read that is P is a negative definite matrix, if and only if that x transpose P of x is less than zero, when x is not equal to null vector. That this is not a null vector. So, again this is a quadratic function and may be a symmetric matrix, may not be symmetric matrix but, the fun that P is, if it is a non-symmetric matrix or symmetric matrix. It is the negative definite if this x transpose P x which is a quadratic form in terms of variables x, x means this, add as a n components of their x 1 x 2 dot dot x n, if this value will be less than 0, for all values of x except this one.

So, next definition is the definition of positive semi-definite, again if P is positive semi-definite, positive semi-definite, semi-definite, in mathematically it is written positive definite, P is greater than 0. I will read it P is positive semi-definite if and only if x transpose P x, that is a scalar value, this value will be greater than equal to 0, when x not equal to a null vector. So, this is the definition of that one. Again is there, we can also derive the definition of negative semi-definite matrix.

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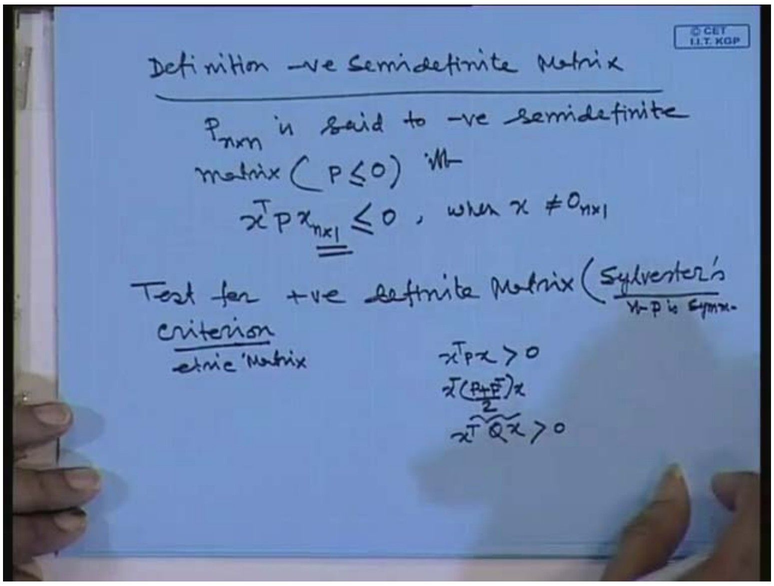

Definition of negative semi-definite matrix, negative semi-definite matrix. So, the matrix P n cross n is said to be negative semi-definite matrix, in short it is written like this way, will read P is semi-definite matrix, negative semi definite matrix. Sorry if and only if the quadratic form, that means that matrix you pre multiply and post multiplied. Pre multiply x transpose and post multiplied by x. x is a vector of dimension n cross 1, if these values, these values ((Refer time: 42:44)) is either negative or equal to zero.

This for when x is not equal to n null vector. x dimension is n cross n. So, this is that. Now question is coming that, there are infinite number of vectors are there exist, then how will check for these things, whether the matrix P is a positive definite matrix, which in turn that x transpose P x is positive definite quadratic function or not.

So, that is infinite number. The test for, now the test for positive definite matrix. This is done for Sylvester criteria. Sylvester criteria, criterion and this is valid, Sylvester criteria you can apply only if matrix P is symmetric matrix. This is for, if P is symmetric matrix, then only you can, symmetric matrix. So a matrix is positive definite or not that test is one can do by using Sylvester criteria, can do within similar provided, this matrix a symmetric matrix. According to the dimensions, according to the definition of symmetric or a positive definite matrix, we know that x transpose P, x transpose P must be x transpose P positive definite matrix this must be greater than zero.

Now, if P is, given P is not what is called symmetric, so we can always express that one I just shown you we can all always write it this one x transpose P by 2, x transpose P plus P transpose by 2 x. Whatever the value, we will get it this one. This value are exactly same, so in other words that if P is not symmetric matrix, we will convert into this form P plus P transpose by 2 and then test with this matrix.

Which matrix is symmetric that P plus P transpose by 2 I am denoting it by Q. So I am testing with this matrix. Q Matrix if Q is symmetric, in turn I can say this function if Q is symmetric, x transpose P greater than 0, will be here by using the Sylvester criteria test. I have to find out whether Q is positive definite or not, if positive definite then, this will get. So, how to check positive definite matrix? That Q. So, form now onwards I will consider our P is a symmetric matrix.

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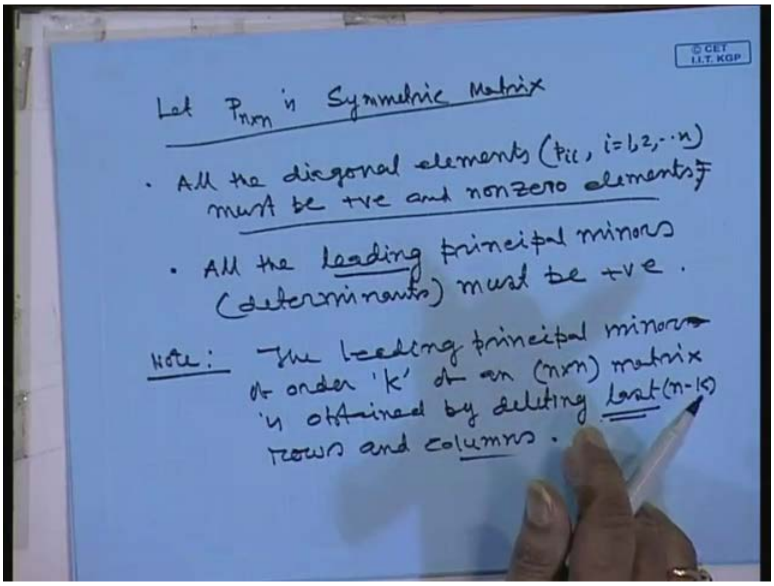

So, Sylvester test for ((Refer Time: 46:00)). Let P n cross n is symmetric matrix, even it is not symmetric matrix, I will convert into a P plus P transpose by 2 that one, I will consider as a P which is symmetric matrix, symmetric matrix. Then, Sylvester theorem tells, even if it is a symmetric matrix P, that P is positive definite provided, first check is all the diagonal elements of P must be positive. All the, all the diagonal elements that means, p ii i is equal to 1 2 dot dot n, must be positive and non-zero, non-zero elements.

This is the first, if this is there all diagonals of p is like, whatever the p matrix is given, convert into symmetric matrix like this way P plus P transpose by 2 and then check all the diagonal matrix are positive and non-zero. If it is so, further you post it like this way or all the leading, all the leading, leading principle minus, means determinate must be positive.

So, let us note what do you mean by the leading principal minors. The leading principal minors, the leading principal, leading principal min or, of order K of an, n cross n matrix is obtained by deleting, by deleting last, mind it last n minus K. K is the order n minus k rows and columns, last n minus rows and columns. So, this is the leading principal minors. So, let us example, take an example and check how this test can be done? Whether a matrix is positive definite matrix or not?

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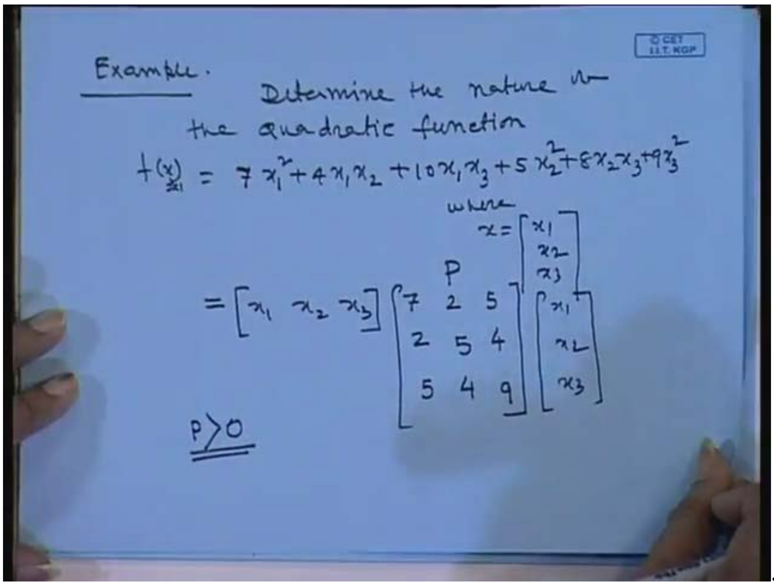

So, if you consider this one example this, determine the nature of the quadratic function, the nature of the quadratic function means, whether the function is positive definite, negative definite or positive semi-definite or negative semi-definite, this nature. So, that function is given once again. I can write into this general form, which is a function of your 3 variables x. So I am writing the dimension x is dimension ((Refer Time: 49:57)) So, seven x 1 square plus 4 x 1 x 2 plus 10 x 1 x 3 plus 5 x 2 square plus 8 x 2 x 3 plus 9 x 3square. Where we have consider x is equal to vector, consist of x1 x2 and x3. 7 x 1 square 10 x 1 x 3 5 x 2 plus 8 x 2 x 3 and 9.

So, this thing I can easily convert into a matrix and vector form, that you use. So x 1 x 2, these are there, then this P matrix, this is the our, this you have to fill up form this information. I told you that x 1 square is 7. So, it will go in 1 1 position, first diagonal elements I will. x 2 square is 5, so it will come here, x 2 2 position. x 3 square coefficient is 9, so it will come 3 3 position. Now you see, x 1 x 2 product of x 1 x 2. So, you can put it here 4, here 0. But, problem is I want this is a symmetric matrix, so that I can test the Sylvester inequality condition whether P is, this P matrix if it is symmetric, this value will be greater than zero, provided P is only positive definite.

So, this I will equally distribute between the a 1 2 position and 2 1 position. So, 4, 2 2 equal, so the, so that it will become symmetric. Then 10, what is that 1 x 1 and x 3, when

1 3 position 5 is here and 5 I am giving here. 1 3, 3 1 position. Then your 2 3 position 2 3 position is 8, this is 4 and this is 3 2 position 4.

So, this is our P matrix and this P matrix is a symmetric matrix. So, if we can show if P is a symmetric matrix, if you can show P is a positive definite matrix, means P greater than 0. If you can show, positive definite, then this function value is always greater than 0 for all values of x, except x is not equal to null vector. So, let us say by using Sylvester inequality condition, Sylvester criteria, then what we have to find out?

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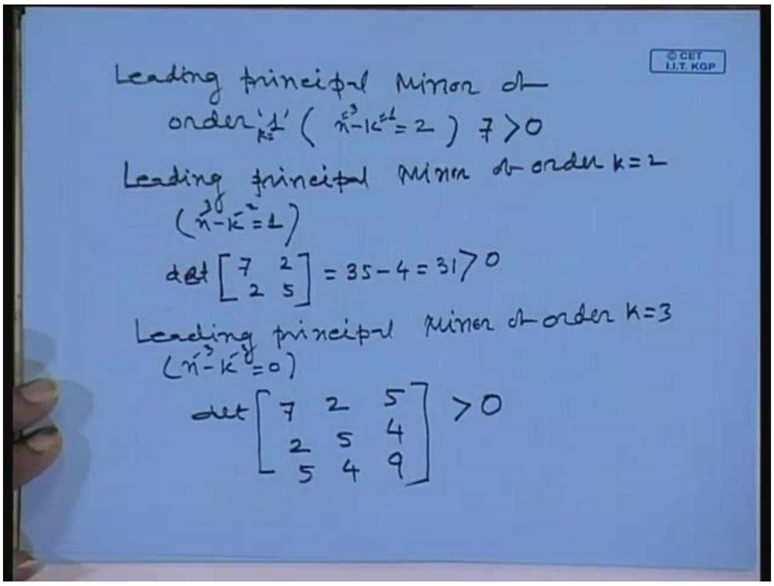

First we have to find out the leading principal minor of order one. Leading principal minor of order one, then what is this? Then our, how many rows and columns we have to delete from the last row you see. I have written n minus K, n is equal to [vocalised-noise] and order is K. Order is one, so I have to delete 2 rows 2 columns from the last rows. 2 rows and 2 columns, so only these elements is left. So, that is 7 order is 7 and that is greater than 0. Next is leading principal minor of order 2, K is equal to here, K is equal to you can write K is equal to 1.

So, this is what? That n minus K, n is equal to 3, k is equal to 2. This is one last row and last column you delete from the matrix P. Last row, last column. So, you have a only this matrix, this matrix is if you see, this matrix, this will be a 7 last row, last column. If you delete it then your matrix is determinate of that one. Determinate of that 7 2 determinate of this matrix 7 2 2 5. And that determinate if you see, this is 35 minus 4 is equal to 31 which is greater than 0.

Last, because if we here, order is an, I have to consider up to n th order minus, leading principal minor of order K is equal to 3, that means it indicates n minus K. n is equal to 3 k is equal to 3, this is no rows, no columns you have to delete, that means you have to take the full matrix. What is P, is given. So, determinate of that, determinate of that matrix, you have to find out this one. That matrix is 7, see this one 7 2 5 2 5 4 5 4 9. So, this determinate, you have to find out. So, that value, if you find out this determinate you will get this determinate value is. You know the, how to find out the determinate value is greater than zero.

So, according Sylvester theorem, if you see this one, he is telling first what is matrix P is if it is a symmetric matrix only. Our case we have formulated this thing, in the symmetric matrix checking this one, that all the diagonal elements are positive or not. These all are positive, then proceed for the all leading minors must be greater than zero. So, all leading minors is order one, we got 7 order greater than 0. Order 2 is we got greater than 0. Order 3 is also greater than 0. That means what? You can say that P is a only positive definite matrix.

Once P is positive definite matrix, we by definition, we know x transpose P, x transpose P, x is always greater than 0. This transpose P is always greater than 0. So, this function quadratic function is a positive definite matrix. Similarly, you can go for what is called positive semi-definite. So, next class will discuss positive semi definite and negative definite, negative semi-definite, how to test using the Sylvester criteria, Sylvester criteria.

Thank you.