Optimal Control

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Lecture - 25
Solution of Programming Problem Using Exterior Penalty Function Method

Last class we have discussed that how to solve the linear programming problem, and quadratic programming problem by using exterior, sorry interior point method. And we have taken a 1 example to illustrate this technique that we could not complete, this problem.

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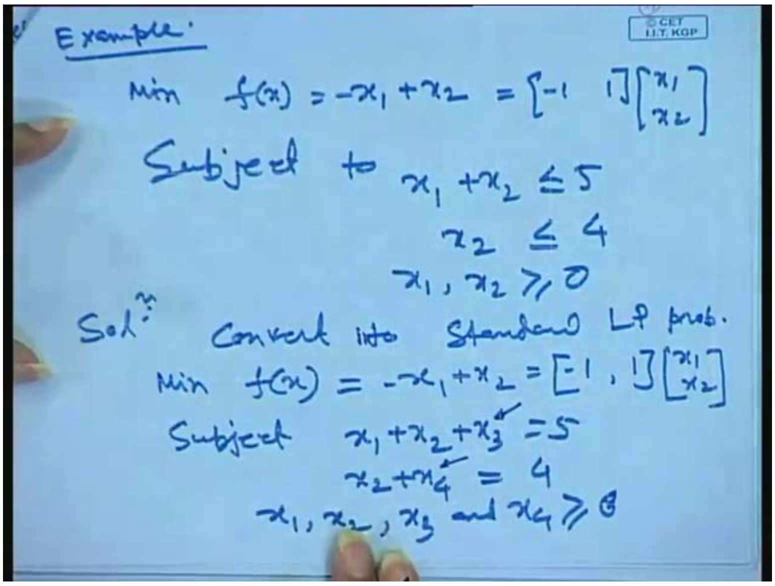

So, we recap this problem once again so we have given this is the function we have given which is linear function subject to some linear constants. This is the constant this is the constant and eligible has x 1 and x 2 are greater than to equal to 0. So, if you want to solve this problem by using the interior point method, first step is convert this linear L P problem into a standard L P problem what is the standard L P problem is there.

So, inequality sign what is there you convert into a equality sign by introducing, the new variable which is x 3, which is artificial variable and x 4 another artificial variable, in this second equation. So, our problem is now you solve this linear equation agree, that linear equation subject to equality constraints and the right hand side of this equality constraints are all we have to make it all positive. And the all variables x 1 and x 2 and the artificial variable x 3 and x 4 are greater than or equal to 0. So, first type is convert to a standard L P problem, next step we know what is the how the way we have defined the f x in the symbol c transpose x and equality sign this constraint with A into x.

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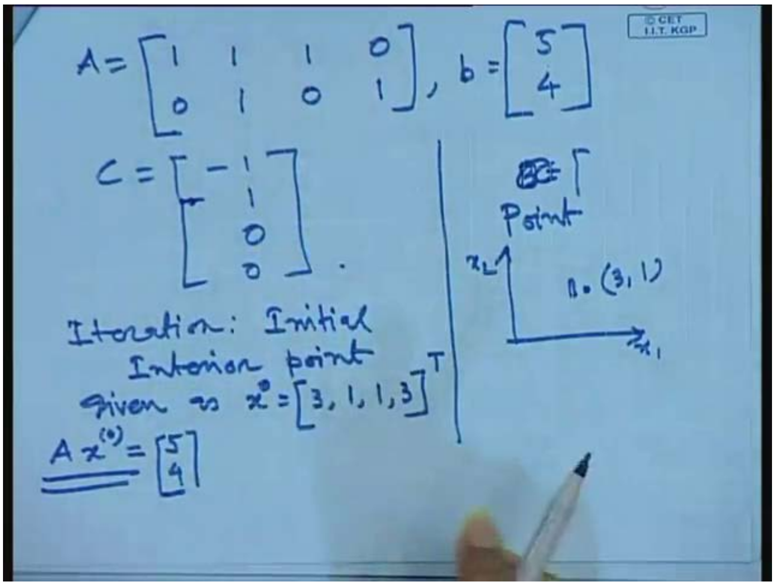

So, according to x this problem our A is this 1 b is the right hand side of the equality constraint, right hand side 5 4, then c is the associated to coefficient associate with the objective function. So, our problem is now to solve using a interior point method, we take a initial guess of let us call 3 coma 1 x 1 is 3 x 2 is 2, but that is not the, if you see we have a additional two points are there x 3 and x 4. And accordingly if you take x 0 is the initial condition according to this 3 x 1 3 x 2 to 1 and x 3, I have considered 1, x 4 is 4 this is the initial condition. We have to choose the initial condition in such a way so that that point is interior region or in the feasible region of this problem, so how to check whether it is in the feasible region or not.

So, we have check the A into x margin equal to b if this condition is satisfy then this initial case is inside the feasible region, or interior point of the region. So, A is this one multiplied by this vector you will get 5 and 4, and these employees that this is inside the region. That means, the feasible refine, so once you take this initial guess initial point interior, initial interior point then you see this point is not equal distance from the x. So, what you have to do we have to change into a new variable by scaling the variables. So, new variables changes are made by using that one.

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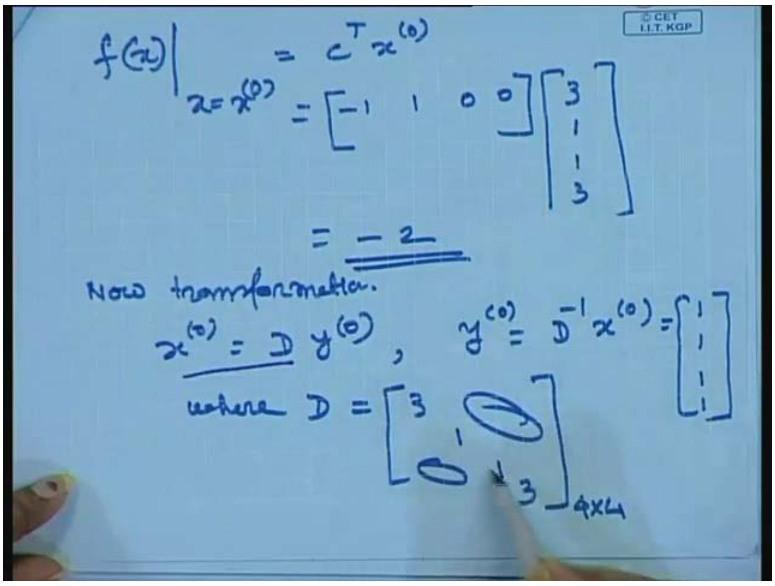

x 0 x superscript 0 means initial condition y superscript of 0, new variables and we have a new variables are 4 also and D is the skilling effects of this 1, or you can said that y 0 is the D inverse from this 1 y 0 is D inverse x 0. You scale the initial interior point in such a way new coordinate variables on new decision variables that this y 1, y 2, y 3, y 4 are equidistance from the access. So, this is the choice of scaling this and the D choice with the whatever the initial point, initial interior point and this D will be with that element in the diagonal form.

And if you multiply d inverse at 0 you will see this will be one, this indicate x y 1 of 0, y 2 of 0, y 3 of 0, y 4 of 0 all are equidistance from the axis, that new variables. So, if you look at this our objective function with the interior point in your original problem interior point, then this value is minus 2 at this movement so our job is to now in transform coordinate axis to move. And you also find that in transform coordinate axis is, what is the objective function value?

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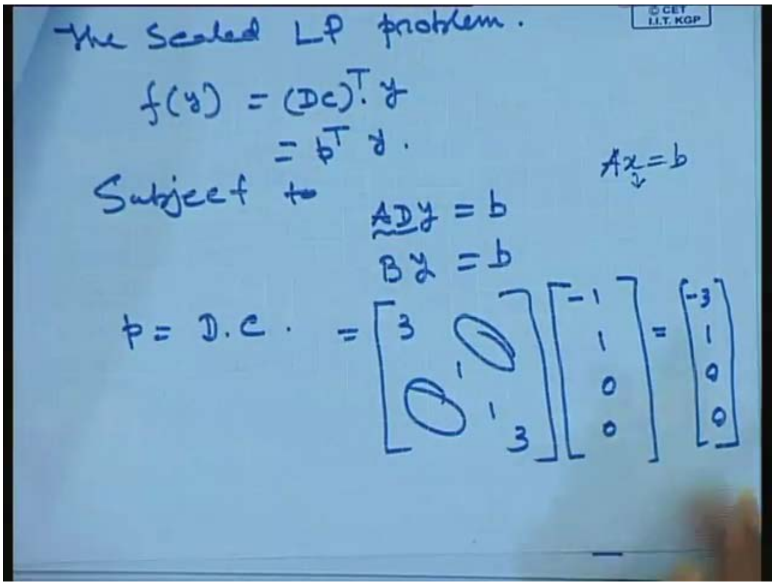

And if you scaled, that means scaled L P problem f(y) is nothing but a c transpose of x and x is what D into y. So, this nothing but a p transfers y and p is Dc and subject to conditions our original problem is A into x is equal to b. We know x has a relation with the new variables with D into y. So, if we put x variable D into y that will be coming D into y, A into d is equal to b we considered that product. So, if you consider that in scaled L P problem, optimal problem not optimum the value of the function at that initial interior point, new initial interior point that value of the function will be equal to once again it is a 2.

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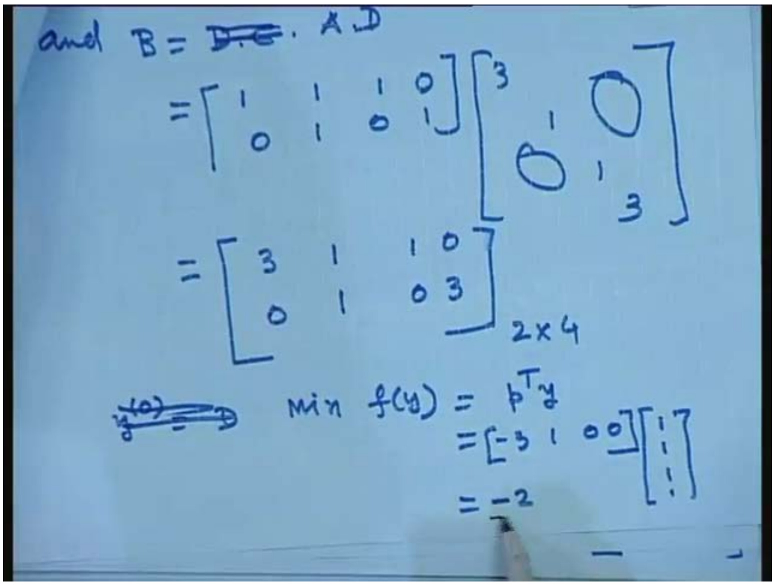

If you see this one that is 2, p transpose y is 2, we have seen A D is what is nothing but b I can easily compare b because I know A I know D. So, I can easily compute b, so this is our b, now once you will know b. That means, I am now in new decision variables that means y from that decision variables, when an interior point and y is y 0 of the super script 0 is the initial interior point in what is the new decision variable coordinate x-x.

So, if you now our job is I have to move in such a direction, so that function value is decreased, so that is our main goal to this one. And that point what is y of 0 must be interior point of this problem. That means, must be in the feasible region of the new decision variables. So, we know what is the direction of this, so that we have derived.

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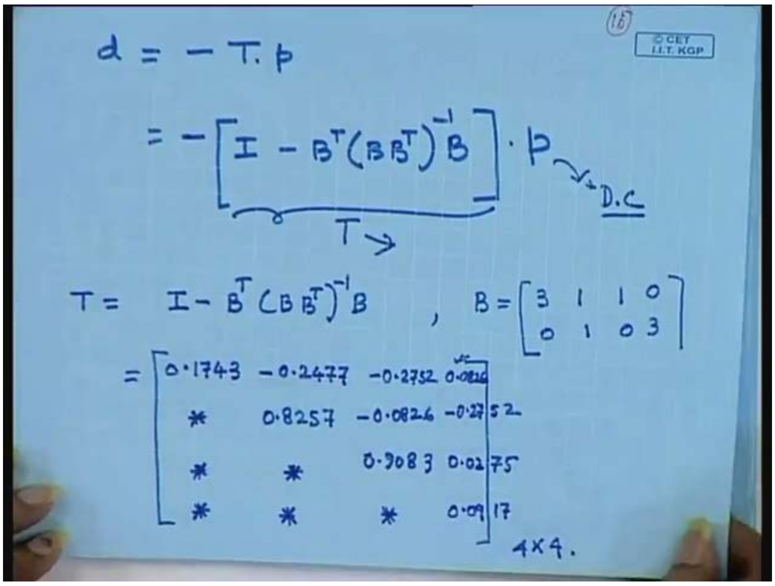

The direction of the vector is nothing but a minus T into P and T is the this matrix is T and this T matrix is a projection matrix, that we have discussed earlier multiply it by P. And P you know, this what is P its nothing but a D c transpose is P transpose. So, P is nothing but D into c. So, I can calculate P is nothing but D into c, so once we know the B matrix, B is what A into D that just now we have told it. I know B, I know P, I know this is an identity matrix of our dimension, I can compute the directions which direction to move it. This direction, which direction I have to, and this direction if you move it, that will give you the what is called dissent direction? That means the function value the new coordinate x is the new decision variables after scaling the L P problem, the function value will decrease.

In other words the original the value problem, then if you go back to the original problem than the original optimization problem, that function value will also decrease from its initial value. Now, we can complete this one if you see this one we can compare that one, that is our this quantity by using the numerical figures of B and P. So, first we have to compute P, so T is equal to i minus B transpose B into B transpose whole inverse B this is our T. So, this so this T value, we know B, B is what just now we have computed 3 1 1 0 then 0 1 0 3 support the value of B here matrix B here then you will get I am not computing in details and one can see this one because i is a symmetric matrix minus another symmetric matrix result, results is the symmetric matrix.

So, see this one B B T is also a symmetric matrix and inverse of this one is a symmetric matrix then I multiplied by both sides B transpose B that is also symmetric results is coming this is a symmetric matrix. So, I am writing the results after putting the value of B in this expression and I am writing the values there 1 7 4 3 minus 0.477 minus 0.2752 then 0.826 then this since this matrix is symmetric this is same as that one. So, star is the corresponding elements of the symmetric matrix is same.

So, next is your 8.257 minus 0.826 then your minus 0.272752, now 1331 elements are same. Since, it is a symmetric matrix that this elements comes here so I am not writing just star means, the corresponding symmetric elements are same. So, next is 3 2 you see 3 2 position is same as 2 3 position. That means this numerical value will come here, so it is this one, then 3 3 position and your 3 3 position is 0.9083 then this values is 0.0275.

So, now 4th row is one-forth this element is same as 4 1 because it is a symmetric matrix star, then 2 4, 4 2 elements are same as 2 4. That means these elements is here this when 4th element is same as 3 4 elements, this element is same as this element. Now, 4 4 elements is 0.0917 so this matrix dimension is 4 by 4. So, our projection matrix T is known to use. So, once it is known to us I can directly find out the direction, which direction the y, coordinate axis I have to move. So, that the function value will reduce from the earlier value from the function value, if it move in that direction. So, at this moment we know T, so once we know T, we can immediately find out the what is our deduction d.

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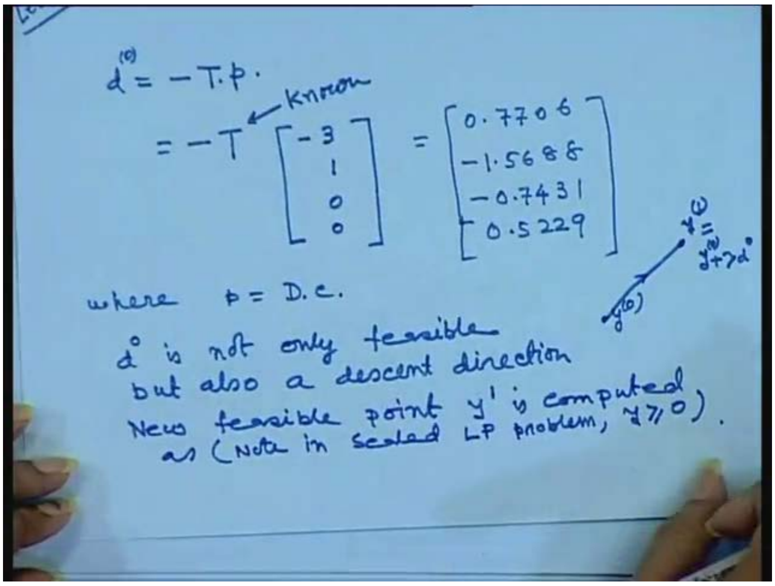

d is equal to minus T into P, so if you calculate the direction of this one P you will know. So, we are now at what is called initial interior point in transform axis. So, initial point is this and from there which direction to move that is minus T into P. And not only that, this is a feasible direction it also gives the descent direction and that means function value will also decrease both feasible and function value also decrease in this direction, if your move. So, this values is T value is known, this is know, P value is just we have calculated the P value is this 1 0 0, P is equal d into c if you see this or P is equal to is nothing but a D into c these value I am writing.

So, this if you calculate that one if you calculate this one it will come 0.7706 minus 1.5688 minus 1.7431 then 0.5229. So, we know the descent direction. So, if you see the T you got it, and P you have calculated, how will you calculate P? Where P is once I am calculating P is equal to D into c that we have calculated. And D you already know that what is the D value we have considered, see this D value what we have considered, that this is the D valued value is that one that 3113, 4 by 4 matrix D into c is the P values, that you have already calculated.

So, I know the descent direction then what we have to do next our that if we this our we are now y 0, initial interior point. So, we have to move in D direction and what up to what? That means our y of super script 1 is equal to y of 0 plus lambda into d 0 superscript d 0. So, in this direction you have to move it, and which in turn the function value will decrease and this point also in the feasible region, that we are procured earlier.

So, let us see that one how to decide the lambda max in the direction of this is the direction lambda max, if you move this one in this direction this direction, we have to move it. So, now I have to find out d 0 is the not only feasible, but also a descent direction. Means, this if you move in the d 0, the point y of 0 it is an interior point and as well as the function value also decrease. For that one we have to choose lambda in such a way so that y of 1 will be in the inside the interior point, so that operation and these things we are now doing and which is we already explained earlier. So, now you see new feasible point y of 1 is computed as note in scaled L P problem, where y is greater than or equal to 0. Now, we will see how u is computed.

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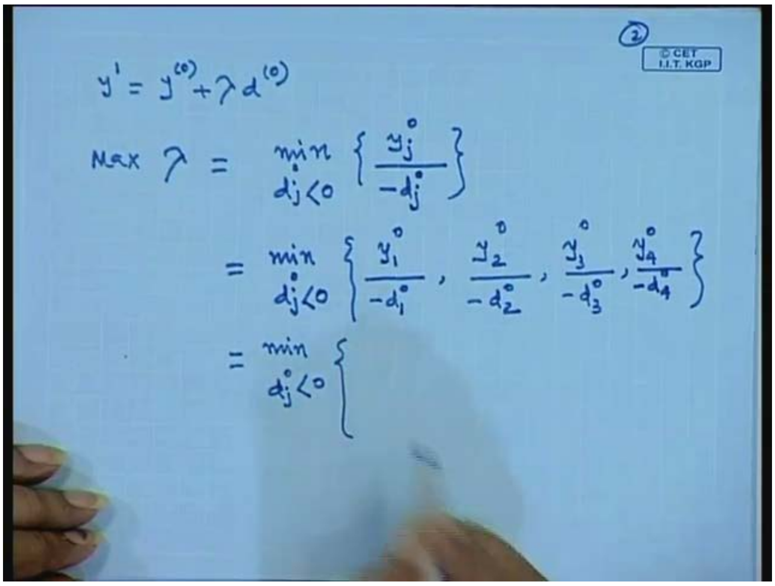

So, if you see this y of 1is equal to y of 0 plus lambda then d of 0. So, this lambda max, max of lambda is computed minimum of dj less than 0, dj means elements of d, mean show many elements of d is there if it is n elements are there d 1, d 2, dot dot dn. The elements which are less than 0 that you compute like this way lambda max y j super script 0 minus d j super script 0. So, for this problem you see what we are getting it here min d j superscript d j 0, d j super script 0 you can write it, then how many y 1, y 1 of superscript of 0 minus d 1 of superscript 0. Another point is y 2 super script 0 minus d 2 super script 0.

Then third one is y 3 super script 0 minus d 3 super script 0, last one y 4, y has the 4 coordinates are there, when we convert the standard L P problem into a scaled L P problem by transformations, then this is equal to d 4 super script 0 minus. So, you completed this one minimum of lambda j super script 0 less than 0, for less than 0 now you see d we have computed here.

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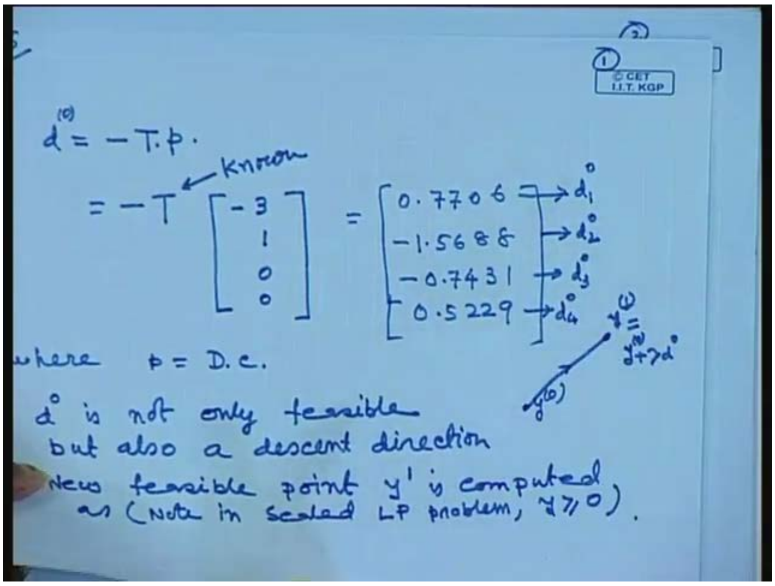

So, this is if we consider this is our d 1 super script 0 this is our d 2 super script 0, this is our d 3 super script 0, this is our d 4 super script 0.

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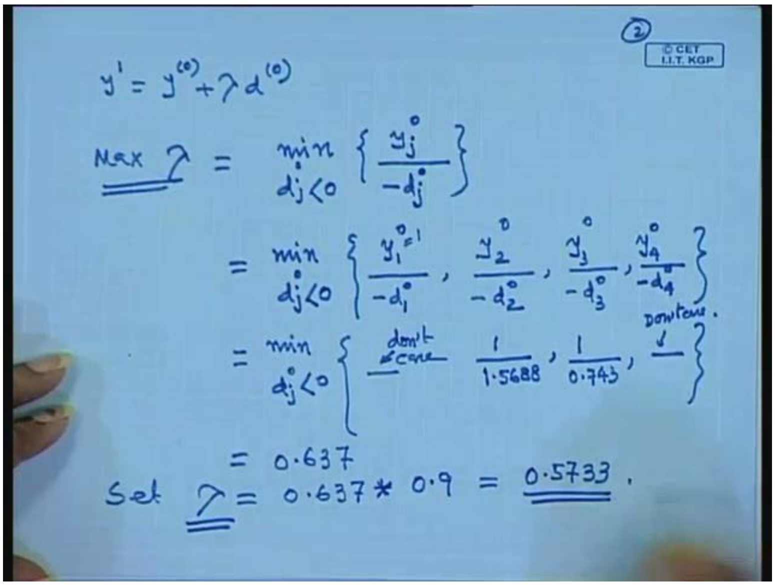

So, we will consider only the less than 0 d j superscript less than 0. So, this d 1 super script value is positive, this quantity will be positive, the ratio is because y 1 0 is equal to 1, this is positive consider with a minus sign. So, this is negative of that one, so the this is dash means don’t care when d j is positive you do not care, when the d 1 0 is positive we do not care this dash indicates do not care, because we are interested about d j less than 0.

And that expression we have given earlier that if it is, now look at this one that if lambda is a your negative quantity, or positive that we have to analyze this first because we compute first that one then this is equal to 1 divide it by d 2, that d 2 value will be minus and precede will be minus, it will be plus plus 1.5688 then d 3 value is same minus and d 3 is preceded it with a minus. So, it will be d 3 0 is one that will be a 0.7431and last one is y 4 0 is 1, d 4 value is positive so result should be a negative one. So, it do not don’t care. So, out of this which one is minimum we have to take into account as a lambda max.

Now, see this one out of this, this is the maximum or a minimum because 1 by this quantity is minimum that small value than this one. So, our value is 0.637 lambda max value will choose 0.637. And if you choose this value exactly we have seen earlier and explained that if we choose lambda x is this one, then there you will get one of the element of this y of 1 is 0. It indicates that you are on the boundary of this region to avoid that one, what we select this one, whatever the lambda max you got it that you multiply by less than 1.

Generally you multiply this 0.9, it assure that all the elements of new point of y 1 will lie inside the region means, it is in the interior point not only the boundary of the region is ensured. So, keeping this in mind then we can write, set lambda is equal to 0.637 into 0.9 which will come 0.5733. So, this is the lambda will select for all our problems, once we select this one then what is our new interior point from y 0 to y 1, what is the new interior point.

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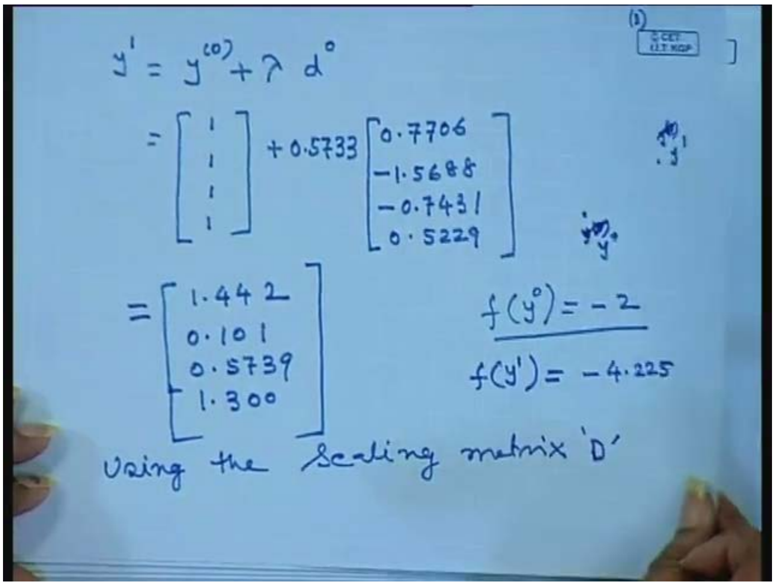

So, y 1 is equal to y of 0 then plus lambda into d of 0 y we have scaled all 1 1 1 1element then plus lambda just now you got it 0.5733. So, this is 0.5733 into d 0, d 0 you got it 0.7760 minus 1.5688 minus 0.7431, 0.5229. So, if you compute this one then finally, you will get 1.442 then you will get it 0.101 then you will get it 0.5739 then 1.300. So, this is the new value of that one.

Now, see what is the function value previously we have seen if you look at this y of 0, that value you have got it minus 2, if you see this one then if you compute this one this value, you will get the value of that one this value if you put it here, what is the value of y of 1? That value you will get it point minus 0.4225 and you see from this point to this point you moved it in proper direction y of 1, this is 1 y super script 0 and super script one. And the function value is reduced from minus 2 to minus 4.225, but once again I have to go back to our original problems. So, we know how the new variable and original variables are they are related. Now, you can write it this one using the scaling matrix D then if we recollect what is this, how they are related?

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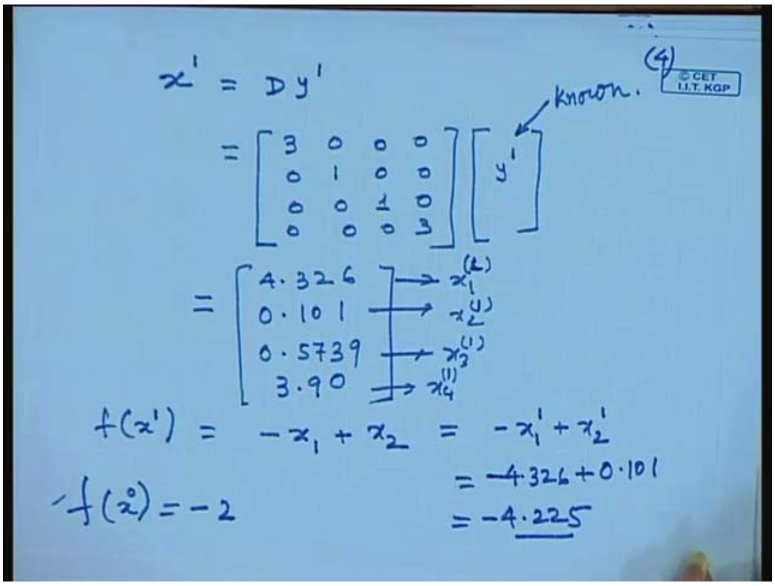

x of 1 is equal to D of 1 either this one with bracket you can write it, with bracket if you write it is a iteration indicates first iteration we have. So, this is the variable decision variable in the original problems that means, x 1 x 2 x 3 x 4 we can compute once I know d and d, if you recollect d is 3 0 0 0 0 1 0 0 0 0 1 0 then 0 0 0 3 and this y 1 you know already, what is the y 1 value put this value here then you will get finally, 4.326 this is known, this values is known 4.36, 0.101 and 0.5739 and 3.90, so it in the original problems with d operations.

That means, with the matrix scaling matrix the we are back to 3 original problem with the decision with variable x of 1. So, previously x of 1 if x of 0 was if your see x of 0 was you that is 3 1 1 3, now it was now it has moved to this point. So, let us see this function below in the original variable function below, this value is minus x 1 if you see this minus x 1 plus x 2, what is x 1 now value minus x 1 of superscript 1 plus x 2 of superscript 1. This superscript indicates that iteration so that below you got x 1 of this is this is x 1 of superscript 1, this is x 2 of superscript 1, this x 3 superscript 1, and this is x 4 superscript 1.

This superscript if you like you can put it in bracket means it is an iteration it indicates the iteration, first iteration after first iteration from x of superscript 0 to x 1 we moved. So, you can put this below this is minus 4.326 plus 0.101 that value is 4.225 and that value previously you look, if you look the expression x of 0 previously that below was 2.

So, you have reduced to minus the function below is decrease from minus 2 to minus 4.225. That means with the choice of the decent direction we able to reduce the function below from minus 2 to this. And you have to repeat this process because we have not reached the optimal value of this one, if you repeat this process once again.

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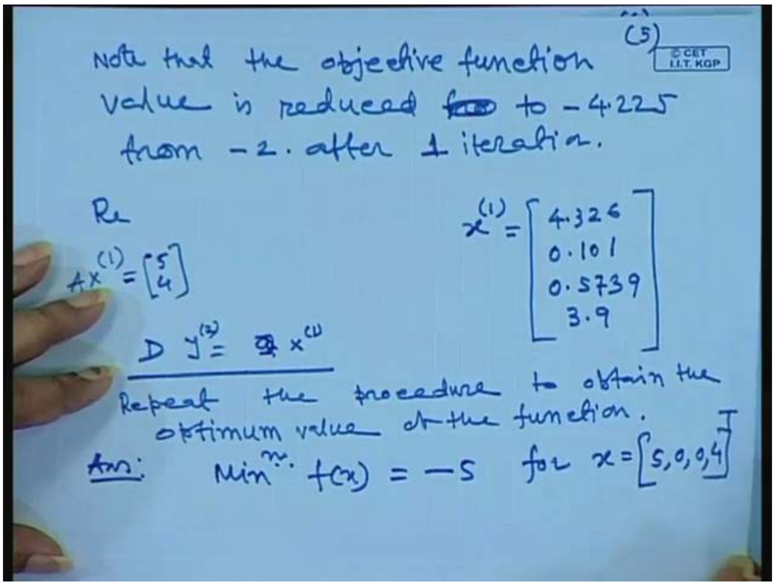

So, I just note that that the objective function value, what I have just told you just heading this one is reduced from reduced to minus 4.225 from minus 2 after 1 iteration. So, next is repeat this process then what we have we to repeat this one, we now we will consider this x of super script 1, if you consider the this x of 1 bracket this 1, this will you know it, what is that value you got it? Just now you calculated, this values I am writing this values at their 4.326, 0.101 then 0.5739, 0.3, 0.9.

So, this is our you can think that this is our initial guess, so this are not equidistant from the what is called this is a obviously this is a interior point, if you want to check it you can check it A x of 1 because we have solved this way only interior point, this equal to this will be A x superscript of 1, this will be equal to you that you see this 1. So, our basic equation if you see x superscript 1 and this x you will do the x 1 and x 2 comes or if you write this, you will get x superscript 1, the values are 5 and 4. I just show you that will be a 5 and 4.

See this one this is the our after computing the standard of the x is this and this if you put the value of x 1 x 2 and x 3 here you should get it 5 because it is an interior point, if you get it 5 and this equation you get it 4 it indicates you are inside the feasible region. So, you can one can check that this will be 5. 4 check. Now, this is our initial guess, so what we have to do we are not equidistant from the x what we have to do? We have to scale saying positive, so we have to do y of that is new value of y 2 is we got it, y 1 then you just got y 2 is again D into your is x of y 1 sorry x of this equal to this x of y 1 is equal to that y of is values. So, by using this one and choice of D is same as the initial values of this diagonal element should be that one.

So, next step is repeat, repeat the procedure, agree? So repeat the procedure to obtain the optimum value of the function, agree? So you can you do another iteration, you will see the minimum value of the function minimum value of the function f of x is equal to minus 5 4, x is equal to 5 0 0 4 transpose. And physically also you can see these that you will get the value of the function is minus 5, from the statement of the problem, let us see the statement of the problem, what is this? If you see this is the our objective function this is our objective function in our original problem and this is our the constant on the problem. So, if you draw this equation and objective function in a rough paper you will see that our problem is like this way.

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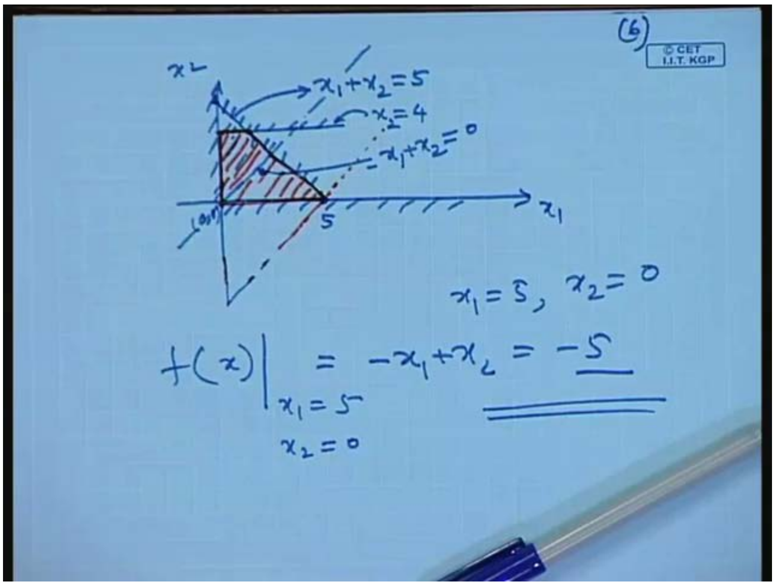

So, when x 1 is greater than equal to 0 x 2 is greater than 0 x 1 greater than 0 this is our x 1, this is our x 2 x 1 is greater than 0 means, it indicates this region and when x 2 is greater than 0 this indicates that region. That means we are in the first quadrant of this coordinate x is, so our first equation x 2 is equal to you see, the first is an x 1 plus x 2 is less than or equal to 5 that we can represent by let us call this is 5, and you call this is 5 so that is the equation. And this is the our region and that is the your x 1 plus x 2 is equal to 5 and less than this one this our region again. The next equation is x 2 is equal to less than 4. That means the next two is that region is that this is the x 2 is equal to 4.

So, this constant to all constant we have presented here, agree? This is origin 0 0 then what is the objective function f of x is equal to minus x 1 plus x 2. So, this is we can write this is the our objective function value is 0, when it is passing through the origin this is the equation of a straight line. So, this is our objective function that x 1 minus x1 plus x 2 is equal to 0. Now, we can see visualize this one, our visible region is if you see look carefully our visible region is that red shaded portion, or the boundaries of this one is that one.

Now, this the objective function f(x), whose value is 0 when passing through the origin. So, if you move parallel to this one, now you see it is the y is equal to m x plus c and c is minus. That means objective function value is decreasing and you can move to parallel to this one, this line parallel with this line. Of course, otherwise the slope will change and objective function description will be changed. So, you can move parallel to this one up to that one, agree?

So, at this point this is a feasible point of this one, so at this point what is the function of value of that. So, this point is our x 1 is equal to 5 x 2 is equal to 0. So, our function value at this point f of x at x 1 is equal to 5 and x 2 is equal to 0, and that will give you the maximum, what is the minimum value of the function. If you put this minus x 1 plus x 2 is equal to minus 5, that graphically also one can see this is the primary.

So, one can draw the conclusion like this way in a linear program problem, if you use the interior point method that point initial guess, you must have to take inside the feasible region. And when you do the iteration slowly it is approaching to the one of the vertices of the boundaries is it will give you the optimum value of this function, in this case this vertices will give you the optimum value of the function.

So, at this moment you know how to solve the linear programming problem, as well as the what is called the linear problem, how to solve using the what is called the interior point method? So next topic what we will cover is this one, that is the basic concept of what is called first we will say how to solve the non-linear programming problem using the exterior penalty function, agree? Then you will see how to solve the same problem non linear problem using interior point method.

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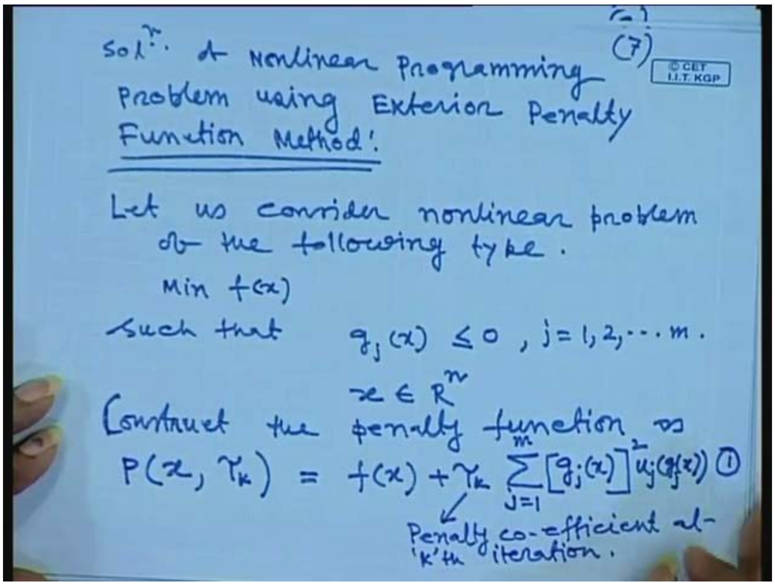

So, first is solution of so that question whatever left over was there last class we have completed the solution of the problem, solution of non-linear programming problem using exterior penalty function method. So, in linear what is linear point method, we have seen if it is if you have solve to if you liked to solve the problems of linear programming problems using ((Refer Time: 43:38)) method you have to take initial condition inside the feasible region initial point then do the scaling, go in the proper direction. So, that the scale L P problem function value is decreased from the initial value. And go back to you what is called the original problem and if you repeat this process ultimately you will see the your optimal point will be one of the vertices of the L P problems, agree?

So that, so this is our next problem that non linear programming problem how to solve the exterior penalty function method. The constant let us call what is the basic statement of the problem, let us consider non linear problem of the following type, so if you an non linear problem and constants also is either linear or non-linear, first you convert that reformulate the problem in to a on constraint, what is called minimization problem UMP non constraint minimization problem, that we have already discussed earlier. But only the point is your initial guess must be the outside the feasible region, and then once you convert into a unconstraint operation problem that problem, you can solve it by using what is called our standard technique, what we have discussed earlier.

Either what is called Stefan Boltzmann method or you can solve with Newton Raphson method or you can do the what is called conjugate gradian method. The advantage of each method is discussed at when earlier, so once you have non linear programming method want to solve this by what is exterior penalty function method, what you have to do convert this problem into a what is tender, what is called unconstant optimization problem, then you can solve it either analytically or numerical method by applying the Newton Raphson method, Stefan Boltzmann method method or the conjugate gradian method.

So, our function is like this way minimize f of x, this function may be non-linear may be linear constant function may be what is called non linear also, such that are subject to such that g j of x is less than equal to 0, agree? And j is equal to 1, 2, dot dot m and x belong to the dimension of x is n dimensional of x. So, what we have to do it first that I told you first you convert into a unconstant optimization problem. So, first you convert into a unconstant optimization problems, so this unconstant optimization problem is obtain by heading a penalty function or term, that measures the distance that is the what is called additional term, what we have adding to the objective function. So, this additional terms you have to give which is with a some factor agree, so the our construct next is construct, the penalty function.

As P stands for penalty function x is the decision variables of optimization problems, and tau is the scalar quantity, that equal to f of x plus tau of k summation of these all inequality constants I converted into this form j is equal to 1 to n g j of x whole square into u j g of g j of x and that bracket close and this is equation number one, this is equation number one. So, this tau of k is called penalty coefficient, penalty coefficient at the k indicate k th iteration at k th iteration.

So, what is the this is the word objective function is given, the constant has given these objective function are no linear. And then construct the penalty function like this way, objective function plus this. Now, I will discuss what is this one, that if the our initial point, which is exterior point that outside the feasible region and you penalize heavily.

That means, you give the punishment from the objective from that function if it inside this one, then you give what is 0 of this one let us call how to do this one.

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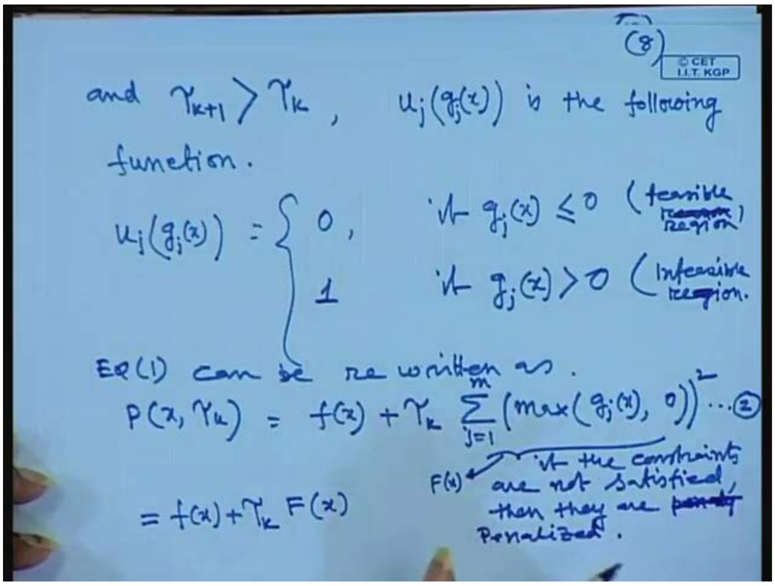

And tau k plus 1 is greater than tau k that, kth iteration is value what is the value of tau k we have considered k plus 1 is next iteration the value of tau k must be greater than at kth iteration, that is the thing. Then u j g j, g j of x this value is a function of that is the following function, what is this I have define this u j, g j of x is equal to 0 if g j of x is less than equal to 0.

Now, see this of statement of the problem, if you see the statement of the problem this is the constraint. And if these constraints are satisfied that means our point is interior point this constraint is when inside the feasible region, and if this constraint is not satisfied then we are exterior point. Means we are not in feasible region outside the feasible region. So, you penalize with one, so you can see this is our in feasible region this is feasible region, and this is infeasible region infeasible region, agree?

Now, what will be the function, now if you just look at this expression the equation number one, if you consider this is the equation number one. So, equation number one can be re-written as P x tau of a is equal to f of x plus tau k summation of j is equal to 1 to m max g j x minus 0 this, then whole square. So, let us call this is equation number two. How I retained this one, see this one when it is the feasible region. That means g j of x less than 0 then this quantity of value is 0 this quantity. So, this will be 0 whole part will be not be there, when there is outside the feasible region this value is one. That means this is g j x whole square into tau k, so that what you have write it now you see the maximum of this equivalently you can write it maximum of this.

So, when it is not in the feasible region this value is greater than 0 isn't not that greater than 0, then what are the value is there then square up this one, this is same as that one. So, this would be positive maximum this one means g j you are taking g j what is the maximum and that square. So, it can write it this one if the constraint are not satisfied then they are penalized, agree? So, this equation I can write at f(x) into tau k into f of x that whole thing I am writing this thing I am writing f of x, and this you know this portion summation of this one.

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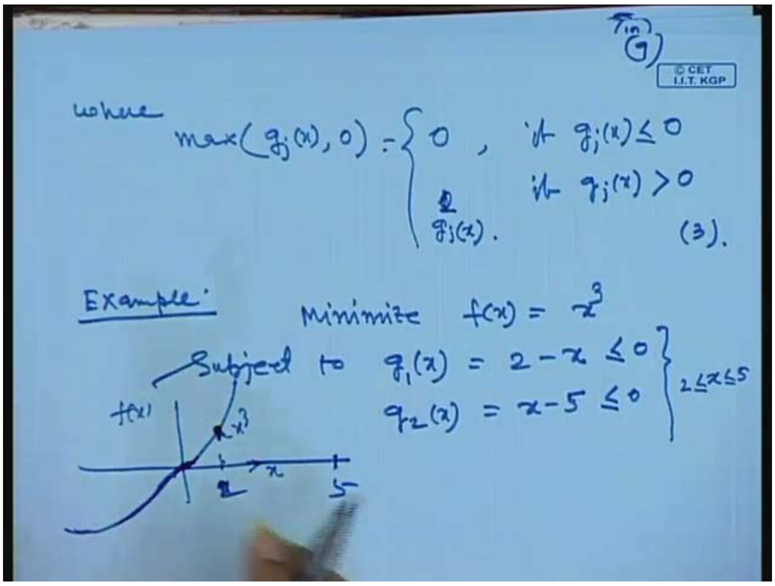

So, where I can write max of g j x 0 is equal to 1, this value because satisfy the our constraints, that means this value is less than 0 maximum of this 1 is 0, out of this because this value will be 0 minus negative quantity, when g of x is less than that that means x is in the feasible region. That means if g j of x is less than equal to 0, and that is equal to 1 this value will be if g j x value this value will be, what is when it is greater than 0, this will be a 1 from this will yes, this will be a your g j of x. Again maximum of that one because that value is greater than this so it will be g j of this. So, this is the equation number three.

So, we will explain this thing, will explain with an example let us call our problem is minimize f of x is equal to x cube, then subject to g 1 of x is equal to 2 minus x is less than equal to 0 then g 2 of x is x minus 5 is less than equal to 5 less than equal to this 2 equation combinely you can see nothing but x is less than 5 and 2. So, it is you see 2 minus x if you bring it to here is less than 0 this one and x minus 5 is less than this 2 things, I combine together and what is the minimum value of function at what point.

Obviously, you see x is to has a minimum value of the function if you plot this 1, x is 0 then strictly increasing like this way and that is what about that would be that one x minus that would be this way strictly it is just cubic form x, this is f of x, agree? So, this function value because its range is given x is given 2 to 5, so its minimum value of this one is here only agree. So, will discuss this in next class that physical interpretation of what is called penalty function, while we will solve the non-linear problem using the exterior point method. So, I will stop it here today.